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Fiducial Approach for the Storage Reliability Assessment of Complex Repairable Systems

YANG Yang · ZHAO Letian · CHEN Siyi · YU Dan

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Abstract In recent years, there has been a growing focus on high-reliability products in industrial and military fields. Storage plays a crucial role in these products, and as a result, research on storage reliability has gained more attention. This paper specifically targets a class of products with complex structures that require long-term storage, regular testing, and maintenance. Firstly, an expression method for system availability is provided based on the reliability structure of the system and the maintenance situation of the constituent equipment in storage. Since the expression of system availability is typically complex and difficult to compute, and the storage life of the system cannot be represented as an explicit function of the reliability indicators of the constituent equipment, it is challenging to evaluate the availability and storage life of repairable systems. To address this, the paper proposes a comprehensive evaluation methodology for assessing the storage reliability of complex repairable systems based on fiducial inference. This approach is employed to derive point estimates and determine the lower fiducial limits for both system availability and storage life. Furthermore, simulation results demonstrate that this comprehensive evaluation method for storage reliability of complex repairable systems is not only convenient but also highly effective.

Keywords Availability, fiducial method, repairable system, storage life.

1 Introduction

In practical applications, numerous high-reliability products (such as various medical emergency equipment, weaponry, and ammunition) utilized across industrial, commercial, and military domains are not immediately put into use upon production. Instead, they are subject to prolonged storage under specific environmental conditions until required for use. Consequently,

YANG Yang
University of Chinese Academy of Sciences, Beijing 100190, *China.* Email: abc9039@sina.com.
ZHAO Letian
Computer Application Institute of Nuclear Industry, Beijing 100142, *China.*
CHEN Siyi · YU Dan (Corresponding author)
Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, *China.*
Email: chensiyi@amss.ac.cn; dyu@amss.ac.cn.
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the assessment of storage reliability represents a crucial quality criterion for such products, garnering increasing attention[1]. These products typically possess intricate configurations, comprising diverse equipment components integrated into a cohesive whole. To enhance the future reliability of these products during their eventual utilization, regular inspections and corresponding maintenance are conducted on the constituent equipment throughout the storage process[2]. Regarding the storage reliability of such complex repairable systems, key indicators reside in system availability and storage life[3]. This paper will delve into a discussion surrounding these two metrics.

Numerous scholars have dedicated their efforts to exploring the storage reliability of repairable systems. Existing literature primarily focuses on two aspects: The maintenance time interval and the number of maintenance occurrences within a specific period[4]. The discussion regarding availability primarily focuses on the inference of steady-state availability, while the discussion regarding instantaneous availability addresses the availability of individual repairable equipment at a given moment[5]. In contrast, there is a dearth of research concerning the evaluation of storage life for systems. One contributing factor to this situation is the lack of a unified definition for the storage life of systems at present. One definition hinges on the longest duration during which the system can meet a specific performance index[6], while another associates it with the life expectancy of the component possessing the shortest life within the system[3]. Furthermore, most existing methods solely pertain to the assessment of system availability[7]. Rendering them inadequate for evaluating the storage life of the system, as it cannot be expressed explicitly as a function of equipment reliability indices[8]. Additionally, for reliability assessment problems that require considering the actual conditions of equipment detection and maintenance during storage, the established mathematical models for system reliability become more complex, leading to significant challenges in statistical and mathematical processing[9]. Therefore, in existing literature, the system is often treated as a whole, analyzing the mechanism of storage reliability variation and establishing predictive models to estimate the storage life[10]. Furthermore, the previous approach of indirectly obtaining the evaluation of storage life through comprehensive reliability assessment results lacks specificity and fails to fully utilize relevant data information from the perspective of storage life itself. As a result, it introduces significant uncertainties and randomness, greatly diminishing the reference value of the evaluation outcomes. These factors contribute to the urgent need for solving the issue of comprehensive assessment of system storage life.

Currently, statistical inference methods employed in comprehensive reliability assessment encompass frequency statistical methods, Bayesian methods, and fiducial inference. Frequency statistical methods are primarily divided into two categories: Exact methods and approximate methods. However, neither of these approaches is applicable for the comprehensive assessment of system storage life[11]. The Bayesian approach utilizes multiple sources of information to acquire prior knowledge about the equipment, enabling more accurate estimation of system reliability indices by combining sample information[12]. For the comprehensive evaluation of the storage reliability indicators determined by the distribution parameters of the system's component equipment and the rules governing detection and maintenance, it is extremely challenging to establish the prior distributions for all the relevant parameters. Efron proposed the Bootstrap method[13], which has also been applied to evaluate the lower confidence limit of repairable system availability, but it requires a large amount of calculations[14]. Fiducial inference was first introduced and studied by Fisher[15], a renowned statistician. This method randomizes parameters based on samples, recognizing that sample randomness leads to inference result uncertainty, with the fiducial distribution describing such uncertainty. It provides a means to identify the true objective prior. Scholars have explored various methods for finding the fiducial distribution. Fraser[16] employed Haar measure to derive the fiducial distribution under the family of transform distributions. Dawid and Stone[17] obtained the fiducial distribution under the simple function model through parametric equation solutions. Xu[12] proposed a general approach for finding the fiducial distribution of parameters based on the family of pivot distributions. In recent years, many scholars have further studied and applied the fiducial inference method[18–20]. Fiducial inference has also been explored in the field of reliability. Grubbs[21] and Pierce[22] investigated approximate confidence limits under two-parameter exponential distributions. By providing the fiducial distribution of the availability of Weibull-type equipment at any given time, Yu, et al.[5] obtained point estimates and lower confidence limits for maintainable Weibull equipment availability. They further extended these conceptual methods[17, 22], which can still make precise inferences for small sample situations. In light of the frequently encountered small sample sizes in complex repairable systems, researchers have utilized the fiducial inference method to assess their storage reliability[23–25].

The primary contributions of this research are as follows. Firstly, we employ the longest duration that the system can consistently meet a specific performance index as the definition of system storage life. Additionally, we establish a storage model for a complex repairable system based on engineering inspection and maintenance rules. Secondly, by utilizing the complete sample (i.e., the specific time points when equipment failure occurs and repairs are required) of the life distribution, we provide the pivotal representation of the equipment's reliability function. We have also demonstrated the precision of the lower fiducial limits for reliability indicators obtained through this approach. Finally, we utilize the fiducial inference method to derive the point estimation and the lower fiducial limits of system availability and storage life.

