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The coalbed methane transport model and its application in the presence of matrix shrinkage

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Based on the theories of surface physical chemistry, theoretical formulations for permeability and porosity are presented which include both stress effect and matrix shrinkage in a single equation. Then, a three-dimensional, dual porosity, non-equilibrium adsorption, pseudosteady state mathematical model for gas and water is established and solved by the fully implicit method and the block preconditioning orthomin algorithm. A history matching for the Qinshui Well TL003 is done. From the results, it is shown that the obvious enhancement of permeability occurs along with the passing time but the reservoir pressure of 15$^{\#}$ coal seam cannot fulfill the critical adsorption pressure as a result of the water recharge of the aquifer. Hence, it is suggested to plug the 15$^{\#}$ coal seam.

coalbed methane, surface energy, matrix shrinkage, permeability, numerical simulation

Unlike the conventional natural gas reservoirs, coal seams are both source rock and reservoir rock. Methane is mainly adsorbed on the surface of micropores controlled by the reservoir pressure. During the depletion of coal reservoirs, a large reduction in permeability will occur due to the increasing effective stress applied to the coal seam. However, this reduction will be offset by the permeability enhancement resulting from the matrix shrinkage associated with methane desorption. The emergence of injection schemes for enhanced coalbed methane recovery has led to a renewed focus on the characteristics of coal seams. The effect of matrix shrinkage on permeability was first quantified by Gray[1]. In 1990 Sawyer et al.[2] published a shrinkage model (ARI) developed for the COMET simulator, but the matrix shrinkage compressibility and the matrix swelling coefficient used in ARI model were difficult to obtain at reservoir condition typically encountered in coalbed methane fields. Although many scholars[3−5] have obtained some experiment values, further laboratory findings are needed for representatives. In 1996 Palmer et al.[6] assumed the porosity $\ll 1$ and presented a theoretical shrinkage model (P&M) that described the

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matrix shrinkage more in terms of strain and coal's mechanical properties. And two years later, they issued another revised model[7]. Since then, a series of theoretical permeability models have been put forward[8,9].

Based on the theories of surface physical chemistry, theoretical formulations for permeability and porosity are presented which include both stress effect and matrix shrinkage in a single equation. Then, a three dimensional, dual porosity, nonequilibrium adsorption, pseudosteady state mathematical model for gas and water is established, and a history matching for the Qinshui Well TL003 is performed. The results have shown that the presented model is reasonable.

1 Theoretical formulations for permeability

Researches indicate that adsorptions of coal matrix to CH_4, N_2 and CO_2 follow Langmuir isotherm, i.e.,

$$V = \frac{V_L bp}{1+bp}, \tag{1}$$

where $b=1/p_L$; p_L is the Langmuir pressure, MPa; V is the total volume of the adsorbed gas per unit mass of the reservoir in equilibrium at pressure p, $\mathrm{m^3/t}$; V_L is the Langmuir volume, $\mathrm{m^3/t}$.

In view of multi-component effect, the adsorption capacity of each gas component is described by the extended Langmuir equation[10] , namely

$$V_i = \frac{V_{Li} b_i y_i p}{1+\sum_{i=1}^{n} b_i y_i p}, \tag{2}$$

where $b_i=1/p_{Li}$; p_{Li} is the Langmuir pressure of component i, MPa; V_{Li} is the Langmuir volume of component i , $\mathrm{m^3/t}$; y_i is the mole fraction of the component i.

Particles on the coal matrix surface possess certain surface energy, resulting in the gas molecules adsorption, and the surface energy will be reduced after gas molecules adsorption [11]. The solid swelling strain is proportional to its surface energy reduction by Bangham[12, 13]:

$$\varepsilon = \rho_c S \Delta\gamma / E, \tag{3}$$

where ε is the solid amount of relative deformation, f; ρ_c is the coal density, $\mathrm{t/m^3}$; S is the surface area of coal matrix, $\mathrm{m^2/t}$; E is Young's modulus of the coal, MPa; $\Delta\gamma$ is the variation of surface energy, $\mathrm{J/m^2}$.

