SCIENCE CHINA
Technological Sciences

August 2010 Vol.53 No.8: 2278–2290
doi: 10.1007/s11431-010-4003-7

Constructal optimization of a vertical insulating wall based on a complex objective combining heat flow and strength

XIE ZhiHui, CHEN LinGen* & SUN FengRui

Postgraduate School, Naval University of Engineering, Wuhan 430033, China

Received March 2, 2010; accepted May 4, 2010

For a vertical insulating wall, a product function of heat flow and strength with power weight is introduced as the complex optimization objective to compromise between insulating performance and mechanical performance. Under the global constraints of fixed external dimensions and safety requirements, the constructal optimization of the wall is carried out by taking the complex function maximization as the objective. It is shown that the maximum of the complex-objective function and its corresponding optimal internal structure design under a certain environmental condition can be obtained by allowing the internal structure of the wall to vary (evolve) freely. The validity, effectivity and applicability of the complex function are proved by the results and the power weight parameter in the range from 0.4 to 4 can compromise between the requirements of insulating and strength simultaneously and preferably. The constructal optimization with coequal attention to heat flow and strength and the corresponding results are discussed in detail. The optimal structure design and the corresponding performance analyses under various environmental conditions of application are presented. When the change of environment is greater and the total Rayleigh number is bigger, the insulating wall with large number of cavities should be employed. When the total Rayleigh number is small, the better performance can be obtained by reasonably employing the insulating wall with small number of cavities. The complex function has better self-adaptability, and the results in the recent literature are special cases of this paper.

constructal theory, insulating wall, multidiscipline, generalized thermodynamic optimization

Citation: Xie Z H, Chen L G, Sun F R. Constructal optimization of a vertical insulating wall based on a complex objective combining heat flow and strength. Sci China Tech Sci, 2010, 53: 2278–2290, doi: 10.1007/s11431-010-4003-7

1 Introduction

Since constructal theory was put forward by Bejan in 1996 [1], it has quickly become a continually developing hot spot of research [2–10]. Its research subjects involve many fields such as engineering [3,7,8], sociology [9], economics [11,12], material science [13], geophysics [14], biology [15], physics [16], etc. It is called constructal optimization that employs constructal theory to guide engineering design, i.e. to search the optimal internal structure, external shape and time rhythm of one thing by taking performance maximization as an objective with given global constraints [2–8,17]. And the researches, which combine entransy dissipation extremum principle [18–54] and finite time thermodynamics [55–67] with constructal theory [1–17,68–95], are the new directions of this field.

Recently, the research of constructal optimization mainly focuses on various single-objective optimization problems, including time minimization [1,68], profit rate maximization [11], cost minimization [11,12], maximum temperature difference minimization [69–74], heat transfer rate maximization [75], fluid flow resistance minimization [76], heat flux maximization [77,78], path length minimization [79], exergy loss minimization [80], electrical resistance minimization [81], power maximization [82], entransy dissipation

*Corresponding authors (email: lgchenna@yahoo.com; lingenchen@hotmail.com)

rate minimization [33,34,36,38,46,49,51–54], etc. The research of multi-objective optimization includes the optimization of convective heat transfer by taking fluid flow resistance minimization and thermal resistance minimization into account simultaneously [83,84] and various tree-shaped heat exchangers [85–87], the optimization of hot water pipe network by taking pumping power minimization and heat loss minimization into account simultaneously [88], the optimization of solid-gas chemical reactor by taking high density of chemical reaction and low pumping power into account simultaneously [89,90], etc. Especially, Lorente and Bejan [91] studied the constructal optimization of an insulating wall by combining heat transfer and strength, and exploited a new direction called multidisciplinary constructal optimization. Gosselin et al. [92] further studied the constructal optimization of a beam under thermal attack by combining heat transfer and strength. Gosselin and Silva [93] studied the nano-scale fluid flow problem by combing heat transfer and power dissipation. Gosselin and Bejan [94] optimized an electromagnet with high conductive material inserted by taking maximum temperature difference minimization as the objective with the fixed magnetic induction. Wei et al. [34,46,95] considered that the power dissipation of electromagnet increases with the increase of working temperature and that the mechanical strength of the solenoid decreases with the increase of the structure temperature, and optimized electromagnets by taking entransy dissipation rate minimization as the objective. Two different complex-objective functions characterizing the magnetic intensity and heat transfer performance of electromagnet synthetically were built. One was based on magnetic induction and maximum temperature difference and the other was based on magnetic induction and entransy dissipation rate. And the constructal optimization of compositive performance combining heat transfer and magnetic effect was conducted and compared by releasing the constraint of the fixed magnetic induction.

The precondition of engineering optimization is to choose a suitable objective function for different application purposes and situations. One may obtain different results, even widely divergent ones, when a different objective function is employed. In practical engineering, some multidisciplinary and multi-objective requirements are involved when an object pursues both its optimal performance and single-objective optimization which always attends to one thing and loses another. Ref. [91] analyzed the competitive relationship between insulating performance (overall thermal resistance) and mechanical performance (strength) of a special internal structure of an insulating wall, and optimized the internal structure by taking the overall thermal resistance as the optimization objective with the prescribed strength serving as an equation constraint. The corresponding optimal internal structures were presented. Its essential was single-objective optimization, and the mechanical performance was not optimized simultaneously in the process of the insulating performance optimization. For giving attention to insulating performance and mechanic performance simultaneously and perfectly, this paper introduces a product function of heat flow and strength with power weight as a complex performance index of a vertical insulating wall, and the constructal optimization of the insulating wall is conducted by taking the new index maximization as the objective, and the optimal internal structures which meet practical engineering requirements are obtained, i.e. the geometric structures with the maximum complex function are obtained by allowing the internal structure of insulating wall to vary (evolve) freely with some given global constraints.

2 Model

Insulating walls, such as energy-saving wall of architecture and refractory wall for high temperature have been used widely in engineering. Arranging cavities with air in an insulating wall is a classical and general measure for enhancing insulating effect. This measure can not only enlarge the thermal resistance of wall to reduce heat transfer rate (e.g. reducing the energy consumptions of heating or cooling system), but also to reduce the mass of solid materials and decrease the weight of wall, consequently the compositive benefits of reducing energy consumption and protecting environment are achieved. The insulating wall has been popularized emphatically because it accords with the sustainable ideal more suitably compared with complete solid wall. Consider the model of the vertical insulating wall shown in Figure 1 [91]. The height, width and thickness $(H \times W \times L)$ are fixed, n vertical air-filled cavities with equal thickness L_a slab the solid material into n+1 plies with equal thickness L_s equidistantly. For characterizing the composite degree of the wall, the cavity volume fraction is defined as

$$\Phi = nL_a / L. \tag{1}$$

And the solid material volume fraction becomes $1-\Phi = (n+1)L_s / L$.

The wall material is isotropy, and heat flows in one dimension which is perpendicular to the wall. The overall temperature difference ΔT between the two outboard planes of the wall is fixed. Because the thermal conductivity difference between general solid materials and air is very large (e.g. the ratio of the thermal conductivity of brick to that of air is $\lambda_s / \lambda_a \approx 20 >> 1$ [91]), one has

$$\Delta T \approx n\Delta T_a, \tag{2}$$

where ΔT_a is the temperature difference between the two outboard planes of a cavity.

The practical A-A cross section of the insulating wall is shown in Figure 1(b), and there are transversal ribs, which

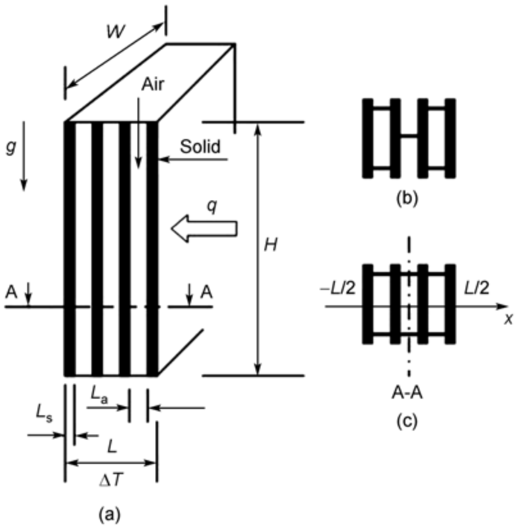

Figure 1 Insulating wall [91].

occupy considerably less material and space between plies. The main role of ribs is to connect the slabs into a whole. For the sake of simplification, the effects of transversal ribs on both insulating performance and mechanic performance of the wall are neglected in various calculation herein, and the staggered arrangement pattern of ribs is also simplified into serial arrangement pattern as shown in Figure 1(c).

