

Application analysis of empirical mode decomposition and phase space reconstruction in dam time-varying characteristic

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In view of some courses of the time-varying characteristics processing in the analysis of dam deformation, the paper proposes a new method to analyze the dam time-varying characteristic based on the empirical mode decomposition and phase space reconstruction theory. First of all, to reduce the influences on the traditional statistical model from human factors and assure the analysis accuracy, response variables of the time-varying characteristic are obtained by the way of the empirical mode decomposition; and then, a phase plane of those variables is reconstructed to investigate their processing rules. These methods have already been applied to an actual project and the results showed that data interpretation with the assists of empirical mode decomposition and phase space reconstruction is effective in analyzing the perturbations of response variables, explicit in reflecting the entire development process, and valid for obtaining the evolution rules of the time-varying characteristic. This methodology is a powerful technical support for people to further master the rules of dam operation.

dam safety monitoring, dam time-varying characteristic, empirical mode decomposition, phase space reconstruction

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1 Introduction

The time-varying characteristic of dam deformation is gradually represented in a long-term and slow development process, in other words, it reflects the development rule of the response variables of dam time-varying characteristic in the mass. The changing phenomenon is influenced not only by external loads, but also by creep, dry shrinkage of concrete and other factors. Furthermore, those influences are irreversible with time. In some cases, the development rule of response variables of dam time-varying characteristic reflects the behavior of dam operation. In general, the re-

sponse variables of dam time-varying characteristic have a drastic change process during the impounding period and tend to be stable in the following years. If the response variables of the time-varying characteristic in dam safety monitoring change dramatically in operation period, it is an evidence of the dam body or dam foundation in abnormal situation [1–3].

At present, the response variables of dam time-varying characteristic can be obtained by the following procedures: first of all, some factors related to the time should be determined, and a statistical model is established based on the actual dam safety monitoring data. Then the correlation coefficients are obtained through the regression analysis method. Finally, the deformation due to dam time-varying characteristic can be gained. This method needs to draw up

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the structural form of the factors of dam time-varying characteristic before computing, and can be affected by human factors [4–8]. In addition, if the water level and other factors are closely related to the factor of dam time-varying characteristic, this correlation affects the regression result obviously. The empirical mode decomposition theory treats any complicated signal as the accumulation of some different, simple and nonlinear signals. From this point, some basic signals can be separated from the complicated signal sequence [9, 10]. The monitoring data of the dam deformation can be viewed as a digital signal sequence consisting of the different frequency components, and the frequency of response variables of dam time-varying characteristic is low, so these low frequency changes may be separated through the empirical mode decomposition theory. The phase space is an abstract space which is made up of the state variable factors. The phase orbit is the line of phase points in the phase space and is the evolution process of the system state over time [11, 12]. This paper analyzes the development rule of the response variables of dam time-varying characteristic based on empirical mode decomposition and phase space reconstruction theory.

2 Application of empirical mode decomposition in the response variables of dam time-varying characteristic

Empirical mode decomposition can separate the complex signal into a number of basic signals from the high-frequency one to the low-frequency one, which are intrinsic mode functions (IMF). Each of these basic signals can be linear, nonlinear or non-stationary, while they have one common characteristic that each IMF has the same number of extreme points and zero-crossing points in the entire signal part. Furthermore, the IMF is symmetrical in the local average and its waveform is similar to the sinusoidal one from the view of shape. The local average is defined by the up and down envelope line of the signal, and based on such a definition, the different IMFs can be separated through the characteristic scales of the signal [13–16]. As a result, the characteristic scales are defined by the time span of the extreme points of the adjacent signals based on the experimental modal decomposition method of the multi-resolution analysis. At any time, a complicated signal can have different IMFs. All the IMFs are mixed to form a complicated signal. Each of these IMFs is independent of each other. And there are no multiple extreme points between the adjacent zeros. The procedure of the IMF decomposing is as follows:

(1) Determine all the local extreme value points of a time sequence $x(t)$, and then link these extreme large and small points by a line respectively, by doing so we get the up-and-down envelope line of the time sequence $x(t)$. Take note of the average value of the up and down envelope line

as $m(t)$.

