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Periodic and chaotic dynamics of composite laminated piezoelectric rectangular plate with one-to-two internal resonance

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The bifurcations and chaotic dynamics of a simply supported symmetric cross-ply composite laminated piezoelectric rectangular plate are studied for the first time, which are simultaneously forced by the transverse, in-plane excitations and the excitation loaded by piezoelectric layers. Based on the Reddy's third-order shear deformation plate theory, the nonlinear governing equations of motion for the composite laminated piezoelectric rectangular plate are derived by using the Hamilton's principle. The Galerkin's approach is used to discretize partial differential governing equations to a two-degree-of-freedom nonlinear system under combined the parametric and external excitations. The method of multiple scales is employed to obtain the four-dimensional averaged equation. Numerical method is utilized to find the periodic and chaotic responses of the composite laminated piezoelectric rectangular plate. The numerical results indicate the existence of the periodic and chaotic responses in the averaged equation. The influence of the transverse, in-plane and piezoelectric excitations on the bifurcations and chaotic behaviors of the composite laminated piezoelectric rectangular plate is investigated numerically.

composite laminated piezoelectric rectangular plate, third-order shear deformation, parametric excitation, bifurcation, chaos

1 Introduction

Nowadays piezoelectric materials, which include piezoelectric lead-zirconate-titanate (PZT) and piezoelectric polyvinylidene fluoride (PVDF), are new type functional materials in engineering fields. Piezoelectric materials can be used as the actuators and sensors in engineering structures. Therefore, composite laminated piezoelectric plates have been widely applied to aircraft, large space station and shuttle in the two past decades[1,2]. With the increased applications of composite laminated piezoelectric plates in engineering fields, for example morphing structures or morphing wings, composite laminated plates with piezoelectric materials can undergo large oscillating deformation, which leads to nonlinear oscillations of plates. Research on the nonlinear dynamics, bifurcations, and chaos of composite laminated piezoelectric plate will play a significant role in engineering applications. However, up to now, only a few studies on the bifurcations and chaotic dynamics of composite laminated piezoelectric plate have been conducted.

Several researchers have focused their attention on investigating the nonlinear dynamic responses of composite laminated plates. Srinivasamurthy and Chia[3] presented a nonlinear shear-deformable theory for dynamic behaviors of generally laminated circular plates with movable and immovable in-plane boundary conditions. Boginich[4] analytically studied the nonlinear oscillations of nonconservative elastic laminated systems by applying the asymptotic method. Demchouk[5] constructed a quasi-three-dimensional theory to investigate

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dynamic thermoelastic problem of laminated composite plates. Chueshov and Lasiecka[6] studied the inertial manifold for the von Karman plate equations and discussed three different dissipative mechanisms including the viscous, structural and thermal damping. Courilleau and Mossino[7] studied the limit characteristics of nonlinear monotone equations in laminated plates. Ye et al.[8] dealt with the nonlinear dynamic behaviors of a parametrically excited, simply supported, symmetric cross-ply laminated rectangular thin plate. The geometric nonlinearity and nonlinear damping were included in the governing equations of motion. Ye et al.[9] investigated nonlinear oscillations and chaotic dynamics of a simply supported antisymmetric cross-ply composite laminated rectangular thin plate under parametric excitation. Lee and Reddy[10] studied the nonlinear responses of laminated composite plates under thermomechanical loading using the third-order shear deformation plate theory. Zhang et al.[11] investigated the nonlinear oscillations and chaotic dynamics of a parametrically excited simply supported symmetric cross-ply laminated composite rectangular thin plate with the geometric nonlinearity and nonlinear damping. Varelis and Saravanos[12] presented a coupled theoretical and computational framework for analyzing the small amplitude-free vibration of composite laminated plates with piezoelectric actuators and sensors. Hao et al.[13] investigated the nonlinear oscillations, bifurcations and chaos of a functionally graded materials (FGM) plate and found that the periodic, quasi-periodic and chaotic motions exist for the FGM rectangular plate under certain conditions.

The responses of composite laminated piezoelectric plates were also considered by investigators in the two past decades. Ye and Tzou[14] studied the responses and distributed control of a laminated piezoelectric semicircular shell in the changing temperature environment. Krommer and Irschik[15] analyzed flexural vibrations of composite piezoelectric plates in which piezoelectric layers are used to generate distributed actuation or perform distributed sensing of strains in plates. They demonstrated that coupling among the mechanical, electrical and thermal fields can be taken into account by means of effective stiffness parameters and an effective thermal loading. Correia et al.[16] developed a semi-analytical axisymmetric shell finite element model with embedded or surface bonded piezoelectric actuators or sensors to study active damping vibration control of structures. Donadon et al.[17] investigated the effect of the in-plane piezoelectric induced stresses on the natural frequencies of composite plates, which are square and clamped along two opposing edges and free along the other two . Lim and Lau[18] investigated the electro-mechanical behaviors of a thick, laminated actuator with piezoelectric and isotropic lamina under externally applied electric loading using a new two-dimensional computational model. Karnaukhov and Tkachenko[19] investigated the active damping of nonstationary vibrations of a hinged rectangular plate with distributed piezoelectric actuators using the dynamic-programming method.

In this paper, we study the bifurcations and chaotic dynamics of a four-edged simply supported composite laminated piezoelectric rectangular plate subject to the transverse, in-plane and piezoelectric excitations. Based on the von Karman-type equations and the Reddy's third-order shear deformation plate theory, the Hamilton's principle is employed to obtain the governing equations of motion for the composite laminated piezoelectric rectangular plate. Because only transverse nonlinear oscillations of the composite laminated piezoelectric rectangular plate are considered, the governing equations of motion can be reduced to a two-degree-freedom nonlinear system under combined parametric and external excitations by using the Galerkin's method. The case of 1:2 internal resonance and primary parametric resonance is considered. The method of multiple scales is used to obtain the averaged equation of the original non-autonomous system. Numerical method is utilized to investigate the bifurcations, periodic and chaotic motions of the composite laminated piezoelectric rectangular plate. The bifurcation diagrams are also obtained by using numerical simulation. It is found from the numerical results that there exist the periodic and chaotic motions of the composite laminated piezoelectric rectangular plate under certain conditions. It is also observed that the chaotic responses are especially sensitive to the forcing and the parametric excitations.

2 Formulation

Consider a composite laminated piezoelectric rectangular plate simply supported at four-edge, where the edge lengths are a and b, respectively and thickness is h. The composite laminated piezoelectric rectangular plate is considered as regular symmetric cross-ply laminates with n layers. Some of these layers are made of the

PVDF piezoelectric materials as actuators, while others are made of fiber-reinforced composite materials. It is assumed that different layers of the symmetric cross-ply composite laminated piezoelectric rectangular plate are perfectly bonded to each other and with piezoelectric actuator layers embedded in the plate. A Cartesian coordinate system *Oxyz* is located in the middle surface of the composite laminated piezoelectric rectangular plate. Assume that (u, v, w) and (u_0, v_0, w_0) represent the displacements of an arbitrary point and a point in the middle surface of the composite laminated piezoelectric rectangular plate in the *x*, *y* and *z* directions, respectively. It is also assumed that the in-plane excitations of the composite laminated piezoelectric rectangular plate are loaded along the *y* direction at $x=0$ and the *x* direction at $y=0$ with the form of $q_0 + q_x \cos\Omega_1 t$ and $q_1 + q_y \cos\Omega_2 t$, respectively. The transverse excitation, which loads to the composite laminated piezoelectric rectangular plate, is represented by $q = q_3 \cos\Omega_3 t$. The dynamic electrical loading is expressed as $E_z = E_z \cos(\Omega_4 t)$.

According to the Reddy's third-order shear deformation theory (TSDT), the displacement field of the composite laminated piezoelectric rectangular plate is assumed to be the following form:

$$u(x, y, z, t) = u_0(x, y, t) + z\phi_x(x, y, t) - z^3 \frac{4}{3h^2}\left(\phi_x + \frac{\partial w_0}{\partial x}\right), \tag{1a}$$

$$v(x, y, z, t) = v_0(x, y, t) + z\phi_y(x, y, t) - z^3 \frac{4}{3h^2}\left(\phi_y + \frac{\partial w_0}{\partial y}\right), \tag{1b}$$

$$w(x, y, z, t) = w_0(x, y, t), \tag{1c}$$

where (u, v, w) are the displacement components along the (x, y, z) directions, (u_0, v_0, w_0) is the deflection of a point on the middle plane ($z = 0$), ϕ_x and ϕ_y respectively represent the rotations of transverse normal of the mid-plane about the *x* and *y* axes.