3 Heat transfer and strength analysis

3.1 The overall heat transfer coefficient

The overall thermal resistance of insulating wall is the sum of the resistances of air and solid layers. Considering the effect of natural convection in air layers, the overall thermal resistance formula is

$$R = \frac{nL_{\mathrm{a}}}{\lambda_{\mathrm{a}} HWNu} + \frac{(n+1)L_s}{\lambda_{\mathrm{s}} HW}, \tag{3}$$

where Nu number expresses the relative heat transfer augmentation effect due to natural convection in a air space, i.e.

$$Nu = \frac{q_{\mathrm{actual}}}{q_{\mathrm{conduction}}}. \tag{4}$$

Based on comprehensive analysis and comparison, a criteria correlation, $Nu = 0.364\frac{L_{\mathrm{a}}}{H} Ra_{H,\Delta T_{\mathrm{a}}}^{1/4}\ (Nu \geqslant 2)$ [96,97], which has more extensive adaptability was chosen in ref. [91], and it was changed to adapt pure conduction case by employing the method presented in ref. [98], i.e.

$$Nu = \left[1 + \left(0.364\frac{L_{\mathrm{a}}}{H} Ra_{H,\Delta T_{\mathrm{a}}}^{1/4}\right)^m\right]^{1/m}, \tag{5}$$

where the dimensionless Rayleigh number $Ra_{H,\Delta T_{\mathrm{a}}} = g\beta H^3 \Delta T_{\mathrm{a}} /(\alpha v)$, g is gravitational acceleration, β is coefficient of volumetric thermal expansion, α is thermal diffusivity, v is kinematic viscosity, and the parameter m is 3 [91].

The thermal resistance across a completely solid wall is $[L/(\lambda_{\mathrm{s}} HW)]$, and the dimensionless overall resistance formula can be built by using it as reference scale,

$$\tilde{R} = \frac{R}{L/(\lambda_{\mathrm{s}} HW)} = \frac{\lambda_{\mathrm{s}}}{\lambda_{\mathrm{a}}} \Phi[1 + (0.364 n^{-5/4} \Phi Ra^*)^m]^{-1/m} + 1 - \Phi, \tag{6}$$

where the global parameter Rayleigh number $Ra^* = \frac{L}{H} Ra_{H,\Delta T}^{1/4}$, which characterizes the intensity of natural convection in cavity and reflects the different environmental condition.

According to the correlation of heat transfer coefficient and thermal resistance, the overall heat transfer coefficient of the insulating wall including area information of heat transfer is the reciprocal $1/\tilde{R}$ of its thermal resistance, and the overall heat transfer coefficient of completely solid wall with the same dimensions is one. Therefore, the overall heat transfer coefficient difference between insulating wall and completely solid wall is $(1-1/\tilde{R})$, and the heat transfer rate reduced, which results from comparing the case of using insulating wall with that of using completely solid wall under the same temperature difference, is

$$\tilde{Q} \sim (1 - 1/\tilde{R}) = f_1(n, \Phi, Ra^*). \tag{7}$$

$\tilde{Q}$ is the reduced heat loss or cooling load, and one has $\tilde{Q} = (1-1/\tilde{R})$ when the temperature difference is the unit. Therefore, the overall heat transfer coefficient difference $(1-1/\tilde{R})$ essentially reflects the income of using insulating wall, and it can be employed to characterize the insulating performance of the wall. The larger $(1-1/\tilde{R})$ is, the less the heat transfer rate of insulating wall compared with that of completely solid wall under the same temperature difference is; the higher the income of using insulating wall is, the better the insulating performance is.

3.2 Strength

The mechanical strength is used as the mechanics index of the wall, i.e. its resistance to bend and buckle in the plane of Figure 1. It is controlled by the area moment of inertia of the horizontal wall cross section

$$I_n = \int_{-L/2}^{L/2} x^2 W \mathrm{d}x. \tag{8}$$

When the insulating wall is composed of solid materials completely, the mechanical strength of the wall approaches its maximum $I_0=L^3W/12$, and the dimensionless mechanical strength formula is built by using it as reference scale [91]

$$\tilde{I}_n = \frac{I_n}{L^3W/12} = f_2(n,\Phi). \tag{9}$$

The dimensionless mechanical strength formulas of the wall with different cavity number are as follows:

$$\tilde{I}_1 = 1-\Phi^3,\ \tilde{I}_2 = 1-\frac{1}{3}\Phi-\frac{1}{3}\Phi^2-\frac{1}{3}\Phi^3,$$
$$\tilde{I}_3 = 1-\frac{1}{2}\Phi-\frac{1}{3}\Phi^2-\frac{1}{6}\Phi^3,\ \tilde{I}_4 = 1-\frac{3}{5}\Phi-\frac{3}{10}\Phi^2-\frac{1}{10}\Phi^3,$$
$$\tilde{I}_5 = 1-\frac{2}{3}\Phi-\frac{4}{15}\Phi^2-\frac{1}{15}\Phi^3,\ \tilde{I}_6 = 1-\frac{5}{7}\Phi-\frac{5}{21}\Phi^2-\frac{1}{21}\Phi^3,.... \tag{10}$$

The requirement of optimal design for mechanical performance is that the larger the mechanical strength is, the better its performance is.

3.3 The complex function of product of heat flow and strength with power weight

The compositive goal of optimal design of insulating wall is to get a good insulating performance (which means $(1-1/\tilde{R})$ is as large as possible) as well as a good mechanical strength (which means I is as large as possible). But from eqs. (6), (7), (9) and (10), there are competitions between the requirements of internal structure design (volume fraction Φ and cavity numbers n) for insulating performance and mechanical performance. As the followings, when Φ is fixed, the mechanical strength of the insulating wall is maximum when all the solid materials are arranged on both outboards of the insulating wall (i.e. n=1), and its mechanical performance is the optimum. However, at the same time, the cavity volume is the maximum. Large cavity can cause the development of natural convection, so that $(1-1/\tilde{R})$ decreases evidently and the insulating performance worsens. If the number n of cavities increases, though insulating performance improves, mechanical performance worsens. When n is fixed, the smaller Φ is, the better the mechanical performance is. But the relation between the insulating performance and the cavity volume fraction is nonmonotonic, and the insulating performance is affected by Ra^*. When Φ decreases, the insulating performance may improve or worsen.

The objectives competition between multidisciplines such as insulating and strength provides an opportunity to utilize constructal theory, i.e., getting a reasonable structure which gives attention to some requirements of every aspect by balancing several kinds of competition under the given global constraints.

In the multi-disciplinary and multi-objective optimization problems, single-objective optimization always attends to one thing and loses another. To meet the requirements of various objectives, one of the methods used most commonly is to integrate incommensurate or incompatible multiple objective functions into one objective function. These methods include weighted sum method, ideal point method, cost-effectiveness analysis method, goal arrangement method and primary goal method, etc. [99]. For the optimization of heat engine, Yan [100] built an objective function ($\eta^\lambda P$) of a product of efficiency and power with power weight. In order to characterize the compositive performance of an insulating wall combining insulating and strength, this paper introduces a complex function of a product of heat flow and strength with power weight, i.e.

$$F = (1-1/\tilde{R})\tilde{I}_n^{\gamma}, \tag{11}$$

where γ is power weight parameter and its value can range from 0 to ∞ in theory.

Power weight γ reflects the attention degree given to insulating performance and mechanical performance of the insulating wall. And the larger γ is, the much more emphasis on strength is; the smaller γ is, the much more emphasis on insulating is. Constructal design is conducted by taking eq. (11) maximization as the objective in this paper, and the more reasonable insulating structure which gives attention to insulating and strength is obtained by allowing the internal structure of insulating wall to vary freely according to the model shown in Figure 1. When $\gamma = 0$, eq. (11) degenerates to an objective function only for insulating performance; when $\gamma \to \infty$, eq. (11) degenerates to an objective function only for mechanical performance; when $\gamma = 1$, eq. (11) is an objective function paying coequal attention to insulating and strength. In engineering design, sometimes the insulating performance needs to be given prominence, and sometimes the mechanical performance needs to be given prominence. Therefore, the value of γ can be chosen according to different requirements of practical application.

