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Adaptive Asymptotic Tracking Control for Stochastic Nonlinear Systems with Unknown Backlash-Like Hysteresis*

WANG Le · SUN Wei · WU Yuqiang

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Abstract In this study, an adaptive asymptotic tracking control problem is considered for stochastic nonlinear systems with unknown backlash-like hysteresis. By utilizing backstepping technology and bound estimation approach, an adaptive asymptotic tracking control scheme is designed, where fuzzy systems are applied to approximate unknown function terms, the effect of hysteresis and stochastic disturbances is compensated appropriately. The proposed scheme ensures that the tracking error can asymptotically converge to zero in probability and all signals of the closed-loop system are bounded almost surely. Finally, the effectiveness of the control scheme is verified by giving a simulation example.

Keywords Adaptive fuzzy control, asymptotic tracking, stochastic nonlinear systems, unknown backlash-like hysteresis.

1 Introduction

Since nonlinear systems have a wide range of engineering application backgrounds, the research on the control of nonlinear systems has never stopped. In recent years, the control methods for nonlinear systems have become more and more extensive, such as backstepping control[1–3], fuzzy control[4–6], and sliding control[7–9]. The adaptive backstepping control is widely used in controller design, especially for strict-feedback nonlinear systems. Many significant results have been obtained[10–13]. Uncertain switched nonlinear systems with nonstrict-feedback form in [12] are studied by employing the backstepping technique, and the proposed

WANG Le · SUN Wei (Corresponding author)
School of Mathematics Science, Liaocheng University, Liaocheng 252000*, China.*
Email: wangle9909@163.com; sunwei@lcu.edu.cn.
WU Yuqiang
School of Engineering, Qufu Normal University, Rizhao 276826*, China.* Email: wyq@qfnu.edu.cn.
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control method achieves asymptotic tracking performance. Nowadays, adaptive fuzzy or neural approaches become the very effective tools to deal with the problems in nonlinear systems, where they are used to approximate or estimate unknown function terms in many researches[14–17]. Chen, et al.[15] proposed a model-based adaptive event-triggered control scheme by using neural networks, which effectively avoids Zeno behavior. In [17], adaptive fuzzy controllers are designed by employing backstepping technique, which guarantees the tracking performance under the constraint of prescribed tracking performance functions.

With the development of research, the problems caused by stochastic disturbances have got more and more attention because they often bring a lot of negative effect, such as affecting systems performance and causing systems instability. Therefore, how to deal with the effect of stochastic disturbances has always been the challenging work. The application of backstepping technology in stochastic systems has got many successful results, such as [18–21]. To the specific, Wu, et al.[18] first studied the stochastic nonlinear systems with Markov switching and design an adaptive backstepping controller. Then, for semi-strict nonlinear systems, Wang, et al.[21] proposed a disturbance observer based-backstepping sliding mode control strategy. In addition, by combining backstepping strategy and adaptive fuzzy or neural network control scheme, many excellent results have been achieved for stochastic nonlinear systems[22–26]. In [23], an asymptotic tracking control scheme is presented for stochastic systems by proposing a novel gain suppressing inequality approach and employing fuzzy logic systems. Sui, et al.[24] studied the problem of finite-time control for nontriangular stochastic nonlinear systems. An adaptive output-feedback neural network control strategy is proposed for a class of stochastic nonlinear time-varying delay systems with unknown control directions in [25]. These studies have brought more solutions to the problems existing in stochastic systems.

As a common phenomenon, hysteresis exists in many physical systems, for example, electric hysteresis and elastic hysteresis. This phenomenon always brings lots of troubles in actual systems, which can cause the control performance of the systems to decrease. Therefore, the study of hysteresis has been a hot topic. For the past few decades, plenty of progress has been made in the problem of hysteresis. Some researchers propose several mathematical models, such as backlash-like model, Bouc-Wen model, and Prandtl-Ishlinskii model. With the help of these models, a lot of outgoing results have been obtained[27–29]. Besides, backstepping method, adaptive fuzzy or neural approaches and command filter technology have been widely used to deal with this type of problem in [30–32]. In [30], the issue of multi-input and multi-output (MIMO) nonlinear systems with actuator hysteresis and full-state constraints is solved by a neural network control approach. The authors in [31] investigated the problem of compensating for rate-dependent hysteresis nonlinearity in nonlinear uncertain systems. The study of MIMO nonlinear systems with hysteresis in [32] is considered by means of fuzzy approach, in which all the signals of the closed-loop system are semi-globally uniformly ultimately bounded.

It is noteworthy that the phenomenon of hysteresis exists not only in determinate nonlinear systems but also in stochastic systems, which makes the research more challenging. Hence, some scholars have designed some control schemes to solve the issue of hysteresis in stochastic systems[33, 34], while the proposed control schemes can't achieve asymptotic tracking. To our knowledge, there are few approaches to reach the goals. Based on the above observations, this study considers an adaptive asymptotic tracking controller for strict-feedback stochastic nonlinear systems with unknown backlash-like hysteresis. The following is a summary of the main contributions of this study:

1) Unlike the determinate nonlinear systems in [30–32], this study considers the phenomenon of backlash-like hysteresis in stochastic nonlinear systems. An adaptive asymptotic tracking controller is designed for a class of systems in strict-feedback form. In design progress, we just need to know the signs of control coefficients functions, which is inspired by a bound estimate method in [35]. Different from [35], the number of adaptive laws is less and the calculation burden is greatly reduced.

2) For the existing researches[33, 34] which consider the issue of hysteresis in stochastic systems, the tracking error can converge to a small neighborhood of the origin. In this study, the tracking error can asymptotically converge to zero based on the proposed control strategy, which achieves better tracking performance.

