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Adaptive Barrier-Lyapunov-Functions Based Control Scheme of Nonlinear Pure-Feedback Systems with Full State Constraints and Asymptotic Tracking Performance*

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Abstract In this paper, the authors propose an adaptive Barrier-Lyapunov-Functions (BLFs) based control scheme for nonlinear pure-feedback systems with full state constraints. Due to the coexist of the non-affine structure and full state constraints, it is very difficult to construct a desired controller for the considered system. According to the mean value theorem, the authors transform the pure-feedback system into a system with strict-feedback structure, so that the well-known backstepping method can be applied. Then, in the backstepping design process, the BLFs are employed to avoid the violation of the state constraints, and neural networks (NNs) are directly used to online approximate the unknown packaged nonlinear terms. The presented controller ensures that all the signals in the closed-loop system are bounded and the tracking error asymptotically converges to zero. Meanwhile, it is shown that the constraint requirement on the system will not be violated during the operation. Finally, two simulation examples are provided to show the effectiveness of the proposed control scheme.

Keywords Asymptotic tracking control, barrier Lyapunov functions, full state constraints, nonlinear pure-feedback systems.

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1 Introduction

During the past few decades, controller design problem for nonlinear systems has become an active study topic[1–9]. In this issue, various methods have been employed such as adaptive backstepping control, sliding mode control, robust control, and so on[10–19]. In particular, adaptive backstepping technique is a powerful method for synthesizing adaptive controller for nonlinear systems with parameter uncertainties. Many remarkable results were obtained when dealing with the controller design problem for nonlinear systems using the backstepping technique[20–22]. For example, an adaptive control scheme for a class of uncertain systems with unknown input time-delay using standard backstepping technique was proposed in [21]. As a challenging control problem, an adaptive backstepping control method was provided in [22] for uncertain switched nonlinear systems. However, the above results require that the uncertain nonlinearities in systems are either known functions with unknown parameters, which are linear with respect to the known functions, or restricted by known nonlinear functions. Therefore, it is difficult to use the design methods in [20–22] when these conditions imposed on uncertain nonlinearities are not available.

Recently, a class of backstepping-based adaptive neural controllers and fuzzy controllers for nonlinear systems were developed, in which NNs and fuzzy logic systems as universal approximators were used to model nonlinear functions without a priori knowledge[23–32]. Meanwhile, a number of significant results on backstepping-based adaptive neural control for nonlinear systems were obtained[33–43]. For example, an adaptive control method based on NNs was studied in [44], which used NNs to approximate unknown nonlinear functions to control pure-feedback systems. In [45], an adaptive neural control strategy was developed to circumvent the unknown nonlinear terms of the multiple input multiple output nonlinear systems. However, it should be noted that most of the existing results for the tracking problem of nonlinear systems can only realize the practical tracking performance[46–52]. The reason is that the existing methods cannot completely eliminate the influence of parameter uncertainties in the system. Furthermore, the effect of repeating differentiations for virtual control laws in the backstepping procedure causes the problem of "explosion of complexity", which can lead to the unapplication problem of the backstepping method for high-order nonlinear systems. In addition, a system can work perfectly only if the system is a stable system. Compared with practical stability, asymptotic stability is more accurate and significance to practical industry. Hence, it is interesting to design an efficiently controller for unknown nonlinear system to achieve the asymptotic tracking performance.

However, all the control results mentioned above ignored the influence of constraints, which usually occur in many practical situations. Apparently, it is more difficult to design a desired controller for systems with constraints compared with the unconstrained ones. In order to constrain the system states to an ideal range, the past studies have proposed various methods, such as the performance control[53], the set invariance notions[54], the extremum-seeking algorithm[55], the reference governors[56], and the model predictive control[57]. In recent years, BLFs have become an effective tool to meet the requirement of state constraints for nonlinear strict-feedback systems. For instance, a novel almost fast finite-time adaptive tracking controller was developed in [58] by using the BLFs, so that the full state constraints are not violated and the tracking error converges to a small compact set. In [59], for a class of strict-feedback nonlinear systems with time-varying full state constraints, the time-varying BLFs were employed to ensure the requirement of the time-varying state constraints. To handle the state and input constraints, the integral barrier Lyapunov function (IBLF) was introduced to the backstepping design procedure in [60]. Unfortunately, the above methods can only be utilized in the design of nonlinear strict-feedback systems rather than more general systems such as pure-feedback nonlinear systems. The authors in [61] presented an adaptive neural control algorithm for nonlinear pure-feedback systems with full state constraints, in which the practical tracking performance was achieved. To the best of our knowledge, the asymptotic tracking issue of nonlinear pure-feedback systems with full state constraints has not been adequately addressed, which motivates us for this study.

In this study, inspired by the aforementioned discussions, an adaptive neural-network based asymptotic tracking control scheme is presented for a class of nonlinear pure-feedback systems with full state constraints. The main contributions of this paper are listed as follows: (i) Different from the existing results[56, 57, 59] which focused on the strict-feedback system with full state constraints, this paper studies a more general class of nonlinear pure-feedback system with full state constraints. By using the mean value theorem, the pure-feedback system is transformed into the system with a strict feedback structure. For the transformed system, the BLFs are integrated into the design of Lyapunov functions to prevent the violation of the full state constraints. (ii) Compared with the previous results in [48, 50–52, 61–63] for the bounded tacking performance, one significant advantage of this paper is that the neural-based control scheme is developed by incorporating the NNs universal approximation capability into the framework of adaptive backstepping design procedure, such that the asymptotic output tracking performance is ensured. (iii) Compared with the adaptive laws that constructed in [18, 27, 37, 58, 60], the adaptive laws are designed skillfully in this paper. Through the clever use of integration technology, only two adaptive parameters need to be updated online and thus the complexity of analytic calculations is effectively depressed.

2 System Description and Problem Statement

Consider the following nonlinear pure-feedback system:

$$\begin{cases} \dot{x}_i = f_i(x_1, x_2, \cdots, x_{i+1}), \quad i = 1, 2, \cdots, n-1, \\ \dot{x}_n = f_n(x, u), \\ y = x_1, \end{cases} \tag{1}$$

where $x = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in R^n$ is the state vector of the system. $y \in R$ and $u \in R$ are the control output and input of the system, respectively. Let $\overline{x}_i = [x_1, x_2, \cdots, x_i]^{\mathrm{T}} \in R^i$, $f_i(\overline{x}_{i+1})$ and $f_n(x, u)$ are the nonlinear smooth functions that are continuously differentiable for $\overline{x}_{i+1}$

and u. All state variables are constrained in a set $|x_i| < l_{c_i}$, where l_{c_i} are positive constants. Let $g_i(\overline{x}_{i+1}) = \partial f_i(\overline{x}_{i+1})/\partial x_{i+1}$, $g_n(x,u) = \partial f_n(x,u)/\partial u$.

From the mean-value theorem in [64], it can be deduced that there exist some points x^0_{i+1} between 0 and x_{i+1} and some points u^0 between 0 and u, such that

$$\begin{cases} f_i(\overline{x}_{i+1}) = f_i(\overline{x}_i) + g_i(\overline{x}_i, x^0_{i+1})x_{i+1}, \quad i = 1, 2, \cdots, n-1, \\ f_n(x,u) = f_n(x) + g_n(x,u^0)u, \\ y = x_1. \end{cases} \tag{2}$$

The control objective of this paper is to construct an adaptive neural state feedback controller $u(t)$ for the system (1), which can guarantee that the system output y can achieve the asymptotic tracking for the reference trajectory y_d in spite of existing unknown nonlinear functions, all the signals in the system can be bounded and the full state constraints are not violated.