(2) Let $h_1(t)$ denote the difference between $x(t)$ and $m(t)$:

$$h_1(t) = x(t) - m(t). \quad (1)$$

Ideally, $h_1(t)$ should be an IMF. But, for the nonlinear data, envelope average value may be different from the real local average value, because of the existence of some asymmetry waves. The filtering process mainly functions in two ways, one is to delete the accumulated waves and the other is to make the waves more symmetric. In order to achieve the previous aim, we need to repeat the previous progress until $h_1(t)$ becomes an IMF, which is marked as $f_1(t) = h_1(t)$.

(3) After decomposing the first IMF from the original sequence, using $x(t)$ subtract $f_1(t)$ we then get the remaining value of the sequence:

$$x_1(t) = x(t) - f_1(t). \quad (2)$$

(4) $x_1(t)$ is regarded as the new “primitive” serial. Repeating the above steps and extracting the second, the third and until the n th basic pattern of component in sequence, then, $x_n(t)$ becomes a monotonous sequence, it does not contain any mode of information, $r_n(t) = x_n(t)$ is the original signal of the remainder term. Thus, the signal is decomposed into $f_i(t) (i=1, 2, \dots, n)$, which are the intrinsic mode functions and the remainder term $x_n(t)$, that is

$$x(t) = \sum_{i=1}^n f_i(t) + r_n(t). \quad (3)$$

A lot of noise is often contained in the data of dam safety detection, some useless frequency components not only increase the number of layers of empirical mode decomposition and decrease the efficiency of decomposition but also accumulate the boundary error because of the excess of decomposition. Decomposition series will have an impact on the accuracy of empirical mode decomposition. Therefore, we will use wavelet analysis for reducing the impact on the analysis of data signals.

3 Application of phase space reconstruction in the response variables of dam time-varying characteristic

The sequence of response variables of dam time-varying characteristic is the appearance characteristic of the entire dam structure and is also the reaction of the system state variables. It reflects the dynamic evolution of response variables of the time-varying characteristic (displacement) under the effects of the reason factors of concrete structures (environment and load, etc.). The phase space reconstruction theory can establish the one-to-one correspondence relationship between the state of system and the points of the phase space. The one point of the phase space means a

state of the system and the orbit of the phase space reflects the evolution process of the state changes with time [17]. Therefore, the phase space reconstruction in the field of nonlinear dynamics theory reconstructs the phase space of the effect quantity of the time-varying characteristic, which can analyze the development regularity of dam time-varying characteristic. At present, phase space reconstruction theory includes the delay reconstruction method and the derivative reconstruction method.

Through delaying time, delay reconstruction method inserts one-dimensional observation sequence $x(t)$ into m -dimensional phase space:

$$X(t) = (x(t), x(t + \tau), \dots, x(t + (m - 1)\tau)), \quad (4)$$

where τ is a positive integer, called the delay time.

Derivative reconstruction method can obtain system topological equivalence reconstructed phase space by some single variables' all-order derivatives. In the reconstructed phase space, vector elements are $\{x(t), \dot{x}(t), \ddot{x}(t), \dots\}$, where $\dot{x}(t), \ddot{x}(t)$ are the first and the second derivatives of $x(t)$ respectively.

Since the phase space derivative reconstruction method can effectively reflect time-varying effect quantity's change rate, in order to display the evolution process of time-varying characteristic better, this paper adopts derivative reconstruction to reconstruct phase space. Dam's irreversible deformation along with time can be expressed as first order nonlinear differential equation, that is:

$$\dot{\delta}_\theta = f(\delta_\theta, a), \quad (5)$$

where $\dot{\delta}_\theta$ is the time derivative of dam time effect deformation δ_θ , namely time effect rate; a could be any parameter.