The nonlinear strain-displacement relations are given as follows:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \ \varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right),$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right),$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \ \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right), \ \varepsilon_{zz} = \frac{\partial w}{\partial z}. \tag{2}$$

Substituting eq. (1) into eq. (2) yields the strains

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^{(0)} \\ \varepsilon_y^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \kappa_x^{(0)} \\ \kappa_y^{(0)} \\ \kappa_{xy}^{(0)} \end{Bmatrix} + z^3 \begin{Bmatrix} \kappa_x^{(2)} \\ \kappa_y^{(2)} \\ \kappa_{xy}^{(2)} \end{Bmatrix},$$
$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{zx}^{(0)} \end{Bmatrix} + z^2 \begin{Bmatrix} \gamma_{yz}^{(2)} \\ \gamma_{zx}^{(2)} \end{Bmatrix}, \tag{3}$$

where

$$\begin{Bmatrix} \varepsilon_x^{(0)} \\ \varepsilon_y^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial x}\right)^2 \\ \dfrac{\partial v_0}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial y}\right)^2 \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} + \dfrac{\partial w_0}{\partial x}\dfrac{\partial w_0}{\partial y} \end{Bmatrix},$$

$$\begin{Bmatrix} \kappa_x^{(0)} \\ \kappa_y^{(0)} \\ \kappa_{xy}^{(0)} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \varphi_x}{\partial x} \\ \dfrac{\partial \varphi_y}{\partial y} \\ \dfrac{\partial \varphi_x}{\partial y} + \dfrac{\partial \varphi_y}{\partial x} \end{Bmatrix}, \ \begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{zx}^{(0)} \end{Bmatrix} = \begin{Bmatrix} \varphi_y + \dfrac{\partial w_0}{\partial y} \\ \varphi_x + \dfrac{\partial w_0}{\partial x} \end{Bmatrix}, \tag{4}$$

$$\begin{Bmatrix} \kappa_x^{(2)} \\ \kappa_y^{(2)} \\ \kappa_{xy}^{(2)} \end{Bmatrix} = -c_1 \begin{Bmatrix} \dfrac{\partial \varphi_x}{\partial x} + \dfrac{\partial^2 w_0}{\partial x^2} \\ \dfrac{\partial \varphi_y}{\partial y} + \dfrac{\partial^2 w_0}{\partial y^2} \\ \dfrac{\partial \varphi_x}{\partial y} + \dfrac{\partial \varphi_y}{\partial x} + 2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix},$$

$$\begin{Bmatrix} \gamma_{yz}^{(2)} \\ \gamma_{zx}^{(2)} \end{Bmatrix} = -c_2 \begin{Bmatrix} \varphi_y + \dfrac{\partial w_0}{\partial y} \\ \varphi_x + \dfrac{\partial w_0}{\partial x} \end{Bmatrix},$$

$$c_1 = \frac{4}{3h^2}, \ c_2 = 3c_1.$$

It is known that the constitutive relations are of the form

$$\sigma_{ij} = \sigma_{ijkl}^{s}\,\varepsilon_{kl} - e_{ijk}\,E_k, \ (i, j, k, l = x, y, z), \tag{5}$$

where E_k is the electric field, and e_{ij} is the piezoelectric moduli.

The stress-strain relationship σ_{ijkl}^s without piezoelectric effect is represented as follows:

$$\begin{Bmatrix} \sigma_{xx}^s \\ \sigma_{yy}^s \\ \tau_{yx}^s \\ \tau_{xz}^s \\ \tau_{xy}^s \end{Bmatrix} = \begin{Bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & 0 \\ \bar{Q}_{21} & \bar{Q}_{22} & 0 & 0 & 0 \\ 0 & 0 & \bar{Q}_{44} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{55} & 0 \\ 0 & 0 & 0 & 0 & \bar{Q}_{66} \end{Bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix}, \tag{6}$$

where

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2}, \quad Q_{12} = Q_{21} = \frac{\nu E}{1-\nu^2},$$
$$Q_{44} = Q_{55} = Q_{66} = \frac{E}{2(1-\nu)}. \tag{7}$$

According to the Hamilton's principle, the nonlinear governing equations of motion for the composite laminated piezoelectric rectangular plate are given as follows:

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 + J_1 \ddot{\phi}_x - c_1 I_3 \frac{\partial \ddot{w}_0}{\partial x}, \tag{8a}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = I_0 \ddot{v}_0 + J_1 \ddot{\phi}_y - c_1 I_3 \frac{\partial \ddot{w}_0}{\partial y}, \tag{8b}$$

$$\frac{\partial \bar{Q}_x}{\partial x} + \frac{\partial \bar{Q}_y}{\partial y} + \frac{\partial}{\partial x}\left(N_{xx} \frac{\partial w_0}{\partial x} + N_{xy} \frac{\partial w_0}{\partial y} \right)$$
$$+ \frac{\partial}{\partial y}\left(N_{xy} \frac{\partial w_0}{\partial x} + N_{yy} \frac{\partial w_0}{\partial y} \right)$$
$$+ c_1 \left(\frac{\partial^2 P_{xx}}{\partial x^2} + 2 \frac{\partial^2 P_{xy}}{\partial x \partial y} + \frac{\partial^2 P_{yy}}{\partial y^2} \right) + q - \gamma \dot{w}_0$$
$$= I_0 \ddot{w}_0 - c_1^2 I_6 \left(\frac{\partial \ddot{w}_0}{\partial x^2} + \frac{\partial \ddot{w}_0}{\partial y^2} \right)$$
$$+ c_1 \left[I_3 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) + I_4 \left(\frac{\partial \ddot{\phi}_x}{\partial x} + \frac{\partial \ddot{\phi}_y}{\partial y} \right) \right], \tag{8c}$$

$$\frac{\partial \bar{M}_{xx}}{\partial x} + \frac{\partial \bar{M}_{xy}}{\partial y} - \bar{Q}_x = J_1 \ddot{u}_0 + k_2 \ddot{\phi}_x - c_1 J_4 \frac{\partial \ddot{w}_0}{\partial x}, \tag{8d}$$

$$\frac{\partial \bar{M}_{xy}}{\partial x} + \frac{\partial \bar{M}_{yy}}{\partial y} - \bar{Q}_y = J_1 \ddot{v}_0 + k_2 \ddot{\phi}_y - c_1 J_4 \frac{\partial \ddot{w}_0}{\partial y}, \tag{8e}$$

where the dot represents the partial differentiation with respect to time t, the comma denotes the partial differentiation with respect to a specified coordinate, γ is the damping coefficient, and all kinds of inertias in eq. (8) are calculated by

$$I_i = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \rho^k z^i \, \mathrm{d}z,$$
$$J_i = I_i - c_1 I_{i+2}, \; K_2 = I_2 - 2 c_1 I_4. \tag{9}$$