2 System Description and Preliminaries

Consider the strict-feedback stochastic nonlinear system with unknown backlash-like hysteresis as follows:

$$\begin{cases} x_i = (g_i(\overline{x}_i)x_{i+1} + f_i(\overline{x}_i))t + h_i(\overline{x}_i)\omega, \quad i = 1, 2, \cdots, n-1, \\ x_n = (g_n(x)u(\nu) + f_n(x))t + h_n(x)\omega, \\ y = x_1, \end{cases} \tag{1}$$

where $\overline{x}_i = [x_1, x_2, \cdots, x_i]^{\mathrm{T}} \in R^i, i = 1, 2, \cdots, n$, $x = \overline{x}_n \in R^n$ are the system state vectors, and $y \in R$ is the system output. ω is an r-dimensional independent standard Wiener process defined on a complete probability space $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P\}$, where Ω is a sample space, $\mathcal{F}$ is a σ-field, $\{\mathcal{F}_t\}_{t\geq 0}$ is a filtration, and P is the probability measure. $g_i(\overline{x}_i), f_i(\overline{x}_i)$ and $h_i(\overline{x}_i)$ are unknown smooth locally Lipschitz nonlinear functions. $u \in R$ as the output of the unknown backlash-like hysteresis can be expressed by

$$\frac{u}{t} = a\left|\frac{\nu}{t}\right|(c\nu - u) + b\frac{\nu}{t}, \tag{2}$$

where ν is the input of the unknown backlash-like hysteresis, a, b and c are unknown constants, $c > 0$ is the slope of the lines and satisfies $c > b$. For example, when the parameters are selected as $a = 1$, $b = 0.345$ and $c = 3.1635$, $\nu(t) = k\sin(2.3t)$ with $k = 2.5$ and $k = 4.5$, and the initial conditions are $u(0) = 0$ and $\nu(0) = 0$, the dynamic of the backlash-like hysteresis model (2) is presented in Figure 1.

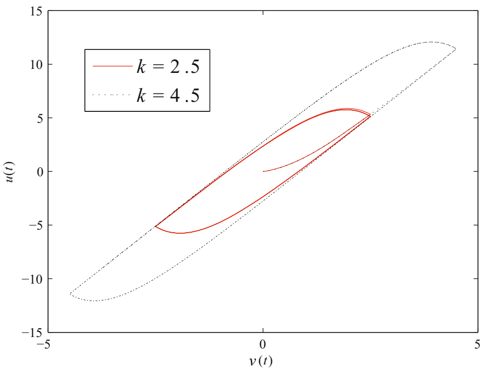

Figure 1 Backlash-like hysteresis curves

Based on [36], (2) can be written as

$$u(t) = c\nu(t) + d(\nu), \tag{3}$$

$$d(\nu) = [u_0 - c\nu_0]\mathrm{e}^{-a(\nu-\nu_0)\mathrm{sgn}\dot{\nu}} + \mathrm{e}^{-a\nu\mathrm{sgn}\dot{\nu}} \int_{\nu_0}^{\nu} [b - c]\mathrm{e}^{a\zeta(\mathrm{sgn}\dot{\nu})}\, d\zeta, \tag{4}$$

where $u_0 = u(0), \nu_0 = \nu(0)$ are the initial values of u and ν, respectively. After analysis, it is not difficult to conclude that $\lim_{\nu\to\infty} |d(\nu)| = |\frac{c-b}{a}|$, that is, there exists a uniform bound D such that $|d(\nu)| \le D$.

In this study, the control objective is to design an adaptive fuzzy controller for the system (1) such that the system output y can asymptotically track the reference signal y_d in probability and all signals of the closed-loop system are bounded almost surely although there exists the phenomenon of hysteresis. To reach the target, we give the following assumptions.

Assumption 1 The reference output y_d and its time derivative up to the nth order $y_d^{(n)}$ are continuous and bounded.

Assumption 2 The signs of g_i are known, $g_i(\overline{x}_i)$ are bounded and satisfy $0 < \underline{g}_i \le |g_i(\overline{x}_i)| \le \overline{g}_i$, $i = 1, 2, \cdots, n$, where $\underline{g}_i$ and $\overline{g}_i$ are unknown positive constants.

Remark 1 Different from the existing results[37–39] where the control coefficients of the systems are required to be known constants or known nonlinear functions, we just need that $g_i(\overline{x}_i)$ are bounded in this study. In addition, Assumption 2 indicates that $g_i(\overline{x}_i)$ are either positive or negative. Without loss of generality, we assume that $g_i(\overline{x}_i) > 0$. In control design, the lower bound $\overline{g_i}$ is added to the Lyapunov function to solve the problem caused by the unknown virtual control coefficients.

Next, the following necessary definitions and lemmas are introduced.

Definition 1 (see [34]) Consider the following stochastic system

$$x = f(x, t)t + h(x, t)\omega,$$

where $x \in R^n$ is the system state, ω is an r-dimensional independent standard Winner progress, $f(x, t)$ and $h(x, t)$ are locally Lipschitz functions. For any given function $V(x, t) \in C^{2,1}$, define the differential operator $\mathcal{L}$ as

$$\mathcal{L}V=\frac{\partial V}{\partial t}+\frac{\partial V}{\partial x}f+\frac{1}{2}\mathrm{Tr}\left\{h^{\mathrm{T}}\frac{\partial^2 V}{\partial x^2}h\right\}\tag{5}$$

with $\mathrm{Tr}(A)$ being the trace of matrix A.

Lemma 1 (see [35]) *For any variable x and positive integrable time-varying function $\sigma(t)$, the following inequality holds:*

$$0\le |x|-\frac{x^2}{\sqrt{x^2+\sigma^2(t)}}<\sigma(t),$$

where $\sigma(t)$ satisfies $\lim_{t\to+\infty}\int_{t_0}^{t}\sigma(s)s\le\overline{\sigma}<\infty$ with $\overline{\sigma}$ being a positive constant.

