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Optimization model and algorithm for mixed traffic of urban road network with flow interference

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In this paper, the problem of interferences between motors and non-motors in urban road mixed traffic network is considered and the corresponding link impedance function is presented based on travel demand. On the base of this, the main factors that influence travelers' traffic choices are all considered and a combined model including flow-split and assignment problem is proposed. Then a bi-level model with its algorithm for system optimization of urban road mixed traffic network is proposed. Finally the application of the model and its algorithm is illustrated with a numerical example.

mixed traffic, impedance function, user equilibrium, optimization

In China, the large population and the increasing number of cars, together with existing bicycles, buses, and pedestrians, have resulted in many urban traffic problems such as congestion, air pollution, and a shortage of fuel. The traffic analysis theories, methods, and models, which were developed based on pure motor traffic and used extensively in developed countries, are thought to be inadequate to solve the problems caused by mixed traffic. In practice, both of the travelers' mode choice and route choice, together with their responses to congestion, result in the flow pattern over a network. So, the distinct characteristic of mixed traffic is represented by interferences between different modes, especially between motor traffic and non-motor traffic. Most of previous research[1−5] focused primarily on motor traffic, which implies they paid primary attention to drivers' route choice problem while the travelers' mode choice was rarely investigated. Some research[6−8] simultaneously considered the travelers' mode choice and route choice, however, the interferences between different traffic flows were not considered in general.

In the last decade, the bi-level programming approach has emerged as an important area for progress in handling complicated traffic problems. Typical examples include continuous equilib-

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rium network design[9], traffic signal optimization[10], discrete network design[11], O-D matrices estimation[12], congestion pricing[13] and so on. In this paper, a bi-level programming model for system optimization of urban mixed traffic network is proposed, where the traffic authority is the decision maker or the leader, and the travelers who can freely choose traffic modes and routes are the followers. The objective of the traffic authority is to reduce the system congestion, while the objective of the travelers is only to minimize their own travel cost. Accordingly, the lower-level problem is a combined model that describes travelers' mode choice and route choice simultaneously, while the upper-level problem represents a system performance optimization problem, the objective of which is to minimize the total travel cost of mixed traffic network.

1 Link impedance function for mixed traffic

In traffic demand modeling, link impedance function is mainly used to describe the relationship between travel time and link flow. Most of previous research used simple function forms such as BPR. However, it is difficult to model mixed traffic by such functions, because they have the following inherent limitations. Firstly, the characteristics of different traffic flows are not considered in such functions. Secondly, the interferences between different traffic modes are ignored in these functions, especially between motor vehicles and non-motor vehicles. In this paper, a new type of link impedance function is proposed to better simulate a mixed traffic system, in which the characteristics of motor and non-motor traffic and their interferences are considered.

Assume that there are K types of motor modes in the mixed traffic network. The standard pcu (passenger car unit) flow of motor mode k on link a can be formulated as

$$v_a^k = x_a^k \cdot \left(\frac{U_k}{A_k}\right), \quad \forall k, a, \tag{1}$$

where v_a^k denotes the standard pcu flow of motor mode k on link a; x_a^k is the number of travelers who choose motor mode k on link a; U_k is pcu conversion coefficient of motor mode k; A_k is occupancy rate of mode k, which means the average number of travelers within each vehicle of motor mode k. Obviously, the total pcu flow of all motor modes, represented by v_a^{motor}, can be written as

$$v_a^{\text{motor}} = \sum_k v_a^k, \quad \forall a. \tag{2}$$

Assume that the non-motor flow on link a, represented by $x_a^{\text{non-motor}}$, is the same as the number of travelers who choose non-motor mode on link a. The link impedance functions for motor traffic and non-motor traffic can be respectively formulated as

$$t_a^k = t_a^{k(0)} \left[1 + \alpha \left(\frac{v_a^{\text{motor}}}{C_a^{\text{motor}}}\right)^{\beta}\right] \left[1 + \gamma \left(\frac{x_a^{\text{non-motor}}}{C_a^{\text{non-motor}}}\right)^{\varphi}\right], \quad \forall k, a, \tag{3}$$

$$t_a^{\text{non-motor}} = t_a^{\text{non-motor}(0)} \left[1 + \alpha \left(\frac{x_a^{\text{non-motor}}}{C_a^{\text{non-motor}}}\right)^{\beta}\right] \left[1 + \gamma \left(\frac{v_a^{\text{motor}}}{C_a^{\text{motor}}}\right)^{\varphi}\right], \quad \forall a, \tag{4}$$

where t_a^k and $t_a^{\text{non-motor}}$ denote the travel time of motor mode k and non-motor mode on link a, respectively; $t_a^{k(0)}$ and $t_a^{\text{non-motor}(0)}$ are the free-flow travel time of motor mode k and non-motor mode on link a; C_a^{motor} and $C_a^{\text{non-motor}}$ are the practical capacities of motor and non-motor modes on link a; α, β, γ and φ are parameters.