The structure of the remaining paper is outlined as follows. Section 2 introduces the storage model for complex repairable systems, along with the expression of its availability function. In Section 3, we present the calculation methods for point estimates and lower fiducial limits of system availability and storage life for these complex repairable systems. In Section 4, a simulation study is conducted, and the estimation results based on both the complete samples and interval-type data (in this paper, refers to the data obtained by periodically monitoring the equipment at fixed intervals during the storage process) are presented separately.

2 The Storage Model for Complex Repairable Systems

2.1 Storage Reliability

Denote X_t as the random state variable of the system at the time t, that is,

$$X_t = \begin{cases} 1, & \text{the system is normal at time } t, \\ 0, & \text{the system fails at time } t. \end{cases}$$

Definition 1[26] Availability of the system at the time t: The probability that the system has the specified function at time t under the specified working (storage) conditions and testing and maintenance rules is called the availability of the product at the time t, which is denoted as $A(t)$, $A(t) = P\{X_t = 1\}$.

Definition 2[6] The storage life of the system: Given the threshold A_0 for the availability detection during the storage phase of a product, the longest duration during which the product's availability remains above or equal to the threshold is called the storage life of the product, which is denoted as T, $T = \sup\{t|A(S) \geq A_0, 0 \leq S \leq t\}$.

2.2 The Storage Model for Complex Repairable Systems

In practicality, the storage and maintenance procedures of products are inherently intricate. In order to enhance the elucidation of the proposed comprehensive assessment methodology for product reliability, we initially undertook a suitable simplification of the storage and maintenance model, subsequently presenting the system's availability function in a comprehensible analytic form.

The complex repairable system, denoted as S discussed in this paper is comprised of M equipment that form an integrated entity. The storage life distributions of these constituent equipment can be fitted by Exponential, Weibull, Normal, and Lognormal distributions, respectively. Accordingly, their storage maintenance models are categorized into three scenarios: "Bad-as-Old," "Good-as-New," and "Mixed Maintenance".

Let $A_1(t), \cdots, A_M(t)$ denote the respective availabilities of the M equipment comprising the system, and let $A_S(t)$ represent the system's availability at time t. For a specific system, it is possible to derive the expression for its system availability based on its reliability structure function and the maintenance conditions of its constituent equipment. However, the expression for system availability is typically complex. In our discussion here, we refer to a generic system that represents a class of complex repairable systems. Due to variations in the reliability structure functions and maintenance approaches among different systems, it is not feasible to determine the specific expression for system availability. Hence, we simply express the system availability as $A_S(t)$ in a generalized manner. In accordance with the definition of the reliability structure function, the system's availability can be expressed in terms of the availabilities of its constituent equipment as follows: $A_S(t) = \varphi(A_1(t), \cdots, A_M(t))$. Here, the symbol φ signifies the reliability structure function, whose specific form is contingent upon the system's structure and the interdependencies among its equipment.

We denote the lifetime distribution function of the equipment as $F(t)$. In previous literature, the storage maintenance model for repairable equipment and their availability function are expressed as follows:

1) "Bad-as-Old" storage maintenance model.

Model Description: Assuming that the equipment commence storage from time zero, subsequent inspections are conducted at fixed intervals of a ($a > 0$). If the inspection outcome indicates that the equipment remains operational, it continues to be stored. Conversely, if the inspection reveals equipment failure, the equipment undergoes maintenance for a fixed duration of b ($0 < b < a$). After the completion of the maintenance period, the equipment is restored to normal functioning and resumes storage. If the result of the maintenance process is equivalent to replacing the equipment with another identical and concurrently stored equipment that remains unimpaired, this model is referred to as the "Bad-as-Old" storage maintenance model.

Let $A(t)$ denote the availability of the equipment at time t, and let $a_j = j \cdot a$ ($j = 0, 1, \cdots$), $b_j = a_j + b$ ($j = 1, 2, \cdots$). The expression for the availability of the equipment at any given time t can be obtained as follows[5]:

$$A(t) = \begin{cases} 1 - F(t), & a_0 < t \leq a_1, \\ \dfrac{1 - F(t)}{1 - F(a_k)} A(a_k), & a_k < t \leq b_k \ (k \geq 1), \\ \dfrac{1 - F(t)}{1 - F(b_k)} (1 - A(a_k)) + \dfrac{1 - F(t)}{1 - F(a_k)} A(a_k), & b_k < t \leq a_{k+1}, \end{cases} \tag{1}$$

among them,

$$\begin{cases} A(a_1) = 1 - F(a_1), \\ A(a_{k+1}) = \dfrac{1 - F(a_{k+1})}{1 - F(b_k)} + \left[\dfrac{1 - F(a_{k+1})}{1 - F(a_k)} - \dfrac{1 - F(a_{k+1})}{1 - F(b_k)} \right] A(a_k). \end{cases}$$

In this manner, the availability $A(t)$ of the "Bad-as-Old" equipment can be expressed as a function of the equipment's storage life distribution $F(t), F(a_j), F(b_j)$, $j = 1, \cdots, k$.

2) "Good-as-New" storage maintenance model.

Model Description: Assuming that the equipment commence storage from time zero, subsequent inspections are conducted at fixed intervals of a ($a > 0$). If the inspection outcome indicates that the equipment remains operational, it continues to be stored. Conversely, if the inspection reveals equipment failure, the equipment undergoes maintenance for a fixed duration of c ($0 < c < a$). After the completion of the maintenance period, the equipment is restored to normal functioning and resumes storage. If the maintenance result is equivalent to replacing the device with a brand new and identical one, then this model is referred to as the "Good-as-New" storage maintenance model.

Let $A(t)$ denote the availability of the equipment at time t, and let $a_j = j \cdot a$ ($j = 0, 1, \cdots$), $c_j = a_j + c$ ($j = 1, 2, \cdots$). The expression for the availability of the equipment at any given time $t(c_m \leq c < c_{m+1}$ ($m = 0, 1, \cdots$)) can be obtained as follows[27]:

$$A(t) = 1 - F_0(t) + \sum_{k=1}^{m} \sum_{j=1}^{k} \upsilon_k^{(j-1)} (1 - F(t - c_k)), \tag{2}$$

among them

$$
\begin{cases}
\nu_j^{(0)} = g_j, & j = 1, 2, \cdots, \\
\nu_j^{(1)} = \sum_{k=1}^{j-1} \nu_k^{(0)} h_{j-k}, & j = 2, 3, \cdots, \\
\vdots \\
\nu_j^{(n)} = \sum_{k=1}^{j-1} \nu_k^{(n-1)} h_{j-k}, & j = n+1, n+2, \cdots.
\end{cases}
$$

$$
g_n = F(a_n) - F(a_{n-1}), h_n = F(a_n - c) - F(a_{n-1} - c), \quad n = 1, 2, \cdots.
$$

In this manner, the availability $A(t)$ of the "Good-as-New" equipment can be expressed as a function of the equipment's storage life distribution $F(t)$, $F(t - c_j)$, $F(a_j)$, $F(a_j - c)$ $(j = 1, 2, \cdots, m)$.