In this paper, the nonlinear smooth functions $f_i(\overline{x}_{i+1})$ and $f_n(x)$ are unknown. Radial basis function neural networks (RBF NNs) will be used as a tool to estimate them, and the estimated function can be written in the following form:

$$f_i(s) = W_i^{\mathrm{T}} \varPsi_i(s) + \delta_i(s), \tag{3}$$

where $s \in R^m$ denotes the input vector of NNs, m represents the dimension of NNs input. W_i are the weight vectors. $\varPsi(s) = [\varphi_1(s), \varphi_2(s), \cdots, \varphi_\tau(s)]^{\mathrm{T}} \in R^\tau$ denotes the basis vector function, τ is the number of neurons in the neural networks, $\varphi_i(s)$ is the base function of the neural network, and $\delta_i, i = 1, 2, \cdots, n$ are the NNs inherent approximation errors which can be adjusted to arbitrary small by choosing ideal bounded weight vector W^*, where

$$W_i^* = \arg \min_{W \subset R^n} \left\{ \sup \left| \underset{s\in\varOmega_s}{f_i(s)} - W_i^{\mathrm{T}} \varPsi_i(s) \right| \right\}, \tag{4}$$

the following Gaussian functions are chosen as the base function of neural networks:

$$\varphi_i(s) = \exp[-(s-\xi_i)^{\mathrm{T}}(s-\xi_i)/\lambda_i^2], \quad i = 1, 2, \cdots, \tau, \tag{5}$$

where ξ_i and λ_i are the center and the width of the base functions $\varphi_i(s)$, respectively.

Assumption 2.1 (see [61]) The unknown gain functions $g_i, i = 1, 2, \cdots, n$, are bounded, there exist constants $b_m > 0$ and $b_M > 0$ such that $0 < b_m < |g_i| < b_M < +\infty$.

Assumption 2.2 (see [61]) Assume the tracking signal y_d satisfy $|y_d(t)| \le M_0 \le l_{c_1}$ and its ith order derivatives $y_d{}^{(i)}(t)$ satisfies $\left|y_d{}^{(i)}(t)\right| \le M_i, i = 1, 2, \cdots, n$, where $M_1, M_2, \cdots, M_n$ are positive constants.

Remark 2.1 Assumption 2.1 is a general assumption which is widely used to deal with the unknown gain functions and the same assumption can be found in [58–60]. Assumption 2.2 is also a common assumption when the tracking control problem under constraints is studied such as the ones in [59–61]. In Assumption 2.2, we assume that $|y_d(t)| \le M_0 \le l_{c_1}$. Due to $x_1 = \chi_1 + y_d(t)$, we have $|x_1| \le |\chi_1| + |y_d| < l_{b_1} + M_0$. Let $l_{b1} = l_{c1} - M_0$ then, $|x_1| < l_{c_1}$.

Lemma 2.1 (see [65]) *Given* $\upsilon \in R$*, there exist* $\rho > 0$ *and a hyperbolic tangent function* $\tanh(\cdot)$ *to make the following inequality holds:* $|\upsilon| \leq \upsilon\tanh(\upsilon/\rho) + \rho\kappa$*, where* $\kappa = 0.2785$.

3 Neural Adaptive Controller Design

In order to avoid complicated operations, let $\theta = \max\left\{\frac{1}{b_m}\|W_1^*\|^2, \frac{1}{b_m}\|W_2^*\|^2, \cdots, \frac{1}{b_m}\|W_n^*\|^2\right\}$, $\varepsilon = \max\left\{\frac{1}{b_m}\delta_1, \frac{1}{b_m}\delta_2, \cdots, \frac{1}{b_m}\delta_n\right\}$; $\widetilde{\theta} = \theta - \widehat{\theta}$; $\widetilde{\varepsilon} = \varepsilon - \widehat{\varepsilon}$; $\widehat{\theta}$ and $\widehat{\varepsilon}$ are the estimates of θ, ε respectively. The adaptive laws are constructed as

$$\begin{aligned}\dot{\widehat{\theta}} &= \sum_{i=1}^{n} l_{\chi_i}\frac{r_1}{2\eta_i}\tanh(l_{\chi_i}/\rho(t))\|\varPsi_i\|^2 - r_1\rho(t)\widehat{\theta},\\ \dot{\widehat{\varepsilon}} &= \sum_{i=1}^{n} l_{\chi_i} r_2\tanh(l_{\chi_i}/\rho(t)) - r_2\rho(t)\widehat{\varepsilon},\end{aligned}\tag{6}$$

where r_1, r_2 are two positive constants and the definition of l_{χ_i} will be specified later.

Step 1 Let $\chi_1 = x_1 - y_d$, the time derivative of χ_1 is given by

$$\dot{\chi}_1 = \dot{x}_1 - \dot{y}_d = f_1(\overline{x}_1) + g_1x_2 - \dot{y}_d.\tag{7}$$

Next, choose the Lyapunov function as

$$V_1 = \frac{1}{2}\log(l_{b_1}^2/(l_{b_1}^2 - \chi_1^2)) + \frac{b_m\widetilde{\theta}^2}{2r_1} + \frac{b_m\widetilde{\varepsilon}^2}{2r_2},\tag{8}$$

where r_1 and r_2 are two positive constants, $l_{b_1} = l_{c_1} - M_0$.

Differentiating V_1 yields

$$\begin{aligned}\dot{V}_1 &= l_{\chi_1}(f_1 + g_1x_2 - \dot{y}_d) - \frac{b_m\widetilde{\theta}\dot{\widehat{\theta}}}{r_1} - \frac{b_m\widetilde{\varepsilon}\dot{\widehat{\varepsilon}}}{r_2}\\ &\leq l_{\chi_1}(\widetilde{f}_1 + g_1x_2) - \frac{b_m\widetilde{\theta}\dot{\widehat{\theta}}}{r_1} - \frac{b_m\widetilde{\varepsilon}\dot{\widehat{\varepsilon}}}{r_2} - \eta_1 l_{\chi_1}\tanh\left(\frac{l_{\chi_1}}{\rho(t)}\right),\end{aligned}\tag{9}$$

where $\widetilde{f}_1 = f_1 - \dot{y}_d + \eta_1\tanh\left(\frac{l_{\chi_1}}{\rho(t)}\right)$, $l_{\chi_1} = \chi_1/(l_{b_1}^2 - \chi_1^2)$. Then, using RBF NNs to estimate function $\widetilde{f}_1$, we have

$$\dot{V}_1 \leq l_{\chi_1}(W_1^{*\mathrm{T}}\varPsi_1(\overline{x}_1) + \delta_1(\overline{x}_1) + g_1x_2) - \frac{b_m\widetilde{\theta}\dot{\widehat{\theta}}}{r_1} - \frac{b_m\widetilde{\varepsilon}\dot{\widehat{\varepsilon}}}{r_2} - \eta_1 l_{\chi_1}\tanh\left(\frac{l_{\chi_1}}{\rho(t)}\right).\tag{10}$$

