SCIENCE CHINA
Technological Sciences

August 2010 Vol.53 No.8: 2122–2128
doi: 10.1007/s11431-009-3172-8

Noise reduction method for nonlinear signal based on maximum variance unfolding and its application to fault diagnosis

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Received November 13, 2009; accepted February 26, 2010

A new noise reduction method for nonlinear signal based on maximum variance unfolding (MVU) is proposed. The noisy signal is firstly embedded into a high-dimensional phase space based on phase space reconstruction theory, and then the manifold learning algorithm MVU is used to perform nonlinear dimensionality reduction on the data of phase space in order to separate low-dimensional manifold representing the attractor from noise subspace. Finally, the noise-reduced signal is obtained through reconstructing the low-dimensional manifold. The simulation results of Lorenz system show that the proposed MVU-based noise reduction method outperforms the KPCA-based method and has the advantages of simple parameter estimation and low parameter sensitivity. The proposed method is applied to fault detection of a vibration signal from rotor-stator of aero engine with slight rubbing fault. The denoised results show that the slight rubbing features overwhelmed by noise can be effectively extracted by the proposed noise reduction method.

nonlinear noise reduction, manifold learning, maximum variance unfolding, fault diagnosis

Citation: Zhang Y, Li B W. Noise reduction method for nonlinear signal based on maximum variance unfolding and its application to fault diagnosis. Sci China Tech Sci, 2010, 53: 2122–2128, doi: 10.1007/s11431-009-3172-8

1 Introduction

The useful information of operating machinery tends to be submerged by the strong noise, so it is necessary to eliminate noise from measured signal effectively before diagnosing machine faults. The mechanical systems are often characterized by non-linear dynamics behaviors, which make the signal have the characteristic of power broadband and pseudo noise. So the conventional linear methods based on band-pass filter in the frequency domain are difficult to separate the signal from noise effectively. Recently, a number of non-linear analysis methods, such as local projection (LP) [1], singular spectrum decomposition (SSD) [2] and kernel principal component (KPCA) [3, 4], are used for signal denoising. The LP and SSD methods are both conducted by performing local iterative modification on all points in the phase space to approximate the true attractor trajectory. However, the two methods are considered as a local linear method, which can not describe the global nonlinear characteristic of the dynamics system. The KPCA-based method maps the nonlinear data in phase space into a higher dimensional feature space by kernel function, and performs linear dimension reduction by PCA to separate the clean signal space from the noise subspace. However, its performance largely depends on its kernel function and parameters which can only be selected empirically.

Manifold learning which is a new effective method of nonlinear dimensionality reduction has attracted more and more attention recently. The purpose of manifold learning is to project the original high-dimensional data into a lower dimensional feature space by preserving the local neighborhood structure. Manifold learning is effective for us to discover the intrinsic low-dimensional manifold structure of

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nonlinear high-dimensional data. At present, the representative methods include isometric mapping (ISOMAP) [5], locally linear embedding (LLE) [6], Laplacian eigenmaps [7] and maximum variance unfolding (MVU) [8]. MVU proposed by Weinberger in 2006 is a new manifold learning method based on the notion of local isometry, which unfolds the underlying data manifold in its reduced space by rotation and translation subject to the constraints that preserve the angles and distances of local neighborhoods. It can not only learn the low-dimensional manifold embedded in the high-dimensional data but also indicate the intrinsic dimension of manifold.

We consider the noise-free attractor trajectory of system as a low-dimensional smooth hypersurface manifold embedded in the phase space, and use the manifold learning method to discover the attractor manifold for noise reduction. In this paper, the noisy signal is firstly embedded into a high-dimensional phase space based on phase space reconstruction theory, and MVU is used to perform nonlinear reduction on the data of phase space in order to separate low-dimensional manifold representing the attractor from noise subspace. Then the noise reduction result is achieved through reconstructing the low-dimensional manifold. However, there is no apparent mapping between the high-dimension space and the reduced space in MVU, and MVU does not provide a method that can reconstruct the manifold from low-dimension space to high-dimension space. Thus the local polynomial regression is used to reconstruct the manifold. The simulation results of Lorenz system show that the proposed MVU-based noise reduction method outperforms the KPCA-based method and has the advantages of simple parameter estimation and low parameter sensitivity. The proposed method is applied to fault detection of a vibration signal from rotor-stator of aero engine with slight rubbing fault. The denoised results show that the slight rubbing features overwhelmed by noise can be effectively extracted by the proposed noise reduction method.