The following FLSs are utilized in the control design.

IF-THEN rules: R_i: If x_1 is F_1^i, x_2 is F_2^i, $\cdots$ and x_n is F_n^i, THEN y is A^i, $i=1,2,\cdots,n$. The FLS is expressed as

$$y(x)=\frac{\sum_{i=1}^{N}\Phi_i\prod_{j=1}^{n}\nu_{F_j^i}(x_j)}{\sum_{i=1}^{N}[\prod_{j=1}^{n}\nu_{F_j^i}(x_j)]}.$$

Choose $\varphi_i(x)=\frac{\prod_{j=1}^{n}\nu_{F_j^i}(x_j)}{\sum_{i=1}^{N}[\prod_{j=1}^{n}\nu_{F_j^i}(x_j)]}$, $\Phi(x)=[\varphi_1(x),\varphi_2(x),\cdots,\varphi_N(x)]^{\mathrm{T}}$ is the basis function vector and $P=[p_1,p_2,\cdots,p_N]^{\mathrm{T}}$ is the weight vector. Then, we have

$$y(x)=P^{\mathrm{T}}\Phi(x).$$

Lemma 2 (see [40]) *For a continuous function $\overline{f}(x)$ defined on a compact set U and any $\varepsilon>0$, there exists an FLS $P^{\mathrm{T}}\Phi(x)$ satisfying*

$$\sup_{x\in U}|\overline{f}(x)-P^{\mathrm{T}}\Phi(x)|\le\varepsilon.$$

3 Adaptive Controller Design

The design process of controller will be presented by using the above knowledge and the adaptive backstepping technology in this section. First, the following coordinate transformations are introduced

$$\begin{aligned}&z_1=x_1-y_d,\\&z_i=x_i-\alpha_{i-1},\quad i=2,3,\cdots,n,\end{aligned}\tag{6}$$

where α_{i-1} represent the virtual controllers. Then, the following constants are defined before the design procedure,

$$\theta=\max\left\{\frac{\|P_i\|}{\underline{g}_i},i=1,2,\cdots,n\right\},$$

$$\rho=\max\left\{\frac{\varepsilon_i}{\underline{g}_i}+\frac{\overline{g}_n}{\underline{g}_n}D,i=1,2,\cdots,n\right\},$$

$\zeta = \frac{1}{c}$. $\widehat{\theta}$, $\widehat{\rho}$ and $\widehat{\zeta}$ are estimations of θ, ρ and ζ, respectively; $\widetilde{\theta} = \theta - \widehat{\theta}$, $\widetilde{\rho} = \rho - \widehat{\rho}$ and $\widetilde{\zeta} = \zeta - \widehat{\zeta}$ are the estimation errors.

Step 1 From (1) and (6), we have

$$z_1 = (g_1 z_2 + g_1 \alpha_1 + f_1 - \dot{y}_d)t + h_1 \omega. \tag{7}$$

Define a Lyapunov function candidate as

$$V_1 = \frac{1}{4\underline{g}_1} z_1^4 + \frac{1}{2r}\widetilde{\theta}^2 + \frac{1}{2\gamma}\widetilde{\rho}^2. \tag{8}$$

The following equality can be obtained from (5) and (7)

$$\mathcal{L}V_1 = \frac{z_1^3}{\underline{g}_1}(g_1 z_2 + g_1 \alpha_1 + f_1 - \dot{y}_d) + \frac{3}{2}\frac{z_1^2}{\underline{g}_1} H_1 - \frac{1}{r}\widetilde{\theta}\dot{\widehat{\theta}} - \frac{1}{\gamma}\widetilde{\rho}\dot{\widehat{\rho}} \tag{9}$$

with $H_1 = h_1^{\mathrm{T}} h_1$. According to Young's inequality and Lemma 1, it follows that

$$\frac{g_1}{\underline{g}_1} z_1^3 z_2 \le \frac{3}{4}\frac{g_1}{\underline{g}_1} z_1^4 + \frac{1}{4}\frac{g_1}{\underline{g}_1} z_2^4, \tag{10}$$

$$\frac{3}{2\underline{g}_1} z_1^2 H_1 \le \frac{3}{2\underline{g}_1}\frac{z_1^4 H_1^{\mathrm{T}} H_1}{\sqrt{z_1^4 H_1^{\mathrm{T}} H_1 + \sigma_1^2}} + \frac{3}{2\underline{g}_1}\sigma_1. \tag{11}$$

Then, substituting (10) and (11) into (9) gives

$$\mathcal{L}V_1 \le \frac{g_1}{\underline{g}_1} z_1^3 \alpha_1 + \frac{1}{\underline{g}_1} z_1^3 \overline{f}_1 + \frac{1}{4}\frac{g_1}{\underline{g}_1} z_2^4 + \frac{3}{2\underline{g}_1}\sigma_1 - \frac{1}{r}\widetilde{\theta}\dot{\widehat{\theta}} - \frac{1}{\gamma}\widetilde{\rho}\dot{\widehat{\rho}}, \tag{12}$$

where $\overline{f}_1(X_1) = f_1(x_1) - \dot{y}_d + \frac{3}{4} g_1 z_1 + \frac{3}{2}\frac{z_1 H_1^{\mathrm{T}} H_1}{\sqrt{z_1^4 H_1^{\mathrm{T}} H_1 + \sigma_1^2}}$, it is obvious that $\overline{f}_1(X_1)$ is a function containing $x_1, y_d, \dot{y}_d$. Next, using Lemma 2, there exists an FLS $P_1^{\mathrm{T}} \Phi_1(X_1)$ such that

$$\overline{f}_1 = P_1^{\mathrm{T}} \Phi_1(X_1) + \delta_1, \quad \|\delta_1\| \le \varepsilon_1,$$

where δ_1 is an approximation error. Further, based on Lemma 1, the following inequality holds

$$\frac{1}{\underline{g}_1} z_1^3 \overline{f}_1 \le \theta \frac{z_1^6 \Phi_1^{\mathrm{T}} \Phi_1}{\sqrt{z_1^6 \Phi_1^{\mathrm{T}} \Phi_1 + \sigma_1^2}} + \theta\sigma_1 + \rho \frac{z_1^6}{\sqrt{z_1^6 + \sigma_1^2}} + \rho\sigma_1. \tag{13}$$