In addition, the stress resultants N_{xx}, N_{yy}, N_{xy}, M_{xx}, M_{yy}, M_{xy}, P_{xx}, P_{yy}, P_{xy}, $\bar{Q}_x$ and $\bar{Q}_y$ are represented as follows:

$$N_{xx} = A_{11} \left[\frac{\partial u_0}{\partial x} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \right]$$
$$+ A_{12} \left[\frac{\partial v_0}{\partial y} + \frac{1}{2} \left(\frac{\partial w_0}{\partial y} \right)^2 \right] - N_x^p, \tag{10a}$$

$$N_{yy} = A_{21} \left[\frac{\partial u_0}{\partial x} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \right]$$
$$+ A_{22} \left[\frac{\partial v_0}{\partial y} + \frac{1}{2} \left(\frac{\partial w_0}{\partial y} \right)^2 \right] - N_y^p, \tag{10b}$$

$$N_{xy} = A_{66} \left(\frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} + \frac{\partial u_0}{\partial y} \right), \tag{10c}$$

$$M_{xx} = \left(D_{11} - c_1 F_{11} \right) \frac{\partial \phi_x}{\partial x} + \left(D_{12} - c_1 F_{12} \right) \frac{\partial \phi_y}{\partial y}$$
$$- c_1 F_{11} \frac{\partial^2 w_0}{\partial x^2} - c_1 F_{12} \frac{\partial^2 w_0}{\partial y^2} - M_{xx}^p, \tag{10d}$$

$$M_{yy} = \left(D_{21} - c_1 F_{21} \right) \frac{\partial \phi_x}{\partial x} + \left(D_{22} - c_1 F_{22} \right) \frac{\partial \phi_y}{\partial y}$$
$$- c_1 F_{21} \frac{\partial^2 w_0}{\partial x^2} - c_1 F_{22} \frac{\partial^2 w_0}{\partial y^2} - M_{yy}^p, \tag{10e}$$

$$M_{xy} = \left(D_{66} - c_1 F_{66} \right) \left(\frac{\partial \phi_x}{\partial x} + \frac{\partial \phi_y}{\partial y} \right) - 2 c_1 F_{66} \frac{\partial^2 w_0}{\partial x \partial y}, \tag{10f}$$

$$P_{xx} = \left(F_{11} - c_1 H_{11} \right) \frac{\partial \phi_x}{\partial x} + \left(F_{12} - c_1 H_{12} \right) \frac{\partial \phi_y}{\partial y}$$
$$- c_1 H_{11} \frac{\partial^2 w_0}{\partial x^2} - c_1 H_{12} \frac{\partial^2 w_0}{\partial y^2}, \tag{10g}$$

$$P_{yy} = \left(F_{21} - c_1 \nu H_{21} \right) \left(\frac{\partial \phi_x}{\partial x} + \frac{\partial \phi_y}{\partial y} \right)$$
$$- c_1 H_{21} \frac{\partial^2 w_0}{\partial x^2} - c_1 H_{22} \frac{\partial^2 w_0}{\partial y^2}, \tag{10h}$$

$$P_{xy} = \left(P_{66} - c_1 H_{66} \right) \left(\frac{\partial \phi_x}{\partial x} + \frac{\partial \phi_y}{\partial y} \right) - 2 c_1 H_{66} \frac{\partial^2 w_0}{\partial x \partial y}, \tag{10i}$$

$$\bar{Q}_x = \left(A_{44} - c_2 D_{44}\right)\phi_x + \left(A_{44} - c_2 D_{44}\right)\frac{\partial w_0}{\partial y}, \quad (10j)$$

$$\bar{Q}_y = \left(A_{55} - c_2 D_{55}\right)\phi_x + \left(A_{55} - c_2 D_{55}\right)\frac{\partial w_0}{\partial x}, \quad (10k)$$

$$N_x^P = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{11}^k\, e_{31}^k\, E_z\,\mathrm{d}z,$$
$$N_y^P = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{22}^k\, e_{32}^k\, E_z\mathrm{d}z, \quad (10l)$$

where $N_i^P = N_i^P \cos(\Omega_4 t)$ $(i = x, y)$ represent the piezoelectric stress resultants, and A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, and H_{ij} respectively are the stiffness elements of the composite laminated piezoelectric rectangular plate, which are denoted as

$$\left(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}\right) = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{ij}^k \left(1, z, z^2, z^3, z^5, z^6\right)\mathrm{d}z,\ (i, j = 1, 2, 6), \quad (11a)$$

$$\left(A_{ij}, D_{ij}, F_{ij}\right) = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \bar{Q}_{ij}^k \left(1, z^2, z^4\right)\mathrm{d}z, (i, j = 4, 5). \quad (11b)$$

Substituting eq. (10) into eq. (8), we obtain the governing equations of motion in terms of generalized displacements $(u_0, v_0, w_0, \phi_x, \phi_y)$ as

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 u_0}{\partial y^2} + \left(A_{12} + A_{66}\right)\frac{\partial^2 v_0}{\partial x \partial y}$$
$$+A_{11}\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x^2} + A_{66}\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2}$$
$$+\left(A_{12} + A_{66}\right)\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial x \partial y} = I_0 \ddot{u}_0 + J_1\ddot{\phi}_x - c_1 I_3 \frac{\partial \ddot{w}_0}{\partial x}, \quad (12a)$$

$$A_{66}\frac{\partial^2 v_0}{\partial x^2} + A_{22}\frac{\partial^2 v_0}{\partial y^2} + \left(A_{21} + A_{66}\right)\frac{\partial^2 u_0}{\partial x \partial y}$$
$$+A_{66}\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial x^2} + A_{22}\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y^2}$$
$$+\left(A_{21} + A_{66}\right)\frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x \partial y} = I_0 \ddot{v}_0 + J_1\ddot{\phi}_y - c_1 I_3 \frac{\partial \ddot{w}_0}{\partial y}, \quad (12b)$$

$$A_{66}\frac{\partial w_0}{\partial x}\frac{\partial^2 u_0}{\partial y^2} - H_{22}c_1^2\frac{\partial^4 w_0}{\partial y^4}$$
$$+c_1\left(2F_{66} + F_{12} - 2H_{66}c_1 - H_{12}c_1\right)\frac{\partial^3 \phi_y}{\partial y \partial x^2}$$
$$+c_1(F_{22} - H_{22}c_1)\frac{\partial^3 \phi_y}{\partial y^3} - H_{11}c_1^2\frac{\partial^4 w_0}{\partial x^4}$$
$$+A_{11}\frac{\partial w_0}{\partial x}\frac{\partial^2 u_0}{\partial x^2} + (F_{44}c_2^2 - 2D_{44}c_2 + A_{44})\frac{\partial \phi_y}{\partial y}$$
$$+c_1(F_{21} + 2F_{66} - H_{21}c_1 - 2H_{66}c_1)\frac{\partial^3 \phi_x}{\partial y^2 \partial x}$$
$$-c_1^2(H_{21} + 4H_{66} + H_{12})\frac{\partial^4 w_0}{\partial y^2 \partial x^2}$$
$$+(A_{44} - N_y^P\cos(\Omega_4 t) + F_{44}c_2^2 - 2D_{44}c_2)\frac{\partial^2 w_0}{\partial y^2}$$
$$+(A_{21} + 4A_{66} + A_{12})\frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y \partial x}$$
$$+c_1(F_{11} - H_{11}c_1)\frac{\partial^3 \phi_x}{\partial x^3} + (A_{21} + A_{66})\frac{\partial w_0}{\partial y}\frac{\partial^2 u_0}{\partial y \partial x}$$
$$+A_{21}\frac{\partial u_0}{\partial x}\frac{\partial^2 w_0}{\partial y^2} + A_{66}\frac{\partial w_0}{\partial y}\frac{\partial^2 v_0}{\partial x^2}$$
$$+A_{22}\frac{\partial w_0}{\partial y}\frac{\partial^2 v_0}{\partial y^2} + \frac{1}{2}(A_{12} + 2A_{66})\left(\frac{\partial w_0}{\partial y}\right)^2\frac{\partial^2 w_0}{\partial x^2}$$
$$+A_{22}\frac{\partial^2 w_0}{\partial y^2}\frac{\partial v_0}{\partial y} + (A_{12} + A_{66})\frac{\partial w_0}{\partial x}\frac{\partial^2 v_0}{\partial y \partial x}$$
$$+\frac{1}{2}(A_{21} + 2A_{66})\frac{\partial^2 w_0}{\partial y^2}\left(\frac{\partial w_0}{\partial x}\right)^2 + \frac{3}{2}A_{11}\left(\frac{\partial w_0}{\partial x}\right)^2\frac{\partial^2 w_0}{\partial x^2}$$
$$+A_{11}\frac{\partial^2 w_0}{\partial x^2}\frac{\partial u_0}{\partial x} + A_{12}\frac{\partial^2 w_0}{\partial x^2}\frac{\partial v_0}{\partial y} + 2A_{66}\frac{\partial^2 w_0}{\partial y \partial x}\frac{\partial v_0}{\partial x}$$
$$+2A_{66}\frac{\partial^2 w_0}{\partial y \partial x}\frac{\partial u_0}{\partial y} + \frac{3}{2}A_{22}\left(\frac{\partial w_0}{\partial y}\right)^2\frac{\partial^2 w_0}{\partial y^2}$$
$$+(A_{55} + q_x\cos(\Omega_1 t) - N_x^P\cos(\Omega_4 t)$$
$$+F_{55}c_2^2 - 2D_{55}c_2)\frac{\partial^2 w_0}{\partial x^2}$$
$$+(F_{55}c_2^2 - 2D_{55}c_2 + A_{55})\frac{\partial \phi_x}{\partial x} - q\cos(\Omega_3 t) + \mu\frac{\partial w_0}{\partial t}$$
$$= I_0\frac{\partial \ddot{w}_0}{\partial t^2} - c_1^2 I_6\left(\frac{\partial^2 \ddot{w}_0}{\partial x^2} + \frac{\partial^2 \ddot{w}_0}{\partial y^2}\right)$$
$$+c_1 I_3\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) + c_1 J_4\left(\frac{\partial \ddot{\phi}_x}{\partial x} + \frac{\partial \ddot{\phi}_y}{\partial y}\right), \quad (12c)$$