Ref. [91] studied the constructal optimization of an insulating wall combining heat transfer and strength, and exploited a new direction called multi-disciplinary constructal optimization. But its essential was single-objective optimization, and that method has some limitations. That is, ref. [48] is the optimization study taking the overall thermal resistance (eq. (6)) maximization as the objective with the constraint of a fixed strength (eq. (9)). Therefore, its essential is a multi-disciplinary optimization with one single-objective in which one objective served as an equation constraint and the other served as the optimization objective, and the mechanical performance was not optimized simultaneously when the insulating performance optimization was carried out. The results obtained were only localized in the constructal design of thermal resistance maximization with the fixed strength. The heat transfer rate reduced, which results from comparing the case of using insulating wall with the case of using completely solid wall, increases with the increase of the overall heat transfer coefficient difference between insulating wall and completely solid wall. Therefore, from the viewpoint of actual gains from using insulating wall, it can reflect more actual effectivity by employing $(1-1/\tilde{R})$ of the insulating wall to characterize the insulating performance in this paper than by employing $\tilde{R}$ to characterize that in ref. [91]. More importantly, the multi-objective complex function introduced in this paper supplies a precondition for optimizing multi-disciplinary indexes simultaneously with releasing an equation constraint of one objective function, and this method avoids those limitations of single-objective optimization in ref. [91]. Complex function (eq. (11)) synthesizes the two indexes of insulating and strength essentially. The optimization employing this complex function can take the two requirements of insulating and strength into account simultaneously, and power weight γ endows flexibility and applicability to the complex function.

4 Optimization analysis

Safety is the fundamental requirement in every engineering design, so the strength of insulating wall must meet a lower limitation. $\tilde{I}_n \geqslant 0.5$ is employed in the optimization process herein. In the following text, the effect of power weight γ on the constructal optimization of the insulating wall will be analyzed and the constructal optimization giving coequal attention to insulating and strength will be studied. All the results obtained belong to the effective solution set of this optimization problem.

4.1 Effect of γ on constructal optimization

4.1.1 Effects of γ on $F_{\max}$–Ra^* characteristic and the corresponding $(1-1/\tilde{R})$ and $\tilde{I}_n$

Figure 2 shows the effect of power weight γ on $F_{\max}$–Ra^* characteristic. As shown in the figure, the curves of $F_{\max}$–Ra^* decrease with the increase of γ as a whole. For any γ, with the increase of Ra^*, $F_{\max}$ remains smooth with a small segment, then decreases quickly, and finally decreases slowly. With the increase of γ, the length of segment which decreases quickly extends and the length of segment which decreases slowly shortens.

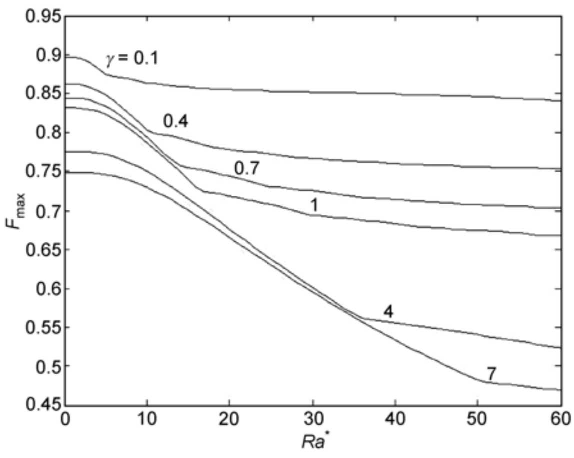

Figure 2 Effect of γ on $F_{\max}$–Ra^* characteristic.

Figures 3 and 4 show $(1-1/\tilde{R})-Ra^*$ characteristic and I–Ra^* characteristic obtained with the method of this paper (solid line) and the method in ref. [91] (dashed line), respectively. From the solid lines shown in Figure 3, the curves of $(1-1/\tilde{R})-Ra^*$ decrease with the increase of γ and this accords with the purpose of building the complex objective function combining heat flow and strength by taking exponent γ as a weight, and also proves the rationality of the complex function and the feasibility of the method herein. With the increase of Ra^*, each $(1-1/\tilde{R})$ curve keeps smooth with a small segment, then exhibits some staged decreasing segments. The reason that causes the staged phenomenon is that the numbers of cavities in insu-

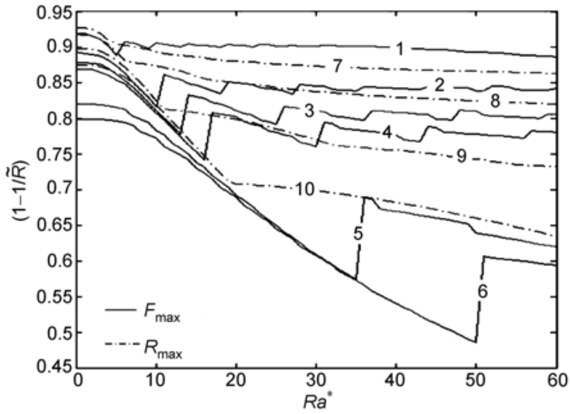

Figure 3 Effect of γ on $(1-1/\tilde{R})-Ra^*$ characteristic when F is at maximum. 1, γ=0.1; 2, γ=0.4; 3, γ=0.7; 4, γ=1; 5, γ=4; 6, γ=7; 7, I=0.7; 8, I=0.8; 9, I=0.9; 10, I=0.95.

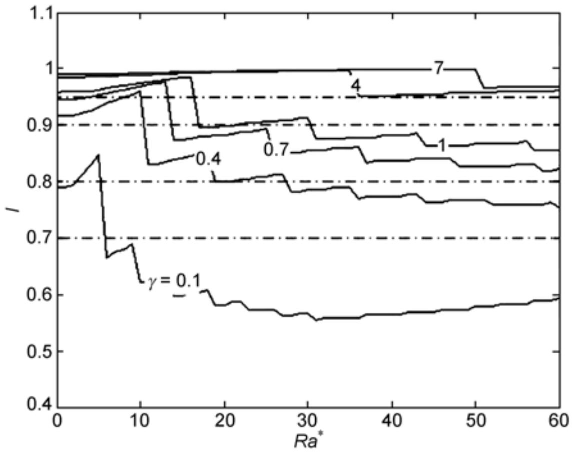

Figure 4 Effect of γ on I–Ra^* characteristic when F is at maximum.

lating wall are discrete integers, not continuous real numbers, therefore its correlative function is discrete. The larger γ is, the larger the upward jump between two adjoining segments is, but the smaller the number of decreasing segments is. The larger Ra^* is, the lower the descending rate of each decreasing segment is. Comparing Figure 4 with Figure 3, one can find that I–Ra^* characteristics are reversed roughly to $(1-1/\tilde{R})-Ra^*$ characteristics when F is at maximum, but the relation between $(1-1/\tilde{R})$ and I is not the absolutely reciprocal relation. The curve of I–Ra^* increases with the increase of γ. With the increase of Ra^*, each I–Ra^* curve remains smooth with a small segment, then exhibits some staged increasing segments. The larger γ is, the larger the downward jump between the two adjoining segments is, but the smaller the number of increasing segments is. The larger Ra^* is, the lower the increasing rate of each increasing segment is.

Synthesizing the analyses of Figures 2–4, one can draw a conclusion that the smaller γ is, the larger the range of application environment in which the function value is higher when the complex function maximization is taken as the objective. If γ is too small, though $(1-1/\tilde{R})$ is higher in the larger range of application environment and insulating performance of the wall is good, the corresponding I_n decreases too quickly (e.g. I_n corresponding to γ=0.1 decreases more largely than that corresponding to γ=0.4), and mechanical performance of the wall is too bad. If γ is too large, though I_n is higher in the larger range of application environment and mechanical performance of the wall is good, the corresponding $(1-1/\tilde{R})$ decreases more largely (e.g. $(1-1/\tilde{R})$ corresponding to γ=4 decreases more largely than that corresponding to γ=1). Specially, in the range of high Ra^*, the insulating performance of the wall is too bad. Moreover, the increase of I_n corresponding to γ=7 is not obvious compared with that corresponding to γ=4. Therefore, γ ranging from 0.4 to 4 can give better attention to the requirements of insulating performance and mechanical performance. This also further approves the rationality and validity of the complex function eq. (11).

4.1.2 Effect of γ on the best performance and corresponding structure when Ra^* is fixed

Figure 5 shows the effect of γ on $F_{\max}$–Φ characteristic with Ra^*=20 and 30. As shown in the figure, for any γ and Ra^*, with the increase of Φ, $F_{\max}$ increases in the beginning and then decreases. However, the larger γ is, the quicker the increase and decrease are, and the smaller the range of peak value with larger $F_{\max}$ is. It is shown that, for special environment, when γ is smaller, the higher value of objective function can be attained in the wider range of Φ; however, when γ is larger, the range of Φ in which the objective function value is higher is reduced, and this indicates that the corresponding precision demand upon structure design is more strict. Comparing the solid lines with the dotted lines in the figure, one can find that, when γ is fixed and Ra^* is smaller, $F_{\max}$ varies more gently with the increase of Φ, but the value of $F_{\max}$ is bigger.