2 MVU algorithm

Suppose the noisy data set $\boldsymbol{X}=(\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n)^{\mathrm{T}}$ is sampled from r-dimensional manifold M embedded in the d-dimensional space $\boldsymbol{R}^d$. Without the knowledge of r and M, the purpose of manifold learning is to find the mapping $f:\boldsymbol{R}^d \to \boldsymbol{R}^r (r \ll d)$ according to the high-dimensional data $\boldsymbol{X}$ and get the low-dimensional manifold coordinate $\boldsymbol{Y}=(\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_n)^{\mathrm{T}}$, $\boldsymbol{y}_i \in \boldsymbol{R}^r$ in one-to-one correspondence with the data set $\boldsymbol{X}$.

The method for maximum variance unfolding is constructed based on a simple intuition. Assumed that the inputs $\boldsymbol{x}_i$ are connected to their k nearest neighbors by rigid rods, the algorithm attempts to pull the inputs apart by maximizing the total sum of their pairwise distances without breaking (or stretching) the rigid rods that connect the nearest neighbors. The outputs are obtained from the final state of this transformation. For example, Figure 1 shows the nonlinear dimensionality reduction results for an S-curve data set by MVU. From the color coding of data, we can see that MVU successfully unfolds the S-curve and preserves its local structure in the reduced space.

The "unfolding" transformation described above can be formulated as a quadratic program. Let $\boldsymbol{W}_{ij} \in \{0,1\}$ denote whether inputs $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ are k-nearest neighbors. The MVU algorithm can be expressed as the following optimization:

$$\begin{aligned}
&\max \sum_{i,j=1}^{n} \left\|\boldsymbol{y}_i-\boldsymbol{y}_j\right\|^2, \\
&\text{s.t.} \ \left\|\boldsymbol{y}_i-\boldsymbol{y}_j\right\|^2 \boldsymbol{W}_{ij} = \left\|\boldsymbol{x}_i-\boldsymbol{x}_j\right\|^2 \boldsymbol{W}_{ij}, \\
&\sum_{i=1}^{n} \boldsymbol{y}_i = 0.
\end{aligned} \tag{1}$$

Here, the first constraint enforces that distances between nearby inputs match distances between nearby outputs, while the second constraint yields a unique solution (up to rotation) by centering the outputs on the origin.

The apparent intractability of this quadratic program can be finessed by a simple change of variables. Note that as written above, the optimization over the outputs $\boldsymbol{y}_i$ is not convex, meaning that it potentially suffers from spurious local minima. Defining the inner product matrix $\boldsymbol{K}_{ij}=\langle \boldsymbol{y}_i, \boldsymbol{y}_j \rangle$, we can reformulate the optimization as a semi-definite program (SDP) over the matrix $\boldsymbol{K}=[\boldsymbol{K}_{ij}]_{n\times n}$, which can be written as

$$\begin{aligned}
&\max \operatorname{trace}(\boldsymbol{K}) \\
&\text{s.t.} \ \boldsymbol{K} \geqslant 0, \\
&\sum_{i,j=1}^{n} \boldsymbol{K}_{ij} = 0, \\
&\boldsymbol{K}_{ii} - 2\boldsymbol{K}_{ij} + \boldsymbol{K}_{jj} = \left\|\boldsymbol{x}_i-\boldsymbol{x}_j\right\|^2, \ \text{if } \boldsymbol{W}_{ij}=1,
\end{aligned} \tag{2}$$

where trace(·) denotes the trace of matrix $\boldsymbol{K}$. The last (additional) constraint needs the matrix $\boldsymbol{K}$ to be positive semi-definite, which can guarantee the data to be from convex set.

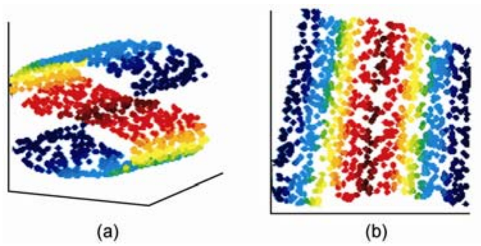

Figure 1 (a) The S-curve data; (b) the dimensionality reduction result for S-curve data.