$$(D_{11} - 2F_{11}c_1 + H_{11}c_1^2)\frac{\partial^2 \phi_x}{\partial x^2} + (D_{66} - 2F_{66}c_1 + H_{66}c_1^2)\frac{\partial^2 \phi_x}{\partial y^2}$$
$$-c_1(F_{11} - H_{11}c_1)\frac{\partial^3 w_0}{\partial x^3} - (F_{55}c_2^2 - 2D_{55}c_2 + A_{55})\frac{\partial w_0}{\partial x}$$
$$+(D_{12} + D_{66} + H_{66}c_1^2 - 2F_{66}c_1 + H_{12}c_1^2 - 2F_{12}c_1)\frac{\partial^2 \phi_y}{\partial y \partial x}$$

$$-c_1(2F_{66}+F_{12}-2H_{66}c_1-H_{12}c_1)\frac{\partial^3 w_0}{\partial y^2\partial x}$$

$$+(2D_{55}c_2-A_{55}-F_{55}c_2^2)\phi_x=J_1\ddot{u}_0+K_2\ddot{\phi}_x-c_1J_4\frac{\partial\ddot{w}_0}{\partial x}, \quad (12d)$$

$$(D_{66}-2F_{66}c_1+H_{66}c_1^2)\frac{\partial^2\phi_y}{\partial x^2}$$

$$-c_1(F_{21}+2F_{66}-H_{21}c_1-2H_{66}c_1)\frac{\partial^3 w_0}{\partial y\partial x^2}$$

$$+(H_{21}c_1^2+D_{66}+D_{21}-2F_{21}c_1+H_{66}c_1^2-2F_{66}c_1)\frac{\partial^2\phi_x}{\partial y\partial x}$$

$$+(H_{22}c_1^2+D_{22}-2F_{22}c_1)\frac{\partial^2\phi_y}{\partial y^2}-c_1(F_{22}-H_{22}c_1)\frac{\partial^3 w_0}{\partial y^3}$$

$$-(F_{44}c_2^2-2D_{44}c_2+A_{44})\frac{\partial w_0}{\partial y}+(2D_{44}c_2-F_{44}c_2^2-A_{44})\phi_y$$

$$=J_1\ddot{v}_0+K_2\ddot{\phi}_y-c_1J_4\frac{\partial\ddot{w}_0}{\partial y}. \quad (12e)$$

The simply supported boundary conditions can be represented as

$$x=0: v=w=\phi_y=N_{xy}=M_{xx}=0;$$
$$x=a: u=v=w=\phi_y=N_{xy}=M_{xx}=0, \quad (13a)$$

$$y=0:\ u=w=\phi_x=N_{xy}=M_{yy}=0;$$
$$y=b:\ u=v=w=\phi_x=N_{xy}=M_{yy}=0, \quad (13b)$$

$$\int_0^h N_{xx}\big|_{x=0}\,\mathrm{d}z=-\int_0^h\left(q_0+q_x\cos\Omega_1 t\right)\mathrm{d}z,$$
$$\int_0^h N_{yy}\big|_{y=0}\,\mathrm{d}z=-\int_0^h\left(q_1+q_y\cos\Omega_2 t\right)\mathrm{d}z. \quad (13c)$$

The boundary condition eq. (13c) also includes the influence of the in-plane load. We consider nonlinear dynamics of the composite laminated piezoelectric rectangular plate in the first two modes of u_0, v_0, w_0, ϕ_x and ϕ_y. It is our desire to choose a suitable mode function to satisfy the boundary condition of the composite laminated piezoelectric rectangular plate. Thus, we write u_0, v_0, w_0, ϕ_x and ϕ_y in the following forms:

$$u_0=u_1(t)\cos\frac{\pi x}{2a}\cos\frac{\pi y}{2b}+u_2(t)\cos\frac{3\pi x}{2a}\cos\frac{\pi y}{2b}, \quad (14a)$$

$$v_0=v_1(t)\cos\frac{\pi y}{2b}\cos\frac{\pi x}{2a}+v_2(t)\cos\frac{\pi y}{2b}\cos\frac{3\pi x}{2a}, \quad (14b)$$

$$w_0=w_1(t)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}+w_2(t)\sin\frac{3\pi x}{a}\sin\frac{\pi y}{b}, \quad (14c)$$

$$\phi_x=\phi_1(t)\cos\frac{\pi x}{a}\sin\frac{\pi y}{b}+\phi_2(t)\cos\frac{3\pi x}{a}\sin\frac{\pi y}{b}, \quad (14d)$$

$$\phi_y=\phi_3(t)\cos\frac{\pi y}{b}\sin\frac{\pi x}{a}+\phi_4(t)\cos\frac{\pi y}{b}\sin\frac{3\pi x}{a}. \quad (14e)$$

By means of the Galerkin method, substituting eq. (14) into eq. (12), integrating and neglecting all inertia terms in eqs. (12a), (12b), (12d) and (12e), we obtain the expressions of u_1, u_2, v_1, v_2, ϕ_1, ϕ_2, ϕ_3 and ϕ_4 via w_1 and w_2 as follows:

$$\begin{aligned}
&u_1=k_1w_1^2+k_2w_2^2+k_3w_1w_2+k_4,\\
&u_2=k_5w_1^2+k_6w_2^2+k_7w_1w_2+k_8,\\
&v_1=k_9w_1^2+k_{10}w_2^2+k_{11}w_1w_2+k_{12},\\
&v_2=k_{13}w_1^2+k_{14}w_2^2+k_{15}w_1w_2+k_{16},\\
&\phi_1=k_{15}w_1,\ \ \phi_2=k_{16}w_2,\ \ \phi_3=k_{17}w_1,\ \ \phi_4=k_{18}w_2.
\end{aligned} \quad (15)$$

In order to obtain the dimensionless governing equations of motion, we introduce the transformations of the variables and parameters:

$$\bar{u}=\frac{u_0}{a},\ \bar{v}=\frac{v_0}{b},\ \bar{w}=\frac{w_0}{h},\ \bar{\phi}_x=\phi_x,\ \bar{\phi}_y=\phi_y,$$

$$\bar{x}=\frac{x}{a},\ \bar{y}=\frac{y}{b},\ \bar{q}=\frac{b^2}{Eh^3}q,\ \bar{q}_x=\frac{b^2}{Eh^3}q_x,$$

$$\bar{q}_y=\frac{b^2}{Eh^3}q_y,\ \ \bar{t}=\pi^2\left(\frac{E}{ab\rho}\right)^{1/2}t,$$

$$\bar{\Omega}_i=\frac{1}{\pi^2}\left(\frac{ab\rho}{E}\right)^{1/2}\Omega_i\ (i=1,2),\ \ \bar{A}_{ij}=\frac{(ab)^{1/2}}{Eh^2}A_{ij}, \quad (16)$$

$$\bar{B}_{ij}=\frac{(ab)^{1/2}}{Eh^3}B_{ij},\ \bar{D}_{ij}=\frac{(ab)^{1/2}}{Eh^4}D_{ij},\ \bar{E}_{ij}=\frac{(ab)^{1/2}}{Eh^5}E_{ij},$$

$$\bar{F}_{ij}=\frac{(ab)^{1/2}}{Eh^6}F_{ij},\ \ \bar{H}_{ij}=\frac{(ab)^{1/2}}{Eh^8}H_{ij},$$

$$\bar{I}_i=\frac{1}{(ab)^{(i+1)/2}\rho}I_i.$$

For simplicity, we drop the overbar in the following analysis. Substituting eqs. (13)—(16) into eq. (12c) and applying the Galerkin procedure, we obtain the governing equations of motion of the composite laminated piezoelectric rectangular plate for the dimensionless as follows:

$$\ddot{w}_1+\mu_1\dot{w}_1+\omega_1^2w_1+(a_2\cos\Omega_1 t+a_3\cos\Omega_2 t+a_4\cos\Omega_4 t)w_1$$
$$+a_5w_1^2w_2+a_6w_2^2w_1+a_7w_1^3+a_8w_2^3=f_1\cos\Omega_3 t, \quad (17a)$$

$$\ddot{w}_2+\mu_2\dot{w}_2+\omega_2^2 w_2+(b_2\cos\Omega_1 t+b_3\cos\Omega_2 t+b_4\cos\Omega_4 t)w_2$$
$$+b_5 w_2^2 w_1+b_6 w_1^2 w_2+b_7 w_2^3+b_8 w_1^3=f_2\cos\Omega_3 t, \quad (17b)$$