3) "Mixed Maintenance" storage maintenance model.

Model Description: Assuming that the equipment commence storage from time zero, subsequent inspections are conducted at fixed intervals of a $(a > 0)$. If the inspection outcome indicates that the equipment remains operational, it continues to be stored. Conversely, if the diagnostic outcome indicates equipment failure, it necessitates the initiation of restorative measures. Assuming that the equipment experiences two modes of failure: Minor repair and major repair, with probabilities of occurrence p and $1 - p$ respectively. If a minor repair failure occurs, the equipment is restored to normal functioning and continues storage after undergoing a "Bad as Old" method with a fixed maintenance time b $(0 < b < a)$. If a major repair failure occurs, the equipment is replaced with a new one after undergoing a "Good-as-New"method with a fixed maintenance time c $(0 < c < a)$, and resumes storage. This model is referred to as the "mixed maintenance" storage maintenance model.

Let $A(t)$ denote the availability of the equipment at time t, and let $a_j = j \cdot a$ $(j = 0, 1, \cdots)$, $b_j = a_j + b$ $(j = 1, 2, \cdots,)$, $c_j = a_j + c$ $(j = 1, 2, \cdots)$. The expression for the availability of the equipment at any given time t can be obtained as follows[28]:

$$
A(t) = \begin{cases}
1 - F(t), & 0 < t < b_1; \\
1 - F(t) + \sum_{k=1}^{m} \sum_{j=1}^{k} \nu_k^{(j-1)} F(t - b_k) & \\
+ \sum_{k=1}^{m-1} \sum_{j=1}^{k} (1-p) \nu_k^{(j-1)} F(t - c_k), & b_m \leq t < c_m; \\
1 - F(t) + \sum_{k=1}^{m} \sum_{j=1}^{k} \nu_k^{(j-1)} F(t - b_k) & \\
+ \sum_{k=1}^{m} \sum_{j=1}^{k} (1-p) \nu_k^{(j-1)} F(t - c_k), & c_m \leq t < b_{m+1};
\end{cases} \tag{3}
$$

among them

$$\begin{cases} \nu_j^{(0)} = g_j, & j = 1, 2, \cdots, \\ \nu_j^{(1)} = \sum_{i=1}^{j-1} \nu_i^{(0)} h_{j-i}, & j = 2, 3, \cdots, \\ \vdots & \\ \nu_j^{(k)} = \sum_{i=k}^{j-1} \nu_i^{(k-1)} h_{j-i}, & j = k+1, k+2, \end{cases}$$

In this manner, the availability $A(t)$ of the "mixed maintenance" equipment can be expressed as a function of the equipment's storage life distribution

$$R(t), R(t-b_j), R(t-c_j), R(a_j), R(a_j-b), R(a_j-c), \quad j = 1, 2, \cdots, m.$$

Further, the system availability can be expressed as a function of its component equipment storage life distribution, and it is a complex function composed of equipment life distribution containing unknown parameters.

3 Fiducial Method for Comprehensive Evaluation of Reliability

3.1 System Availability Distribution Function Based on the Fiducial Method

3.1.1 Pivot Quantity Representation of Equipment Storage Life Distribution

In the subsequent analysis, the pivotal variable is employed to symbolize various representative life distributions, specifically Exponential, Weibull, Normal, and Lognormal distributions[28], and we prove that the lower limit of equipment reliability obtained by this method is accurate.

(i) Case One: Exponential Distribution.

Suppose that the storage life random variable T of equipment can be fitted with an exponential distribution, $T \sim F(t|\theta) = 1 - \mathrm{e}^{-t/\theta}$.

Let $t_1, t_2, \cdots, t_n$ be the i.i.d. samples of T, and denote the total test time $T_n = \sum_{k=1}^{n} t_k$. In previous studies, the pivot quantity representation of $R(t)$ has been obtained as $R(t) \overset{d}{\sim} \mathrm{e}^{-t/\theta}$, $\theta \overset{d}{\sim} \frac{2T_n}{\chi^2_{2n}}$.

Theorem 3.1 *The α quantile of fiducial distribution $R(t) \overset{d}{\sim} \mathrm{e}^{-t/\theta}$ has property $P_\theta\{R \geq R_L\} = 1 - \alpha$, for any θ.*

Proof Since $\frac{2T_n}{\theta} \sim \chi^2_{2n}$, it's easy to get the lower confidence limit under the confidence degree α of reliability $R(t) = \mathrm{e}^{-t/\theta}$, that is, $R_\alpha(t) = \exp\{-\frac{t \cdot \chi^2_{2n}(\alpha)}{2T_n}\}$.

Therefore, the confidence limits derived from the fiducial distribution obtained through the pivotal quantity method are evidently precise.

By leveraging the relationship $F(t) = 1 - R(t)$ between the distribution of lifetime and reliability, we can obtain the pivotal quantity representation

$$F(t) \overset{d}{\sim} 1 - \mathrm{e}^{-t/\theta(t)} \tag{4}$$

for the distribution of storage lifetime of the equipment.

(ii) Case Two: Weibull Distribution.

Suppose that the storage life random variable T of equipment can be fitted with a Weibull distribution $T \sim F(t) = 1 - \mathrm{e}^{-(t/\eta)^m}$, $t > 0$, m $(m > 0)$ is the shape parameter, η $(\eta > 0)$ is the scale parameter.

Let $t_1, t_2, \cdots, t_n$ be the i.i.d. samples of T, and denote $x_i = \ln t_i$, $i = 1, \cdots, n$, $\overline{x} = \sum_{i=1}^{n} x_i/n$, $s = \sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2/n}$. Let $W_i = m(x_i - \ln \eta)$. W_i follows the standard extreme value distribution. In previous studies, the pivot quantity representation of $R(t)$ has been obtained as $R(t) \overset{d}{\sim} \mathrm{e}^{-\mathrm{e}^{\theta(t)}}$, $\theta(t) \overset{d}{\sim} \overline{W} - \frac{\overline{x}-\ln(t)}{s}V$. ∎

Theorem 3.2 *The α quantile of fiducial distribution $R(t) \overset{d}{\sim} \mathrm{e}^{-\mathrm{e}^{\theta(t)}}$ has property $P_\theta\{R \geq R_L\} = 1 - \alpha$, for any θ.*

Proof From $E[x] = \mu - \gamma\sigma$, $\mathrm{var}[x] = \frac{\pi^2}{6} \cdot \sigma^2$, where Euler's constant $\gamma = 0.5772$.