From Lemma 2.1 and Young's inequality, we have

$$\begin{aligned}l_{\chi_1}W_1^{*\mathrm{T}}\varPsi_1 &\leq |l_{\chi_1}|\,W_1^{*\mathrm{T}}\varPsi_1\\ &\leq |l_{\chi_1}|\frac{1}{2\eta_1}\|\varPsi_1\|^2 b_m\theta + |l_{\chi_1}|\,\eta_1\\ &\leq l_{\chi_1}\tanh\left(\frac{l_{\chi_1}}{\rho(t)}\right)\frac{1}{2\eta_1}\|\varPsi_1\|^2 b_m\theta + \frac{1}{2\eta_1}\|\varPsi_1\|^2 b_m\theta\kappa\rho(t)\end{aligned}$$

$$
\begin{aligned}
&+\eta_1 l_{\chi_1}\tanh\left(\frac{l_{\chi_1}}{\rho(t)}\right)+\eta_1\kappa\rho(t)\\
=&l_{\chi_1}\tanh\left(\frac{l_{\chi_1}}{\rho(t)}\right)\frac{1}{2\eta_1}\|\varPsi_1\|^2 b_m\theta+l_{\chi_1}\tanh\left(\frac{l_{\chi_1}}{\rho(t)}\right)\eta_1\\
&+\left(\frac{1}{2\eta_1}\|\varPsi_1\|^2 b_m\theta+\eta_1\right)\kappa\rho(t),
\end{aligned}\tag{11}
$$

$$
l_{\chi_1}\delta_1\le|l_{\chi_1}|\,b_m\varepsilon\le l_{\chi_1}\tanh\left(\frac{l_{\chi_1}}{\rho(t)}\right)b_m\varepsilon+b_m\varepsilon\kappa\rho(t),\tag{12}
$$

where bounded function $\rho(t)$ is positive uniform continuous. It is easy to known that there is a positive constant ρ_1, which satifies $\lim_{t\to\infty}\int_0^t\rho(\xi)d\xi\le\rho_1<+\infty$.

Using the inequalities (11) and (12), it has

$$
\begin{aligned}
\dot V_1\le&l_{\chi_1}g_1\chi_2+l_{\chi_1}g_1\alpha_1+l_{\chi_1}\tanh\left(\frac{l_{\chi_1}}{\rho(t)}\right)\frac{1}{2\eta_1}\|\varPsi_1\|^2 b_m\theta+l_{\chi_1}\tanh\left(\frac{l_{\chi_1}}{\rho(t)}\right)b_m\varepsilon\\
&+\left(\frac{1}{2\eta_1}\|\varPsi_1\|^2 b_m\theta+\eta_1+b_m\varepsilon\right)\kappa\rho(t)-\frac{b_m\widetilde\theta\dot{\widehat\theta}}{r_1}-\frac{b_m\widetilde\varepsilon\dot{\widehat\varepsilon}}{r_2}.
\end{aligned}\tag{13}
$$

The virtual control signal α_1 is designed as

$$
\alpha_1=-\frac{\tanh(l_{\chi_1}/\rho(t))\|\varPsi_1\|^2\widehat\theta}{2\eta_1}-\tanh(l_{\chi_1}/\rho(t))\widehat\varepsilon-\frac{G_1\chi_1^2}{l_{\chi_1}}.\tag{14}
$$

Using (14), we have

$$
\begin{aligned}
\dot V_1\le&\left(\frac{\|\varPsi_1\|^2 b_m\theta}{2\eta_1}+\eta_1+b_m\varepsilon\right)\kappa\rho(t)+\frac{1}{r_1}b_m\widetilde\theta\left(\frac{r_1}{2\eta_1}l_{\chi_1}\tanh\left(\frac{l_{\chi_1}}{\rho(t)}\right)-\dot{\widehat\theta}\right)\\
&+\frac{1}{r_2}b_m\widetilde\varepsilon\left(r_2 l_{\chi_1}\tanh\left(\frac{l_{\chi_1}}{\rho(t)}\right)-\dot{\widehat\varepsilon}\right)-Q_1\chi_1^2+l_{\chi_1}g_1\chi_2,
\end{aligned}\tag{15}
$$

where $Q_i=G_ib_m,i=1,2,\cdots,n$.

Step 2 Let $\chi_2=x_2-\alpha_1$, the time derivative of χ_2 is given by

$$
\dot\chi_2=f_2(\overline{x}_2)+g_2x_3-\dot\alpha_1.\tag{16}
$$

Next, Lyapunov functions V_2 is designed as

$$
V_2=V_1+\frac12\log(l_{b_2}^2/(l_{b_2}^2-\chi_2^2)),\tag{17}
$$

where $|\chi_2|\le l_{c_2}$, $l_{b_2}=l_{c_2}-\overline{\alpha}_1$.

The time derivative of V_2 is

$$
\begin{aligned}
\dot V_2=&\dot V_1+l_{\chi_2}\dot\chi_2\\
\le&\dot V_1+l_{\chi_2}(\widetilde f_2+g_2x_3)-\eta_2 l_{\chi_2}\tanh\left(\frac{l_{\chi_2}}{\rho(t)}\right),
\end{aligned}\tag{18}
$$

where $\widetilde{f}_2 = f_2 - \dot{\alpha}_1 + \eta_2 \tanh\left(\frac{l_{\chi_2}}{\rho(t)}\right) + \frac{l_{\chi_1} g_1 \chi_2}{l_{\chi_2}}$, $l_{\chi_2} = \chi_2/(l_{b_2}^2 - \chi_2^2)$. Then, using RBF NNs to estimate function $\widetilde{f}_2$, we have

$$\dot{V}_2 \leq \dot{V}_1 + l_{\chi_2}(W_2^{*\mathrm{T}} \Psi_2 + \delta_2 + g_2 x_3) - \eta_2 l_{\chi_2} \tanh\left(\frac{l_{\chi_2}}{\rho(t)}\right) - l_{\chi_1} g_1 \chi_2. \tag{19}$$

From Lemma 2 and Young's inequality, we have

$$\begin{aligned} l_{\chi_2} W_2^{*\mathrm{T}} \Psi_2 \leq & |l_{\chi_2}| W_2^{*\mathrm{T}} \Psi_2 \\ \leq & |l_{\chi_2}| \frac{1}{2\eta_2} \| \Psi_2 \|^2 b_m \theta + |l_{\chi_2}| \eta_2 \\ \leq & l_{\chi_2} \tanh\left(\frac{l_{\chi_2}}{\rho(t)}\right) \frac{1}{2\eta_2} \| \Psi_2 \|^2 b_m \theta + \frac{1}{2\eta_2} \| \Psi_2 \|^2 b_m \theta \kappa \rho(t) \\ & + l_{\chi_2} \tanh\left(\frac{l_{\chi_2}}{\rho(t)}\right) \eta_2 + \eta_2 \kappa \rho(t) \\ = & l_{\chi_2} \tanh\left(\frac{l_{\chi_2}}{\rho(t)}\right) \frac{1}{2\eta_2} \| \Psi_2 \|^2 b_m \theta \\ & + l_{\chi_2} \tanh\left(\frac{l_{\chi_2}}{\rho(t)}\right) \eta_2 + \left(\frac{1}{2\eta_2} \| \Psi_2 \|^2 b_m \theta + \eta_2\right) \kappa \rho(t), \end{aligned} \tag{20}$$

$$l_{\chi_2} \delta_2 \leq |l_{\chi_2}| b_m \varepsilon \leq l_{\chi_2} \tanh\left(\frac{l_{\chi_2}}{\rho(t)}\right) b_m \varepsilon + b_m \varepsilon \kappa \rho(t). \tag{21}$$