There are several efficient general-purpose toolboxes for solving semi-definite programming problems, such as the SeDuMi [9] and CSDP [10] toolboxes.

Let $\boldsymbol{K}^*$ denote the optimal solution of $\boldsymbol{K}$. The spectral decomposition of $\boldsymbol{K}^*$ can be written as

$$\boldsymbol{K}_{ij}^* = \sum_{\alpha=1}^{n} \lambda_\alpha \boldsymbol{V}_{\alpha i} \boldsymbol{V}_{\alpha j}, \tag{3}$$

where $\boldsymbol{V}_{\alpha i}$ denotes the *i*-th element of the eigenvector corresponding to the α-th largest eigenvalue λ_α, if the eigenvalues are sorted from the largest to the smallest.

An *n*-dimensional mapping $\boldsymbol{y}_i^*$ of the inputs $\boldsymbol{x}_i$ is given by identifying the α-th element of $\boldsymbol{y}_i^*$ as

$$\boldsymbol{y}_{\alpha i}^* = \sqrt{\lambda_\alpha} \boldsymbol{V}_{\alpha i}. \tag{4}$$

The $r(r << d)$-dimensional mapping $\boldsymbol{y}_i$ is obtained by reserving the first *r* elements of $\boldsymbol{y}_i^*$ and truncating the rest elements.

3 Noise reduction method for nonlinear signal based on MVU

Suppose $\boldsymbol{z} = [z_1, z_2, \cdots, z_N]$ is one-dimensional noisy nonlinear signal. By embedding it into an *m*-dimensional phase space, we can obtain the following data matrix:

$$\boldsymbol{X} = \begin{bmatrix} z_1 & z_2 & \cdots & z_{N-(m-1)\times\tau} \\ z_{1\times\tau+1} & z_{1\times\tau+2} & \cdots & z_{N-(m-2)\times\tau} \\ \vdots & \vdots & \ddots & \vdots \\ z_{(m-1)\times\tau+1} & z_{(m-1)\times\tau+2} & \cdots & z_N \end{bmatrix}, \tag{5}$$

where m denotes embedding dimension and satisfies $m \geqslant 2D+1$, D is the fractal dimension of attractor, τ denotes delay time. $\boldsymbol{x}_i = [z_i, z_{1\times\tau+i}, \cdots, z_{(m-1)\times\tau+i}]^{\mathrm{T}}$, which is a column vector of matrix $\boldsymbol{X}$ corresponding to one data point of phase space. So there are $N_0 = N-(m-1)\times\tau$ data points in all. Takens has proven that the attractor trajectory of system can be recovered by reconstructing the phase space with a proper embedding dimension. In the m-dimensional phase space, the system attractor is located on a low-dimensional subspace, whereas the white noise in signal distributes in every dimension of the phase space. If the noise-free attractor track of the system is considered as a low-dimensional smooth hypersurface manifold embedded in the phase space, then we can use the manifold learning method to separate the attractor manifold for noise reduction according to the different distributions of signal and noise in the phase space.

According to the analysis above, the MVU algorithm is used to perform nonlinear mapping for the data in the phase space. The optimal kernel matrix is learned by solving (2), and then the N_0-dimensional mappings $\boldsymbol{y}_1^*, \boldsymbol{y}_2^*, \cdots, \boldsymbol{y}_{N_0}^*$ are obtained by solving (3) and (4).