At any time, system's $\dot{\delta}_\theta = f(\delta_\theta, a)$ represents the motion state which is called the phase that has been characterized by $\dot{\delta}_\theta$ and δ_θ . Their value corresponds to a plane point namely phase point, and the plane $(\delta_\theta, \dot{\delta}_\theta)$ is called phase plane (two-dimensional phase space) where abscissa axis is for dam time-varying response variable δ_θ , ordinate axis for dam time effect change rate $\dot{\delta}_\theta$.

According to literature, time-varying monitoring model for concrete dam deformation is

$$\delta(t) = f(H, T, \theta, t) = \delta_H(t) + \delta_T(t) + \delta_\theta(t), \quad (6)$$

where $\delta_H(t)$ is water component; $\delta_T(t)$ is temperature component; $\delta_\theta(t)$ is time effect component.

For a certain time t , by the use of expression (6) the calculated value $\hat{\delta}(t)$ can be obtained, after making a comparison between the calculated value $\hat{\delta}(t)$ and the measured value $\delta(t)$, $|\hat{\delta}(t) - \delta(t)|$ can be obtained too. According to probability and statistic theory, the probability for $|\hat{\delta}(t) - \delta(t)|$ falling into the interval $[0, 2S]$ is 95.5%, and $[0, 3S]$ is 99.7% where S is the residual standard deviation of this model. Hereby, the method which applies time-varying monitoring model to judge anomaly occurrence is put forward, that is:

(1) Normal: $|\hat{\delta}(t) - \delta(t)| \leq 2S$, dam structure is under normal operation;

(2) Abnormal: $2S < |\hat{\delta}(t) - \delta(t)| \leq 3S$, dam structure is under abnormal operation, but the measured value sequence of dam deformation has a tendency variation;

(3) Dangerous: $|\hat{\delta}(t) - \delta(t)| > 3S$, dam structure is under abnormal operation, at this time, its causes should be analyzed on the basis of present or earlier period environmental change, as well as whether there is anomaly occurrence in dam's own internal structure.

The response variable of dam time-varying characteristic reflects the tendency variation of dam deformation, in addition, it is also the important index measuring dam structure behavior. By the application of empirical mode decomposition method combined with time-varying monitoring model for dam deformation, dam time-dependent deformation $\delta_\theta(t)$ is studied and the change forms of dam time-varying characteristic are distinguished.

The development of dam deformation's time-varying characteristic response variable can generally divided into three forms: (1) time-varying characteristic response variable increases gradually and tends to be stable, its time effect change rate $d\delta_\theta/dt > 0$, moreover, $d^2\delta_\theta/dt^2 < 0$, as shown in Figure 1(a); (2) time-varying characteristic response variable maintains a constant speed to grow, its time effect change rate $d\delta_\theta/dt > 0$, moreover, $d^2\delta_\theta/dt^2 = 0$, as shown in Figure 1(b); (3) time-varying characteristic response variable develops at a increasing rate, its time effect change rate

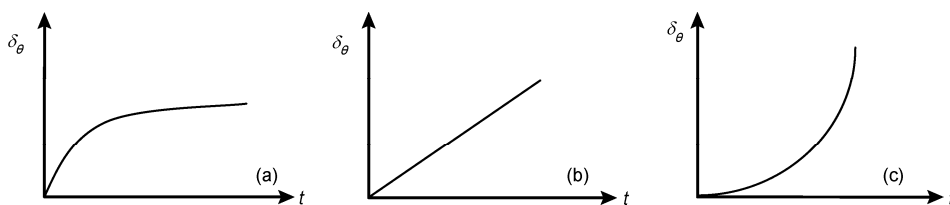


Figure 1 Evolution forms of the response variable of time-varying characteristic.

$d\delta_\theta/dt > 0$, moreover, $d^2\delta_\theta/dt^2 > 0$, as shown in Figure 1(c).