The surface energy variation of coal matrix caused by gas adsorption is given by Gibbs equation[12,13] as follows:

$$\Delta\gamma = \gamma_0 - \gamma = \int_0^p \Gamma RT \,\mathrm{d}\ln p, \tag{4}$$

where γ_0 is the coal surface energy under vacuum condition, $\mathrm{J/m^2}$; γ is the coal surface energy after its adsorbing gas, $\mathrm{J/m^2}$; Γ is the difference between surface concentration and bulk phase concentration, $\Gamma=V/(V_0 S)$, $\mathrm{mol/m^2}$; R is the universal gas constant, 8.3143 J/(mol·K); T is the absolute temperature, K; p is the real gas pressure, MPa; V_0 is the gas molar volume under standard condition, 22.4 L/mol.

Substituting eq. (4) into eq. (3) gives

$$\varepsilon(p) = \frac{\rho_c RT}{V_0 E} \int_0^p \frac{V}{p} \mathrm{d}p. \tag{5}$$

On the contrary, the coal matrix shrinks as a result of gas desorption. When the reservoir pressure declines from the critical desorption pressure p_r to p, the coal matrix shrinkage is calculated as

$$\Delta\varepsilon = \varepsilon(p_r) - \varepsilon(p) = \frac{\rho_c RT}{V_0 E}\int_p^{p_r} \frac{V}{p}\mathrm{d}p. \tag{6}$$

Substitution of eq. (1) into eq. (6) gives the coal matrix shrinkage under single component gas desorption condition:

$$\Delta\varepsilon = \frac{\rho_c V_L RT}{V_0 E}\left[\ln(1+bp_r) - \ln(1+bp)\right]. \tag{7}$$

Similarly, substituting eq. (2) into eq. (6) yields the coal matrix shrinkage under multi-component gas adsorption condition:

$$\Delta\varepsilon = \sum_{i=1}^{n} \frac{V_{Li} b_i y_i \rho_c RT}{EV_0 \sum_{j=1}^{n} b_j y_j}\left[\ln\left(1 + p_r \sum_{k=1}^{n} b_k y_k\right) - \ln\left(1 + p\sum_{k=1}^{n} b_k y_k\right)\right]. \tag{8}$$

Supposing that the relationship between the coal matrix and the fracture network in coal seams can be described by the matchstick model, the following equation is obtained according to the Seidle model[3]

$$\frac{\Delta\phi_f}{\phi_{fr}} = \left(1 + \frac{2}{\phi_{fr}}\right)\Delta\varepsilon, \tag{9}$$

where $\Delta\phi_f$ is the fracture porosity variation induced by the matrix shrinkage, f; ϕ_{fr} is the critical fracture porosity, defined as the fracture porosity when methane starts to be desorbed, f.

On the other hand, the coal framework will be deformed by the increase in effective stress resulting from the reservoir pressure reduction. The stress-induced changes in both porosity and permeability have been expressed by Schwerer and Pavone[14] as

$$\phi_f = \phi_{f0}\mathrm{e}^{c_f(p-p_0)}, \tag{10}$$

$$k_f = k_{f0}\left(\frac{\phi_f}{\phi_{f0}}\right)^3, \tag{11}$$

where c_f is the pore volume compressibility, 1/MPa; k_f is the fracture permeability, 10^{-3} μm^2; k_{f0} is the fracture permeability at initial time, 10^{-3} μm^2.

By superimposing the effects of pore compressibility and matrix shrinkage, the final expressions for porosity and permeability are

$$\phi_f = \phi_{f0}\mathrm{e}^{c_f(p-p_0)} + \phi_{fr}\left(1 + 2/\phi_{fr}\right)\Delta\varepsilon, \tag{12}$$

$$\frac{k_f}{k_{f0}} = \left(\frac{\phi_f}{\phi_{f0}}\right)^3. \tag{13}$$

2 Coalbed methane reservoir simulation model

From the existing models and field practices in China, a three dimensional, dual porosity, two-phase mathematical model for gas and water is presented with the following assumptions.