Take the torque estimation of parameters μ and σ as $\widehat{\mu} = \overline{x} + \gamma \cdot \frac{\sqrt{6}}{\pi} \cdot s$, and $\widehat{\sigma} = \frac{\sqrt{6}}{\pi} \cdot s$.

Take the torque estimation of reliability R as $\widehat{R}_x = \exp\{-\exp\{-\frac{\widehat{\mu}-\ln t}{\widehat{\sigma}}\}\}$.

Then the reliability point estimation can be rewritten as follows:

$$\begin{aligned}
\widehat{R}_x =& \exp\left\{-\exp\left\{-\frac{\overline{x} + \frac{\sqrt{6}}{\pi} \cdot \gamma \cdot s - \ln t}{\frac{\sqrt{6}}{\pi} \cdot s}\right\}\right\} \\
=& \exp\left\{-\exp\left\{-\gamma - \frac{\pi}{\sqrt{6}} \cdot \frac{\overline{x} - \ln t}{s}\right\}\right\} \\
=& \exp\left\{-\exp\left\{-\gamma - \frac{\pi}{\sqrt{6}} \cdot \frac{\sigma \cdot \overline{w} + \mu - \ln t}{\sigma \cdot V}\right\}\right\} \\
=& \exp\{-\exp\left\{-\gamma + \frac{\pi}{\sqrt{6}} \cdot \frac{-\frac{\mu - \ln t}{\sigma} - \overline{w}}{V}\right\}\} \\
=& \exp\left\{-\exp v\left\{-\gamma + \frac{\pi}{\sqrt{6}} \cdot \frac{\log\log R^{-1} - \overline{w}}{V}\right\}\right\}.
\end{aligned}$$

$\widehat{R}_X$ stands for replacing the sample observation value $\widehat{R}_x$ with the sample X.

Denotes $G(x, \theta) = P_\theta\{\widehat{R}_X \geq \widehat{R}_x\}$, $\theta = (\mu, \sigma)$.

Then $G(x, \theta) = \widetilde{G}(x, R) \overset{\text{def}}{=} P_R\{\overline{w} - V \cdot \frac{\overline{x}-\ln t}{s} \geq \log\log R^{-1}\}$, $R = \mathrm{e}^{-\mathrm{e}^{-\frac{\mu - \ln t}{\sigma}}}$.

According to the sample space ranking method, for a given confidence degree α, the lower confidence limit of reliability is $R_L(x) = \inf_R\{R : \widetilde{G}(x, R) > \alpha, 0 \leq R \leq 1\}$.

Since

$$\begin{aligned}
R_L(x) &= \inf_R\{R : \widetilde{G}(x, R) > \alpha\} \\
&= \inf_R\left\{R : P_R\left\{\overline{w} - V \cdot \frac{\overline{x} - \ln t}{s} \geq \log\log R^{-1}\right\} > \alpha\right\} \\
&= \inf_R\left\{R : P_R\left\{\exp\left\{-\exp\left\{\overline{w} - V \cdot \frac{\overline{x} - \ln t}{s}\right\}\right\} \leq R\right\} > \alpha\right\}.
\end{aligned}$$

$R_L(x)$ is precisely the α-fraction of the random variable $\exp\{-\exp\{\overline{w} - V \cdot \frac{\overline{x}-\ln t}{s}\}\}$ under the given statistic $(\overline{x}, s^2)$.

By leveraging the relationship $F(t) = 1 - R(t)$ between the distribution of lifetime and reliability, we can obtain the pivotal quantity representation

$$F(t) \overset{d}{\sim} 1 - \mathrm{e}^{-\mathrm{e}^{\theta(t)}} \tag{5}$$

for the distribution of storage lifetime of the equipment.

(iii) Case Three: Normal Distribution.

Suppose that the storage life random variable X of equipment can be fitted with a Normal distribution $X \sim N(\mu, \sigma^2)$, $\mu(\mu \in R)$ is the mean of the distribution and σ $(\sigma^2 > 0)$ is the variance of the distribution.

Let $x_1, x_2, \cdots, x_n$ be the i.i.d. samples of X, denote $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, $s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$, and denote $Y_i = \frac{x_i - \mu}{\sigma}$, $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$, $V^2 = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \overline{Y})^2$. In previous studies, the pivot quantity representation of equipment reliability $R(t)$ has been obtained as $R(t) \overset{d}{\sim} 1 - \phi(\overline{Y} + V\frac{t-\overline{x}}{s})$. ∎

Theorem 3.3 *The α quantile of fiducial distribution $R(t) \overset{d}{\sim} 1 - \phi(\overline{Y} + V\frac{t-\overline{X}}{S})$ has property $P_\theta\{R \geq R_L\} = 1 - \alpha$, for any θ.*

Proof The point estimate of reliability R is $\widehat{R}_x = 1 - \phi(\frac{t-\overline{x}}{s})$.

By simple derivation, we can obtain $\widehat{R}_x = 1 - \phi(\frac{t-\mu-\sigma\overline{Y}}{\sigma V}) = 1 - \phi(\frac{\frac{t-\mu}{\sigma} - \overline{Y}}{V}) = 1 - \phi(\frac{\lambda - \overline{Y}}{V})$, $\lambda = \frac{t-\mu}{\sigma}$. In this case, $R_x = 1 - \phi(\lambda)$. Therefore, calculating the lower confidence limit for the reliability R is equivalent to finding the upper confidence limit for parameter λ at the same confidence level α.

$\widehat{R}_X$ stands for replacing the sample observation value $\widehat{R}_x$ with sample X, using the sample space ranking method, it can be seen that $\lambda_\alpha = \inf\{\lambda : P_\theta\{\widehat{R}_x \geq \widehat{R}_X\} \leq \alpha\}$.