By the application of derivative reconstruction method, the reconstructed phase space can be made up which corresponds to the response variable of dam time-varying characteristic's expression form (Figure 2). As shown in Figure 2, when the effect quantity of dam time-varying characteristic tends to be stable along with time, the change rate of its time effect also rapidly reduces and its corresponding path decreases in line; when the response variable of dam time-varying characteristic maintains a constant speed to grow along with time, its corresponding path shows as an horizontal line; when the effect quantity of dam time-varying characteristic develops at an increasing rate along with time, its corresponding path increases in line. Thus, phase space can visually display the evolution law of the response variable of dam time-varying characteristic along with time [18].

From the above, and according to dam deformation safety monitoring data, the response variable of dam time-varying characteristic can be obtained by utilization of empirical mode decomposition theory and phase space reconstruction can be made through phase space reconstruction technique, and by obtaining path development the change law of the response variable of dam time-varying characteristic can be analyzed, in this case, not only its evolution and devel-

opment process can be clearly studied, but also the practical working behavior of dam operation can be better reflected.

4 Practical project analysis

Taking the radial displacement of a measuring point at the top of a gravity arch dam as an example, we analyzed the result by comparing the result of direct decomposing by the wavelet with that of the comprehensive analysis based on wavelet and EMD. From March 28, 2001 to December 31, 2007, there were 2370 values, almost each day had a value, the interval of monitoring is $\Delta\tau = 1d$, the curve of the monitoring data is shown in Figure 3.

The time series of dam safety monitoring data is a digital signal sequence with different frequency components. The temperature of dam body often cycles with the air temperature, as well as the monitoring value. In addition, reservoir water level also cycles with the alternating of seasons due to the nature inflow. So the portion varying with environment conditions (reservoir water level, temperature) has a certain cycle: annual period with higher frequency which is notable; the random portion influenced by random factor and monitoring error vibrates with high frequency; the other portion is the buzz developed by the random factor and monitoring

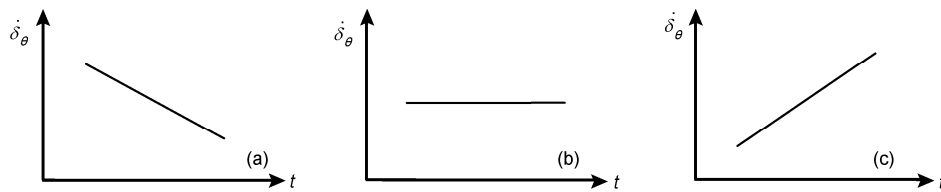


Figure 2 Phase plane corresponding with evolution forms of the response variable of time-varying characteristic.