Using the inequalities (20) and (21), (19) becomes

$$\begin{aligned} \dot{V}_2 \leq & \dot{V}_1 + l_{\chi_2} \tanh\left(\frac{l_{\chi_2}}{\rho(t)}\right) \frac{1}{2\eta_2} \| \Psi_2 \|^2 b_m \theta + l_{\chi_2} \tanh\left(\frac{l_{\chi_2}}{\rho(t)}\right) b_m \varepsilon + l_{\chi_2} g_2 x_3 \\ & + \left(\frac{1}{2\eta_2} \| \Psi_2 \|^2 b_m \theta + \eta_2 + b_m \varepsilon\right) \kappa \rho(t) - l_{\chi_1} g_1 \chi_2. \end{aligned} \tag{22}$$

The virtual control law α_2 is designed as

$$\alpha_2 = -\frac{\tanh(l_{\chi_2}/\rho(t)) \| \Psi_2 \|^2 \widehat{\theta}}{2\eta_2} - \tanh(l_{\chi_2}/\rho(t)) \widehat{\varepsilon} - \frac{G_2 \chi_2^2}{l_{\chi_2}}. \tag{23}$$

Using (23), we have

$$\begin{aligned} \dot{V}_2 \leq & \left(\frac{\| \Psi_1 \|^2 b_m \theta}{2\eta_1} + \frac{\| \Psi_2 \|^2 b_m \theta}{2\eta_2} + \eta_1 + \eta_2 + 2 b_m \varepsilon\right) \kappa \rho(t) - Q_1 \chi_1^2 - Q_2 \chi_2^2 \\ & + \frac{1}{r_1} b_m \widetilde{\theta} \left(\frac{r_1}{2\eta_1} l_{\chi_1} \tanh\left(\frac{l_{\chi_1}}{\rho(t)}\right) + \frac{r_1}{2\eta_2} l_{\chi_2} \tanh\left(\frac{l_{\chi_2}}{\rho(t)}\right) - \dot{\widehat{\theta}}\right) + l_{\chi_2} g_2 \chi_3 \\ & + \frac{1}{r_2} b_m \widetilde{\varepsilon} \left(l_{\chi_1} r_2 \tanh\left(\frac{l_{\chi_1}}{\rho(t)}\right) + l_{\chi_2} r_2 \tanh\left(\frac{l_{\chi_2}}{\rho(t)}\right) - \dot{\widehat{\varepsilon}}\right). \end{aligned} \tag{24}$$

Step *k* Let $\chi_k = x_k - \alpha_{k-1}$, the time derivative of χ_k is given by

$$\dot{\chi}_k \leq f_k + g_k x_{k+1} - \dot{\alpha}_{k-1}. \tag{25}$$

Next, define a positive Lyapunov function V_k as

$$V_k = \sum_{i=1}^{k-1} V_i + \frac{1}{2}\log(l_{b_k}^2/(l_{b_k}^2 - \chi_k^2)), \tag{26}$$

where $|x_k| \le l_{c_k}$, $l_{b_k} = l_{c_k} - \overline{\alpha}_{k-1}$.

The time derivative of V_k is

$$\begin{aligned}\dot{V}_k &= \sum_{i=1}^{k-1} \dot{V}_i + l_{\chi_k}\dot{\chi}_k \\ &\le \sum_{i=1}^{k-1} \dot{V}_i + l_{\chi_k}(\widetilde{f}_k + g_k x_{k+1}) - \eta_k l_{\chi_k} \tanh\left(\frac{l_{\chi_k}}{\rho(t)}\right),\end{aligned} \tag{27}$$

where $\widetilde{f}_k = f_k - \dot{\alpha}_{k-1} + \eta_k \tanh\left(\frac{l_{\chi_k}}{\rho(t)}\right) + \frac{l_{\chi_{k-1}} g_{k-1}\chi_k}{l_{\chi_k}}$, $l_{\chi_k} = \chi_k/(l_{b_k}^2 - \chi_k^2)$. Then, using RBF NNs to estimate function $\widetilde{f}_k$, we have

$$\dot{V}_k \le \sum_{i=1}^{k-1} \dot{V}_i + l_{\chi_k}({W_k}^{*\mathrm{T}} \Psi_k + \delta_k + g_k x_{k+1}) - \eta_k l_{\chi_k} \tanh\left(\frac{l_{\chi_k}}{\rho(t)}\right) - l_{\chi_{k-1}} g_{k-1}\chi_k. \tag{28}$$

From Lemma 2.1 and Young's inequality, we have

$$\begin{aligned} l_{\chi_k} {W_k}^{*\mathrm{T}} \Psi_k &\le |l_{\chi_k}|\, {W_k}^{*\mathrm{T}} \Psi_k \\ &\le |l_{\chi_k}| \frac{1}{2\eta_k} \|\Psi_k\|^2 b_m\theta + |l_{\chi_k}|\,\eta_k \\ &\le l_{\chi_k} \tanh\left(\frac{l_{\chi_k}}{\rho(t)}\right)\frac{1}{2\eta_k}\|\Psi_k\|^2 b_m\theta + \frac{1}{2\eta_k}\|\Psi_k\|^2 b_m\theta\kappa\rho(t) \\ &\quad + \eta_k l_{\chi_k} \tanh\left(\frac{l_{\chi_k}}{\rho(t)}\right) + \eta_k\kappa\rho(t) \\ &= l_{\chi_k} \tanh\left(\frac{l_{\chi_k}}{\rho(t)}\right)\frac{1}{2\eta_k}\|\Psi_k\|^2 b_m\theta + l_{\chi_k} \tanh\left(\frac{l_{\chi_k}}{\rho(t)}\right)\eta_k \\ &\quad + \left(\frac{1}{2\eta_k}\|\Psi_k\|^2 b_m\theta + \eta_k\right)\kappa\rho(t), \end{aligned} \tag{29}$$

$$l_{\chi_k}\delta_k \le |l_{\chi_k}|\, b_m\varepsilon \le l_{\chi_k} \tanh\left(\frac{l_{\chi_k}}{\rho(t)}\right) b_m\varepsilon + b_m\varepsilon\kappa\rho(t). \tag{30}$$

Combing (28), (29) and (30), we have

$$\begin{aligned}\dot{V}_k \le & l_{\chi_k} \tanh\left(\frac{l_{\chi_k}}{\rho(t)}\right)\frac{1}{2\eta_k}\|\Psi_k\|^2 b_m\theta + \left(\frac{1}{2\eta_k}\|\Psi_k\|^2 b_m\theta + \eta_k + b_m\varepsilon\right)\kappa\rho(t) \\ & + l_{\chi_k} \tanh\left(\frac{l_{\chi_k}}{\rho(t)}\right) b_m\varepsilon - l_{\chi_{k-1}} g_{k-1}\chi_k + \sum_{i=1}^{k-1}\dot{V}_i + l_{\chi_k} g_k x_{k+1}.\end{aligned} \tag{31}$$

The virtual control law α_k is designed as

$$\alpha_k = -\frac{\tanh(l_{\chi_k}/\rho(t))\|\Psi_k\|^2\widehat{\theta}}{2\eta_k} - \tanh(l_{\chi_k}/\rho(t))\widehat{\varepsilon} - \frac{G_k\chi_k^2}{l_{\chi_k}}. \tag{32}$$