Figures 6 and 7 show the $\Phi_{\rm opt}$–n and $n_{\rm opt}$–Φ characteristics corresponding to the performance curves shown in Figure 5. As shown in Figure 6, when $\gamma \leqslant 1$ and for any Ra^*, with the increase of n, $\Phi_{\rm opt}$ increases in the beginning and then decreases slowly; the smaller γ is, the bigger the increasing amplitude is, but all of the decreasing amplitude are small. When $\gamma > 1$, for any Ra^*, $\Phi_{\rm opt}$ decreases with the increase of n monotonously. When n is small, $\Phi_{\rm opt}$ corresponding to Ra^*=20 is bigger than that corresponding to Ra^*=30; the two $\Phi_{\rm opt}$ tend to be the same with the increase of n, and when γ is larger, the two $\Phi_{\rm opt}$ at the smaller n tend to be the same.

As shown in Figure 7, when γ is fixed, the range of $n_{\rm opt}$ with Ra^*=30 is wider than that corresponding to Ra^*=20. When γ< 4, for any γ and Ra^*, with the increase of Φ, $n_{\rm opt}$ increases in the beginning and then decreases slowly. When

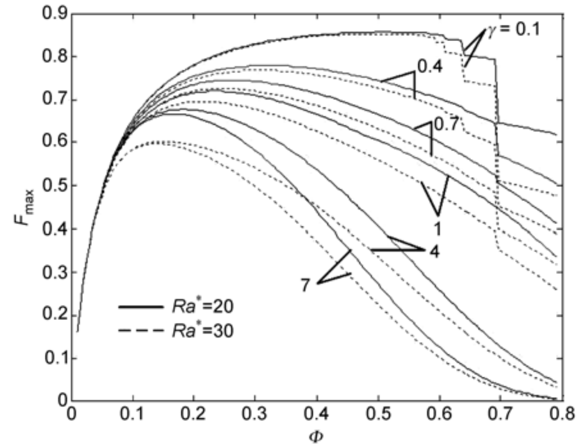

Figure 5 Effect of γ on $F_{\max}$–Φ characteristic.

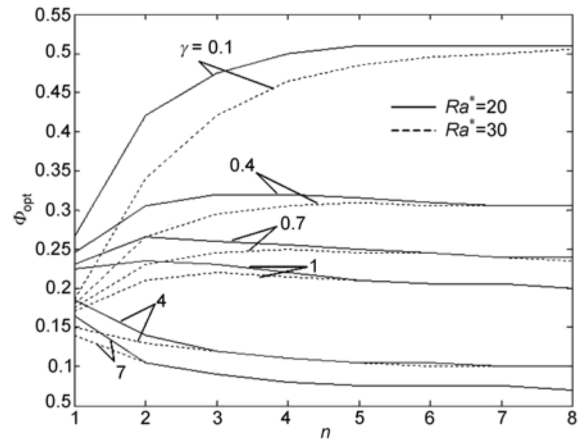

Figure 6 Effect of γ on $\Phi_{\rm opt}$–n characteristic.

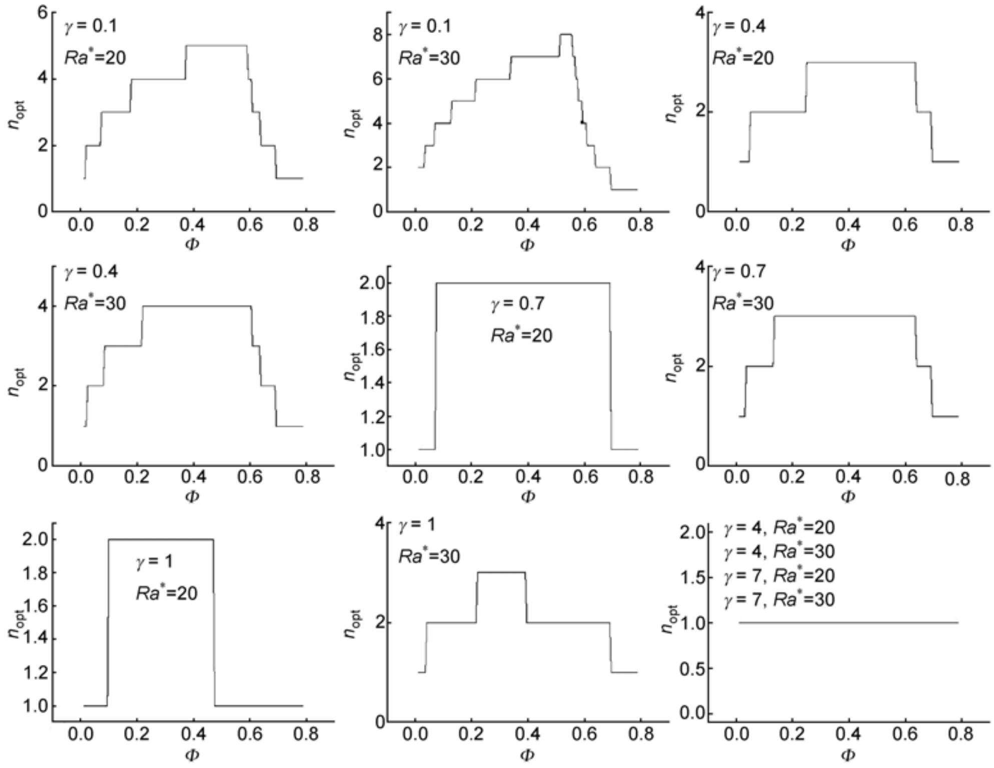

Figure 7 Effect of γ on n_{opt}–Φ characteristic.

$\gamma \geqslant 4$, n_{opt}=1. For any Ra^*, the range of n_{opt} is reduced with the increase of γ. When Ra^* is fixed, the bigger γ is, the smaller the range of n_{opt} is. When Ra^* is bigger, the range of n_{opt} is reduced more slowly.

4.2 Optimization analysis when γ=1

4.2.1 Comparative analysis between fixing n and releasing n

Figure 8 shows three-dimensional characteristic of F–Φ–Ra^* with n=3. As shown in F–Φ characteristic in the figure, for any Ra^*, with the increase of Φ, F increases in the beginning and then decreases slowly, and there exist the sole F_{max} and Φ_{opt}. Both Φ_{opt} and F_{max} decrease with the increase of Ra^*. F_{opt}=0.24 and F_{max}=0.7041 at Ra^*=0; Φ_{opt}=0.215 and F_{max}=0.6938 at Ra^*=30; F_{opt}=0.18 and F_{max}=0.6544 at Ra^*=60. As shown in each F–Ra^* characteristic in the figure, when Φ<0.486, F holds the same roughly with the increase of Ra^*; when $0.486 \leqslant \Phi \leqslant 0.639$, F decreases little with the increase of Ra^*. When Φ<0.11, the effect of Ra^* on F is not larger than 2.25%; when Ra^*<20, the effect of Φ on F is not larger than 3.51%.

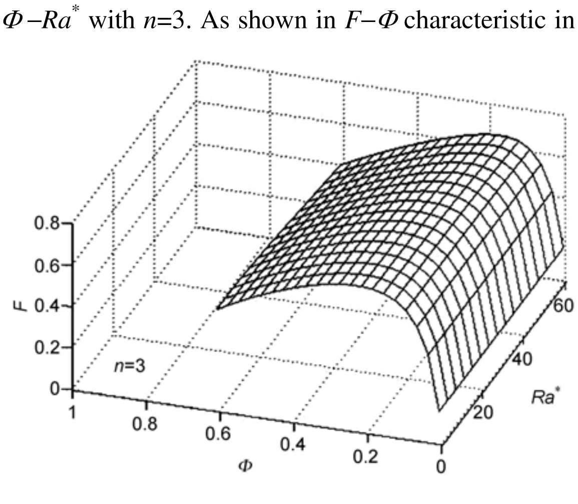

Figure 8 Three-dimensional characteristic of F–Φ–Ra^* with n=3.