How to specify the reduced dimension r (determined by the fractal dimension of attractor) is a crucial problem. It is difficult to compute the fractal dimension of attractor accurately due to the influence of noise [11]. So it is not appropriate to specify *r* by the fractal dimension of attractor. MVU can indicate the intrinsic dimension of manifold. MVU unfolds the data manifold in the reduce space and hence $\boldsymbol{y}_1^*, \boldsymbol{y}_2^*, \cdots, \boldsymbol{y}_{N_0}^*$ are very likely to lie on a linear subspace of the reduce space. Since spectral decomposition of kernel matrix $\boldsymbol{K}$ is performed in the reduce space and eigenvalues of $\boldsymbol{K}$ are proportional to the variance along principal components, it is considered that the largest r eigenvalues are associated with variances of manifold and the rest smallest $N_0 - r$ eigenvalues are associated with the variances of noise. Weinberger [8], who proposed MVU, also pointed that a large gap in the eigenvalue spectrum between the *r*-th and *r*+1-th eigenvalue indicates that the data lie on or near a manifold of dimensionality *r*. So the intrinsic dimension of data can be effectively determined according to the eigenvalues of $\boldsymbol{K}$ learned by MVU. For example, Figure 2 shows the normalized eigenvalues of the kernel matrix learned by MVU and the kernel matrix of KPCA for the S-curve data set shown in Figure 1 (only the largest 15 normalized eigenvalues are shown). The 3rd largest normalized eigenvalue from MVU which suddenly decreases to near zero indicates the correct intrinsic dimension (*r*=2) of the S-curve data set.

Thus we choose the largest *r* eigenvalues and their corresponding eigenvectors to compute the manifold's coordinates $\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_{N_0}$ in the *r*-dimensional space by solving (4). So the $(N_0 - r)$-dimensional noise subspace is separated by choosing *r*, and $\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_{N_0}$ represent the clean attractor manifold.

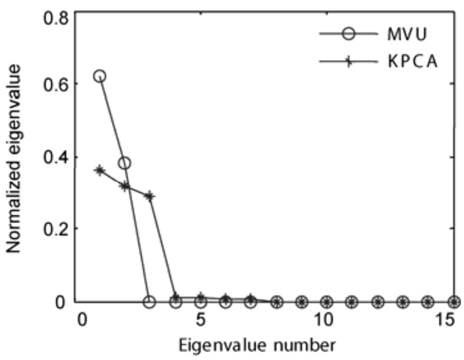

Figure 2 The normalized eigenvalues of the kernel matrix learned by MVU and the kernel matrix of KPCA for the S-curve data set.

The one-dimensional noise-reduced signal can be obtained through reconstructing the noise-reduced manifold from reduced space to the high-dimensional phase space. However, MVU does not provide a reconstructing method for the low-dimension manifold. There is no apparent mapping between high-dimension space and reduced space in MVU. We define the reconstruction problem by

$$\boldsymbol{x}_i = f(\boldsymbol{y}_i) + \boldsymbol{\varepsilon}_i,\ \ i = 1,\ 2,\ \cdots,\ N_0, \tag{6}$$

where $\boldsymbol{x}_i \in \mathbf{R}^m$ denotes the high-dimensional data point in the phase space, $f(\cdot)$ denotes the nonlinear mapping function from low-dimensional data to the high-dimensional data, and $\boldsymbol{\varepsilon}_i$ denotes the noise. The purpose of reconstruction is to recover the attractor manifold $f(\boldsymbol{y}_i)$ in the high-dimensional phase space. Eq. (6) can be considered as a non-parametric regression problem, thus we apply local polynomial regression to $\{(\boldsymbol{x}_i, \boldsymbol{y}_i)\}_{i=1}^{N_0}$ to construct the manifold $f(\boldsymbol{y}_i)$ underlying the set of points $\boldsymbol{x}_i$. The detail of local polynomial regression algorithm is referred to ref. [12].

The MVU-based noise reduction method is summarized in the following.

1) Choose the proper embedding dimension m and delay time τ to embed the one-dimensional noisy signal into an m-dimensional phase space.

2) Choose the proper number of nearest neighbors k to construct the $N \times N$ binary adjacency matrix $\boldsymbol{W}$. Set $\boldsymbol{W}_{ij} = 1$ if $\boldsymbol{x}_i$ is a k-nearest neighbor of $\boldsymbol{x}_j$, otherwise, set $\boldsymbol{W}_{ij} = 0$. $\boldsymbol{x}_i$ denotes the data point in the phase space and N denotes the number of data points.

3) Learn the optimal kernel matrix $\boldsymbol{K}$ by solving (2).

4) Perform spectrum decomposition for $\boldsymbol{K}$ and set the reduced dimension to be the intrinsic dimension r determined according to the eigenvalues of $\boldsymbol{K}$. Then choose the largest r eigenvalues and their corresponding eigenvectors to compute the manifold's coordinates $\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_{N_0}$ in the r-dimensional space by solving (4).

5) Use local polynomial regression to reconstruct the attractor manifold from reduced space to the high-dimensional phase space.