2 Travelers' choice behaviors in mixed traffic network

Firstly, the travelers' mode choices in urban mixed traffic network are considered in this section. The general travel costs of motor mode k and non-motor between O-D pair w, represented by T_k^w and $T_{\text{non-motor}}^w$, respectively, can be described as

$$T_k^w = t_k^w + \varepsilon_k^w, \quad \forall w, k, \tag{5}$$

$$T_{\text{non-motor}}^w = t_{\text{non-motor}}^w + \varepsilon_{\text{non-motor}}^w, \ \forall w, \tag{6}$$

where t_k^w and $t_{\text{non-motor}}^w$ are the measurable travel costs of motor mode k and non-motor mode between O-D pair w which may consist of time, fee, convenience, comfort, etc; ε_k^w and $\varepsilon_{\text{non-motor}}^w$ are stochastic errors. Usually, the measurable travel costs can be described as

$$t_k^w = \mu_k^w + aM_k^w - bE_k^w, \ \forall w, k, \tag{7}$$

$$t_{\text{non-motor}}^w = \mu_{\text{non-motor}}^w - bE_{\text{non-motor}}^w, \ \forall w, \tag{8}$$

where μ_k^w and $\mu_{\text{non-motor}}^w$ are the travel-time of motor mode k and non-motor between O-D pair w at steady state respectively; M_k^w is the potential fee of motor mode k between O-D pair w; E_k^w and $E_{\text{non-motor}}^w$ denote the service levels of motor mode k and non-motor between O-D pair w; a and b are weight parameters.

The probability of a mode chosen by travelers can be described as the likelihood that this mode is perceived as the one with the lowest travel cost within all modes. The probabilities of motor mode and non-motor mode depend on the distributions of ε_k^w and $\varepsilon_{\text{non-motor}}^w$. If they are assumed to be independent and follow Gumbel distributions, then the following logit model can be used to calculate the probability of a traveler's choice between motor and non-motor modes.

$$P_k^w = \frac{\mathrm{e}^{-\theta t_k^w}}{\sum_i \mathrm{e}^{-\theta t_i^w} + \mathrm{e}^{-\theta t_{\text{non-motor}}^w}}, \ \forall w, k, \tag{9}$$

$$P_{\text{non-motor}}^w = \frac{\mathrm{e}^{-\theta t_{\text{non-motor}}^w}}{\sum_i \mathrm{e}^{-\theta t_i^w} + \mathrm{e}^{-\theta t_{\text{non-motor}}^w}}, \ \forall w, \tag{10}$$

where P_k^w and $P_{\text{non-motor}}^w$ are the choice probabilities of motor mode k and non-motor mode, respectively; θ is the dispersion parameter.

Assume that the total travel demand between O-D pair w, represented by q^w, is fixed. The travel demands of different traffic modes should satisfy the following constrains:

$$q_k^w = q^w P_k^w, \ \forall k, w, \tag{11}$$

$$q_{\text{non-motor}}^w = q^w P_{\text{non-motor}}^w, \ \forall w, \tag{12}$$

$$\sum_k q_k^w + q_{\text{non-motor}}^w = q^w,\ \forall w, \tag{13}$$

where q_k^w and $q_{\text{non-motor}}^w$ are the travel demands of motor mode k and non-motor mode between O-D pair w, respectively.