Based on the work shown in Figure 8, releasing the constraint of n, the three-dimensional characteristics of F–Φ–Ra^* with different n are calculated and analyzed. The three-dimensional characteristic of F_{max}–Φ–Ra^* is shown in Figure 9. Comparing the rules shown in Figure 9 with those shown in Figure 8, one can find that, the bigger F can be attained by giving much more evolutional freedom degree of structure to the insulating wall, i.e. the bigger F can be attained by adding the freedom degree of cavity number n. Corresponding to different Ra^*, with the increase of Φ, F_{max} increases quickly in the beginning and then decreases slowly, and there also exists a sole maximum, i.e. Φ still has an optimization opportunity (Φ_{opt}) to approach a bigger F. Φ_{opt} and F_{max} both decrease with the increase of Ra^*. Φ_{opt}=0.365 and F_{max}=0.8315 at Ra^*=0; Φ_{opt}=0.215 and F_{max}=0.6941 at Ra^*=30; F_{max} increases by 18.09% and

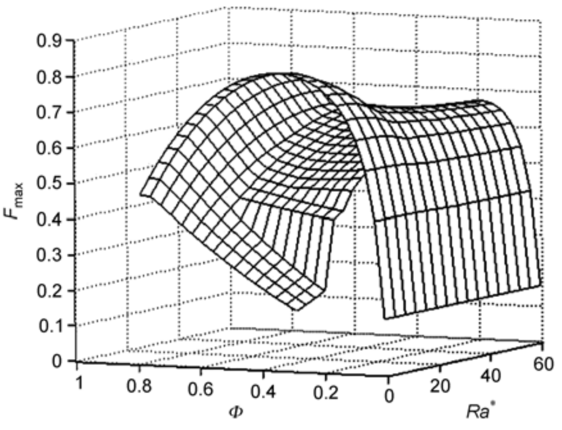

Figure 9 Three-dimensional characteristic of F_{max}–Φ–Ra^*.

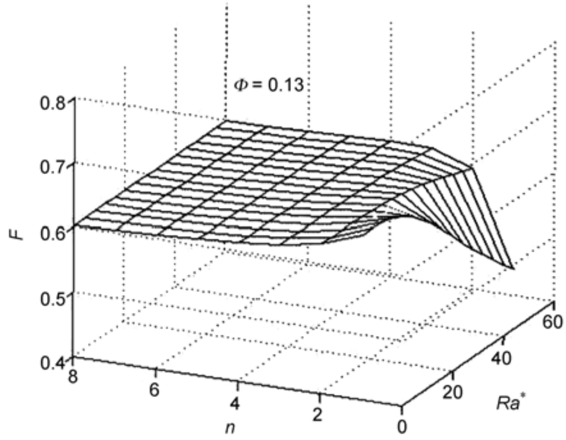

Figure 10 Three-dimensional characteristic of F–n–Ra^* with Φ=0.13.

0.04%, respectively, compared with the results shown in Figure 8. Therefore the smaller Ra^* is, the more notable effect of releasing the constraint of n to increase F is. Comparing the $F_{max}-\Phi$ characteristics corresponding to different Ra^* with each other, one can find that the maximum of F_{max} is reduced obviously with the increase of Ra^* in the beginning, and F_{max} gradually becomes smooth when Ra^*>16. When Φ<0.15, F_{max} corresponding to different Ra^* keep unchanged roughly with the increase of Ra^*; when $0.15 \leqslant \Phi < 0.748$, with the increase of Ra^*, F_{max} corresponding to different Ra^* are reduced in the beginning, and then become gentle quickly; when $0.748 \leqslant \Phi \leqslant 0.793$, F_{max} corresponding to different Ra^* decrease with the increase of Ra^*, and the bigger Φ is, the larger the decreasing amplitude of F_{max} is. When Ra^*>30 and Φ>0.69, F_{max} of various insulating walls are reduced sharply because the thermal resistance decreases obviously.

4.2.2 Comparative analysis between fixing Φ and releasing Φ

Figure 10 shows the three-dimensional characteristic of F–n–Ra^* with Φ=0.13. As shown in the figure, F–n characteristics are obviously different from $F-\Phi$ characteristics, and F–n characteristics corresponding to different Ra^* have different monotonicity. When Ra^*<15, F decreases with the increase of n and gradually becomes smooth; when $15 \leqslant Ra^* \leqslant 60$, with the increase of n, F increases in the beginning and then become gentle. These rules show that there are n_{opt} for different Ra^* with the fixed Φ, and n_{opt} increases with the increase of Ra^*. As shown in the figure, the F–Ra^* characteristics are different according to different n. When $n \leqslant 3$, F decreases with the increase of Ra^*, and the smaller n is, the larger the decreasing amplitude of F is; when n>3, F holds the same roughly with the increase of Ra^*.

Synthesizing the analyses of Figures 8 and 10, one can draw a conclusion that the different design requirements (fixing n or Φ) have different effects on the complex function of $F_{\gamma=1}$. But whatever design requirement is employed, there is the optimal structure design (Φ_{opt} or n_{opt}). Therefore, the method presented herein is feasible, i.e. the structure paying coequal attention to insulating and strength can be obtained by allowing the internal structure of the wall to vary (evolve) freely, and the optimal structure may be different when the application condition (Ra^* value) is different.

Based on the work of Figure 10, releasing the Φ constraint, the three-dimensional characteristics of F–n–Ra^* with different Φ are calculated and analyzed, and the three-dimensional characteristic of F_{max}–Φ–Ra^* is obtained by comparison, as shown in Figure 11. Comparing Figure 10 with Figure 11, one can see that a larger value of F can be obtained by adding the evolutionary Φ degree of freedom for the internal structure of insulating wall. However, corresponding to different n and Ra^*, the effect of releasing the Φ constraint to improve F is different. The smaller n and Ra^* are, the more obvious the effect is. Therefore, the internal structure of the insulating wall with a smaller number of cavities has a more obvious effect on its performance. As shown in Figures 10 and 11, corresponding to different Ra^*, the characteristics of F_{max}–n have different monotonicity; when $Ra^* \leqslant 16$, F_{max} decreases with the increase of n; when $16 < Ra^* \leqslant 60$, with the increase of n, F_{max} increases quickly in the beginning, and then decreases slowly; however, when $n \geqslant 5$, F_{max} approaches to its stability, and the values of F_{max} are close to each other in this case. From F_{max}–Ra^* characteristic, one can see that when n<5, F_{max} decreases monotonously with the increase of Ra^*, and the smaller n is, the larger the decreasing amplitude of F_{max} is. When $n \geqslant 5$, F_{max} holds the same roughly with the increase of Ra^*. It indicates that the performance of the insulating wall with a smaller number of cavities is greatly affected by application environment; while the performance of the insulating wall with a larger number of cavities is

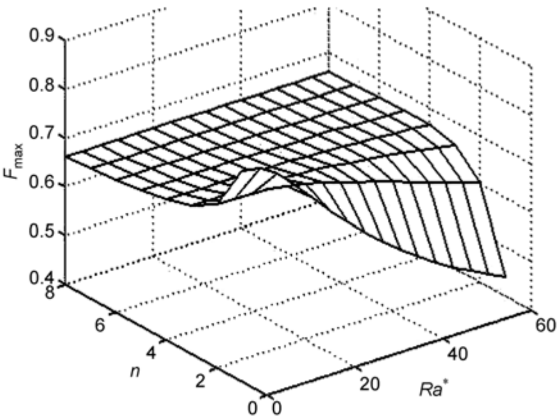

Figure 11 Three-dimensional characteristic of F_{max}–n–Ra^*.

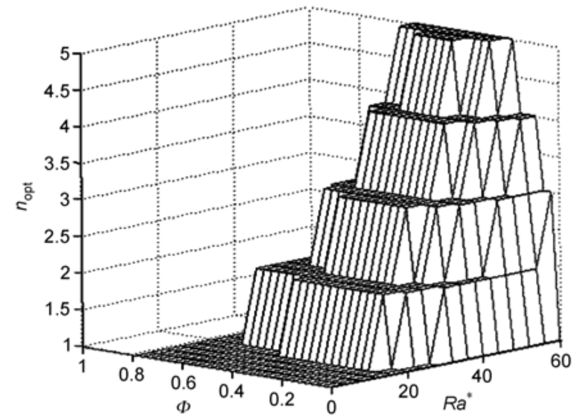

Figure 12 Three-dimensional characteristic of n_{opt}–Φ–Ra^*.

little affected by application environment and its performance is more stable.

Synthesizing the analyses of Figures 9 and 11, one can draw a conclusion that the internal structure with better compositive performance combining insulating and strength can be obtained by releasing constraints of the internal structure of the insulating wall (n and Φ) and allowing it to vary (evolve) freely along the direction of the complex function combining heat flow and strength maximization.