The aforementioned governing equation, which includes the cubic terms, parametric and transverse excitations, describes the nonlinear transverse oscillations of the composite laminated piezoelectric rectangular plate subject to the in-plane and transverse excitations and with the PVDF piezoelectric materials in the first two modes. In the following analysis, we will employ the method of multiple scales to look for the approximate solution of eq. (17).

3 Perturbation analysis

We utilize the method of multiple scales[20] to find the uniform solutions of eq. (17) in the following form:

$$w_1(t,\varepsilon)=x_0(T_0,T_1)+\varepsilon x_1(T_0,T_1)+\cdots, \quad (18)$$

$$w_2(t,\varepsilon)=y_0(T_0,T_1)+\varepsilon y_1(T_0,T_1)+\cdots, \quad (19)$$

where $T_0=t$ and $T_1=\varepsilon t$.

Then, we have the differential operators

$$\frac{\mathrm{d}}{\mathrm{d}t}=\frac{\partial}{\partial T_0}\frac{\partial T_0}{\partial t}+\frac{\partial}{\partial T_1}\frac{\partial T_1}{\partial t}+\cdots=D_0+\varepsilon D_1+\cdots, \quad (20)$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}=(D_0+\varepsilon D_1+\cdots)^2=D_0^2+2\varepsilon D_0 D_1+\cdots, \quad (21)$$

where $D_0=\partial/\partial T_0$ and $D_1=\partial/\partial T_1$.

We study the case of primary parametric resonance and 1∶2 internal resonance. In this resonant case, there are the following resonant relations:

$$\omega_1^2=\frac{\omega^2}{4}+\varepsilon\sigma_1,\ \omega_2^2=\omega^2+\varepsilon\sigma_{12},\ \Omega_1=\omega,$$
$$\Omega_2=\Omega_3=\Omega_4=\omega,\quad \omega_2\approx 2\omega_1. \quad (22)$$

Substituting eqs. (18)—(22) into eq. (17) and balancing the coefficients of corresponding powers of ε on the left-hand and right-hand sides of equations, the differential equations are obtained as follows:

Order ε^0:

$$D_0^2 x_0+\frac{1}{4}\omega^2 x_0=0, \quad (23a)$$

$$D_0^2 y_0+\omega^2 y_0=0. \quad (23b)$$

Order ε^1:

$$D_0^2 x_1+\frac{1}{4}\omega^2 x_1$$
$$=\mu_1 D_0 x_0+(a_2+a_3+a_4)\cos(\omega t)x_0-\sigma_1 x_0+a_5 x_0^2 y_0$$
$$+a_6 x_0 y_0^2+a_7 x_0^3+a_8 y_0^3+f_1\cos(\omega t)+2D_0 D_1 x_0, \quad (24a)$$

$$D_0^2 y_1+\omega^2 y_1=\mu_2 D_0 y_0$$
$$+(b_2+b_3+b_4)\cos(\omega t)y_0-\sigma_1 y_0+b_5 y_0^2 x_0$$
$$+b_6 y_0 x_0^2+b_7 y_0^3+b_8 x_0^3+f_2\cos(\omega t)+2D_0 D_1 y_0. \quad (24b)$$

The solution of eq. (23) in the complex form can be written as

$$x_0=A(T_1)\mathrm{e}^{\frac{1}{2}\omega T_0}+\bar{A}(T_1)\mathrm{e}^{-\frac{1}{2}\omega T_0}, \quad (25a)$$

$$y_0=B(T_1)\mathrm{e}^{\omega T_0}+\bar{B}(T_1)\mathrm{e}^{-\omega T_0}, \quad (25b)$$

where $\bar{A}$ and $\bar{B}$ are the complex conjugates of A and B, respectively.

Substituting eq. (25) into eq. (24) yields

$$D_0^2 x_1+\frac{1}{4}\omega^2 x_1=\Big[-\sigma_1 A+\frac{1}{2}(a_2+a_3+a_3)\bar{A}-\mathrm{i}\omega D_1 A$$
$$+i\frac{1}{2}\mu_1\omega A+2a_6 AB\bar{B}+3a_7 A^2\bar{A}\Big]\mathrm{e}^{\mathrm{i}\frac{1}{2}\omega T_0}+\mathrm{cc}+NST, \quad (26a)$$

$$D_0^2 y_1+\omega^2 y_1=\Big(-\sigma_2 B-2i\omega D_1 B+\mathrm{i}\mu_2\omega B+2b_6 A\bar{A}B$$
$$+\frac{1}{2}f_2+3b_7 B^2\bar{B}\Big)\mathrm{e}^{\mathrm{i}\omega T_0}+\mathrm{cc}+NST, \quad (26b)$$

where cc represents the parts of the complex conjugates of the function on the right-hand side of eq. (26) and *NST* represents the terms that do not produce secular terms.

Eliminating the secular terms from eq. (26) yields

$$D_1 A=\frac{1}{2}\mu_1 A+\frac{1}{2}\mathrm{i}\sigma_1 A-\mathrm{i}a_6 AB\bar{B}-\frac{1}{4}\mathrm{i}a_2\bar{A}$$
$$-\frac{1}{4}\mathrm{i}a_3\bar{A}-\frac{3}{2}\mathrm{i}a_7 A^2\bar{A}-\frac{1}{4}\mathrm{i}a_4\bar{A}, \quad (27a)$$

$$D_1 B=\frac{1}{2}\mu_2 B+\frac{1}{4}\mathrm{i}\sigma_2 B-\frac{1}{2}\mathrm{i}A\bar{A}B$$
$$-\frac{3}{4}\mathrm{i}b_7 B^2\bar{B}-\frac{1}{8}\mathrm{i}f_2. \quad (27b)$$

The functions A and B may be expressed in the Cartesian form

$$A=x_1+\mathrm{i}x_2,\quad B=x_3+\mathrm{i}x_4. \quad (28)$$

Substituting eq. (28) into eq. (27), the averaged equations in the Cartesian form are obtained as follows:

$$\dot{x}_1=\frac{1}{2}\mu_1 x_1-\frac{1}{4}(2\sigma_1+a_2+a_3+G_0)x_2$$
$$+a_6 x_2(x_3^2+x_4^2)+\frac{3}{2}a_7 x_2(x_1^2+x_2^2), \quad (29a)$$

$$\dot{x}_2 = \frac{1}{2}\mu_1 x_2 + \frac{1}{4}(2\sigma_1 - a_2 - a_3 - G_0)x_1 - a_6 x_1(x_3^2 + x_4^2) - \frac{3}{2}a_7 x_1(x_1^2 + x_2^2), \quad (29b)$$

$$\dot{x}_3 = \frac{1}{2}\mu_2 x_3 - \frac{1}{4}\sigma_2 x_4 + \frac{1}{2}b_6 x_4(x_1^2 + x_2^2) + \frac{3}{4}b_7 x_4(x_3^2 + x_4^2), \quad (29c)$$

$$\dot{x}_4 = \frac{1}{2}\mu_2 x_4 + \frac{1}{4}\sigma_2 x_3 - \frac{1}{8}f_2 - \frac{3}{4}b_7 x_3(x_3^2 + x_4^2) - \frac{1}{2}b_6 x_3(x_1^2 + x_2^2). \quad (29d)$$

4 Numerical simulation

In the following research, the fourth-order Runge-Kutta algorithm is employed to numerically analyze the periodic and chaotic responses of the simply supported symmetric cross-ply composite laminated piezoelectric rectangular plate subject to the electric and mechanical loads for the case of 1∶2 internal resonance and primary parametric resonance. We choose the averaged eq. (29) for numerical simulation. We use the parametric excitation a_2, forcing excitation f_2 and piezoelectric excitation G_0 as the controlling parameters when the periodic and chaotic responses of the composite laminated piezoelectric rectangular plate are investigated. Through analyzing the bifurcation diagrams, the complicated nonlinear dynamics, including periodic and chaotic motions, may be observed globally from a range of parameter values. The two-dimensional phase portrait, waveform, three-dimensional phase portrait and Poincare map are depicted to illustrate the nonlinear dynamic behaviors of the composite laminated piezoelectric rectangular plate. It can be clearly found from the numerical results and the bifurcation diagrams that the periodic and chaotic motions occur in the composite laminated piezoelectric rectangular plate.