1) The coal seams are generally characterized as a dual porosity nature composed of a micropore system and a macropore system.

2) The coal seams are compressible, and besides, the fracture system is heterogeneous and anisotropy.

3) In the original state, coal seams are fully saturated by water, and methane is stored on the internal surface of coal matrix in adsorbed state, regardless of free gas and solution gas.

4) Water is treated as a slightly compressible fluid, and the coal matrix, as a result of its small pore diameter, is inaccessible to water.

5) The free gas in the fracture system is assumed to behave like a real gas and obey percolation and diffusion mechanisms while only percolation mechanism for water. Furthermore, percolation and diffusion obey Darcy's law and Fick's first law, respectively.

6) The isothermal flow is assumed in coal seams and the effects of gravity and capillary pressure are considered.

7) The gas diffusion through coal matrix is described as a pseudosteady-state nonequilibrium process and obeys Fick's first law.

2.1 Transport equations in fracture system[15, 16]

Based on the motion equations, continuity equations and the state equation for real gas, the differential equations describing gas-water flow in coal macropore system are given as follows.

$$\nabla \cdot \left[\alpha \frac{k_{rg} k_f}{\mu_g B_g} \nabla \Phi_g + D_{\mathrm{f}} \nabla \left(\frac{S_g}{B_g} \right) \right] + q_{vm} - q_{vg} = \frac{\partial}{\partial t} \left(\frac{\phi_f S_g}{B_g} \right), \tag{14}$$

$$\nabla \cdot \left[\alpha \frac{k_{rw} k_f}{\mu_w B_w} \nabla \Phi_w \right] - q_{vw} = \frac{\partial}{\partial t} \left(\frac{\phi_f S_w}{B_w} \right), \tag{15}$$

where $\Phi_l = p_{fl} - \rho_l g H$, $l = g, w$; α is the units conversion factor, constant; p_f is the fluid pressure, MPa; μ is the fluid viscosity, mPa·s; B is the fluid formation volume factor, f; k_r is the relative permeability to fluid, f; ρ is the fluid density, t/m^3; H is the elevation, m; D_f is the gas diffusion coefficient, m^2/d; S is the fluid saturation, f; q_v is the volumetric fluid flow rate per unit volume reservoir, m^3/(m^3·d); the subscripts g and w indicate gas and water respectively; q_{vm} is the gas flow rate through the coal matrix per unit volume reservoir, m^3/(m^3·d).

For solving eqs. (14) and (15), two auxiliary equations are given below

$$S_w + S_g = 1, \tag{16}$$

$$p_c = p_{fg} - p_{fw}, \tag{17}$$

where p_c is the gas-water capillary pressure, MPa.

2.2 Transport equation in matrix system

The desorbed gas diffusion through the coal matrix to the cleat can be described mathematically by Fick's first law, so the migration velocity of desorbed gas is assumed as a result of concentration gradients in the matrix[11], i.e.,

$$\frac{\partial \overline{V_m}}{\partial t} = -\frac{1}{\tau} \left[\overline{V_m} - V_E(p_{fg}) \right], \tag{18}$$

and

$$q_{vm} = -\frac{\rho_c}{B_g}\frac{\partial \overline{V_m}}{\partial t}, \tag{19}$$

where $\overline{V_m}$ is the average matrix gas concentration, $\mathrm{m^3/t}$; τ is the sorption time, d; $V_E(p_{fg})$ is the equilibrium methane concentration described by Langmuir isotherm, $\mathrm{m^3/t}$.