It is reasonable to assume that the traffic flow pattern in the urban mixed traffic network follows Wardrop's equilibrium principle, which can be mathematically formulated as

$$\mu_k^w - c_k^{wn} \begin{cases} = 0 \\ \leqslant 0 \end{cases}, \quad \text{if} \quad \begin{matrix} f_k^{wn} > 0 \\ f_k^{wn} = 0 \end{matrix}, \ \forall w,k,n, \tag{14}$$

$$\mu_{\text{non-motor}}^w - c_{\text{non-motor}}^{wn} \begin{cases} = 0 \\ \leqslant 0 \end{cases}, \text{if} \begin{matrix} f_{\text{non-motor}}^{wn} > 0 \\ f_{\text{non-motor}}^{wn} = 0 \end{matrix}, \ \forall w,n, \tag{15}$$

where c_k^{wn} and $c_{\text{non-motor}}^{wn}$ are the travel times of motor mode k and non-motor mode on the route n between O-D pair w; f_k^{wn} and $f_{\text{non-motor}}^{wn}$ are the travel demands of motor mode k and non-motor mode on route n between O-D pair w, respectively.

In addition, the travel demands in the urban mixed traffic network should satisfy the following constraints:

$$\sum_n f_k^{wn} = q_k^w,\ \forall w,k, \tag{16}$$

$$x_a^k = \sum_w \sum_n f_k^{wn} \delta_{a,n}^w,\ \forall a,k, \tag{17}$$

$$\sum_n f_{\text{non-motor}}^{wn} = q_{\text{non-motor}}^w,\quad \forall w, \tag{18}$$

$$x_a^{\text{non-motor}} = \sum_w \sum_n f_{\text{non-motor}}^{wn} \delta_{a,n}^w,\ \forall a,k, \tag{19}$$

$$\sum_k q_k^w + q_{\text{non-motor}}^w = q^w,\quad \forall w, \tag{20}$$

where $\delta_{a,n}^w$ is path and link incidence variable, and if link a is on the route n between O-D pair w, then $\delta_{a,n}^w$=1, otherwise $\delta_{a,n}^w$=0.

3 Combined model for urban mixed traffic network

In this paper, we propose the following variation inequality (VI) model to describe the flow-split and assignment problem for urban mixed traffic network: to find $(\boldsymbol{x}^*,\boldsymbol{q}^*) \in \boldsymbol{\Omega}$ such that

$$\boldsymbol{t}(\boldsymbol{x}^*)^{\mathrm{T}}(\boldsymbol{x}-\boldsymbol{x}^*) + \boldsymbol{g}(\boldsymbol{q}^*)^{\mathrm{T}}(\boldsymbol{q}-\boldsymbol{q}^*) \geqslant \boldsymbol{0}, \tag{21}$$

where

$$\begin{aligned}\boldsymbol{\Omega} = \{(\boldsymbol{x},\boldsymbol{q}) \mid &\sum_n f_k^{wn} = q_k^w, \sum_n f_{\text{non-motor}}^{wn} = q_{\text{non-motor}}^w, \sum_k q_k^w + q_{\text{non-motor}}^w = q^w, f_k^{wn} \geqslant 0, f_{\text{non-motor}}^{wn} \geqslant 0, \\ &x_a^k = \sum_w \sum_n f_k^{wn}\delta_{a,n}^w, x_a^{\text{non-motor}} = \sum_w \sum_n f_{\text{non-motor}}^{wn}\delta_{a,n}^w, \forall k,w,n,a\},\end{aligned} \tag{22}$$

$\boldsymbol{x}$ and $\boldsymbol{q}$ are the following vectors respectively:

$$\boldsymbol{x} = [x_1^1,\cdots,x_a^1,x_1^2,\cdots,x_a^k,x_1^{\text{non-motor}},\cdots,x_a^{\text{non-motor}}]^{\mathrm{T}}, \tag{23}$$

$$\boldsymbol{q}=[q_1^1,\cdots,q_1^w,q_2^1\cdots,q_k^w,q_{\text{non-motor}}^1,\cdots,q_{\text{non-motor}}^w]^{\mathrm{T}}, \tag{24}$$

$\boldsymbol{t}$ and $\boldsymbol{g}$ are the vectors of link impedance functions and general cost functions of traffic modes respectively, given by

$$\boldsymbol{t}(\boldsymbol{x})=[t_1^1(\boldsymbol{x}),\cdots,t_a^1(\boldsymbol{x}),t_1^2(\boldsymbol{x}),\cdots,t_a^k(\boldsymbol{x}),t_1^{\text{non-motor}}(\boldsymbol{x}),\cdots,t_a^{\text{non-motor}}(\boldsymbol{x})]^{\mathrm{T}}, \tag{25}$$

$$\boldsymbol{g}(\boldsymbol{q})=[g_1^1(\boldsymbol{q}),\cdots,g_1^w(\boldsymbol{q}),g_2^1(\boldsymbol{q}),\cdots,g_k^w(\boldsymbol{q}),g_{\text{non-motor}}^1(\boldsymbol{q}),\cdots,g_{\text{non-motor}}^w(\boldsymbol{q})]^{\mathrm{T}}. \tag{26}$$