Since $\widehat{R}_x \geq \widehat{R}_X \Longleftrightarrow \phi(\frac{\lambda-\overline{Y}}{V}) \leq 1 - \widehat{R}_X \Longleftrightarrow \phi^{-1}(\widehat{R}_X) \leq -\frac{\lambda-\overline{Y}}{V}$, then $P_\theta\{\widehat{R}_x \geq \widehat{R}_X\} \geq \alpha \Longleftrightarrow \widehat{P}_\theta\{\lambda \leq -\phi^{-1}(\widehat{R}_X)V + \overline{Y}\} \geq \alpha$,

The above equation shows that λ_α is precisely the α-fraction of the random variable $-\phi^{-1}(\widehat{R}_X)$ $V + \overline{Y}$.

In addition, it can be obtained from $\widehat{R}_X = \phi(-\frac{t-\overline{x}}{s})$ to $\phi^{-1}(\widehat{R}_X)V + \overline{Y} = \frac{t-\overline{x}}{s}V + \overline{Y}$.

By comparing this formula with the fiducial distribution obtained by the pivot method, we know that the corresponding fiducial lower limit is the exact confidence limit.

By leveraging the relationship $F(t) = 1 - R(t)$ between the distribution of lifetime and reliability, we can obtain the pivotal quantity representation

$$F(t) \overset{d}{\sim} \phi\left(\overline{Y} + V\frac{t - \overline{x}}{s}\right) \tag{6}$$

for the distribution of storage lifetime of the equipment.

(iv) Case Four: Lognormal Distribution.

Suppose that the storage life random variable X of equipment can be fitted with a Lognormal distribution $\ln X \sim N(\mu, \sigma^2)$, μ $(\mu \in R)$ is the mean of the distribution and σ $(\sigma^2 > 0)$ is the variance of the distribution.

Let $x_1, x_2, \cdots, x_n$ be the i.i.d. samples of X, denote $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, $s^2 = \frac{1}{n}\sum_{i=1}^{n}(\ln x_i - \overline{x})^2$, and denote $Y_i = \frac{\ln x_i - \mu}{\sigma}$, $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$, $V^2 = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \overline{Y})^2$. In previous studies, the pivot quantity representation of equipment reliability $R(t)$ has been obtained as $R(t) \overset{d}{\sim} 1 - \phi(\overline{Y} + V\frac{\ln t - \overline{x}}{s})$. ▮

Theorem 3.4 *The α quantile of fiducial distribution $R(t) \overset{d}{\sim} 1-\phi(\overline{Y}+V\frac{\ln t-\overline{X}}{S})$ has property $P_\theta\{R \geq R_L\} = 1 - \alpha$, for any θ.*

Proof The point estimate of reliability R is $\widehat{R}_x = 1 - \phi(\frac{\ln t - \overline{x}}{s})$.

By simple derivation, we can obtain $\widehat{R}_x = 1-\phi(\frac{\ln t-\mu-\sigma\overline{Y}}{\sigma V}) = 1-\phi(\frac{\frac{\ln t-\mu}{\sigma}-\overline{Y}}{V}) = 1-\phi(\frac{\lambda-\overline{Y}}{V})$, $\lambda = \frac{\ln t-\mu}{\sigma}$. In this case, $R_x = 1 - \phi(\lambda)$. Therefore, calculating the lower confidence limit for the reliability R is equivalent to finding the upper confidence limit for parameter λ at the same confidence level α.

$\widehat{R}_X$ stands for replacing the sample observation value $\widehat{R}_x$ with sample X, using the sample space ranking method, it can be seen that $\lambda_\alpha = \inf\{\lambda : P_\theta\{\widehat{R}_x \geq \widehat{R}_X\} \leq \alpha\}$.

Since $\widehat{R}_x \geq \widehat{R}_X \Longleftrightarrow \phi(\frac{\lambda-\overline{Y}}{V}) \leq 1 - \widehat{R}_X \Longleftrightarrow \phi^{-1}(\widehat{R}_X) \leq -\frac{\lambda-\overline{Y}}{V}$, so $P_\theta\{\widehat{R}_x \geq \widehat{R}_X\} \geq \alpha \Longleftrightarrow \widehat{P}_\theta\{\lambda \leq -\phi^{-1}(\widehat{R}_X)V + \overline{Y}\} \geq \alpha$. The above equation shows that λ_α is precisely the α-fraction of the random variable $-\phi^{-1}(\widehat{R}_X)V + \overline{Y}$.

In addition, from $\widehat{R}_X = \phi(-\frac{\ln t-\overline{x}}{s})$ to $\phi^{-1}(\widehat{R}_X)V + \overline{Y} = \frac{\ln t-\overline{x}}{s}V + \overline{Y}$. By comparing this formula with the fiducial distribution obtained by the pivot method, the corresponding fiducial lower limit is the exact confidence limit.

By leveraging the relationship $F(t) = 1 - R(t)$ between the distribution of lifetime and reliability, we can obtain the pivotal quantity representation

$$F(t) \overset{d}{\sim} \phi\left(\overline{Y} + V\frac{\ln t - \overline{x}}{s}\right) \tag{7}$$

for the distribution of storage lifetime of the equipment.

Generally, life distributions belonging to the 'location-scale' distribution family and the logarithmic 'location-scale' distribution family have concise pivot quantity expressions. Moreover, the lower fiducial limit for reliability obtained using the aforementioned method possesses the property $P_\theta\{R \geq R_L\} = 1 - \alpha$, which holds for any value of θ[12]. ▮

3.1.2 Fiducial Distribution of System Availability

By substituting Equations (4)–(7) from Subsection 3.1.1 into the expression (1)–(3) of the availability function for each piece of equipment in Subsection 2.2, we can derive the pivotal representation of the equipment's availability. This pivotal representation is then substituted into the reliability structure function $A_S(t) = \varphi(A_1(t), \cdots, A_M(t))$ of the system to obtain the fiducial distribution of the system's availability.

Since it is difficult to provide the specific expression of the fiducial distribution function, their distributions can be approximated using Monte Carlo methods.

3.2 Fiducial Method for Comprehensive Evaluation of System Storage Reliability

Let the system S be composed of M equipment, and $X = \{\overrightarrow{X_1}, \cdots, \overrightarrow{X_M}\}$ be a set of complete samples of the life distribution of its component equipment.

3.2.1 System Availability

The algorithm for computing the point estimate and the lower fiducial limit of the system availability is presented below.

Step 1 Based on the complete sample $X = \{\overrightarrow{X_1}, \cdots, \overrightarrow{X_M}\}$ of M equipment, construct the pivot quantity $\Phi = \{\overrightarrow{\phi_1}, \cdots, \overrightarrow{\phi_M}\}$ corresponding to the life distribution parameter $\Theta = \{\overrightarrow{\theta_1}, \cdots, \overrightarrow{\theta_M}\}$ of each kit by using the pivot quantity representation method provided in Subsection 3.1.1.