4.2.3 Analysis of optimal structure with double free variables n and Φ

Figures 12 and 13 show the three-dimensional characteristics of n_{opt}–Φ–Ra^* and Φ_{opt}–n–Ra^*. From Figure 12, when $0 \leqslant Ra^* \leqslant 60$, n_{opt} is located between 1 and 5; when $Ra^* \leqslant 16$ or Φ>0.687, n_{opt} is 1. As shown in the figure, for the fixed Φ, n_{opt} increases with the increase of Ra^* in a step way; and n_{opt} always reaches to a new higher step (i.e. adding a cavity) in a middle region of Φ. From n_{opt}–Φ characteristic, one can find that when Ra^*>16, with the increase of Φ, n_{opt} always increases in the beginning, and then decreases to its initial value; the larger Ra^* is, the larger the increasing amplitude and decreasing amplitude of n_{opt} are. From Φ_{opt}–Ra^* characteristic shown in Figure 13, one can see that for the different n, Φ_{opt} roughly decreases with the increase of Ra^*; and the smaller n is, the larger the decreasing amplitude of Φ_{opt} is; when $n \geqslant 5$, the decreasing amplitude of Φ_{opt} is small and Φ_{opt} is robust. From Φ_{opt}–n characteristic shown in Figure 13, one can see that when $Ra^* \leqslant 17$ and $n \leqslant 5$, Φ_{opt} decreases monotonously with the increase of n; when Ra^*>17 and $n \leqslant 5$, Φ_{opt} increases monotonously with the increase of n; when $n \leqslant 5$, Φ_{opt} gradually approaches to its stability no matter what Ra^* is.

Synthesizing the analyses of Figures 11–13, one can find that the performance of insulating wall with a larger number of cavities is little affected by Ra^*, and its adaptability is better and the optimal internal structure is robust. Especially,

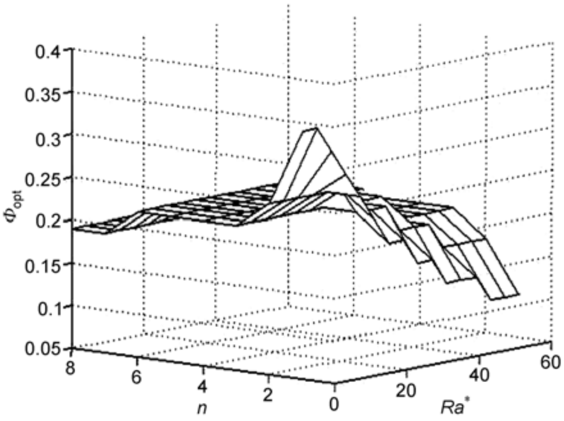

Figure 13 Three-dimensional characteristic of Φ_{opt}–n–Ra^*.

it has better performance for a higher value of Ra^*. Therefore, when the change of the application environment is obvious and Ra^* is high, it is suggested to use the insulating wall with a larger number of cavities. When Ra^* is small, the performance of insulating wall with a smaller number of cavities is better. However, its disadvantage is that its performance is affected by Ra^* more greatly and its adaptability is weak, and the differences of the optimal structures according to different Ra^* are also obvious. Therefore, when using the insulating wall with a smaller number of cavities, one should choose a reasonable structure of the insulating wall according to special application environment with a region of Ra^* to display the performance advantage of the insulating wall in a certain region of Ra^*. Therefore, in the practical engineering design or choosing insulating wall, one should synthetically take various factors such as design requirements and application environment into account.

4.3 Comparative analysis compared with the results of ref. [91]

From eqs. (6), (7), (9) and (10), both the strength $\tilde{I}_n$ of the insulating wall and the thermal resistance $\tilde{R}$ of the insulating wall are the functions of n and Φ. The mechanics constraint is introduced by giving the equation constraint of the prescribed strength in ref. [91], and the internal structure of the insulating wall is optimized by taking $\tilde{R}$ maximization as the objective. Comparing the solid lines (results of this paper) and the dashed lines (resulting from the method in ref. [91]) shown in Figures 3 and 4, one can find that when the value of $(1-1/\tilde{R})$ in the solid lines are higher than those in the dashed lines shown in Figure 3, then the values of I in the corresponding solid line are lower than those in the dashed line shown in Figure 4. For example, when γ=1, comparing $(1-1/\tilde{R})$ obtained by the method of this paper with that obtained by the method in ref. [91] for I=0.9, with the increase of Ra^*, $(1-1/\tilde{R})$ goes through a process of low-high-low-high, and the corresponding I goes through a process of high-low-high–low. It illustrates that complex objective introduced in this paper has a well self-adaptability, that is, it can meet the lowest strength requirement in the practical engineering design. The complex objective can satisfy the two requirements of insulating and strength automatically and coordinately, and the maximum of complex objective can be obtained. However, the strength is fixed in ref. [91], and its essential is the optimal design for pursuing the optimal insulating ability meeting the fixed mechanical strength, and the objective is single in itself rather than compositive. The results obtained are only localized in the constructal design of thermal resistance maximization with the fixed strength (carrying the insulating performance optimization without optimizing the mechanical performance), and they are the optimization points for the given environment conditions and strength.

The paper synthetically considers the competitions between the insulating performance and mechanical performance and builds the complex function F (eq. (11)) of the product of heat flow and strength with power weight as the performance index for balancing the above competition. One can choose a reasonable power weight value γ in the engineering design, and the complex objective has a well applicability. About optimization method, this paper releases the equation constraint of strength employed in ref. [91] and consequently avoids the limitation of that optimization method. However, from the view of practical engineering design, an inequation constraint of strength $(\tilde{I}_n \geqslant 0.5)$ is given out and it is more reasonable. The result obtained in this paper is the constructal design of optimal region corresponding to the simultaneous optimization of both insulating performance and mechanical performance based on the requirement of engineering safety and in a wider calculation region (i.e. satisfying the requirement of the inequation constraint of strength), and it has a wider application region. By using the optimization method herein, one can obtain the optimal region design satisfying a lower limit of strength, and the results obtained in ref. [91] are special cases of this paper.

5 Conclusion

In practical engineering, some multi-disciplinary and multi-objective requirements are involved when an object pursues its optimal performance and single-objective optimization always attends to one thing and loses another. For a vertical insulating wall, considering the competitive relationship between insulating performance and mechanical performance, this paper introduces a product function of heat flow and strength with power weight to serve as the complex performance index for balancing the competition of insulating and strength of the wall. Under the global constraints of fixed external dimensions and safety requirement, the constructal optimization of the wall is conducted by taking the complex function maximization as the objective, and the numerical results prove the rationality of the complex function and the feasibility of the optimization method herein.

The study about the effect of power weight on constructal optimization shows that, the smaller the power weight is, the larger the region of application environment with a higher objective value is. But if the power weight is too small or too large, the optimization cannot compromise insulating and strength preferably, and the range from 0.4 to 4 is appropriate for power weight. For special environment, when power weight is smaller, the higher objective function value can be attained in the wider range of cavity volume fraction; however, when power weight is larger, the range of cavity volume fraction in which the objective function value is higher is reduced, and this indicates that the corresponding precision demand of structure design is more strict.

When heat flow and strength are of coequality (power weight is one), the performance of insulating wall with a larger number of cavities is little affected by total Rayleigh number, and its adaptability is better and the optimal internal structure is robust. When total Rayleigh number is bigger, its performance is better. Therefore, when the change of environment is great and the total Rayleigh number is big, the insulating wall with large number of cavities should be employed. When the total Rayleigh number is small, the insulating wall with a smaller number of cavities is better. But the optimal structures according to different total Rayleigh number and the corresponding performance are affected by total Rayleigh number more greatly and its adaptability is weak. In practical engineering, one should choose a reasonable structure of the insulating wall according to special application environment with a region of total Rayleigh number to display its performance advantage in a certain region of total Rayleigh number.

Ref. [91] is the single-objective optimization study of thermal resistance maximization with the constraint of strength equation, and the results obtained are the optimal points. This paper is multi-objective optimization study giving attention to the two requirements of insulating and strength, and the results obtained are the optimal regions and can provide some guidelines for multi-objective optimization in engineering design. The results in ref. [91] are special cases of this paper. This paper only studies the effect of power weight on constructal optimization of insulating wall, and the further study of the quantitative (qualitative) correlation between power weight parameter and the requirements of practical engineering will be conducted in the future.

This paper was supported by the National Natural Science Foundation of China (Grant No. 10905093), the Program for New Century Excellent Talents in University of China (Grant No. NCET-04-1006) and the Foundation for the Author of National Excellent Doctoral Dissertation of China (Grant No. 200136). The authors wish to thank the reviewers for their careful, unbiased and constructive suggestions, which led to this revised manuscript.