At present, the popular algorithm of VI is the diagonalization method[2]. This algorithm is based on a series of iterations to solve the optimization problem. Correspondingly, at ith iteration, the optimization problem can be described as

$$\min Z(\boldsymbol{x},\boldsymbol{q})=\sum_k\sum_a\int_0^{x_a^k}t_a^k(x_a^{1(i)},x_a^{2(i)},\cdots,\omega,x_a^{\text{non-motor}(i)})\mathrm{d}\omega+\sum_w\sum_k\int_0^{q_k^w}g_k^w(\omega)\mathrm{d}\omega$$

$$+\sum_a\int_0^{x_a^{\text{non-motor}}}t_a^{\text{non-motor}}(x_a^{1(i)},x_a^{2(i)},\cdots,x_a^{k(i)},\omega)\mathrm{d}\omega+\sum_w\int_0^{q_{\text{non-motor}}^w}g_{\text{non-motor}}^w(\omega)\mathrm{d}\omega, \tag{27a}$$

$$\text{s.t.}\quad (\boldsymbol{x},\boldsymbol{q})\in\boldsymbol{\Omega}. \tag{27b}$$

In this paper, g_k^w and $g_{\text{non-motor}}^w$ should be defined as follows in order to transfer the above flow-split and assignment problem into a logit model.

$$g_k^w(x)=\frac{1}{\theta}\ln x+t_k^w,\quad \forall k,w, \tag{28a}$$

$$g_{\text{non-motor}}^w(x)=\frac{1}{\theta}\ln x+t_{\text{non-motor}}^w,\quad \forall k,w. \tag{28b}$$

It has been proved that the optimum solution of programming model (eq. (27a) and (27b)) satisfies the logit based split model and UE condition[14].

4 System optimization model for mixed traffic network

The traffic authority has to coordinate different traffic modes and optimize the structure of transportation system in order to successfully manage a mixed urban traffic network. Commonly, the traffic authority can use policies to influence travelers' mode and route choice to improve the performance of urban mixed traffic networks. This problem can be described as the following model:

$$\min Z(\boldsymbol{\lambda})=\sum_a\{\sum_k x_a^k(\boldsymbol{\lambda})\cdot t_a^k[\boldsymbol{x}(\boldsymbol{\lambda})]+x_a^{\text{non-motor}}(\boldsymbol{\lambda})t_a^{\text{non-motor}}[\boldsymbol{x}(\boldsymbol{\lambda})]\}, \tag{29}$$

where $\boldsymbol{\lambda}=[\lambda_1^1,\cdots,\lambda_1^w,\lambda_2^1,\cdots,\lambda_k^w,\lambda_{\text{non-motor}}^1,\cdots,\lambda_{\text{non-motor}}^w]^{\mathrm{T}}$ represents the policy indices like traffic pricing.

The traffic authority influences travelers' choice behaviors by making traffic policies. On the other hand, travelers make travel decisions independently based on their own experiences and traffic policies. This process can be described as the following bi-level programming model:

$$\min Z(\boldsymbol{\lambda})=\sum_a\{\sum_k x_a^k(\boldsymbol{\lambda})\cdot t_a^k[\boldsymbol{x}(\boldsymbol{\lambda})]+x_a^{\text{non-motor}}(\boldsymbol{\lambda})t_a^{\text{non-motor}}[\boldsymbol{x}(\boldsymbol{\lambda})]\}, \tag{30}$$

where the response function, $\boldsymbol{x}(\boldsymbol{\lambda})$, is given by the following VI model with parameters.

$$\boldsymbol{t}(\boldsymbol{x}^*,\boldsymbol{\lambda})^{\mathrm{T}}(\boldsymbol{x}-\boldsymbol{x}^*)+\boldsymbol{g}(\boldsymbol{q}^*,\boldsymbol{\lambda})^{\mathrm{T}}(\boldsymbol{q}-\boldsymbol{q}^*)\geqslant\boldsymbol{0}, \tag{31a}$$

$$(\boldsymbol{x}^*, \boldsymbol{q}^*) \in \Omega. \tag{31b}$$

The objective of the upper level problem is to minimize the total travel time of a mixed urban traffic network, while the objective of the lower level problem is to minimize individual travel cost under different traffic policies as specified by the upper level.