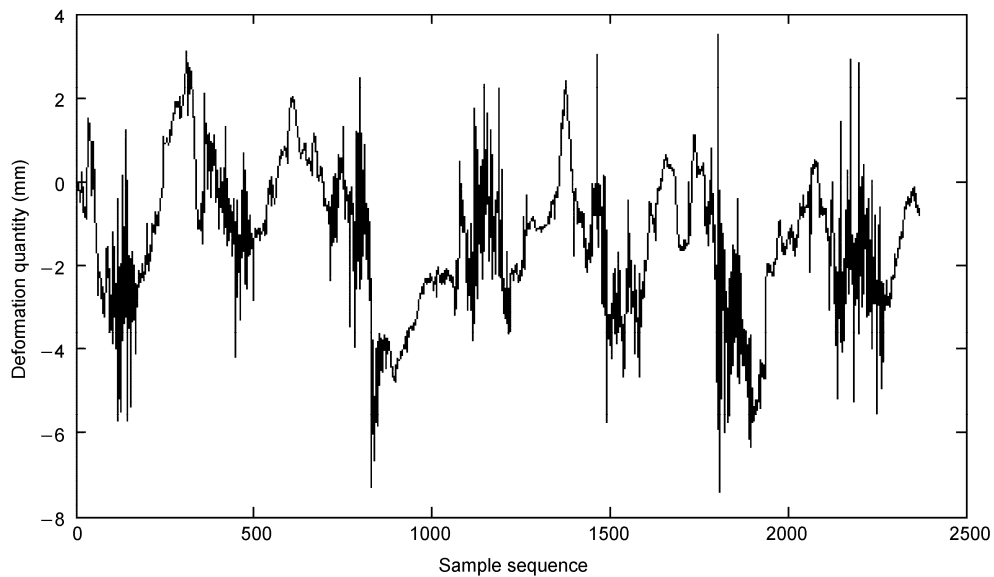


Figure 3 Curve of the monitoring data.

error.

With the help of the computer calculation procedures, the response variable of dam time-varying characteristic can be separated from the dam safety monitoring data through the empirical mode decomposition method, whose form of the development and changes is shown in Figure 4.

Development and change trend of time response variable separated from dam deformation safety monitoring data by application of statistical model is shown in Figure 5.

The selected model of time effect deformation component, which is separated from the response variable of dam time-varying characteristic by application of statistical model, is a linear term plus logarithmic term. That is, firstly select time related factor, and then make regression analysis of dam safety monitoring data to obtain time effect quantity. This kind of method which drafts time effect factor beforehand has certain artificial influence, and then its precision is affected. Moreover, when reservoir water level, temperature and other factors have close correlation with time effect, regression analysis and separating result's effectiveness may be influenced by this kind of correlation.

Contrasting time-varying characteristic response variable's development and change form in Figure 4 with time effect component separated by statistical model, it is shown that the overall changed forms are consistent, but have slight difference in the change law. Development and change form of time-varying characteristic response variable separated by empirical mode decomposition is relatively smooth and gentle in two ends, intermediate curve slope has a great change and its change rate is greater than the change

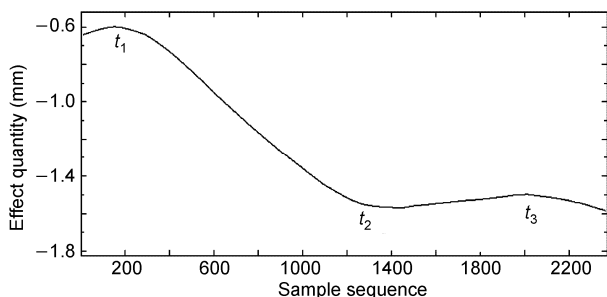


Figure 4 Effect quantity of time-varying characteristic separated by empirical mode decomposition.

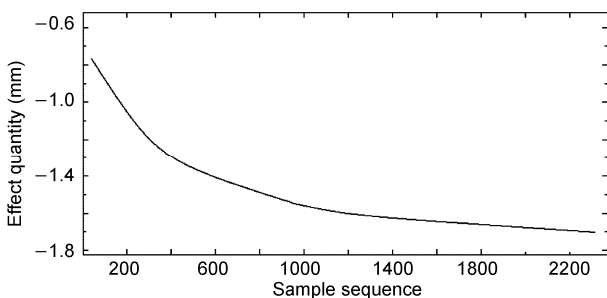


Figure 5 Response variable of time-varying characteristic separated by statistical model.

rate in two ends; whereas time effect component separated by statistical model changes relatively quickly in earlier period and then relatively slowly in the later period, it is determined by the nature of time effect component model.