1 Bejan A. Street network theory of organization in nature. J Advanced Transportation, 1996, 30(2): 85–107
2 Bejan A. Shape and Structure, from Engineering to Nature. Cambridge, UK: Cambridge University Press, 2000
3 Bejan A, Lorente S. Thermodynamic optimization of flow geometry in mechanical and civil engineering. J Non-Equilib Thermodyn, 2001, 26(4): 305–354
4 Bejan A, Lorente S. The Constructal Law (La Loi Constructale). Paris: L'Harmatan, 2005
5 Bejan A, Lorente S. Constructal theory of generation of configuration in nature and engineering. J Appl Phys, 2006, 100(4): 041301
6 Reis A H. Constructal theory: From engineering to physics, and how flow systems develop shape and structure. Appl Mech Rev, 2006, 59(5): 269–282
7 Bejan A, Lorente S, Miguel A, et al. Along with Constructal Theory. In :Hernandez J, Cosinschi M, eds. UNIL FGSE Workshop Series No. 1. University of Lausanne, Faculty of Geosciences and the Environment, Switzerland, 2006
8 Bejan A, Lorente S. Design with Constructal Theory. New Jersey: Wiley, 2008
9 Bejan A, Merkx G W. Constructal Theory of Social Dynamics. New York: Springer, 2007
10 Bejan A, Lorente S, Miguel A F, et al. Constructal Human Dynamics, Security & Sustainability. Amsterdam: IOS Press, 2009
11 Bejan A, Badescu V, De Vos A. Constructal theory of economics. Appl Energy, 2000, 67(1): 37–60
12 Zhou S, Chen L, Sun F. Optimization of constructal economics for volume to point transport. Appl Energy, 2007, 84(5): 505–511
13 Bejan A, Dincer I, Lorente S, et al. Porous and Complex Flow Structures in Modern Technologies. New York: Springer, 2004
14 Bejan A, Reis A H. Thermodynamic optimization of global circulation and climate. Int J Energy Res, 2005, 29(4): 303–316
15 Bejan A, Marden J H. Unifying Constructal theory for scale effects in running, swimming and flying. J Experimental Biology, 2006, 209(2): 238–248
16 Dai W, Bejan A, Tang X, et al. Optimal temperature distribution in a three dimensional triple-layered skin structure with embedded vasculature. J Appl Phys, 2006, 99(10): 104702
17 Lorente S. Constructal view of electrokinetic transfer through porous media. J Phys D: Appl Phys, 2007, 40(9): 2941–2947
18 Guo Z, Cheng X, Xia Z. Least dissipation principle of heat transport potential capacity and its application in heat conduction optimization. Chin Sci Bull, 2003, 48(4): 406–410
19 Meng J, Liang X, Li Z. Field synergy optimization and enhanced heat transfer by multi-longitudinal vortices flow in tube. Int J Heat Mass Transfer, 2005, 48(16): 3331–3337
20 Cheng X, Meng J, Guo Z. Potential capacity dissipation minimization and entropy generation minimization in heat conduction optimization (in Chinese). J Eng Thermophys 2005, 26(6): 1034–1036
21 Han G, Zhu H, Cheng X, et al. Transfer similarity among heat conduction, elastic motion and electric conduction (in Chinese). J Eng Thermophys, 2005, 26(6): 1022–1024
22 Guo Z, Zhu H, Liang X. Entransy—A physical quantity describing heat transfer ability. Int J Heat Mass Transfer, 2007, 50(13-14): 2545–2556
23 Zhu H, Chen J, Guo Z. Electricity and thermal analogous experimental study for entransy dissipation extreme principle (in Chinese). Prog Natural Sci, 2007, 17(12): 1692–1698
24 Han G, Guo Z. Two different thermal optimization objective functions: dissipation of heat transport potential capacity and entropy production (in Chinese). J Eng Thermophys, 2007, 27(5): 811–813
25 Han G, Guo Z. Physical mechanism of heat conduction ability dissipation and its analytical expression (in Chinese). Proc CSEE, 2007, 27(17): 98–102
26 Liu X, Meng J, Guo Z. Entropy generation extremum and entransy dissipation extremum for heat exchanger optimization. Chin Sci Bull, 2009, 54(6): 943–947
27 Chen Q, Ren J. Generalized thermal resistance for convective heat transfer and its relation to entransy dissipation. Chin Sci Bull, 2008, 53(23): 3753–3761
28 Chen Q, Ren J, Guo Z. Field synergy analysis and optimization of decontamination ventilation designs. Int J Heat Mass Transfer, 2008, 51(3-4): 873–881
29 Zhang Y, Liu S. The optimization model of the heat conduction structure. Prog Natl Sci, 2008, 18(6): 665–670
30 Liu X, Guo Z, Meng J. Analyses for entransy dissipation and heat resistance in heat exchangers (in Chinese). Prog Natl Sci, 2008, 18(10): 1186–1190
31 Cheng X, Xu X, Liang X. Homogenization of temperature field and temperature gradient field. Sci China Ser E-Tech Sci, 2009, 52(10): 2937–2942
32 Wu J, Liang X. Application of entransy dissipation extremum principle in radiative heat transfer optimization. Sci China Ser E-Tech Sci, 2008, 51(8): 1306–1314
33 Wei S, Chen L, Sun F. "Volume-point" heat conduction constructal optimization with entransy dissipation minimization objective based on rectangular element. Sci China Ser E-Tech Sci, 2008, 51(8): 1283–1295
34 Wei S, Chen L, Sun F. Constructal multidisciplinary optimization of electromagnet based on entransy dissipation minimization. Sci China Ser E-Tech Sci, 2009, 52(10): 2981–2989
35 Wang S, Chen Q, Zhang B, et al. A general theoretical principle for single-phase convective heat transfer enhancement. Sci China Ser E-Tech Sci, 2009, 52(12): 3521–3526
36 Xie Z, Chen L, Sun F. Constructal optimization for geometry of cavity by taking entransy dissipation minimization as objective. Sci China Ser E-Tech Sci, 2009, 52(12): 3504–3513
37 Chen Q, Ren J, Guo Z. The extremum principle of mass entransy dissipation and its application to decontamination ventilation designs in space station cabins. Chin Sci Bull, 2009, 54(16): 2862–2870
38 Xie Z, Chen L, Sun F. Constructal optimization on T-shaped cavity based on entransy dissipation minimization. Chin Sci Bull, 2009, 54(23): 4418–4427
39 Xia S, Chen L, Sun F. Optimization for entransy dissipation minimization in heat exchanger. Chin Sci Bull, 2009, 54(19): 3587–3595
40 Wang S, Chen Q, Zhang B. An equation of entransy and its applica-