Combining (12) and (13), one can obtain

$$\mathcal{L}V_1 \le -k_1 z_1^4 + \frac{g_1}{4\underline{g}_1} z_2^4 + \frac{g_1}{\underline{g}_1} z_1^3 \alpha_1 + z_1^3 v_1 + \theta\sigma_1 + \rho\sigma_1 + \frac{3\sigma_1}{2\underline{g}_1} + \frac{1}{r}\widetilde{\theta}\Big(\tau_1 - \dot{\widehat{\theta}}\Big) + \frac{1}{\gamma}\widetilde{\rho}\Big(\iota_1 - \dot{\widehat{\rho}}\Big) \tag{14}$$

with $v_1 = k_1 z_1 + \widehat{\theta}\frac{z_1^3 \Phi_1^{\mathrm{T}} \Phi_1}{\sqrt{z_1^6 \Phi_1^{\mathrm{T}} \Phi_1 + \sigma_1^2}} + \widehat{\rho}\frac{z_1^3}{\sqrt{z_1^6 + \sigma_1^2}}$. τ_1 and ι_1 are denoted as

$$\begin{cases} \tau_1 = r\dfrac{z_1^6 \Phi_1^{\mathrm{T}} \Phi_1}{\sqrt{z_1^6 \Phi_1^{\mathrm{T}} \Phi_1 + \sigma_1^2}}, \\ \iota_1 = \gamma\dfrac{z_1^6}{\sqrt{z_1^6 + \sigma_1^2}}. \end{cases}$$

Thus, the virtual control input α_1 is designed as

$$\alpha_1 = -\frac{z_1^3 v_1^2}{\sqrt{z_1^6 v_1^2 + \sigma_1^2}}. \tag{15}$$

Next, applying Lemma 1, (14) an be rewritten as

$$\mathcal{L}V_1 \le -k_1 z_1^4 + \frac{1}{4}\frac{g_1}{\underline{g}_1} z_2^4 + \sigma_1\left(1 + \theta + \rho + \frac{3}{2\underline{g}_1}\right) + \frac{1}{r}\widetilde{\theta}(\tau_1 - \dot{\widehat{\theta}}) + \frac{1}{\gamma}\widetilde{\rho}(\iota_1 - \dot{\widehat{\rho}}). \tag{16}$$

Step i **(**$i = 2, 3, \cdots, n-1$**)**. It follows from (1) and (6) that

$$z_i = (g_i z_{i+1} + g_i \alpha_i + f_i - \Psi_i)t + \left(h_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} h_j\right)\omega, \tag{17}$$

where

$$\begin{aligned}\Psi_i = {} & \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}(g_j x_{j+1} + f_j) + \frac{\partial \alpha_{i-1}}{\partial \widehat{\theta}}\dot{\widehat{\theta}} + \frac{\partial \alpha_{i-1}}{\partial \widehat{\rho}}\dot{\widehat{\rho}} + \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(j)}} y_d^{(j+1)} \\ & + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \sigma_j}\dot{\sigma}_j + \frac{1}{2}\sum_{p,q=1}^{i-1} \frac{\partial^2 \alpha_{i-1}}{\partial x_p \partial x_q} h_p^{\mathrm{T}}(\overline{x}_p) h_q(\overline{x}_q).\end{aligned}$$

Construct a Lyapunov function candidate as

$$V_i = V_{i-1} + \frac{1}{4\underline{g}_i} z_i^4. \tag{18}$$

Then, by using the formulas (5) and (17), it can be obtained that

$$\begin{aligned}\mathcal{L}V_i \le {} & -\sum_{j=1}^{i-1} k_j z_j^4 + \sum_{j=1}^{i-1} \sigma_j\left(1 + \theta + \rho + \frac{3}{2\underline{g}_j}\right) + \frac{1}{r}\widetilde{\theta}(\tau_{i-1} - \dot{\widehat{\theta}}) + \frac{1}{\gamma}\widetilde{\rho}(\iota_{i-1} - \dot{\widehat{\rho}}) \\ & + \frac{g_i}{\underline{g}_i} z_i^3 z_{i+1} + \frac{g_i}{\underline{g}_i} z_i^3 \alpha_i + \frac{1}{\underline{g}_i} z_i^3\left(f_i - \Psi_i + \frac{1}{4}\frac{\underline{g}_i g_{i-1}}{\underline{g}_{i-1}} z_i\right) + \frac{3}{2\underline{g}_i} z_i^2 H_i\end{aligned} \tag{19}$$

with $H_i = (h_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} h_j)^{\mathrm{T}}(h_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} h_j)$. Similarly, from Young's inequality and Lemma 1, we can obtain

$$\frac{g_i}{\underline{g}_i} z_i^3 z_{i+1} \le \frac{3}{4}\frac{g_i}{\underline{g}_i} z_i^4 + \frac{1}{4}\frac{g_i}{\underline{g}_i} z_{i+1}^4, \tag{20}$$

$$\frac{3}{2\underline{g}_i} z_i^2 H_i \le \frac{3}{2\underline{g}_i}\frac{z_i^4 H_i^{\mathrm{T}} H_i}{\sqrt{z_i^4 H_i^{\mathrm{T}} H_i + \sigma_i^2}} + \frac{3}{2\underline{g}_i}\sigma_i. \tag{21}$$