Combing (31) and (32), we have

$$\begin{aligned}\dot{V}_k \leq & \Big(\sum_{i=1}^{k} \frac{\| \Psi_i \|^2 b_m \theta}{2\eta_k} + \sum_{i=1}^{k} \eta_i + k b_m \varepsilon \Big) \kappa\rho(t) + \frac{1}{r_1} b_m \widetilde{\theta} \Big(\sum_{i=1}^{k} \frac{r_1}{2\eta_k} l_{\chi_i} \tanh\left(\frac{l_{\chi_i}}{\rho(t)}\right) - \dot{\widehat{\theta}} \Big) \\ & - \sum_{i=1}^{k} Q_i \chi_i^2 + \frac{1}{r_2} b_m \widetilde{\varepsilon} \Big(\sum_{i=1}^{k} l_{\chi_i} r_2 \tanh\left(\frac{l_{\chi_i}}{\rho(t)}\right) - \dot{\widehat{\varepsilon}} \Big) + l_{\chi_k} g_k \chi_k. \end{aligned} \tag{33}$$

Step n Let $\chi_n = x_n - \alpha_{n-1}$, the time derivative of χ_n is given by

$$\dot{\chi}_n \leq f_n + g_n u - \dot{\alpha}_{n-1}. \tag{34}$$

Next, the Lyapunov function V_n is constructed as following:

$$V_n = \sum_{i=1}^{n-1} V_i + \frac{1}{2} \log(l_{b_n}^2 / (l_{b_n}^2 - \chi_n^2)), \tag{35}$$

where $|x_n| \leq l_{c_n}$, $l_{b_n} = l_{c_n} - \overline{\alpha}_{n-1}$.

The time derivative of V_n is

$$\begin{aligned}\dot{V}_n &= \sum_{i=1}^{n-1} \dot{V}_i + l_{\chi_n} \dot{\chi}_k \\ &\leq \sum_{i=1}^{n-1} \dot{V}_i + l_{\chi_n} f_n + l_{\chi_n} g_n u - l_{\chi_n} \dot{\alpha}_{n-1} \\ &\leq \sum_{i=1}^{n-1} \dot{V}_i + l_{\chi_n} (\widetilde{f}_n + g_n u) - \eta_n l_{\chi_n} \tanh\left(\frac{l_{\chi_n}}{\rho(t)}\right), \end{aligned} \tag{36}$$

where $\widetilde{f}_n = f_n - \dot{\alpha}_{n-1} + \eta_n \tanh\left(\frac{l_{\chi_n}}{\rho(t)}\right) + \frac{l_{\chi_{n-1}} g_{n-1} \chi_n}{l_{\chi_n}}$, $l_{\chi_n} = \chi_n / (l_{b_n}^2 - \chi_n^2)$. Then, using RBF NNs to estimate function $\widetilde{f}_n$, we have

$$\dot{V}_\mathrm{n} \leq l_{\chi_n} ({W_n}^{*\mathrm{T}} \Psi_n(\overline{x}_n) + \delta_n(\overline{x}_n) + g_n u) - \eta_n l_{\chi_n} \tanh\left(\frac{l_{\chi_n}}{\rho(t)}\right) - l_{\chi_{n-1}} g_{n-1} \chi_n + \sum_{i=1}^{n} \dot{V}_i. \tag{37}$$

From Lemma 2.1 and Young's inequality, we have

$$\begin{aligned} l_{\chi_n} {W_n}^{*\mathrm{T}} \Psi_n &\leq |l_{\chi_n}| \, {W_n}^{*\mathrm{T}} \Psi_n \\ &\leq |l_{\chi_n}| \frac{1}{2\eta_n} \| \Psi_n \|^2 b_m \theta + |l_{\chi_n}| \, \eta_n \\ &\leq l_{\chi_n} \tanh\left(\frac{l_{\chi_n}}{\rho(t)}\right) \frac{1}{2\eta_n} \| \Psi_n \|^2 b_m \theta + \frac{1}{2\eta_n} \| \Psi_n \|^2 b_m \theta \kappa \rho(t) \\ &\quad + l_{\chi_n} \tanh\left(\frac{l_{\chi_n}}{\rho(t)}\right) \eta_n + \eta_n \kappa \rho(t) \\ &= l_{\chi_n} \tanh\left(\frac{l_{\chi_n}}{\rho(t)}\right) \frac{1}{2\eta_n} \| \Psi_n \|^2 b_m \theta \\ &\quad + l_{\chi_n} \tanh\left(\frac{l_{\chi_n}}{\rho(t)}\right) \eta_n + \left(\frac{1}{2\eta_n} \| \Psi_n \|^2 b_m \theta + \eta_n \right) \kappa \rho(t), \end{aligned} \tag{38}$$

$$l_{\chi_n} \overline{\delta}_n \leq |l_{\chi_n}| \, b_m \varepsilon \leq l_{\chi_n} \tanh\left(\frac{l_{\chi_n}}{\rho(t)}\right) b_m \varepsilon + b_m \varepsilon \kappa \rho(t). \tag{39}$$

Using the inequalities (38) and (39), (37) becomes

$$\begin{aligned}\dot{V}_n \leq & l_{\chi_n}\tanh\left(\frac{l_{\chi_n}}{\rho(t)}\right)\frac{1}{2\eta_n}\|\varPsi_n\|^2 b_m\theta+\left(\frac{1}{2\eta_n}\|\varPsi_n\|^2 b_m\theta+\eta_n+b_m\varepsilon\right)\kappa\rho(t)\\ &+l_{\chi_n}\tanh\left(\frac{l_{\chi_n}}{\rho(t)}\right)b_m\varepsilon-l_{\chi_{n-1}}g_{n-1}\chi_n+\sum_{i=1}^{n}\dot{V}_i+l_{\chi_n}g_n u. \end{aligned} \tag{40}$$

We select the feedback controller u as

$$u=-\frac{\tanh(l_{\chi_n}/\rho(t))\|\varPsi_n\|^2\widehat{\theta}}{2\eta_n}-\tanh(l_{\chi_n}/\rho(t))\widehat{\varepsilon}-\frac{G_n\chi_n^2}{l_{\chi_n}}. \tag{41}$$

Combing (40) and (41), we have

$$\begin{aligned}\dot{V}_n \leq & \left(\sum_{i=1}^{n}\frac{\|\varPsi_i\|^2 b_m\theta}{2\eta_i}+\sum_{i=1}^{n}\eta_i+nb_m\varepsilon\right)\kappa\rho(t)+\frac{1}{r_1}b_m\widetilde{\theta}\left(\sum_{i=1}^{n}\frac{r_1}{2\eta_i}l_{\chi_i}\tanh\left(\frac{l_{\chi_i}}{\rho(t)}\right)-\dot{\widehat{\theta}}\right)\\ &+\frac{1}{r_2}b_m\widetilde{\varepsilon}\left(\sum_{i=1}^{n}l_{\chi_i}r_2\tanh\left(\frac{l_{\chi_i}}{\rho(t)}\right)-\dot{\widehat{\varepsilon}}\right)-\sum_{i=1}^{n}Q_i\chi_i^2. \end{aligned} \tag{42}$$

Using (6), we have

$$\begin{aligned}\dot{V}_n \leq & \left(\sum_{i=1}^{n}\frac{\|\varPsi_i\|^2 b_m\theta}{2\eta_i}+\sum_{i=1}^{n}\eta_i+nb_m\varepsilon\right)\kappa\rho(t)+\frac{b_m}{r_1}\widetilde{\theta}\widehat{\theta}\rho(t)\\ &+\frac{b_m}{r_2}\widetilde{\varepsilon}\widehat{\varepsilon}\rho(t)-\sum_{i=1}^{n}Q_i\chi_i^2. \end{aligned} \tag{43}$$