6) Obtain the noise-reduced one-dimensional signal by the inverse transform of phase space reconstruction.

7) Repeat the above steps until having a good noise reduction effect.

4 Simulation and analysis

A numerical experiment on Lorenz system was conducted to evaluate the performance of the proposed MVU-based noise reduction method. The Lorenz system is a typical nonlinear dynamic system whose equation is given by

$$\begin{cases} \dot{x} = -\omega(x - y), \\ \dot{y} = -xz + rz - y, \\ \dot{z} = xy - bz. \end{cases} \tag{7}$$

The system is in chaotic when ω=10, r=28, b=3/8. The fourth-order Runge Kutta algorithm with the step size set to 0.01 was used to compute eq. (7). The 2000 data points of x were taken as testing signal, to which 15 dB white Gaussian noise was added. Then we used KPCA-based method and our proposed MVU-based method to perform noise reduction on the noisy testing signal, respectively. The signal to noise ratio (SNR) and mean square error (MSE) were used as performance indexes defined as

$$\mathrm{SNR} = 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_w^2}\right), \tag{8}$$

where σ_x^2 is the square of signal, and σ_w^2 is the square of noise.

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(x(i) - \hat{x}(i))^2, \tag{9}$$

where n is the length of signal, $x(i)$ is the noisy signal, and $\hat{x}(i)$ is the noise-reduced signal.

In the phase space reconstruction, the same parameters were chosen for two method, that is, embedding dimension m=6 and delay time τ=1. For MVU, the number of nearest neighbors k was set to 4. For KPCA, the Gaussian kernel $k(\boldsymbol{a}, \boldsymbol{b}) = \exp(-\|\boldsymbol{a} - \boldsymbol{b}\|^2 / \delta)$ was used and the kernel parameter δ was set to 30 by cross-validation scheme. The normalized eigenvalues of the kernel matrix learned by MVU and the kernel matrix of KPCA are shown in Figure 3 (only the largest 15 normalized eigenvalues were shown). We can see that the third largest normalized eigenvalue of the matrix learned by MVU suddenly decreased to near zero. This indicates that the intrinsic dimension of attractor is 2, which is very close to the true Lorenz system attractor's fractal dimension 2.06. However, we cannot draw a similar conclusion on intrinsic dimension according to the eigenvalues from KPCA because there is no sharp "turning point" in the plot. The reduced dimension of KPCA is set to 5 by counting the eigenvalues larger than the average eigenvalue. This rule was recommended for KPCA by Lee [13].

The two-dimensional phase trajectory of the noisy Lorenz signal is shown in Figure 4. The two-dimensional phase trajectory of noise-reduced Lorenz signal by MVU and KPCA are shown in Figures 5 and 6, respectively. The noise reduction results of the two methods are shown in Table 1. It can

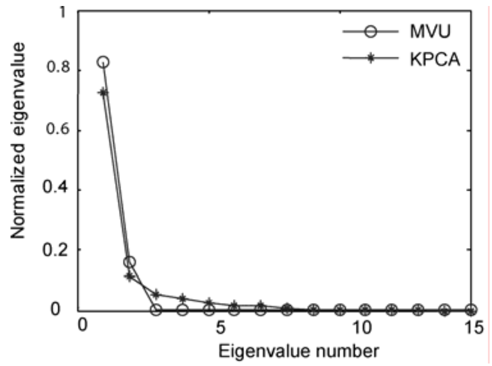

Figure 3 The normalized eigenvalues of the kernel matrix learned by MVU and the kernel matrix of KPCA.

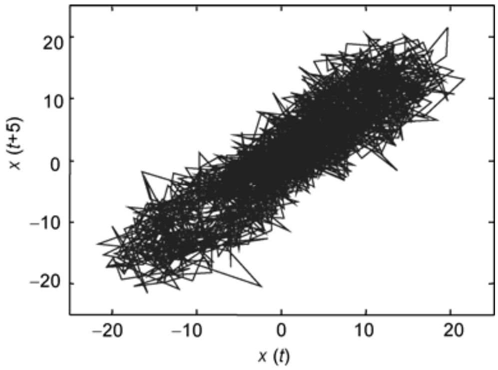

Figure 4 The two-dimensional phase trajectory of the noisy Lorenz signal.