Then, substituting (20) and (21) into (19) produces

$$\begin{aligned}\mathcal{L}V_i \le {} & -\sum_{j=1}^{i-1} k_j z_j^4 + \sum_{j=1}^{i-1} \sigma_j\left(1 + \theta + \rho + \frac{3}{2\underline{g}_j}\right) + \frac{1}{r}\widetilde{\theta}(\tau_{i-1} - \dot{\widehat{\theta}}) \\ & + \frac{1}{\gamma}\widetilde{\rho}(\iota_{i-1} - \dot{\widehat{\rho}}) + \frac{g_i}{\underline{g}_i} z_i^3 \alpha_i + \frac{1}{\underline{g}_i} z_i^3 \overline{f}_i + \frac{1}{4}\frac{g_i}{\underline{g}_i} z_{i+1}^4 + \frac{3}{2\underline{g}_i}\sigma_i,\end{aligned} \tag{22}$$

where $\overline{f}_i(X_i) = f_i(\overline{x}_i) - \Psi_i + \frac{3}{4}g_i z_i + \frac{1}{4}\frac{g_i g_{i-1}}{\underline{g}_{i-1}} z_i + \frac{3}{2}\frac{z_i H_i^{\mathrm{T}} H_i}{\sqrt{z_i^4 H_i^{\mathrm{T}} H_i + \sigma_i^2}}$. In view of Lemma 2, there exists an FLS $P_i^{\mathrm{T}} \Phi_i(X_i)$ such that

$$\overline{f}_i = P_i^{\mathrm{T}} \Phi_i(X_i) + \delta_i, \quad \|\delta_i\| \le \varepsilon_i,$$

where δ_i is an approximation error. According to Lemma 1, the following inequality gives

$$\frac{1}{\underline{g}_i} z_i^3 \overline{f}_i \le \theta \frac{z_i^6 \Phi_i^{\mathrm{T}} \Phi_i}{\sqrt{z_i^6 \Phi_i^{\mathrm{T}} \Phi_i + \sigma_i^2}} + \theta\sigma_i + \rho \frac{z_i^6}{\sqrt{z_i^6 + \sigma_i^2}} + \rho\sigma_i. \tag{23}$$

Substituting (23) into (22), the following inequality yields

$$\begin{aligned} \mathcal{L}V_i \le & -\sum_{j=1}^{i} k_j z_j^4 + \sum_{j=1}^{i-1} \sigma_j \left(1 + \theta + \rho + \frac{3}{2\underline{g}_j}\right) + \frac{1}{r}\widetilde{\theta}(\tau_i - \dot{\widehat{\theta}}) + \frac{1}{\gamma}\widetilde{\rho}(\iota_i - \dot{\widehat{\rho}}) \\ & + \frac{g_i}{\underline{g}_i} z_i^3 \alpha_i + z_i^3 v_i + \theta\sigma_i + \rho\sigma_i + \frac{1}{4}\frac{g_i}{\underline{g}_i} z_{i+1}^4 + \frac{3}{2\underline{g}_i}\sigma_i, \end{aligned} \tag{24}$$

where $v_i = k_i z_i + \widehat{\theta}\frac{z_i^3 \Phi_i^{\mathrm{T}} \Phi_i}{\sqrt{z_i^6 \Phi_i^{\mathrm{T}} \Phi_i + \sigma_i^2}} + \widehat{\rho}\frac{z_i^3}{\sqrt{z_i^6 + \sigma_i^2}}$. τ_i and ι_i are expressed as

$$\begin{cases} \tau_i = \tau_{i-1} + r\dfrac{z_i^6 \Phi_i^{\mathrm{T}} \Phi_i}{\sqrt{z_i^6 \Phi_i^{\mathrm{T}} \Phi_i + \sigma_i^2}}, \\ \iota_i = \iota_{i-1} + \gamma \dfrac{z_i^6}{\sqrt{z_i^6 + \sigma_i^2}}. \end{cases}$$

Therefore, the virtual control signal α_i is constructed as

$$\alpha_i = -\frac{z_i^3 v_i^2}{\sqrt{z_i^6 v_i^2 + \sigma_i^2}}. \tag{25}$$

Applying Lemma 1, there holds

$$\mathcal{L}V_i \le -\sum_{j=1}^{i} k_j z_j^4 + \frac{1}{4}\frac{g_i}{\underline{g}_i} z_{i+1}^4 + \sum_{j=1}^{i} \sigma_j \left(1 + \theta + \rho + \frac{3}{2\underline{g}_j}\right) + \frac{1}{r}\widetilde{\theta}(\tau_i - \dot{\widehat{\theta}}) + \frac{1}{\gamma}\widetilde{\rho}(\iota_i - \dot{\widehat{\rho}}). \tag{26}$$

Step n In view of (1), (3) and (4), the following can be obtained

$$x_n = (g_n c\nu + g_n d + f_n)t + h_n \omega. \tag{27}$$

Based on (6), it can be calculated that

$$z_n = (g_n c\nu + g_n d + f_n - \Psi_n)t + \left(h_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} h_j\right)\omega, \tag{28}$$

where

$$\Psi_n = \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}(g_j x_{j+1} + f_j) + \frac{\partial \alpha_{n-1}}{\partial \widehat{\theta}}\dot{\widehat{\theta}} + \frac{\partial \alpha_{n-1}}{\partial \widehat{\rho}}\dot{\widehat{\rho}} + \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(j)}} y_d^{(j+1)}$$

$$+\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\sigma_j}\dot{\sigma}_j+\frac{1}{2}\sum_{p,q=1}^{n-1}\frac{\partial^2\alpha_{n-1}}{\partial x_p\partial x_q}h_p^{\mathrm{T}}(\overline{x}_p)h_q(\overline{x}_q).$$