4 Stability Analysis

Theorem 4.1 *Consider the nonlinear pure-feedback systems* (1) *with full state constraints under Asumptions* 2.1 *and* 2.2*, and supposed the adaptive rates* $\dot{\widehat{\theta}}$ *and* $\dot{\widehat{\varepsilon}}$ *in* (6)*, the virtual controllers* $\alpha_i, i=1,2,\cdots,n-1$ *are constructed in* (14), (23) *and* (32)*, the actual controller* u *is designed in* (41)*. If the design parameters* r_1, r_2, $G_i, i=1,2,\cdots,n$ *are chosen to satisfy* $l_{c_{i+1}}-\overline{\alpha}_i>l_{b_{i+1}}, \overline{\alpha}_i=\max\left|\alpha_i(\overline{x}_i,\widehat{\theta}_i,\widehat{\delta}_i,y_d{}^{(k)},k=1,\cdots,i)\right|$*, the control scheme designed in this paper can guarantee:* 1) *When the time tends to infinity, the tracking error gradually tends to zero.* 2) *All signals are semi-globally uniformly bounded in the closed-loop systems.* 3) *All state variables are constrained to the specified ranges i.e.,*$|x_i|\leq l_{c_i}$.

Proof Combing (15), (24), (33) and (43), it yields

$$\dot{V}_n \leq \left(\sum_{i=1}^{n}\frac{\|\varPsi_i\|^2 b_m\theta}{2\eta_i}+\sum_{i=1}^{n}\eta_i+nb_m\varepsilon\right)\kappa\rho(t)+\frac{b_m}{r_1}\widetilde{\theta}\widehat{\theta}\rho(t)+\frac{b_m}{r_2}\widetilde{\varepsilon}\widehat{\varepsilon}\rho(t)-\sum_{i=1}^{n}Q_i{\chi_i}^2. \tag{44}$$

Consider the following inequalities:

$$\begin{aligned}&\frac{b_m}{r_1}\widetilde{\theta}\widehat{\theta}\rho(t)\leq -b_m\rho(t)\widetilde{\theta}^2+b_m\rho(t)\widetilde{\theta}\theta\leq\frac{b_m}{2r_1}\rho(t)\theta^2,\\ &\frac{b_m}{r_2}\widetilde{\varepsilon}\widehat{\varepsilon}\rho(t)\leq\frac{b_m}{2r_2}\rho(t)\varepsilon^2,\end{aligned} \tag{45}$$

and considering $\|\psi_i\| \leq \sqrt{\tau_i}$ with $\tau_i > 0$ being the number of nodes in the NNs, we have

$$\dot{V}_n \leq -\sum_{i=1}^{n} Q_i \chi_i^2 + \left(\sum_{i=1}^{n} \frac{\tau_i b_m \theta}{2\eta_i} + \sum_{i=1}^{n} \eta_i + n\varepsilon\right)\kappa\rho(t) + \frac{b_m}{2r_1}\rho(t)\theta^2 + \frac{b_m}{2r_2}\rho(t)\varepsilon^2. \tag{46}$$

Let

$$\gamma = \left(\sum_{i=1}^{n} \frac{\tau_i b_m \theta}{2\eta_i} + \sum_{i=1}^{n} \eta_i + nb_m\varepsilon\right)\kappa + \frac{b_m}{2r_1}\theta^2 + \frac{b_m}{2r_2}\varepsilon^2. \tag{47}$$

Combing (46) and (47), it yields

$$\dot{V}_n \leq -\sum_{i=1}^{n} Q_i \chi_i^2 + \rho(t)\gamma. \tag{48}$$

Integrating (48) over $[0, t]$ yields that

$$\begin{aligned} V_n(t) &\leq V_n(0) + \gamma \int_0^t \rho(\nu) d\nu - \int_0^t \sum_{i=1}^{n} Q_i \chi_i^2(\nu) d\nu \\ &\leq V_n(0) + \gamma\rho_1. \end{aligned} \tag{49}$$

This means that χ_i, χ_n, x_i, x_n, and $\alpha_i, i = 1, \cdots, n-1$ are bounded. What's more, from the inequality (49), we have

$$\int_0^t \sum_{i=1}^{n} Q_i \chi_i^2(\nu) d\nu \leq V_n(0) + \gamma\rho_1. \tag{50}$$

By applying the Barbalat lemma in [66], it is concluded that

$$\lim_{t\to\infty} \chi_1 = 0. \tag{51}$$

From $x_1 = \chi_1 + y_d$ and $|y_d| \leq M_0$, we can have $|x_1| \leq |\chi_1| + |y_d| < l_{b_1} + M_0$. Let $l_{b_1} = l_{c_1} - M_0$ and then, $|x_1| < l_{c_1}$. According to the definition of α_1 in (14), we can get that α_1 is a function of $\widehat{W}$, $\widehat{\delta}$, x_1, χ_1, g_1 and $\dot{y}_d$. Because the boundedness of $\widehat{W}$, $\widehat{\delta}$, x_1, χ_1, g_1, $\dot{y}_d$, α_1 is bounded and satisfies $|\alpha_1| < \overline{\alpha}_1$. Then, $|x_2| \leq |\alpha_1| + |\chi_2| \leq \overline{\alpha}_1 + l_{b_2}$. This implies that $|x_2| < l_{c_2}$ if $l_{b_2} = l_{c_2} - \overline{\alpha}_1$. Similarly, it can be proven that $|x_{i+1}| < l_{c_{i+1}}, i = 2, 3, \cdots, n-1$, when $l_{b_{i+1}} = l_{c_{i+1}} - \overline{\alpha}_i$. It can be known from the definition in (41) that u is a function of $\widehat{W}$, $\widehat{\delta}$, x, χ_n, g_n and $\dot{y}_d, {y_d}^{(2)}, \cdots, {y_d}^{(n)}$. Due to the boundedness of $\widehat{W}$, $\widehat{\delta}$, x, χ_n, g_n and $\dot{y}_d$, the controller u is bounded. From the above analysis, we can draw the conclusion that all the signals in the system are bounded and the constraints of the state variables in the system (1) are not violated. ∎

Remark 4.2 It can be seen from (48), that the bigger values of Q_i are and the smaller value of γ is, the faster the function V_n is going to go down. This means that we can adjust the design parameters in Q_i and γ to get better control performance. Due to $l_{b_i} = l_{c_i} - \overline{\alpha}_{i-1}$, the design parameters l_{b_i} are affected by l_{c_i} and $\overline{\alpha}_{i-1}$. The final sizes are also determined by

l_{c_i} and $\overline{\alpha}_{i-1}$. The design parameters η_i can be used to adjust the size of the virtual control α_i. The selection of η_i are dynamic, it can adjust the value of α_i to the ideal ranges. In general, according to the existing knowledge, there is no specific way to adjust these parameters, and constantly debugging is needed according to the quality of the simulation results.

5 Simulation Study

Example 5.1 In order to illustrate the effectiveness of the proposed scheme, the following nonlinear pure-feedback system is considered:

$$\begin{cases} \dot{x}_1 = x_1 + 2x_2 + x_2^2/3, \\ \dot{x}_2 = x_1 x_2^2 + 2x_2 + 0.2u, \\ y = x_1, \end{cases} \tag{52}$$

where the constraints of x_i are $|x_1| < 1.5$, $|x_2| < 1.5$ and the reference signal is given as $y_d = \sin(t)$.