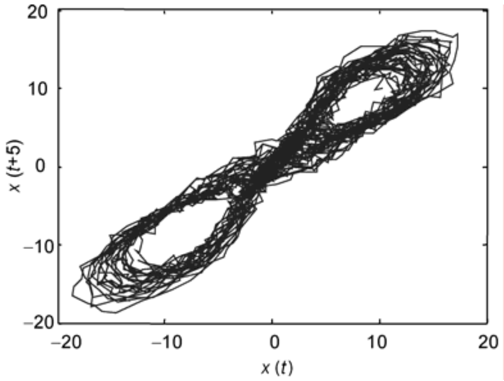

Figure 5 The two-dimensional phase trajectory of the noise-reduced Lorenz signal by KPCA.

be seen that the MVU-based method has good performance for noise reduction. Compared to the KPCA-based method, the SNR of MVU-based method is increased by 3.09 dB and the MSE decreased by 0.0115, which means the MVU-based method outperforms the KPCA-based method. Another advantage of the MVU-based noise reduction method is that it requires only one parameter, the number of nearest neighbors k, which can be estimated simply. In order to analyze the influence of parameter k on the noise reduction efficiency, we chose different k to perform noise reduction on the noisy Lorenz signal above. The experiment result is shown in Figure 7. we can see that the change of SNR of the noise-reduced signal was about 1 dB when k varied from 2 to 10. Thus it is thought that the choice of k has little influence on the noise reduction efficiency in the experiment.

5 Application

Rotor rubbing is the common fault mode of aero engine. The vibration signal generated by rubbing is nonlinear, whose spectrum includes fractional, double, triple frequencies and so on. Since the energy of slight rubbing is small, the slight rub features tend to be overwhelmed by the high background noise and are difficult to be extracted.

Figure 8 shows the time-domain curve and spectrum of a vibration signal from rotor-stator of aero engine with slight rubbing fault. The rotor speed is 390 Hz and the sampling frequency is 5120 Hz. From the order spectrum, we can only find the power frequency and double frequency, which

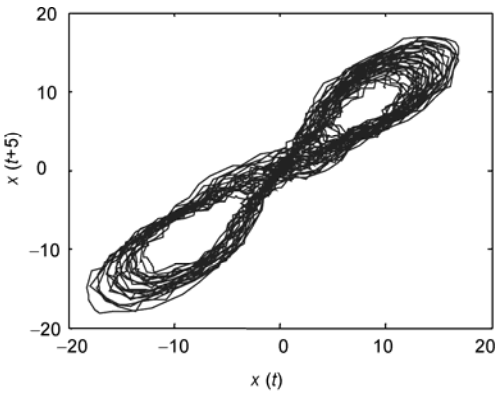

Figure 6 The two-dimensional phase trajectory of the noise-reduced Lorenz signal by MVU.

Table 1 SNR and MSE of noise-reduced signal by MVU and KPCA

Noise reduction method	SNR (dB)	MSE
KPCA	31.16	0.0542
MVU	34.25	0.0427

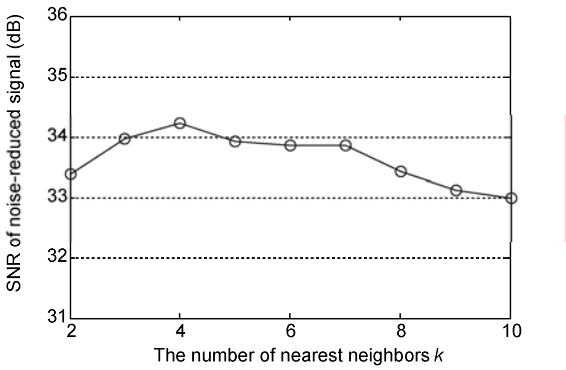

Figure 7 SNR of noise-reduced signal with different parameters k.