5 Solving algorithm

Due to the intrinsic complexity of model formulation, the bi-level programming problem has been recognized as one of the most difficult, yet challenging problems for global optimality in transportation system. In the past, researchers developed alternative solution algorithms for this problem. For example, Abdulaal and LeBlanc[9] applied the Hook-Jeeves heuristic algorithm for direct search of the solution to the network design problem. Fisk[15] developed an alternative single level optimization model for the optimal signal control problem using a gap function. Suwansirikul et al.[16] developed an alternative heuristic method referred to as the equilibrium decomposed optimization algorithm by approximating the derivative of the objective function in the upper-level problem. However, sensitivity analysis methods proposed by Tobin and Friesz[17], which make use of the derivatives of the equilibrium link flows with respect to perturbation parameters, are widely used for network equilibrium problems[18−20]. Accordingly, the sensitive analysis based algorithms is used to solve the bi-level programming model proposed in this paper.

It is necessary to derive the derivatives of the decision variables with respect to the perturbation parameters in the sensitivity analysis approach. In our proposed problem, we need to calculate the derivatives of equilibrium link flows with respect to the policies index. By assuming the initial data, $\boldsymbol{\lambda}^0$, is given and other conditions are fixed, the link flow matrix for a mixed traffic network, $\boldsymbol{x}^*(\boldsymbol{\lambda}^0)$, can be obtained by solving the lower level of the model. Through conducting a sensitivity analysis of VI model (eq. (31a) and (31b)), the approximate differential coefficient, $\nabla_{\boldsymbol{\lambda}} \boldsymbol{x}$, can be obtained. Then the response function can be approximated by the Taylor expansions. That is,

$$\boldsymbol{x}(\boldsymbol{\lambda}) \approx \boldsymbol{x}^*(\boldsymbol{\lambda}^0) + (\nabla_{\boldsymbol{\lambda}} \boldsymbol{x})^{\mathrm{T}} (\boldsymbol{\lambda} - \boldsymbol{\lambda}^0). \tag{32}$$

By substituting eq. (32) into the upper level problem, the whole optimization model can be simplified as one-level optimization problem. The solution of this one-level optimization will then be input into the lower level of the model to run the next iteration. By repeating the iteration process, it is possible to obtain an optimum solution for the above bi-level programming model. This process can be summarized as the following steps:

Step 1: Set the initial value $\boldsymbol{\lambda}^{(0)}$, and set the number of iterations to $i = 1$.

Step 2: Find the solution of the lower level of the model, $\boldsymbol{x}^{(0)}$, which is the link flow matrix for different modes.

Step 3: Find the linear equation of the matrix, $\boldsymbol{x}(\boldsymbol{\lambda}^{(i-1)})$, through sensitivity analysis[17] and Taylor expansion.

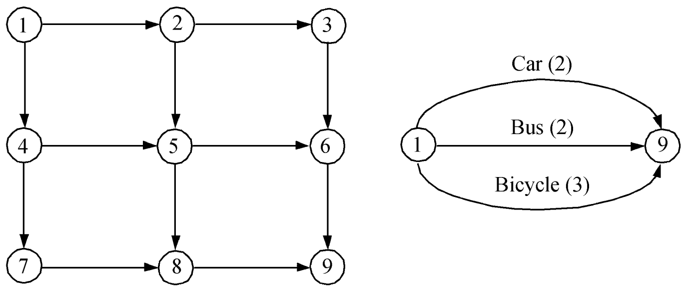
Step 4: Put the linear equation of the matrix into the upper level of the model to update the parameter value of $\boldsymbol{\lambda}^{(i)}$ by solving upper level problem.

Step 5: Examine the convergence. If $\boldsymbol{\lambda}^{i} \approx \boldsymbol{\lambda}^{(i-1)}$ or $i = N$, then iteration stops, where N is the maximum number of iterations. Otherwise, set $i = i+1$ and start a new iteration.

Note that Step 2 and Step 3 are the key steps of the above algorithm. Step 2 solves a VI model, and the simplified diagonalization method[3] can be applied as solution algorithm. In each iteration, the method of successive average (MSA)[3] is employed to solve the flow-split and assignment problem (eq. (27a) and (27b)).