tion. Chin Sci Bull, 2009, 54(19): 3572–3578
41 Guo J, Cheng L, Xu M. Entransy dissipation number and its application to heat exchanger performance evaluation. Chin Sci Bull, 2009, 54(15): 2708–2713
42 Liu X, Guo Z. A novel method for heat exchanger analysis (in Chinese). Acta Phys Sin, 2009, 58(7): 4766–4771
43 Chen Q, Wang M, Pan N, et al. Irreversibility of heat conduction in complex multiphase systems and its application to the effective thermal conductivity of porous media. Int J Nonlinear Sci Numer Simul, 2009, 10(1): 57–66
44 Chen Q, Wang M, Pan N, et al. Optimization principles for convective heat transfer. Energy, 2009, 34(9): 1199–1206
45 Chen L, Chen Q, Li Z, et al. Optimization for a heat exchanger couple based on the minimum thermal resistance principle. Int J Heat Mass Transfer, 2009, 52(21-22): 4778–4784
46 Wei S. Constructal entransy dissipation rate minimization for heat conduction. Ph D Thesis. Wuhan: Naval University of Engineering, 2009
47 Li Z, Guo Z. Field Synergy Principle of Heat Convection Optimization (in Chinese). Beijing: Science Press, 2010
48 Guo Z, Liu X B, Tao W Q, et al. Effectiveness-thermal resistance method for heat exchanger design and analysis. Int J Heat and Mass Transfer, 2010, 53(13-14): 2877–2884
49 Wei S, Chen L, Sun F. Constructal entransy dissipation minimization for 'volume-point' heat conduction without the premise of optimized last-order construct. Int J Exergy, 2010, 7(5): 627–639
50 Xia S, Chen L, Sun F. Optimal paths for minimizing entransy dissipation during heat transfer processes with generalized radiative heat transfer law. Appl Math Model, 2010, 34(8): 2242–2255
51 Xia S, Chen L, Sun F. Entransy dissipation minimization for liquid-solid phase processes. Sci China: Tech Sci, 2010, 53(4): 960–968
52 Wei S, Chen L, Sun F. Constructal optimization of discrete and continuous variable cross-section conducting path based on entransy dissipation rate minimization. Sci China: Tech Sci 2010, 53, doi: 10.1007/s11431- 010-0121-5
53 Wei S, Chen L, Sun F. Constructal entransy dissipation minimization for 'volume-point' heat conduction based on triangular element. Thermal Sci, 2010, (in press)
54 Xiao Q, Chen L, Sun F. Constructal entransy dissipation rate minimization for "disc-point" heat conduction. Chin Sci Bull, 2010, (in press)
55 Chen L. Finite Time Thermodynamic Analysis of Irreversible Processes and Cycles (in Chinese). Beijing: Higher Education Press, 2005
56 Wang J, He J, Mao Z. Performance of a quantum heat engine cycle working with harmonic oscillator systems. Sci China Ser G-Phys Mech & Astron, 2007, 50(2): 163–176
57 Song H, Chen L, Sun F. Optimal configuration of a class of endoreversible heat engines for maximum efficiency with radiative heat transfer law. Sci China Ser G-Phys Mech & Astron, 2008, 51(9): 1272–1286
58 Li J, Chen L, Sun F. Optimal configuration for a finite high-temperature source heat engine cycle with complex heat transfer law. Sci China Ser G-Phys Mech & Astron, 2009, 52(4): 587–592
59 Xia S, Chen L, Sun F. The optimal path of piston motion for Otto cycle with linear phenomenological heat transfer law. Sci China Ser G-Phys Mech & Astron, 2009, 52(5): 708–719
60 He J, He X, Tang W. The performance characteristics of an irreversible quantum Otto harmonic cycles. Sci China Ser G-Phys Mech & Astron, 2009, 52(9): 1317–1323
61 Xia S, Chen L, Sun F. Maximum power output of a class of irreversible non-regeneration heat engines with a non-uniform working fluid and linear phenomenological heat transfer law. Sci China Ser G-Phys Mech & Astron, 2009, 52(12): 1961–1970
62 Liu X, Chen L, Wu F, et al. Ecological optimization of an irreversible harmonic oscillators Carnot heat engine. Sci China Ser G-Phys Mech & Astron, 2009, 52(12): 1976–1988
63 Xia D, Chen L, Sun F. Optimal performance of a generalized irreversible four-reservoir isothermal chemical potential transformer. Sci China Ser B-Chem, 2008, 51(10): 958–970
64 Shu L, Chen L, Sun F. The minimal average heat consumption for heat-driven binary separation process with linear phenomenological heat transfer law. Sci China Ser B-Chem, 2009, 52(8): 1154–1163
65 Ding Z, Chen L, Sun F. Thermodynamic characteristic of a Brownian heat pump in a spatially periodic temperature field. Sci China: Phys Mech & Astron, 2010, 53(5): 876–885, doi: 10.1007/s11433-010-0181-3
66 Ma K, Chen L, Sun F. Optimal paths for a light-driven engine with linear phenomenological heat transfer law. Sci China: Chem, 2010, 53(4): 917–926
67 Xia S, Chen L, Sun F. Maximum work configurations of finite potential reservoir chemical engines. Sci China: Chem, 2010, 53(5): 1168–1176, doi: 10.1007/s11426-010-0132-x
68 Dan N, Bejan A. Constructal tree networks for the time-dependent discharge of finite-size volume to one point. J Appl Phys, 1998, 84(6): 3042–3050
69 Bejan A. Constructal-theory network of conducting paths for cooling a heat generating volume. Trans ASME J Heat Transfer, 1997, 40(4): 799–816
70 Zhou S, Chen L, Sun F. Optimization of constructal volume-point conduction with variable cross-section conducting path. Energy Convers Mgmt, 2007, 48(1): 106–111
71 Wu W, Chen L, Sun F. On the "area to point" flow problem based on constructal theory. Energy Convers Mgmt, 2007, 48(1): 101–105
72 Wu W, Chen L, Sun F. Improvement of tree-like network constructal method for heat conduction optimization. Sci China Ser E-Tech Sci, 2006, 49(3): 332–341
73 Wei S, Chen L, Sun F. The volume-point constructal optimization for discrete variable cross-section conducting path. Appl Energy, 2009, 86(7-8): 1111–1118
74 Chen L, Wu W, Sun F. Constructal re-optimization of heat conduction with the triangular elemental area. Rev Mexi Fis, in press
75 Bejan A, Almogbel M. Constructal T-shaped fins. Int J Heat Mass Transfer, 2000, 43(12-15): 2101–2115
76 Bejan A, Rocha L A O, Lorente S. Thermodynamic optimization of geometry: T- and Y- shaped constructs of fluid streams. Int J Thermal Sci, 2000, 39(9-11): 949–960
77 Bejan A, Fautrelle Y. Constructal multi-scale structure for maximal heat transfer density. Acta Mechanica, 2003, 163(1-2): 39–49
78 da Silva A K, Bejan A. Constructal multi-scale structure for maximal heat transfer density in natural convection. Int J Heat Fluid Flow, 2005, 26(1): 34–44
79 Lorente S, Wechsatol W, Bejan A. Tree-shaped flow structures designed by minimizing path lengths. Int J Heat Mass Transfer, 2002, 45(16): 3299–3312
80 Vargas J V C, Bejan A. Thermodynamic optimization of finned crossflow heat exchangers for aircraft environmental control systems. Int J Heat Fluid Flow, 2001, 22 (6): 657–665
81 Morega A M, Bejan A. A constructal approach to the optimal design of photovoltaic cells. Int J Green Energy, 2005, 2(3): 233–242
82 Vargas J V C, Ordonez J C, Bejan A. Constructal PEM fuel cell stack design. Int J Heat Mass Transfer, 2005, 48(21-22): 4410–4427
83 Bejan A, Errera M R. Convective trees of fluid channels for volumetric cooling. Int J Heat Mass Transfer, 2000, 43(17): 3105–3118
84 Wechsatol W, Lorente S, Bejan A. Dendritic convection on a disc. Int J Heat Mass Transfer, 2003, 46(23): 4381–4391
85 da Silva A K, Bejan A. Constructal multi-scale structure for maximal heat transfer density in natural convection. Int J Heat Fluid Flow, 2005, 26(1): 34–44
86 Zimparov V D, da Silva A K, Bejan A. Thermodynamic optimization of tree-shaped flow geometries. Int J Heat Mass Transfer, 2006, 49(9-10): 1619–1630
87 Luo L, Fan Y, Zhang W, et al. Integration of constructal distributors to a mini crossflow heat exchanger and their assembly configuration optimization. Chem Eng Sci, 2007, 62(13): 3605–3619

88 Wechsatol W, Lorente S, Bejan A. Tree-shaped insulated designs for the uniform distribution of hot water over an area. Int J Heat Mass Transfer, 2001, 44(16): 3111–3123
89 Azoumah Y, Neveu P, Mazet N. Constructal design combined with entropy generation minimization for solid–gas reactors. Int J Thermal Sci, 2006, 45(7): 716–728
90 Zhou S, Chen L, Sun F. Constructal optimization for solid-gas reactors based on triangular element. Sci China Ser E-Tech Sci, 2008, 51(9): 1554–1562
91 Lorente S, Bejan A. Combined 'flow and strength' geometric optimization: internal structure in a vertical insulating wall with air cavities and prescribed strength. Int J Heat Mass Transfer, 2002, 45(16): 3313–3320
92 Gosselin L, Bejan A, Lorente S. Combined 'heat flow and strength' optimization of geometry: mechanical structures most resistant to thermal attack. Int J Heat Mass Transfer, 2004, 47(14-16): 3477–3489
93 Gosselin L, da Silva A K. Combined 'heat transfer and power dissipation' optimization of nanofluid flows. Appl Phys Letter, 2004, 85(18): 4160–4162
94 Gosselin L, Bejan A. Constructal thermal optimization of an electromagnet. Int J Thermal Sci, 2004, 43(4): 331–338
95 Wei S, Chen L, Sun F. Constructal complex-objective optimization of electromagnet based on magnetic induction and maximum temperature difference. Rev Mexi Fis, 2010, in press
96 Bejan A. Convection Heat Transfer, 2nd ed. New York: Wiley, 1995
97 Bejan A. Note on Gill's solution for free convection in a vertical enclosure. J Fluid Mech, 1979, 90: 561–568
98 Churchill S W, Usagi R. A standardized procedure for the production of correlations in the form of a common empirical equation. Indust Eng Chem Fund, 1974, 13(1): 39–44
99 Chen L, Zhang J. MCDM and the introduction of optimal model selection and design application of steam turbine in warship (in Chinese). Turbine Tech, 1986, 28(5): 58–70
100 Yan Z. η and P of a Carnot engine at maximum ($\eta^2 P$) (in Chinese). J Xiamen Univ, 1986, 25(3): 279–286