Choose a Lyapunov function candidate as

$$V_n=V_{n-1}+\frac{1}{4\underline{g}_n}z_n^4+\frac{c}{2\eta}\widetilde{\zeta}^2. \tag{29}$$

It follows from (5) and (28) that

$$\begin{aligned}\mathcal{L}V_n\le&-\sum_{j=1}^{n-1}k_jz_j^4+\sum_{j=1}^{n-1}\sigma_j\left(1+\theta+\rho+\frac{3}{2\underline{g}_j}\right)+\frac{1}{r}\widetilde{\theta}(\tau_{n-1}-\dot{\widehat{\theta}})+\frac{1}{\gamma}\widetilde{\rho}(\iota_{n-1}-\dot{\widehat{\rho}})\\&+\frac{g_n}{\underline{g}_n}z_n^3c\nu+\frac{g_n}{\underline{g}_n}z_n^3d+\frac{1}{\underline{g}_n}z_n^3\left(f_n-\Psi_n+\frac{1}{4}\frac{\underline{g}_ng_{n-1}}{\underline{g}_{n-1}}z_n\right)+\frac{3}{2\underline{g}_n}z_n^2H_n-\frac{c}{\eta}\widetilde{\zeta}\dot{\widehat{\zeta}}\end{aligned} \tag{30}$$

with $H_n=(h_n-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}h_j)^{\mathrm{T}}(h_n-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}h_j)$. As the same of Step i, by using Lemma 1, it can prove that

$$\frac{3}{2\underline{g}_n}z_n^2H_n\le\frac{3}{2\underline{g}_n}\frac{z_n^4H_n^{\mathrm{T}}H_n}{\sqrt{z_n^4H_n^{\mathrm{T}}H_n+\sigma_n^2}}+\frac{3}{2\underline{g}_n}\sigma_n. \tag{31}$$

By combining (31), we get

$$\begin{aligned}\mathcal{L}V_n\le&-\sum_{j=1}^{n-1}k_jz_j^4+\sum_{j=1}^{n-1}\sigma_j\left(1+\theta+\rho+\frac{3}{2\underline{g}_j}\right)+\frac{1}{r}\widetilde{\theta}(\tau_{n-1}-\dot{\widehat{\theta}})\\&+\frac{1}{\gamma}\widetilde{\rho}(\iota_{n-1}-\dot{\widehat{\rho}})+\frac{g_n}{\underline{g}_n}z_n^3c\nu+\frac{\overline{g}_n}{\underline{g}_n}z_n^3D+\frac{1}{\underline{g}_n}z_n^3\overline{f}_n+\frac{3}{2\underline{g}_n}\sigma_n-\frac{c}{\eta}\widetilde{\zeta}\dot{\widehat{\zeta}},\end{aligned} \tag{32}$$

where $\overline{f}_n(X_n)=f_n(x)-\Psi_n+\frac{1}{4}\frac{\underline{g}_ng_{n-1}}{\underline{g}_{n-1}}z_n+\frac{3}{2}\frac{z_nH_n^{\mathrm{T}}H_n}{\sqrt{z_i^4H_n^{\mathrm{T}}H_n+\sigma_n^2}}$. Based on Lemma 2, there exists an FLS $P_n^{\mathrm{T}}\Phi_n(X_n)$ such that

$$\overline{f}_n=P_n^{\mathrm{T}}\Phi_n(X_n)+\delta_n,\quad \|\delta_n\|\le\varepsilon_n,$$

where δ_n denotes an approximation error. Next, from Lemma 1, the following inequality holds

$$\frac{1}{\underline{g}_n}z_n^3\overline{f}_n+\frac{\overline{g}_n}{\underline{g}_n}z_n^3D\le\theta\frac{z_n^6\Phi_n^{\mathrm{T}}\Phi_n}{\sqrt{z_n^6\Phi_n^{\mathrm{T}}\Phi_n+\sigma_n^2}}+\theta\sigma_n+\rho\frac{z_n^6}{\sqrt{z_n^6+\sigma_n^2}}+\rho\sigma_n. \tag{33}$$

Substituting (33) into (32) gives

$$\begin{aligned}\mathcal{L}V_n\le&-\sum_{j=1}^{n}k_jz_j^4+\sum_{j=1}^{n-1}\sigma_j\left(1+\theta+\rho+\frac{3}{2\underline{g}_j}\right)+\frac{1}{r}\widetilde{\theta}(\tau_n-\dot{\widehat{\theta}})\\&+\frac{1}{\gamma}\widetilde{\rho}(\iota_n-\dot{\widehat{\rho}})+\frac{g_n}{\underline{g}_n}z_n^3c\nu+z_n^3v_n+\sigma_n\left(\theta+\rho+\frac{3}{2\underline{g}_n}\right)-\frac{c}{\eta}\widetilde{\zeta}\dot{\widehat{\zeta}},\end{aligned} \tag{34}$$

where $v_n=k_nz_n+\widehat{\theta}\frac{z_n^3\Phi_n^{\mathrm{T}}\Phi_n}{\sqrt{z_n^6\Phi_n^{\mathrm{T}}\Phi_n+\sigma_n^2}}+\widehat{\rho}\frac{z_n^3}{\sqrt{z_n^6+\sigma_n^2}}$. τ_n and ι_n are defined as

$$\begin{cases}\tau_n=\tau_{n-1}+r\dfrac{z_n^6\Phi_n^{\mathrm{T}}\Phi_n}{\sqrt{z_n^6\Phi_n^{\mathrm{T}}\Phi_n+\sigma_n^2}},\\ \iota_n=\iota_{n-1}+\gamma\dfrac{z_n^6}{\sqrt{z_n^6+\sigma_n^2}}.\end{cases}$$