6 Numerical example

A simple numerical example is used to illustrate the effectiveness of the model. The traffic network used in this study is shown in Figure 1, which consists of one O-D pair (1-9), 9 nodes, 12 links, two motor modes (car and bus) and one non-motor mode (bicycle).



(a) (b)

Figure 1 Traffic network structure (a) and traffic mode structure (b).

The link input data are presented in Table 1, while the pcu conversion coefficient, the average occupancy rate, potential user fee, and corresponding service level, which are pertinent to different traffic modes, are illustrated in Table 2.

Table 1 Data of different links

Links	$t_a^{1(0)}$ (h)	$t_a^{2(0)}$ (h)	$t_a^{3(0)}$ (h)	C_a^{motor} $(\text{p}\cdot\text{h}^{-1})$	$C_a^{\text{non-motor}}$ $(\text{p}\cdot\text{h}^{-1})$
(1,2)	0.111	0.178	0.261	1000	600
(2,3)	0.128	0.194	0.278	700	400
(1,4)	0.100	0.167	0.250	1500	800
(2,5)	0.106	0.172	0.256	700	400
(3,6)	0.089	0.156	0.239	700	400
(4,5)	0.078	0.144	0.228	1000	600
(5,6)	0.094	0.161	0.244	1000	600
(4,7)	0.133	0.200	0.283	900	500
(5,8)	0.111	0.178	0.261	700	400
(6,9)	0.144	0.211	0.294	700	400
(7,8)	0.094	0.161	0.244	900	500
(8,9)	0.100	0.167	0.250	900	500

Table 2 Data of different models

Modes	PCU coefficient	Average passengers	Potential fee (RMB)	Service level
Car	1	4	20	10
Bus	1.5	20	5	5
Bicycle	–	–	–	1

The parameters used in this example take the values of α =0.15, β =4, γ =0.1, φ =4, a=0.2, b =0.1 and θ= 0.5, respectively. In general, the city government can use various policies to encourage or discourage travelers' mode and route choices to improve the performance of mixed traffic network, which are expressed here by different $\boldsymbol{\lambda}$ values. In this example, the policies index is assumed to be traffic pricing only, that is $\boldsymbol{\lambda}=[M_{\text{car}}, M_{\text{bus}}]^{\text{T}}$.

The total travel demand, q^w, is used to describe congestion level of the urban mixed traffic network. Two scenarios are considered here: (1) $q^{rs}=10000\ \text{p}\cdot\text{h}^{-1}$, representing a network without any congestion; and (2) $q^{rs}=30000\ \text{p}\cdot\text{h}^{-1}$, representing a network that has reached the congestion saturation point.

Table 3 shows equilibrium link flows and the corresponding travel times of different traffic modes under the initial conditions with a total demand of $q^w=10000\ \text{p}\cdot\text{h}^{-1}$.

Table 3 The results of link demands and its travel time

Links	x_a^1 (p·h^{-1})	x_a^2 (p·h^{-1})	x_a^3 (p·h^{-1})	t_a^1 (h)	t_a^2 (h)	t_a^3 (h)
(1,2)	30.92	864.30	1067.61	0.2225	0.3560	0.6537
(2,3)	0.84	190.07	497.68	0.1584	0.2410	0.3776
(1,4)	2248.90	4330.89	1457.38	0.2140	0.3567	0.6711
(2,5)	30.08	674.23	569.93	0.1491	0.2432	0.4135
(3,6)	0.84	190.07	497.68	0.1102	0.1928	0.3248
(4,5)	1896.29	2723.42	737.19	0.0985	0.1830	0.3121
(5,6)	52.49	1462.81	621.06	0.1053	0.1796	0.2865
(4,9)	352.61	1607.47	720.19	0.1908	0.2862	0.4664
(5,8)	1873.88	1934.85	686.06	0.2256	0.3610	0.6355
(6,9)	53.33	1652.87	1118.74	1.0285	1.5032	2.9974
(7,8)	352.61	1607.47	720.19	0.1352	0.2306	0.4024
(8,9)	2226.49	3542.32	1406.25	0.8016	1.3359	2.7773

Now we examine the total system travel time by varying the prices of car and bus with different levels of total demand. Figure 2(a) shows when $q^{rs}=10000\ \text{p}\cdot\text{h}^{-1}$, the relationship between the total travel time of network and car and bus pricing. It can be seen that the total travel time of the whole network will be reduced as car/bus pricing decreases. This indicates that if there are not congestion over a mixed traffic network, making cars or buses more attractive would improve system performance. Furthermore, it can be found from the figure that the decreasing rate of total travel time is more sensitive to the pricing of the bus than that of the car. Figure 2(b) shows the relationship between the total travel time and car and bus pricing over a congested network. It can be seen that the total travel time of the whole network can be reduced only by decreasing bus pricing, which implies that decreasing bus fare would be the most effective way to reduce the total travel time of network under the condition of high congestion.