Figure 1 illustrates the bifurcation diagram of the composite laminated piezoelectric rectangular plate when the forcing excitation f_2 is located in the interval 30—130. The parameters and the initial conditions are respectively chosen as $\mu_1 = 0.2$, $\mu_2 = 0.2$, $\sigma_1 = 4.31$, $\sigma_2 = 1.23$, $a_2 = 27.59$, $a_3 = -25$, $G_0 = -25$, $a_6 = 3.37$, $a_7 = 4.39$, $b_6 = -13.68$, $b_7 = 11.16$, $x_{10} = 0.62$, $x_{20} = -0.82$, $x_{30} = -1.59$, and $x_{40} = -1.36$. It is observed from Figure 2 that the external excitation f_2 has significant effect on the nonlinear dynamic responses of the composite laminated piezoelectric rectangular plate. In Figure 1, the longitudinal coordinate denotes the deflection of the plate, while the abscissa denotes the external excitation f_2. From Figure 1, it is seen that the motions of the composite laminated piezoelectric rectangular plate changes from chaotic motion to multiple periodic motion, and then from the multiple periodic motion to one periodic motion with the increase of the external excitation amplitude f_2.

In the following investigation, we may change the external excitation amplitude f_2 to find the periodic and chaotic motions of the composite laminated piezoelectric rectangular plate based on Figure 1. Figure 2 indicates the existence of the chaotic motion for the composite laminated piezoelectric rectangular plate when the forcing excitation f_2 is 30.0. Here other parameters are the same as those in Figure 1. Figures 2(a) and (c) represent the phase portraits on the planes (x_1, x_2) and (x_3, x_4), respectively. Figures 2(b) and (d) give the waveforms on the planes (t, x_1) and (t, x_3), respectively. Figures 2(e) and (f) represent the Poincare map on the plane (x_1, x_2) and three-dimensional phase portrait in the space (x_1, x_2, x_3), respectively.

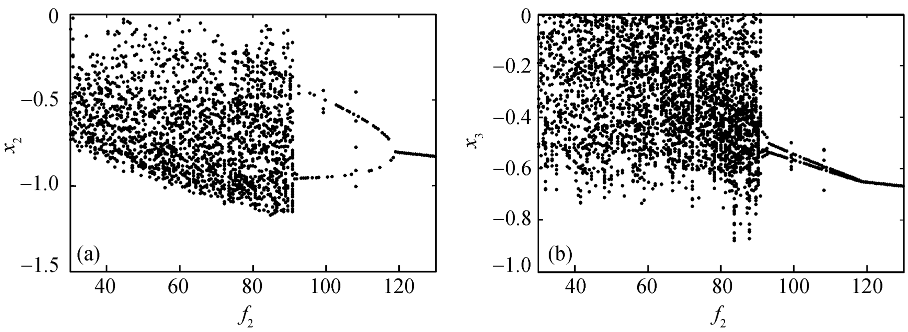

Figure 1 The bifurcation diagram for the transverse excitation f_2.

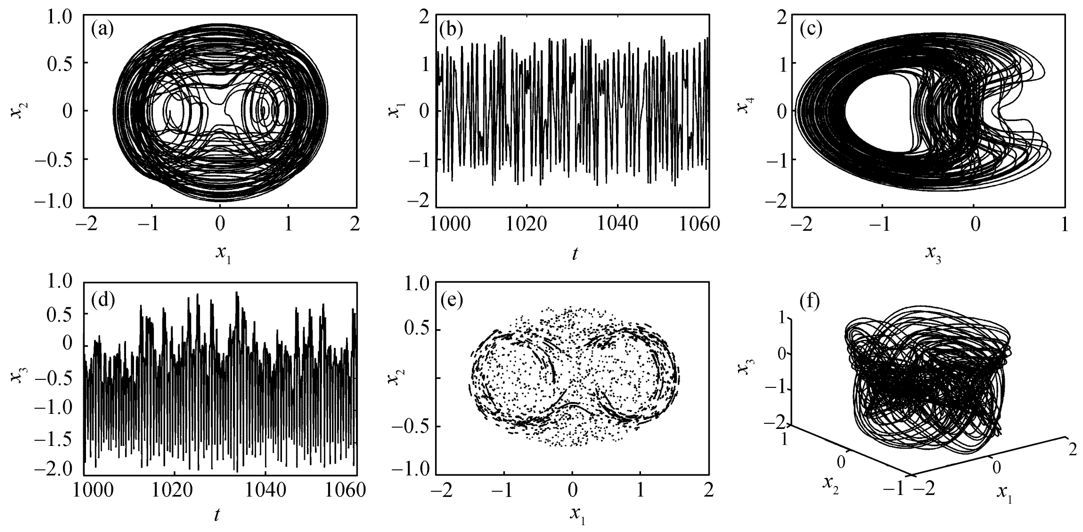

Figure 2 The chaotic motion of the system exists when f_2=30.

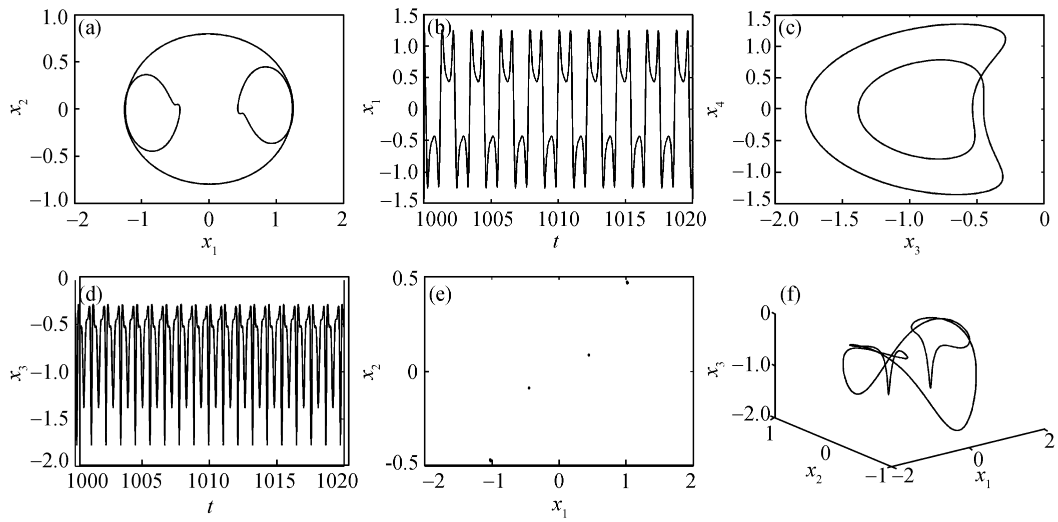

Figure 3 The periodic motion of the system exists when f_2=51.2.

When the external excitation changes to f_2=51.2, the periodic motion of the composite laminated piezoelectric rectangular plates is observed, as shown in Figure 3.