Step 2 According to the method provided in Subsection 3.1.1, the pivot quantity Φ is substituted into the life distribution function of each piece of equipment to obtain the pivot quantity representation of the storage life distribution function:

$$\{F_1(t|\overrightarrow{\theta_1}), \cdots, F_M(t|\overrightarrow{\theta_M})\} \implies \{F_1(t|\overrightarrow{\phi_1}), \cdots, F_M(t|\overrightarrow{\phi_M})\}.$$

Step 3 According to the method provided in Subsection 2.2, through the relationship between the equipment life distribution function and the equipment availability function, the pivot quantity representation of the equipment availability function can be expressed as follows:

$$\{F_1(t|\overrightarrow{\phi_1}), \cdots, F_M(t|\overrightarrow{\phi_M})\} \implies \{A_1(t|\overrightarrow{\phi_1}), \cdots, A_M(t|\overrightarrow{\phi_M})\}.$$

Step 4 Substitute the pivot quantity representation of the availability function of each equipment into the system availability function $A_S(t) = \varphi(A_1(t), \cdots, A_M(t))$ to obtain the pivot quantity representation of the system availability function:

$$\{A_1(t|\overrightarrow{\phi_1}), \cdots, A_M(t|\overrightarrow{\phi_M})\} \implies A(t|\Phi) = \varphi(\{A_1(t|\overrightarrow{\phi_1}), \cdots, A_M(t|\overrightarrow{\phi_M})\}).$$

Step 5 Sample the pivot quantity representation corresponding to the storage life distribution parameters of each of piece equipment for N times to obtain $\Phi^{(n)} = \{\overrightarrow{\phi_1}^{(n)}, \cdots, \overrightarrow{\phi_M}^{(n)}\}$, $n = 1, \cdots, N$, and then substitute it into Steps 2, 3, and 4 successively to get $A^{(m)}(t|\Phi), m = 1, \cdots, N$ as the approximate fiducial distribution of system availability.

Step 6 Take the mean value of $A^{(m)}(t|\Phi), m = 1, \cdots, N$ obtained in Step 5 as the point estimate value of availability. For a given level $1-\alpha$, the α quantile $A_L(\alpha)$ of the approximate fiducial distribution $A^{(m)}(t|\Phi), m = 1, \cdots, N$ is calculated as the lower fiducial limit of the system availability.

Figure 1 is a flow chart for calculating the fiducial distribution of system availability.

This paper presents a methodology for calculating the lower fiducial limit of system availability at a specific time t. The "t moment" can refer to any given point in time, thereby enabling the theoretical derivation of the lower fiducial limit for system reliability at each moment within a given period and facilitating the construction of an associated curve. In practical applications, it is possible to calculate the lower fiducial limit of system availability at densely distributed time points during the relevant time period. These calculated values can then be plotted on a graph and compared with the corresponding graph representing the true system availability, enabling a comprehensive analysis of the assessment results' efficacy at different points in time.

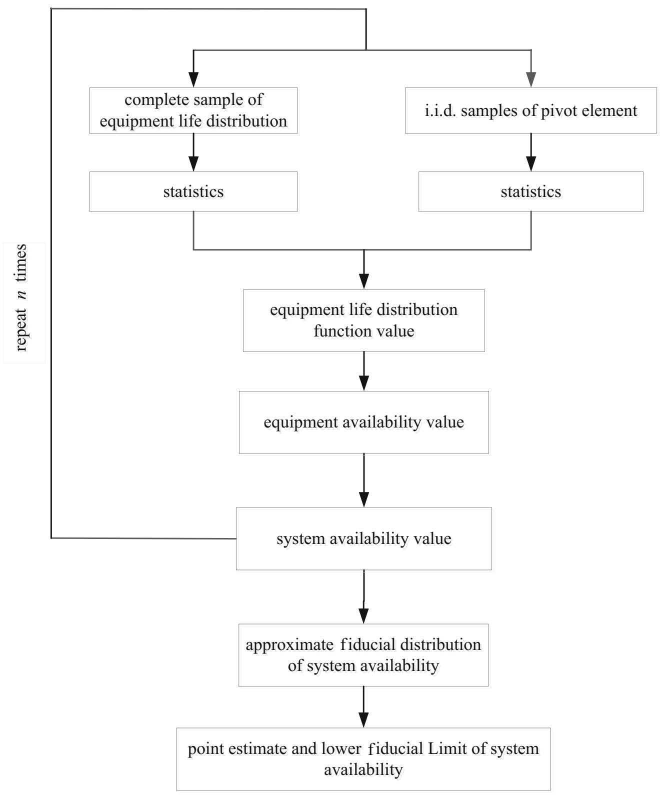

Figure 1 Flowchart of the algorithm for lower fiducial limit of system availability

3.2.2 System Storage Life

The algorithm for computing the point estimate and the lower fiducial limit of the system storage life is presented below.

Step 1 According to the method provided in Subsection 3.2.1, take a sufficiently small time interval c, calculate $A_s^{(i)}(nc|\Phi)$ successively, and compare it with A_0, $n = 1, 2, \cdots$. When $A_s^{(i)}(nc|\Phi)$ is less than A_0 for the first time, denote $T_s^{(i)} = (n-1)c$ as an observed value of system storage life.

Step 2 Repeat Step 1 N times to obtain an approximate fiducial distribution $T_s^{(i)}, i = 1, \cdots, N$ for the storage life of the system.

Step 3 Take the mean value of $T_s^{(i)}, i = 1, \cdots, N$ obtained in Step 2 as the point estimate value of storage life. For a given level $1-\alpha$, the α quantile $T_L(\alpha)$ of the approximate fiducial distribution $T_s^{(i)}, i = 1, \cdots, N$ is calculated as the lower fiducial limit of the system storage life.

Figure 2 is a flow chart for calculating the fiducial distribution of system storage life.