Figure 3 shows the change trend of total travel time with the variations of the prices of car and bus under non-congestion and congestion conditions, respectively without considering the interference between motors and non-motors. We can learn that the change tendency of total travel time is similar to the case of considering interference between motor and non-motor, but the total travel time will be lower if there are no interferences between motor and non-motor over a mixed traffic network. Furthermore, the larger the travel demand is, the more rapidly the total travel time grows up.

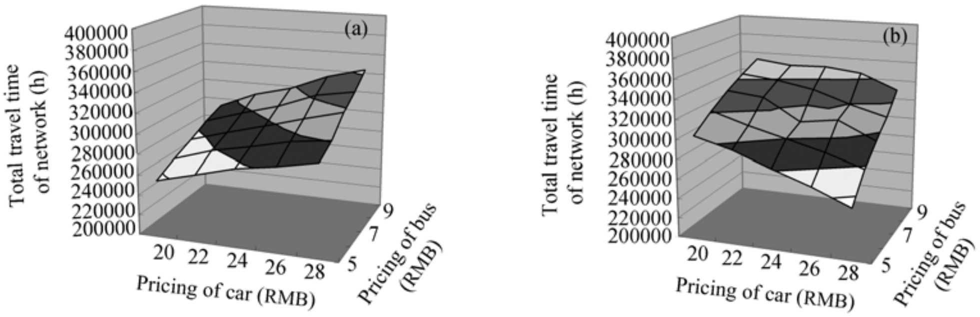


Figure 2 The changes of total travel time with pricing of car and bus when considering interference at q^w=10000 p · h^{-1} (a) and q^w =30000 p · h^{-1} (b).

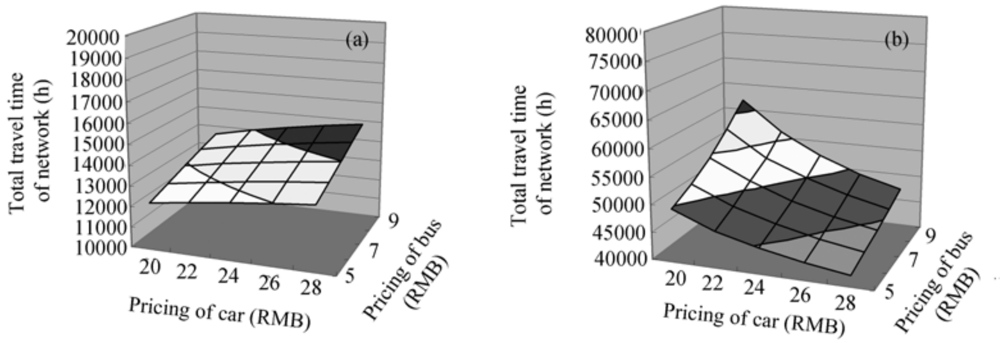


Figure 3 The changes of total travel time with pricing of car and bus when not considering interference at q^w =10000 p · h^{-1} (a) and q^w =30000 p · h^{-1} (b).

7 Conclusion

In this paper, the basic factors that influence travelers' travel choice behaviors are discussed and a new type of link impedance function is developed for the mixed urban traffic network, in which the characteristics of motor and non-motor traffic and the interference between them are taken in to account. The VI assignment model for urban mixed traffic network is developed and a system optimization model for mixed traffic network is studied. The effectiveness of the proposed model is illustrated with a numerical example. Results show that expanding the amount of private cars or bicycles would all result in increased total travel time for highly congested mixed traffic network, which would be prevalent in most Chinese big cities, which in turn recommends that developing and enhancing the public transport should be the most effective way to solve the traffic problem.

It should be noted that this numerical example considers just one optimization target—total system travel time. In actual urban transportation optimization tasks, there are extra goals that need to be considered, such as environment issue and investment budget. In such a case, a more complicated optimization model considering all these factors should be developed and used.

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