Practical monitoring value sequence's change curve in Figure 3 cannot clearly reflect the change and development trend of the effect quantity of dam time-varying characteristic along with time. Its time-varying effect quantity's change has been covered by water level and temperature influence. Dam's deformation appears to change steadily, however, its time-varying effect quantity still has a slow evolution with time. The change form of time-varying response variable in Figure 4, which is obtained through empirical mode decomposition theory, can plainly display the evolution process of the response variable of dam time-varying characteristic along with time. It is seen that dam deformation monitoring points' change has obvious convergence trend, and the deformation tends to be stable. But in Figure 4 only visual embodiment and the overall trend of the response variable of dam time-varying characteristic's evolution process have been reflected, to get further understanding of its specific development and change process, phase space of the response variable of dam time-varying characteristic should be reconstructed, as shown in Figure 6. Change interval above zero coordinate line (horizontal dashed line in Figure 6) is time effect deformation increasing interval where its change rate $\dot{\delta}_\theta$ is positive, and change interval below zero coordinate line is time effect deformation decreasing interval where its change rate $\dot{\delta}_\theta$ is negative. If the absolute value of time effect change rate $\dot{\delta}_\theta$ is bigger, then time effect change speed is higher, while the absolute value's reducing means time effect change rate gets slower. Horizontal axis δ_θ has reflected dam time-varying effect quantity value's change process.

In the analysis of Figure 6, t_0 is the initial monitoring time of the response variable of dam time-varying charac-

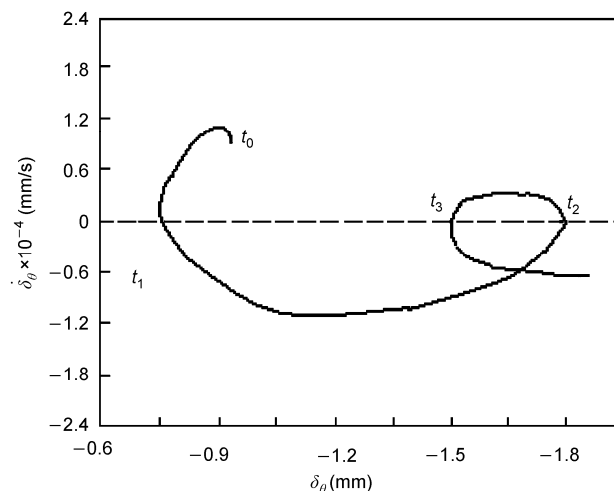


Figure 6 Phase plane of response variable of time-varying characteristic.

teristic, and t_1 , t_2 , t_3 are the corresponding time of intersection points of time effect's phase path and zero coordinate line, that is the typical time when dam time-varying characteristic change law undergoes a change. First, time effect deformation of the response variable of dam time-varying characteristic increases in the interval $[t_0, t_1]$, from about -0.73 mm to about -0.58 mm, and its change rate decreases gradually; time effect deformation decreases in the interval $[t_1, t_2]$, from about -0.58 mm to about -1.60 mm, and its change rate increases at first and then decreases; after t_2 , time-varying effect quantity starts to increase again, what is more, its change rate gradually speeds up at first and seems to slow down when getting to a certain degree; when arriving at t_3 , time-varying effect quantity has already increased to -1.40 mm, after t_3 , it begins to decrease again and its reducing rate speeds up, when time-varying response variable decreases to about -1.60 mm, the reducing rate of dam time-varying effect quantity also slows down and the change tends to be stable.

5 Conclusion

In summary, by use of empirical mode decomposition theory and based on dam deformation safety monitoring measured data, the response variable of dam time-varying characteristic can be separated, which has less impact on the separated result compared with traditionally statistical model because of artificial factor, and therefore it can improve the precision of further analysis in the response variable of dam time-varying characteristic; by application of nonlinear dynamic phase space reconstruction technique and conducting time effect deformation phase space reconstruction to the separated response variable of dam time-varying characteristic, the phase path of the response variable of dam time-varying characteristic is obtained, by which not only tiny fluctuation phenomenon that time-varying characteristic response variable changes with time can be clearly reflected, namely enlarging an tiny change, but also the whole development and change process of the response variable of dam time-varying characteristic can be fully studied. Then dam time-varying characteristic evolution law is analyzed, and the degree that dam time-varying characteristic affects dam safe operation is studied, which provides technical support for further analysis of dam practical working behavior.

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