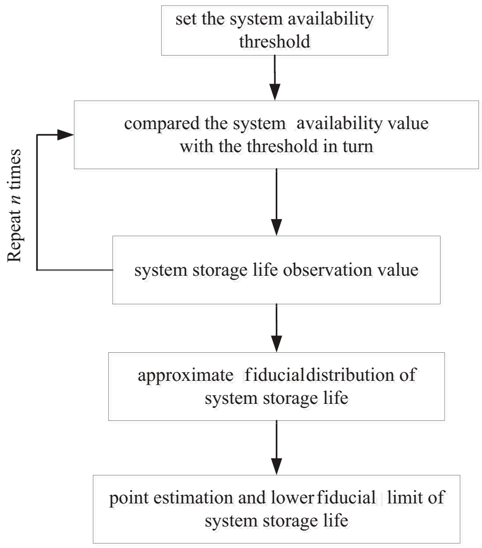

Figure 2 Flowchart of the algorithm for lower fiducial limit of system storage life

3.2.3 Preprocessing of Equipment Life Test Data

The aforementioned fiducial inference method requires the experimental data to be in the form of complete samples. However, in practical scenarios, the obtained storage life test data for equipment are typically interval-censored data obtained by periodically examining them at fixed intervals during the storage process. In such cases, the approach proposed in this paper relies on having a complete sample of equipment life test data. Here, a complete sample refers to the specific time point at which the equipment fails and requires repair. In this study, complete samples are employed for evaluation purposes, showcasing and analyzing the method's ideal effectiveness. Nonetheless, in practical applications, equipment is typically tested at fixed intervals, providing information about the failure occurrence within a certain time period but not the precise time of failure. To address this limitation, preprocessing of the obtained interval data is necessary to generate virtual complete samples. The quantile filling algorithm, based on the moment invariant criterion, can be employed to convert the data into realistic complete samples in advance[24, 29, 30]. Subsequently, the fiducial inference method can be applied to assess the storage reliability of the system.

4 Simulation Results and Analysis

4.1 Simulation Example of System Availability at a Given Time t

The system under consideration consists of 9 equipment, and its system structure is illustrated in Figure 3. The life distribution and maintenance status of the equipment that comprise the system are presented in Table 1. During the storage procedure, initiating from time zero, regular inspections are conducted on each constituent equipment of the system with a periodicity of $a = 1.0$ year. If the test results indicate normal functioning, the equipment is retained for storage. Conversely, if the test results reveal any signs of failure, the corresponding maintenance mode is implemented to rectify the issue. The “Bad as Old” maintenance approach requires a minor repair and maintenance period of $b = 0.25$ years, while the “Good as New” repair methodology necessitates a comprehensive overhaul and maintenance period of $c = 0.5$ years. In cases where “mixed maintenance” is adopted, minor repairs are performed with a probability of p, while significant repairs occur with a probability of $1 - p$. We designate the temporal reference point for the system as $t = 20$ years, with confidence levels set at $\gamma = 0.7, 0.8, 0.9$ respectively. For each equipment, the sample sizes for storage life experiments (complete samples and interval-type data) are set as $n = 15,\ 30,\ 50$. We approximate the fiducial distribution $A_S(t)$ using empirical distributions based on $N = 1000$ corresponding values of $A_S^{(i)}(t), i = 1, 2, \cdots, N$. The simulation is repeated $N^* = 2000$ times.

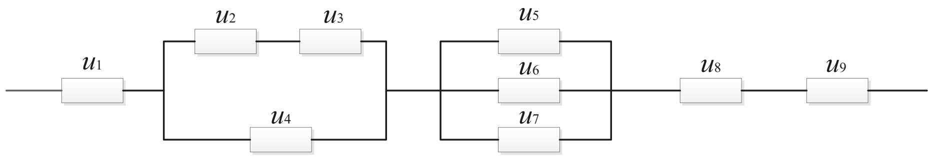

Figure 3 System structure diagram composed of 9 equipment

(i) The system structure.

(ii) Equipment information.

Table 1 System component equipment information

Equipment	Life distribution	Parameter value	Maintenance situation
u_1	Normal distribution	$\mu = 77, \sigma = 15$	bad-as-old
u_2	Weibull distribution	$m = 80, \eta = 3$	mixed maintenance ($p = 0.5$)
u_3	Lognormal distribution	$\mu = 3.8, \sigma = 0.4$	good-as-new
u_4	Normal distribution	$\mu = 90, \sigma = 10$	bad-as-old
u_5	Exponential distribution	$\theta = 100$	bad-as-old
u_6	Lognormal distribution	$\mu = 3.5, \sigma = 0.6$	bad-as-old
u_7	Exponential distribution	$\theta = 100$	good-as-new
u_8	Weibull distribution	$m = 15, \eta = 2.9$	good-as-new
u_9	Normal distribution	$\mu = 90, \sigma = 20$	bad-as-old

4.1.1 Analysis of Simulation Results

According to the data presented in Table 2 and Table 3, when dealing with complete samples, the disparity between the coverage and the intended confidence degree γ gradually diminishes as the sample size increases. The coverage surpasses the desired confidence degree, indicating a conservative estimation. The estimated standard deviation is relatively small. The estimated lower limit mean has a minimal deviation from the true value of system availability, and as the sample size increases, the estimated lower limit mean also increases. This indicates that the estimation becomes more precise with larger sample sizes. In the case of interval-type data, the deviation between coverage and the target confidence degree γ is more noticeable compared to complete samples. However, some values slightly fall below the desired confidence degree, suggesting that the estimations may be slightly aggressive. Similar to complete samples, the standard deviation of the estimate is small and decreases as the sample size increases. The estimates of the lower limit mean demonstrate good agreement with the actual system availability values, with larger sample sizes leading to more accurate estimations.

Table 2 List of availability estimates of the system at a given time (complete sample)

$t = 20, a = 1.0, b = 0.25, c = 0.5, A_0 = 0.9306$					
Point estimation	Degree of confidence	Sample size	Lower mean	Standard deviation	Coverage rate
0.9311		15	0.9273	0.0070	0.8250
0.9305	0.7	30	0.9289	0.0042	0.7745
0.9407		50	0.9294	0.0030	0.7140
0.9311		15	0.9245	0.0072	0.9100
0.9305	0.8	30	0.9274	0.0042	0.8900
0.9407		50	0.9283	0.0031	0.8570
0.9311		15	0.9202	0.0079	0.9735
0.9305	0.9	30	0.9250	0.0045	0.9600
0.9407		50	0.9267	0.0033	0.9525

Table 3 List of availability estimates of the system at a given time (interval-type data)

$t = 20, a = 1.0, b = 0.25, c = 0.5, A_0 = 0.9306$					
Point estimation	Degree of confidence	Sample size	Lower mean	Standard deviation	Coverage rate
0.9359		15	0.9286	0.0045	0.7660
0.9322	0.7	30	0.9298	0.0024	0.6775
0.9287		50	0.9301	0.0014	0.6855
0.9359		15	0.9266	0.0053	0.8980
0.9322	0.8	30	0.9287	0.0028	0.8460
0.9287		50	0.9294	0.0016	0.8795
0.9359		15	0.9230	0.0067	0.9730
0.9322	0.9	30	0.9268	0.0035	0.9585
0.9287		50	0.9282	0.0020	0.9730