In Figure 4, we depict the bifurcation diagram for x_2 and x_4 via the parametric excitation a_2. In this case, the parameters and the initial conditions are respectively chosen as $\mu_1 = 0.2$, $\mu_2 = 0.2$, $\sigma_1 = 4.31$, $\sigma_2 = 1.23$, $f_2 = 50$, $a_3 = -25$, $G_0 = -25$, $a_6 = 3.37$, $a_7 = 4.39$, $b_6 = -13.68$, $b_7 = 11.16$, $x_{10} = 0.62$, $x_{20} = -0.82$, x_{30}= −1.59, and x_{40}= −1.36. Figure 4 illustrates the effect of the in-plane excitation on the nonlinear dynamic responses of the composite laminated piezoelectric rectangular plate. The longitudinal coordinate denotes the deflection of the plate and the abscissa denotes the in-plane excitation. It can be seen from Figure 4 that the motions of the composite laminated piezoelectric rectangular plate change from the one periodic motion to the chaotic motion, and then from the multiple periodic motion to the periodic motion with the increase of the in-plane excitation a_2.

Figure 5 illustrates the existence of the one periodic motion of the composite laminated piezoelectric rectangular plate when the in-plane excitation is a_2=0, where other parameters are the same as those in Figure 4. When the in-plane excitation changes to a_2=9.0, the chaotic motion of the composite laminated piezoelectric rectangular plate occurs, as shown in Figure 6.

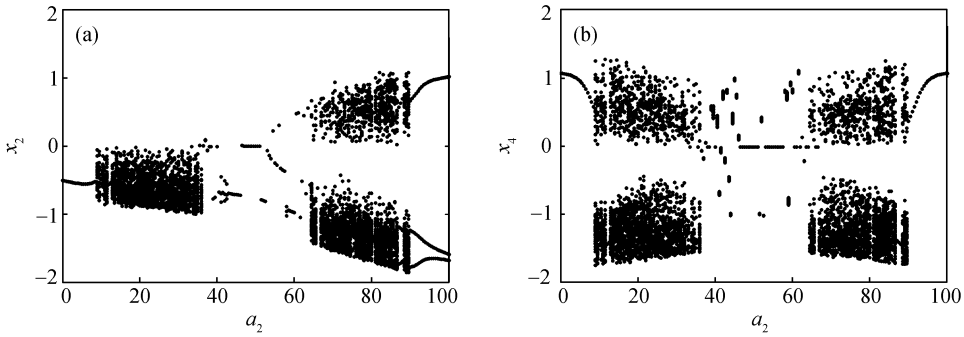

Figure 4 The bifurcation diagram for the parametric excitation a_2.

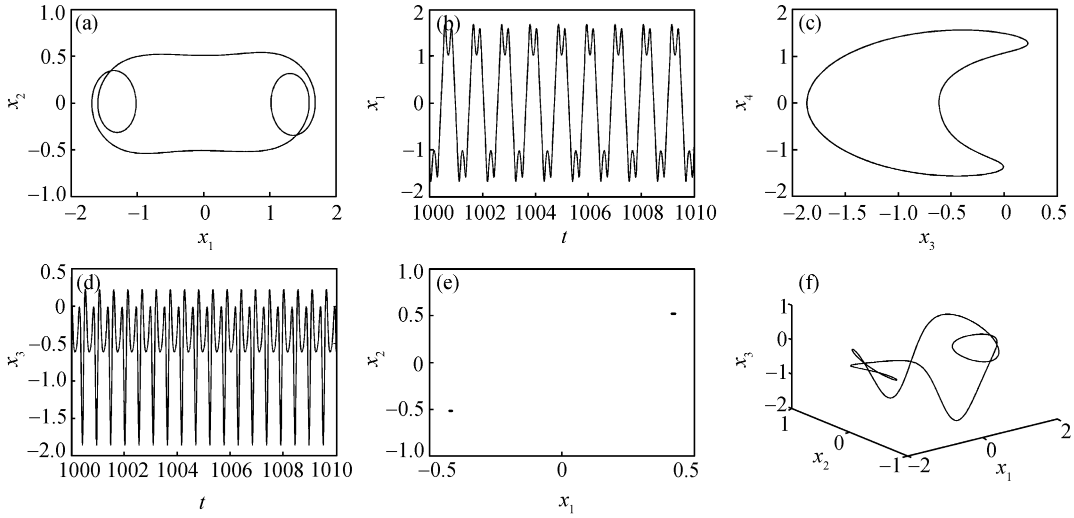

Figure 5 The periodic motion of the system exists when a_2=0.

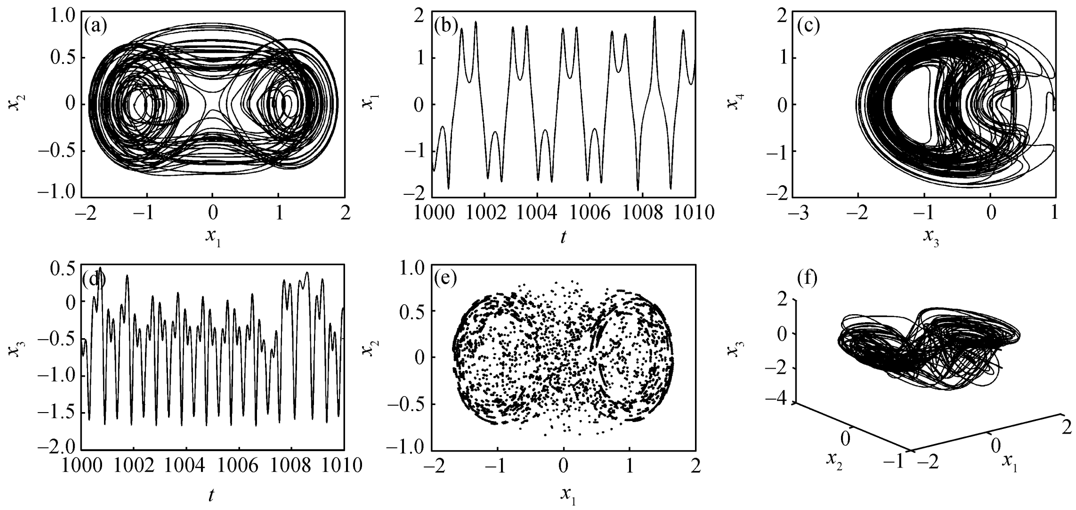

Figure 6 The chaotic motion of the system exists when a_2=9.0.

At last, we give the bifurcation diagrams for x_2 and x_4 via the piezoelectric excitation G_0, as shown in Figure 7. In this situation, the parameters and the initial conditions are respectively chosen as μ_1=0.2, μ_2=0.2, σ_1=4.31, σ_2=1.23, f_2=50, a_2=–25, a_3=–25, a_6=3.37, a_7=4.39, b_6= −13.68, b_7=11.16, x_{10}=0.62, x_{20}=−0.82, x_{30}= −1.59, and x_{40}= −1.36. Figure 7 demonstrates the influence of the piezoelectric excitation on the nonlinear dynamic response of the composite laminated piezoelectric rectangular plate. It is observed from Figure 7 that the motions

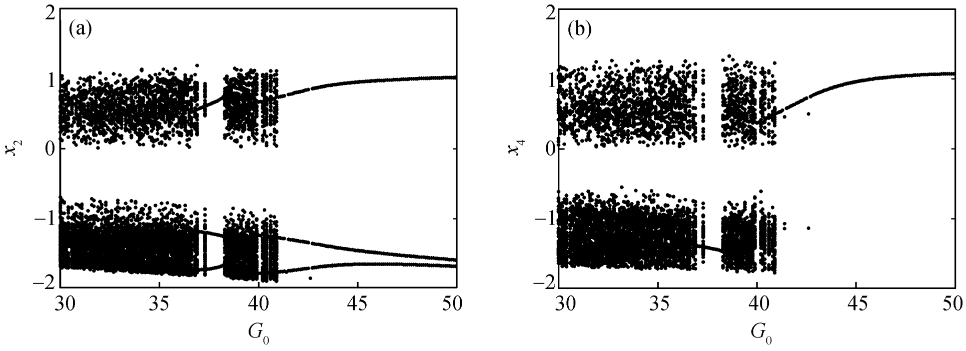

Figure 7 The effect of the piezoelectric excitations G_0.

of the composite laminated piezoelectric rectangular plate change from the chaotic motion to the periodic motion, and then from the periodic motion to the chaotic motion with the increase of the piezoelectric excitation amplitude G_0.