From which, the final controller is designed as

$$\nu = -\frac{z_n^3 v_n^2 \widehat{\zeta}^2}{\sqrt{z_n^6 v_n^2 \widehat{\zeta}^2 + \sigma_n^2}} \tag{35}$$

and adaptive laws are constructed as

$$\begin{cases} \dot{\widehat{\theta}} = \tau_n - \sigma_n r \widehat{\theta}, \\ \dot{\widehat{\rho}} = \iota_n - \sigma_n \gamma \widehat{\rho}, \\ \dot{\widehat{\zeta}} = \eta z_n^3 v_n - \sigma_n \eta \widehat{\zeta}. \end{cases} \tag{36}$$

Further, combining (34)–(36) and Lemma 1 gets

$$\begin{aligned} \mathcal{L}V_n \le & -\sum_{j=1}^{n} k_j z_j^4 + \sum_{j=1}^{n-1} \sigma_j \left(1 + \theta + \rho + \frac{3}{2\underline{g}_j}\right) \\ & + \sigma_n \left(c + \theta + \rho + \frac{3}{2\underline{g}_n}\right) + \sigma_n(\widetilde{\theta}\widehat{\theta} + \widetilde{\rho}\widehat{\rho} + c\widetilde{\zeta}\widehat{\zeta}). \end{aligned} \tag{37}$$

According to the complete square formula, the following inequalities hold

$$\begin{cases} \widetilde{\theta}\widehat{\theta} = \widetilde{\theta}(\theta - \widetilde{\theta}) = -\widetilde{\theta}^2 + \widetilde{\theta}\theta \le \dfrac{\theta^2}{4}, \\ \widetilde{\rho}\widehat{\rho} = \widetilde{\rho}(\rho - \widetilde{\rho}) = -\widetilde{\rho}^2 + \widetilde{\rho}\rho \le \dfrac{\rho^2}{4}, \\ \widetilde{\zeta}\widehat{\zeta} = \widetilde{\zeta}(\zeta - \widetilde{\zeta}) = -\widetilde{\zeta}^2 + \widetilde{\zeta}\zeta \le \dfrac{\zeta^2}{4}. \end{cases} \tag{38}$$

Finally, substituting (38) into (37) shows

$$\mathcal{L}V_n \le -\sum_{j=1}^{n} k_j z_j^4 + \sum_{j=1}^{n} \sigma_j \lambda_j. \tag{39}$$

4 Stability Analysis

Theorem 1 *For the stochastic nonlinear system* (1) *with unknown backlash-like hysteresis* (2)*, under Assumptions* 1–2*, there exist the controllers* (15)*,* (25)*,* (35) *and the adaptive updating laws* (36)*, such that the system output y can asymptotically track the reference signal y_d in probability and all signals of the closed-loop system are bounded almost surely.*

Proof Based on Lemma 2 in [41], there exists a unique solution to the stochastic system (1), it is defined as $e(t) = (z_1(t), z_2(t), \cdots, z_n(t), \widetilde{\theta}(t), \widetilde{\rho}(t), \widetilde{\zeta}(t))$. It follows from (39) that $\mathcal{L}V_n \le -\sum_{j=1}^{n} k_j z_j^4 + \sum_{j=1}^{n} \sigma_j \lambda_j$, then by employing Lemma 4 in [41], we have

$$\begin{aligned} & E(V_n(e(\sigma_r \wedge t))) \\ & \le V_n(e(t_0)) - E\int_{t_0}^{\sigma_r \wedge t} \sum_{j=1}^{n} k_j z_j^4(s) s + E\int_{t_0}^{\sigma_r \wedge t} \sum_{j=1}^{n} \sigma_j(s) \lambda_j s \end{aligned}$$

$$\leq V_n(e(t_0)) + \sum_{j=1}^{n} \overline{\sigma}_j \lambda_j,$$

where $\sigma_r := \inf\{t \geq 0 : |x(t)| \geq r\}$, and $\sigma_r \wedge t$ is the minimum of σ_r and t. It can be noted that $\inf_{|e|\geq R} V_n(e) \to \infty$ as $R \to \infty$. In view of Lemma 2 in [41], we can get a conclusion that $e(t)$ is bounded almost surely on $[0, \infty)$, which implies that $z_1(t), z_2(t), \cdots, z_n(t)$, $\widetilde{\theta}(t)$, $\widetilde{\rho}(t)$ and $\widetilde{\zeta}(t)$ are bounded almost surely on $[0, \infty)$. Combining Assumption 1, the boundedness of y_d determines the boundedness of x_1 on $[0, \infty)$. In addition, it can be obtained that $\widehat{\theta}$, $\widehat{\rho}$ and $\widehat{\zeta}$ are bounded on $[0, \infty)$ due to the boundedness of $\widetilde{\theta}(t)$, $\widetilde{\rho}(t)$ and $\widetilde{\zeta}(t)$. Next, α_1 is bounded almost surely on $[0, \infty)$ because it contains x_1, y_d, $\dot{y}_d$, $\widehat{\theta}$ and $\widehat{\rho}$. From Equation (6), it can be seen that x_2 is bounded almost surely on $[0, \infty)$. By using the same analysis, it can be concluded that x_i, α_i and ν are bounded almost surely on $[0, \infty)$. Consequently, all the signals of the system are bounded almost surely on $[0, \infty)$. From (40), it follows that

$$E \int_{t_0}^{\sigma_r \wedge t} \sum_{j=1}^{n} k_j z_j^4(s) s \leq V_n(e(t_0)) + \sum_{j=1}^{n} \overline{\sigma}_j \lambda_j.$$