The controller of this system is designed as follows

$$u = -\frac{\tanh(l_{\chi_2}/\rho(t))\| \Psi_2\|^2\widehat{\theta}}{2\eta_2} - \tanh(l_{\chi_2}/\rho(t))\widehat{\varepsilon} - \frac{G_2\chi_2^2}{l_{\chi_2}}, \tag{53}$$

where

$$\begin{aligned} &\varphi_i(s) = \exp\left[-(s-\xi_i)^{\mathrm{T}}(s-\xi_i)/\lambda^2\right], \quad i = 1, 2, \cdots, n, \\ &\Psi_1(s) = [\varphi_1(s), \varphi_2(s), \cdots, \varphi_4(s)]^{\mathrm{T}}, \\ &\alpha_1 = -\frac{\tanh(l_{\chi_1}/\rho(t))\| \Psi_1\|^2\widehat{\theta}}{2\eta_1} - \tanh(l_{\chi_1}/\rho(t))\widehat{\varepsilon} - \frac{G_1\chi_1^2}{l_{\chi_1}}. \end{aligned} \tag{54}$$

The adaptive laws are designed as

$$\begin{aligned} &\dot{\widehat{\theta}} = \sum_{i=1}^{2} l_{\chi_i}\tanh(l_{\chi_i}/\rho(t))\frac{r_1}{2\eta_i}\| \Psi_i\|^2 - r_1\rho(t)\widehat{\theta}, \\ &\dot{\widehat{\varepsilon}} = \sum_{i=1}^{2} l_{\chi_i} r_2 \tanh(l_{\chi_i}/\rho(t)) - r_2\rho(t)\widehat{\varepsilon}. \end{aligned} \tag{55}$$

The initial values of the adaptive laws and states are $\widehat{\theta}(0) = 0.2$, $\widehat{\varepsilon}(0) = 0.5$, $x_1(0) = 0.1$, $x_2(0) = 0.3$. Other design parameters are selected as follows: $G_1 = 10$, $G_2 = 10$, $r_1 = 15$, $r_2 = 15$, $\lambda_1 = 2$, $\lambda_2 = 2$. Similar to [53], by using the Matlab routine, we can obtain $l_{b_1} = 1.0$, $l_{b_2} = 0.447$.

From Figure 1 and Figure 2, we can see that the trajectory of y has good tracking performance with the trajectory of the reference signal y_d. Figure 3 shows that the individual state variables in the system are constrained in the specified ranges. The trajectories of the virtual controller α_1 and the controller u are illustrated in Figure 4 and Figure 5, respectively.

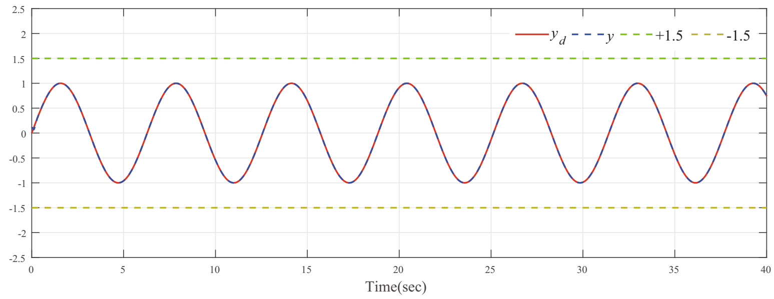

Figure 1 Output tracking perormance of Example 5.1

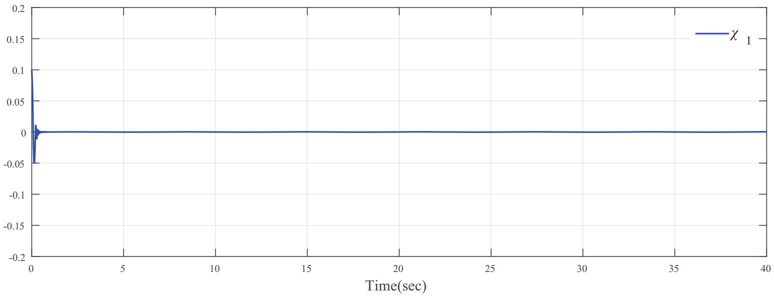

Figure 2 Output tracking error of Example 5.1

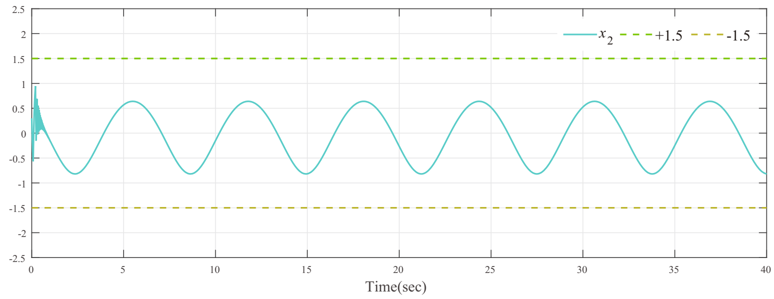

Figure 3 The trajectory of x_2 of Example 5.1

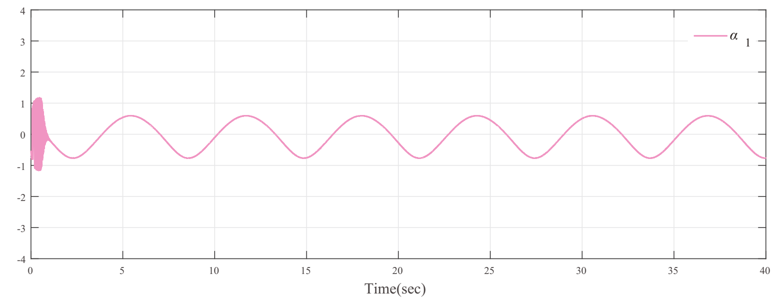

Figure 4 The trajectory of virtual controller α_1 of Example 5.1

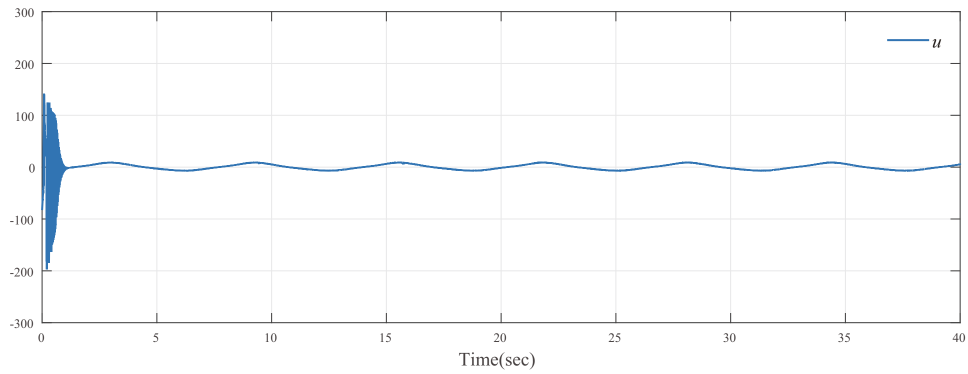

Figure 5 The trajectory of controller u of Example 5.1

Example 5.2 Consider the following nonlinear pure-feedback single-link robot which taken from [52]

$$\begin{cases} H\ddot{\varsigma} + \dfrac{1}{2}mgl\sin\varsigma = u, \\ y = \varsigma, \end{cases} \tag{56}$$

where $g(\mathrm{m/s^2})$ is the acceleration of gravity. $m(\mathrm{kg})$ and $\varsigma(^\circ)$ are the gravity and the angle of the system respectively. u is the input torque of the system; $H(\mathrm{kg}\cdot\mathrm{m}^2)$ is the moment of inertia; $l(\mathrm{m})$ is the length. The system (1) can be expressed as:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -mgl\sin x_1/2H + u/H, \\ y = x_1, \end{cases} \tag{57}$$

where $x_1 = \varsigma$, $x_2 = \dot{\varsigma}$ and the states are constrained in $|x_i| < 1, i = 1, 2$. The reference signal is $y_d = 0.5\sin(t)$. The initial values of the states are chosen as $x_1(0) = 0.15$, $x_2(0) = 0.1$. From Theorem 4.1, other design parameters are selected as $G_1 = 1$, $G_2 = 1$, $r_1 = 15$, $r_2 = 15$, $\lambda_1 = 2$, $\lambda_2 = 2$. Similar to [62], by using the Matlab routine, we can obtain $l_{b_1} = 1.0$, $l_{b_2} = 0.982$.