When the piezoelectric excitation changes to G_0=37.0, there exists the periodic motion of the composite laminated piezoelectric rectangular plate, as shown in Figure 8.

5 Conclusions

We investigate the bifurcations and chaotic dynamics of a four-edge simply supported composite laminated piezoelectric rectangular plate subject to the in-plane, transverse and piezoelectric excitations. Based on the von Karman type equations and the Reddy's third-order shear deformation plate theory, the Hamilton's principle is utilized to establish the governing equations of motion for the composite laminated piezoelectric rectangular plate. Because only transverse nonlinear oscillations of the composite laminated piezoelectric rectangular plate are considered, the governing equations of motion can be reduced to a two-degree-freedom nonlinear system under combined parametric and external excitations by using the Galerkin's method. The case of 1:2 internal resonance and primary parametric resonance is considered. The method of multiple scales is used to obtain the averaged equation of the original non-autonomous system. Numerical method is used to investigate the bifurcations, periodic and chaotic motions of the composite laminated piezoelectric rectangular plate. The bifurcation diagrams are also obtained by using numerical simulation. It is found from the numerical results that there exist the periodic and chaotic motions of the composite laminated piezoelectric rectangular plate under certain conditions.

We obtain three type bifurcation diagrams of the composite laminated piezoelectric rectangular plate, including the bifurcation diagrams for x_2 and x_4 via the

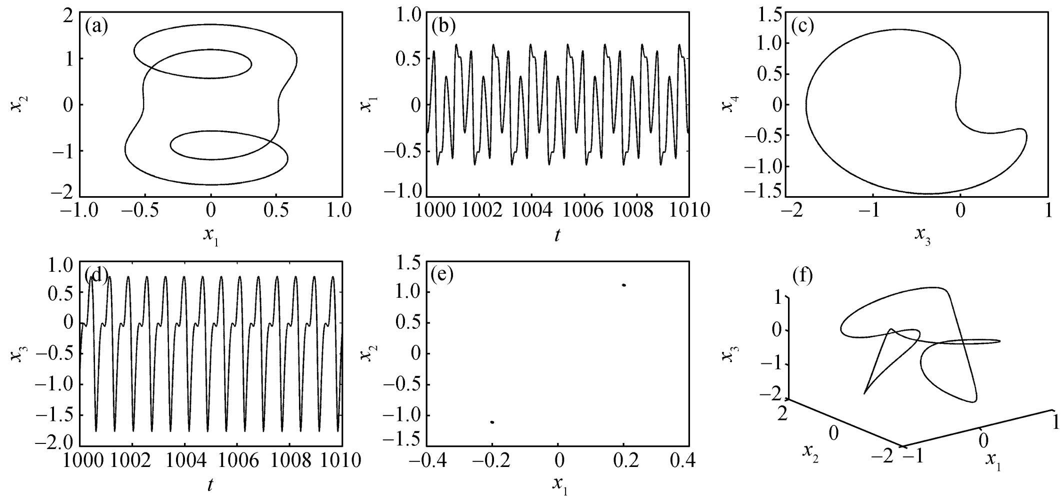

Figure 8 The periodic motion of the system exists when G_0=37.0.

forcing excitation f_2, the bifurcation diagrams for x_2 and x_4 via the parametric excitation a_2 and the bifurcation diagrams for x_2 and x_4 via the piezoelectric excitation G_0, respectively. From Figure 1, it is found that the varying procedure for the motions of the composite laminated piezoelectric rectangular plate is as follows: the periodic motion → quasi-periodic motion → the multiple periodic motion → the chaotic motion with the increase of the external excitation amplitude f_2. Based on Figure 4, it is observed that the varying procedure for the motions of the composite laminated piezoelectric rectangular plate is as follows: the chaotic motion → the multi-periodic motion → the periodic motion with the increase of the in-plane excitation a_2. It is also illustrated from Figure 7 that the varying procedure for the motions of the composite laminated piezoelectric rectangular plate is as follows: the multiple periodic motion → the chaotic motion → the periodic motion with the increase of the piezoelectric excitation amplitude G_0.

The influence of the piezoelectric excitation on the nonlinear oscillations, bifurcations and chaos of the composite laminated rectangular plates is considered in this paper. The piezoelectric excitation can be considered to be a controlling parameter, which may control the nonlinear dynamic responses of the composite laminated piezoelectric rectangular plate. It is found that the forcing and the parametric excitations also have significant influence on the nonlinear dynamic behaviors of the composite laminated piezoelectric rectangular plate. Therefore, we can control the nonlinear dynamic responses of the composite laminated piezoelectric rectangular plate from the chaotic motion to the periodic motion by changing the piezoelectric, forcing and the parametric excitations, respectively.

1 Hurlebaus S, Gaul L. Smart structure dynamics. Mech Syst Sign Proc 20, 2006. 255—281

2 Sheta E F, Moses R W, Huttsell L J. Active smart material control system for buffet alleviation. J Sound Vib, 2006, 292(4): 854—868

3 Srinivasamurthy K, Chia C Y. Nonlinear dynamic and static analysis of laminated anisotropic thick circular plates. Acta Mech, 1990, 82(1): 135—150

4 Boginich O E. Analysis of oscillations of a plate under harmonic excitations of various types with dissipation of energy in materials. Strength Mat, 1997, 29(3): 408—417

5 Demchouk O N. Quasi-three-dimensional theory for solution of dynamic thermoelastic problem of laminated composite plates. Mech Compos Mater, 1998, 34(2): 253—262

6 Chueshov I, Lasiecka I. Inertial manifolds for von karman plate equations. Appl Math Opt, 2002, 46(2): 179—206

7 Courilleau P, Mossino J. Compensated compactness for nonlinear homogenization and reduction of dimension. Calc Var Partial Dif, 2004, 20(1): 65—91

8 Ye M, Lu J, Zhang W, et al. Local and global nonlinear dynamics of a parametrically excited rectangular symmetric cross-ply laminated composite plate. Chaos Solit Fract, 2005, 26(2): 195—213

9 Ye M, Sun Y H, Zhang W, et al. Nonlinear oscillations and chaotic dynamics of an antisymmetric cross-ply composite laminated rectangular thin plate under parametric excitation. J Sound Vibra, 2005, 287(4): 723—758

10 Lee S J, Reddy J N. Non-linear response of laminated composite plates under thermomechanical loading. Int J Non-Linear Mech, 2005, 40(4): 971—985

11 Zhang W, Song C Z, Ye M. Further studies on nonlinear oscillations and chaos of a rectangular symmetric cross-by laminated plate under parametric excitation. Int J Bifur Chaos, 2006, 16(2): 325—347

12 Varelis D, Saravanos D A. Small-amplitude free-vibration analysis of piezoelectric composite plates subject to large deflections and initial stresses. J Vibra Acoust, 2006, 128(1): 41—49

13 Hao Y X, Chen L H, Zhang W, et al. Nonlinear oscillations, bifurcations and chaos of functionally graded materials plate. J Sound Vibra, 2008, 312(4): 862—892

14 Ye R, Tzou H S. Control of adaptive shells with thermal and mechanical excitations. J Sound Vibra, 2000, 5(6): 1321—1338

15 Krommer M, Irschik H. A Reissner-Mindlin-type plate theory including the direct piezoelectric and the pyroelectric effect. Int J Solids Struct, 2000, 141(1): 51—69

16 Correia I F, Mota Soares C M, Mota Soares C A, et al. Active control of axisymmetric shells with piezoelectric layers: a mixed laminated theory with a high order displacement field. Comput Struct, 2002, 80(10): 2265—2275

17 Donadon M V, Almeida S F M, Faria A R D. Stiffening effects on the natural frequencies of laminated plates with piezoelectric actuators. Compos Part B-Eng, 2002, 33(2): 335—342

18 Lim C W, Lau C W H. A new two-dimensional model for electro-mechanical response of thick laminated piezoelectric actuator. Int J Solids Struct, 2005, 42(11): 5589—5611

19 Karnaukhov V G, Tkachenko Ya V. Damping the vibrations of a rectangular plate with piezoelectric actuators. Int Appl Mech, 2008, 44(1): 78—84

20 Nayfeh A H, Mook D T. Nonlinear Oscillations. Wiley: New York, 1979