3 Example analysis

The Well TL003 in the Southeast Qinshui Basin drills through the Quaternary system (Q_4), the Permian Shangshihezi (P_2^1), Xiashihezi (P_1^2) and Shanxi (P_1^1) Formations, the Pennsylvanian Taiyuan (C_{3t}), Benxi (C_{2t}) and Fengfeng (O_{2f}) Formations. The $3^{\#}$ coal seam in the Permian Shanxi Formation and the $15^{\#}$ coal seam in the Pennsylvanian Taiyuan Formation are the main coal-bearing sequences, and both are high rank anthracite coal. The thicknesses of these coal seams are 6.33 m and 0.90 m, respectively and the burial depths of them are 472.37 m and 583.26 m, respectively. Moreover, there is about 100 m of section between the $3^{\#}$ and the $15^{\#}$ coal seams but no hydrodynamic connection vertically. The roof and floor of the $3^{\#}$ coal seam are composed of mudstone and siltstone with low permeability. The roof of the $15^{\#}$ coal seam is mainly composed of Pennsylvanian limestone with a thickness of 9.14 m, however, fractures are well developed in the roof limestone and filled with water. There is hydrodynamic connection through fractures with the $15^{\#}$ coal seam, hence the roof limestone can be regarded as a single aquifer to participate in simulation. While the floor of the $15^{\#}$ coal seam is generally composed of clayey mudstone and bauxitic mudstone with better confining performance. Based on the acquisition of relevant reservoir parameters by means of injection-drawdown tests, coal sample desorption and adsorption experiments, complete data for gas production and water drainage were obtained after reservoir stimulation such as well completion and fracturing during the primary recovery. The presented mathematical model was solved simultaneously by the fully implicit method and the block preconditioning orthomin algorithm[17], and a relevant reservoir simulator was also developed to simulate the Well TL003. The permeability and the porosity are calculated by eqs. (12) and (13). The simulation parameters are listed in Table 1.

Table 1 Parameters for coalbed methane reservoir simulation

		$3^{\#}$ coal seam	K_2 limestone	$15^{\#}$ coal seam
Reservoir pressure (MPa)		3.36	4.30	4.30
Critical pressure (MPa)		2.53	–	1.61
Langmuir pressure (MPa)		3.17	–	2.27
Langmuir volume ($\mathrm{m^3/t}$)		44.27	–	48.92
Porosity (f)		0.02	0.05	0.02
Absolute permeability ($10^{-3}\mu\mathrm{m}^2$)	k_{fx}	3.40	21.0	1.20
	k_{fy}	1.70	21.0	0.80
	k_{fz}	0.00	2.10	0.01
Pore compressibility ($\mathrm{MPa^{-1}}$)		0.062	0.0029	0.062
Skin factor		–3.20	–3.05	–4.55
Coal density ($\mathrm{t/m^3}$)		1.375	–	1.435

History matching was done with the production data for the Qinshui Well TL003 from 16 March 1998 to 11 April 1999, and during that time the well was closed for a week because of well workover. The $3^{\#}$ and $15^{\#}$ coal seams are both infinite on plane and saturated by water in natural situation, thus boundary conditions, consisting of constant pressure and saturation at external boundary and specified pressure for production well, were used to the simulation. The bottomhole flowing pressure along with time is shown in Figure 1.

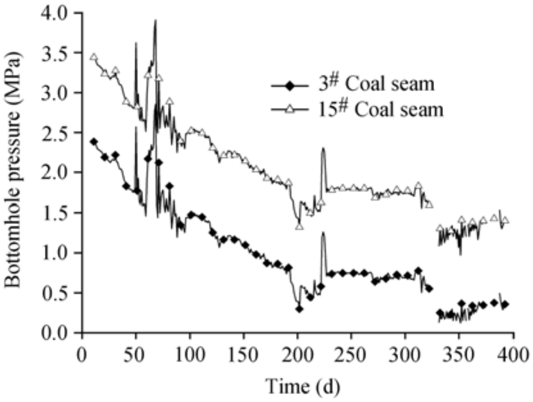

Figure 1 Observed bottomhole pressure along with time.