4.2 Simulation Example of System Storage Life

The system under consideration consists of 5 equipment, and its system structure is illustrated in Figure 4. The life distribution and maintenance status of the equipment that comprise the system are presented in Table 4. During the storage procedure, initiating from time zero, regular inspections are conducted on each constituent equipment of the system with a periodicity of $a = 1.0$ year. If the test results indicate normal functioning, the equipment is retained for storage. Conversely, if the test results reveal any signs of failure, the corresponding maintenance mode is implemented to rectify the issue. The "Bad as Old" maintenance approach requires a minor repair and maintenance period of $b = 0.25$ years, while the "Good as New" repair methodology necessitates a comprehensive overhaul and maintenance period of $c = 0.5$ years. In cases where "Mixed Maintenance" is adopted, minor repairs are performed with a probability of p, while significant repairs occur with a probability of $1 - p$. We establish the threshold for system storage availability as $A_0 = 0.85$, selecting a calculation interval of $d = 0.05$ years to determine the system's availability. Utilizing this information, we can derive the theoretical value of the system's storage lifespan as $t_0 = 93.95$ years. The confidence levels are respectively established as $\gamma = 0.7, 0.8, 0.9$. For each equipment, the sample sizes for storage life experiments (complete samples and interval-type data) are set as $n = 15, 30, 50$. We approximate the fiducial distribution T_S using empirical distributions based on $N = 100$ corresponding values of T_S with $T_S^i, i = 1, 2, \cdots, N$. The simulation is repeated $N^* = 300$ times.

(i) The system structure.

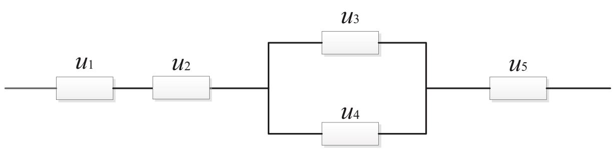

Figure 4 System structure diagram composed of 5 equipment

(ii) Equipment information.

Table 4 System component equipment information

Equipment	Life distribution	Parameter value	Maintenance situation
u_1	Lognormal distribution	$\mu = 10, \sigma = 0.5$	bad-as-old
u_2	Normal distribution	$\mu = 95, \sigma = 1$	bad-as-old
u_3	Exponential distribution	$\theta = 60$	mixed maintenance ($p = 0.9$)
u_4	Weibull distribution	$m = 80, \eta = 1.5$	good-as-new
u_5	Normal distribution	$\mu = 90, \sigma = 30$	good-as-new

4.2.1 Analysis of Simulation Results

Under the aforementioned specific model, the data in Table 5 and Table 6 reveal that, in the case of complete samples, there is a small deviation between the coverage rate and the confidence degree γ for the lower fiducial limit. Initially, with a smaller sample size, the estimated standard deviation exhibits more significance. However, as the sample size increases, the estimated standard deviation gradually diminishes. The mean estimation of the lower limit closely aligns with the theoretical value of the system's storage life showcasing minimal disparity. In the case of interval-type data, a similar trend can be observed. The deviation between the coverage and the confidence degree γ remains small. As the sample size increases, the estimated standard deviation decreases. The mean value estimation of the lower limit also aligns closely with the theoretical value of the system's storage life, albeit with minor differences.

Table 5 List of system storage life estimates (complete sample)

$a = 1.0, b = 0.25, c = 0.5, A_0 = 0.85, T_0 = 93.95$					
Point estimation	Degree of confidence	Sample size	Lower mean	Standard deviation	Coverage rate
93.7521		15	93.8532	0.3056	0.7367
94.0262	0.7	30	93.9218	0.1953	0.6900
93.9957		50	93.9118	0.1215	0.7467
93.7521		15	93.7443	0.3165	0.8300
94.0262	0.8	30	93.8617	0.1823	0.7733
93.9957		50	93.8722	0.1241	0.8400
93.7521		15	92.9650	7.6371	0.8933
94.0262	0.9	30	93.7812	0.2001	0.8700
93.9957		50	93.8200	0.1303	0.9067

Table 6 List of system storage life estimates (interval-type data)

$a = 1.0, b = 0.25, c = 0.5, A_0 = 0.85, T_0 = 93.95$					
Point estimation	Degree of confidence	Sample size	Lower mean	Standard deviation	Coverage rate
93.4213		15	93.6813	5.4400	0.5367
93.8603	0.7	30	93.9477	0.1962	0.6100
94.0215		50	93.9430	0.1590	0.6300
93.4213		15	92.9533	9.3198	0.6500
93.8603	0.8	30	93.8857	0.1962	0.7400
94.0215		50	93.9037	0.1590	0.7167
93.4213		15	89.9660	18.3816	0.8033
93.8603	0.9	30	93.4723	5.4197	0.8800
94.0215		50	93.8382	0.1637	0.8367

5 Conclusions

For the three storage maintenance models of "Bad as Old", "Good as New", and "mixed maintenance", availability emerges as the paramount measure, which can be epitomized by the function of the storage life distribution at a series of specific temporal instances. The life (or storage life) distributions employed in the realm of reliability engineering all belong to the 'location-scale' distribution family and the logarithmic 'location-scale' distribution family. They all possess a concise pivot quantity representation. By substituting distinct values from these distributions into the expression governing the equipment availability function and the system reliability structure-function, one can attain the pivot quantity representation of the system's availability function. Subsequently, the fiducial inference method can be employed to derive its approximate distribution through simulation, facilitating the evaluation of its reliability.

Given that the practical data obtained through periodic detection consist of interval data, a vital consideration arises: The fiducial method proposed in this paper relies on complete sample data as its foundational requirement. Consequently, it becomes imperative to preprocess the data and convert them into fully virtual samples prior to practical implementation. By employing this approach, one can effectively apply the quantile filling algorithm, which operates based on the moment invariant criterion. Furthermore, it is worth noting that while the model under discussion primarily pertains to the repairable storage model, it is indeed plausible to regard the unrepairable system as a distinctive case within this framework.

Based on the simulation findings, it is evident that the utilization of the fiducial inference method enables the assessment of storage reliability for the system, encompassing both complete sample and interval data scenarios. Notably, more precise outcomes are achieved when employing full samples.

Conflict of Interest

The authors declare no conflict of interest.

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