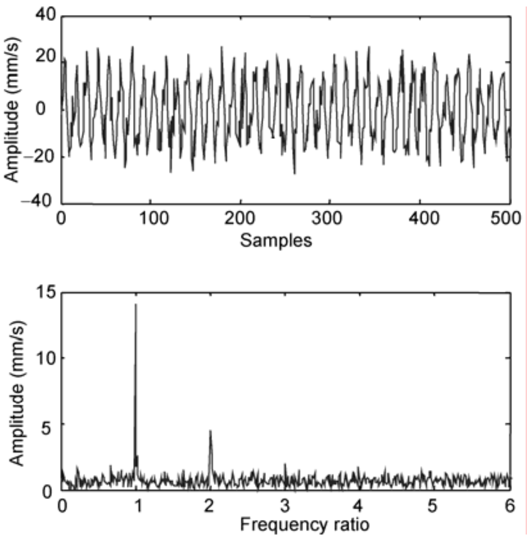

Figure 8 The time-domain curve and spectrum of a vibration signal from rotor-stator of aero engine with slight rubbing fault.

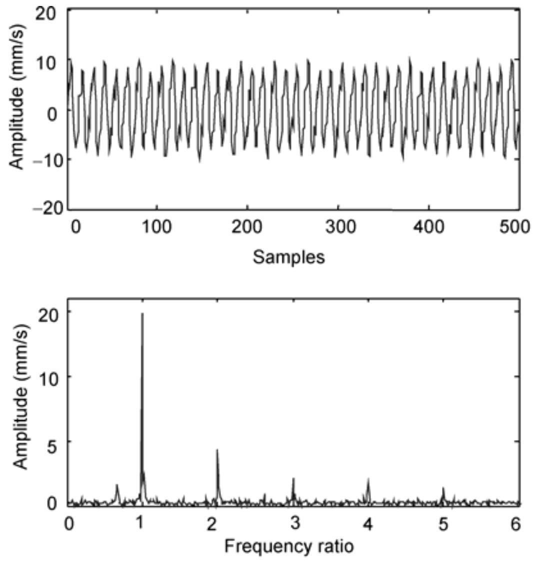

Figure 9 The time-domain curve and spectrum of the noise-reduced vibration signal.

means the rubbing fault features are unobvious. In order to extract the fault feature, we used the proposed MVU-based method to reduce the noise contained in the vibration signal. Among the method parameters, the embedding dimension m, delay time τ and the number of nearest neighbors k were set to 8, 1 and 5, respectively. Figure 9 shows the time-domain curve and spectrum of the noise-reduced vibration signal. From the spectrum, we can find the triple, fourfold, fivefold and fractional frequencies which represent the fault characteristic of slight rotor rubbing. Thus it can be seen that the proposed MVU-based method can reduce the noise in the nonlinear rotor rubbing vibration signal effectively, which is helpful to improve the accuracy of fault diagnosis.

Moreover, we computed the autocorrelation coefficient of the vibration signal to evaluate the performance of MVU-based method. The noise makes the autocorrelation coefficient decrease, so if the autocorrelation coefficient is larger, it means the noise is suppressed more efficiently. The autocorrelation coefficients of the neighbor two sampling moment are shown in Table 2. It can be seen that the autocorrelation coefficient of the noise-reduced vibration signal increased, which means the noise was eliminated efficiently. Compared to the KPCA-based method, the autocorrelation coefficient of noise-reduced signal by MVU-based method was larger, and the autocorrelation coefficient of reduced noise was smaller, which illustrates our proposed MVU-based noise reduction method has better performance.

Table 2 The autocorrelation coefficients of the neighbor two sampling moment

Noise redution method	Autocorrelation coefficient		
	Noisy signal	Noise-reduced signal	Eliminated noise
KPCA	0.8852	0.9224	0.0542
MVU	0.8852	0.9436	0.0218

6 Conclusion

A new noise reduction method for nonlinear signal based on maximum variance unfolding (MVU) was proposed. The noisy signal is firstly embedded into a high-dimensional phase space, and then the manifold learning algorithm MVU performs nonlinear dimensionality reduction on the data of phase space in order to separate low-dimensional manifold representing the attractor from noise subspace. Finally, the noise-reduced signal is obtained through reconstructing the low-dimensional manifold. Simulation results have shown that the proposed MVU-based noise reduction method outperforms the KPCA-based method and has the advantages of simple parameter estimation and low parameter sensitivity. The proposed method was applied to fault detection of a vibration signal from rotor-stator of aero engine with slight rubbing fault. The denoised results have shown that the slight rubbing features overwhelmed by noise can be effectively extracted by the proposed noise reduction method.

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