From Figure 6 and Figure 7, we can see that the trajectory of the y has good tracking performance with the trajectory of the reference signal y_d. Figure 8 shows that the individual state variables in the system are constrained in the specified ranges. Figure 9 and Figure 10 show the trajectories of virtual controller α_1 and the controller u. Figure 11 is provided to show the comparison results between our method and the existing methods. Compared our method with methods used in [58, 63], it can be clearly seen that our method has better tracking performance.

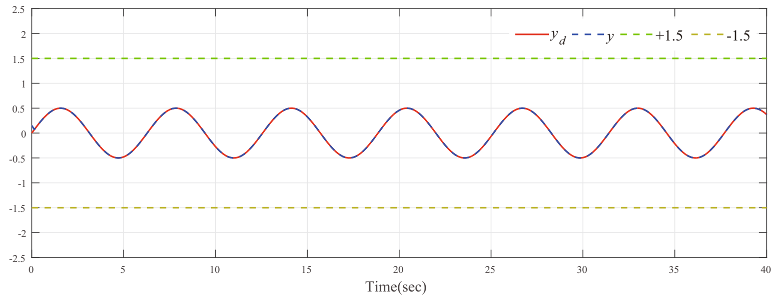

Figure 6 Output tracking performance of Example 5.2

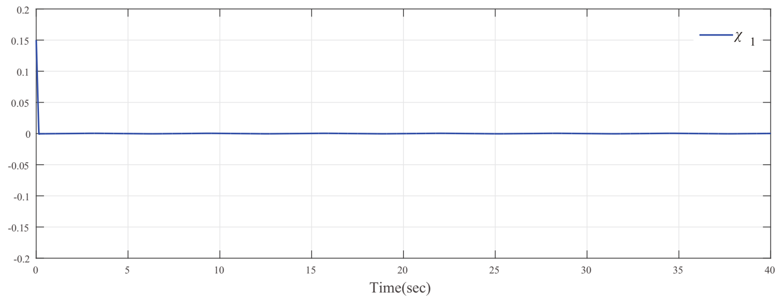

Figure 7 Output tracking error using the scheme of Example 5.2

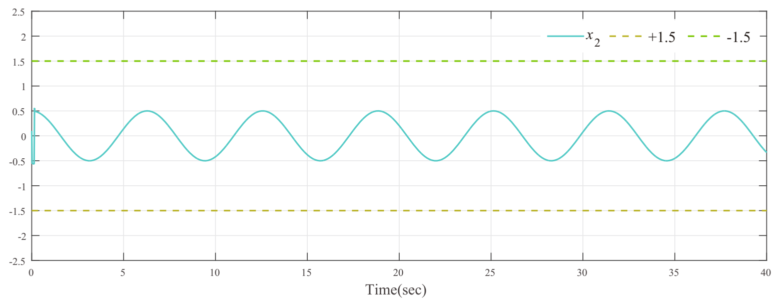

Figure 8 The trajectory of x_2 of Example 5.2

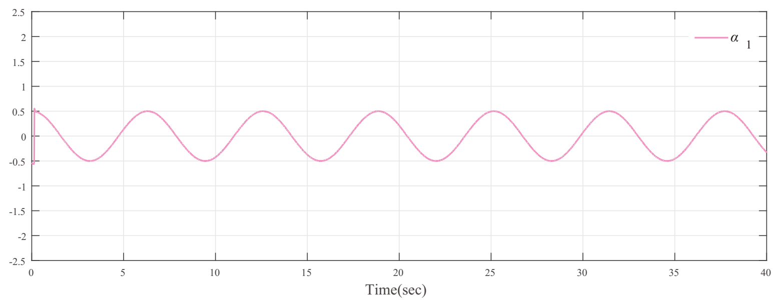

Figure 9 The trajectory of virtual controller α_1 of Example 5.2

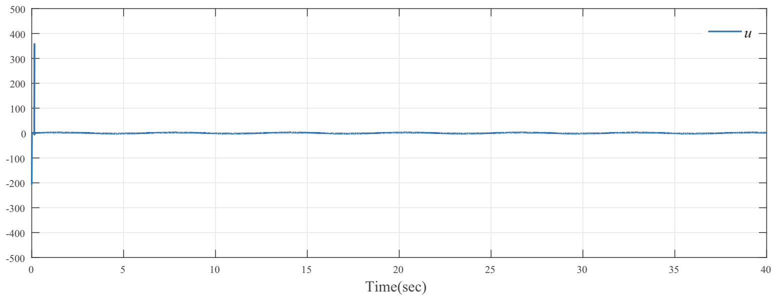

Figure 10 The trajectory of controller u of Example 5.2

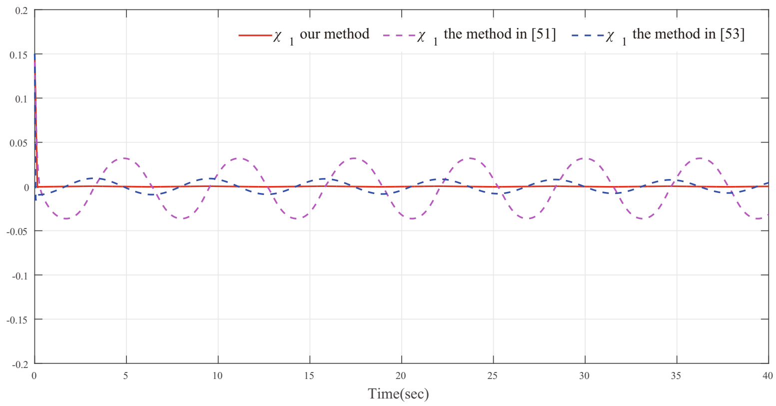

Figure 11 The comparison with the existing results of Example 5.2

6 Conclusions

This paper has studied the controller design problem for a class of nonlinear pure-feedback system with full state constrains to ensure the asymptotic tracking performance. The nonlinear functions in the systems are completely unknown and are not linearly parameterized. In order to apply the backstepping method, we first transformed the pure-feedback system into a system with strict-feedback structure, and NNs were used to online approximate the unknown nonlinear terms. In addition, the BLFs were employed in all steps of the backstepping design such that the requirements of full state constraints were satisfied. The proposed design scheme can guarantee that all the signals in the closed-loop system remain bounded and the tracking error asymptotically converges to zero. Two simulation examples are presented to show the effectiveness of the proposed control scheme in the end. In the fulture, we will concentrate on the controller design problem for pure-feedback interconnected nonlinear systems with full state constraints.

Conflict of Interest

The authors declare no conflict of interest.

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