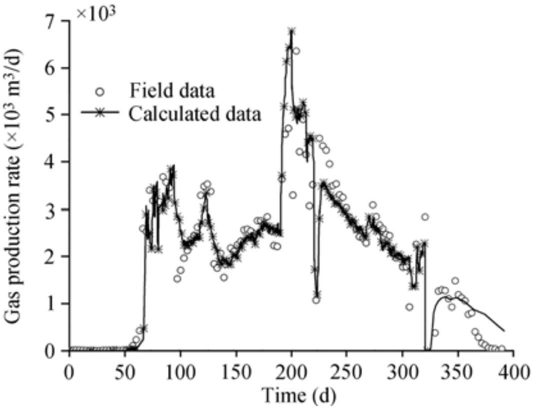

Figure 2 History match curves of gas production rate for the Well TL003.

The simulated gas production curves, as shown in Figures 2 and 3, are consistent with the field observations, which have shown better dynamics during primary coalbed methane recovery. And what is more, the reservoir simulation has indicated that the producing water of the Well TL003 mainly comes from the $15^{\#}$ coal seam and the roof limestone. Owing to the groundwater recharge from the limestone strata, the reservoir pressure of the $15^{\#}$ coal seam remains a high level and cannot fulfill the critical adsorption pressure at all. Therefore, the $15^{\#}$ coal seam should be plugged in order to solely exploit the $3^{\#}$ coal seam. A precipitous decline of the water production will occur in this way, and also an obvious improvement of the gas production.

In Figure 4, the effective stress plays a leading role at the beginning of drainage and results in continuous reduction in permeability. Subsequently, the obvious permeability enhancement occurs due to the leading role of matrix shrinkage.

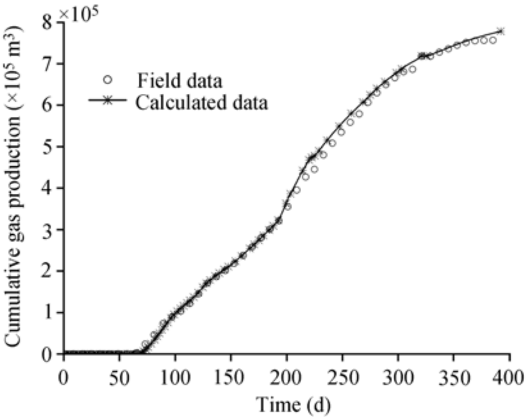

Figure 3 History match curves of cumulative gas production for the Well TL003.

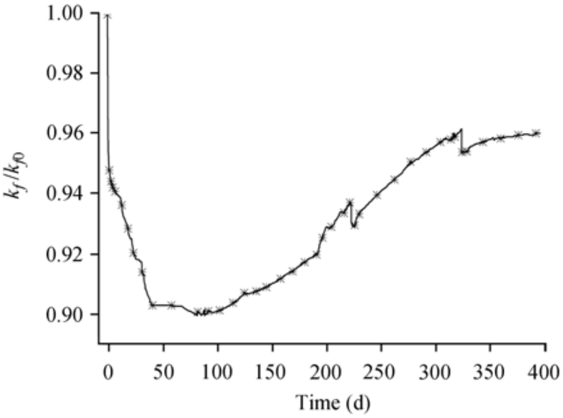

Figure 4 The well block permeability of the $3^{\#}$ coal seam along with time.

4 Conclusions

1) Theoretical formulations for permeability and porosity were presented which included both

stress effect and matrix shrinkage in a single equation.

2) In view of the effect of matrix shrinkage on the porosity and permeability, a three dimensional, dual porosity, nonequilibrium adsorption, pseudosteady state mathematical model for gas and water was established, and relevant numerical simulator was developed.

3) A history matching was done by the above simulator; the results have shown the presented model is reasonable.

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