Since $\lim_{r\to+\infty} \lim_{t\to+\infty} (\sigma_r \wedge t) = \infty$, using Fatou's lemma gets

$$E \int_{t_0}^{\infty} \sum_{j=1}^{n} k_j z_j^4(s) s \leq V_n(e(t_0)) + \sum_{j=1}^{n} \overline{\sigma}_j \lambda_j < +\infty.$$

Then, from Stochastic Barbălat's Lemma on [41], it can prove that $P\big(\lim_{t\to\infty} \|z_i(t)\| = 0\big) = 1$, so the system output y can asymptotically track the reference signal y_d in probability. ∎

5 Simulation Example

In this section, to prove the effectiveness of the proposed strategy, we will give a simulation example which is described in the following form:

$$\begin{cases} x_1 = (10x_2 \cos x_1 + 0.01 \cos(2x_1))t + 0.001\mathrm{e}^{-x_1}\omega, \\ x_2 = (30u(\nu) \cos x_2 + 2x_1 \sin(x_2))t + 0.001\mathrm{e}^{-0.2x_2}\omega, \\ y = x_1, \end{cases}$$

where u is the output of unknown hysteresis with $a = 1$, $c = 3.1635$, and $b = 0.345$. The target signal y_d is expressed as $y_d = 0.6 \sin t$. For fuzzy control, choose the following membership functions as $\nu_{F_j^i} = \mathrm{e}^{-\frac{1}{2}(x_j+\xi_i)^2}$ with $\xi_i = 10, 8, 6, 4, 2, 0, -2, -4, -6, -8, -10$, $i = 1, 2, \cdots, 11$.

In the simulation, choose the parameters as $k_1 = 800$, $k_2 = 45$, $r = 30$, $\gamma = 35$, $\eta = 50$, $\sigma_1 = 0.0001\mathrm{e}^{-0.3t}$, $\sigma_2 = 0.001\mathrm{e}^{-0.25t}$, and the initial conditions are $x(0) = [-0.01, -0.01]^{\mathrm{T}}$, $\widehat{\theta}(0) = 1$, $\widehat{\rho}(0) = 0.5$, $\widehat{\zeta}(0) = 1$. Figures 2–5 verify the conclusions of Theorem 1. The system output y, the reference signal y_d and the tracking error z_1 are presented in Figure 2, in which we can see that the tracking error can asymptotically converge to zero. Figure 3 indicates the curves of the states x_1 and x_2. The control signals ν and u are displayed in Figure 4. And the trajectories of adaptive parameters are plotted in Figure 5. From Figures 2–5, we obtain that all signals of the closed-loop system are bounded. Namely, the control objective of this study can be achieved.

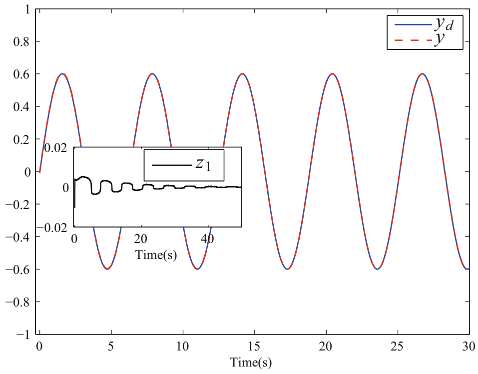

Figure 2 System output y and y_d

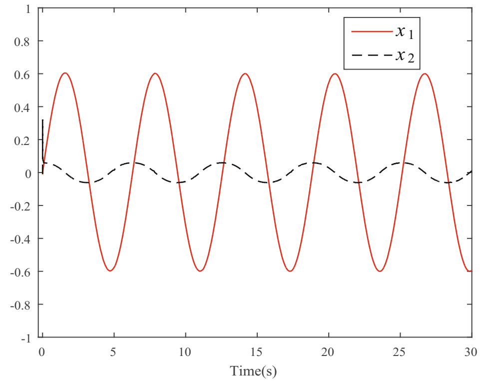

Figure 3 States x_1 and x_2

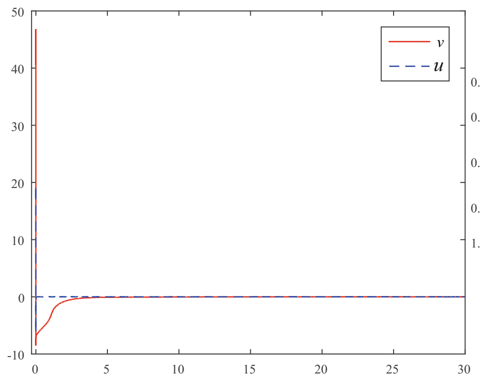

Figure 4 Control signals ν and u

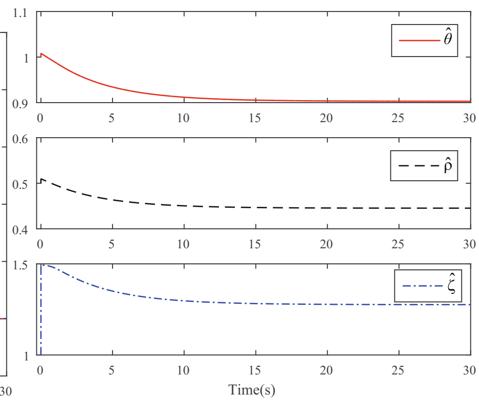

Figure 5 Adaptive parameters $\widehat{\theta}$, $\widehat{\rho}$, and $\widehat{\zeta}$

6 Conclusions

This study has investigated the issue of adaptive asymptotic tracking control for a class of uncertain stochastic nonlinear systems with unknown backlash-like hysteresis. An adaptive asymptotic tracking control scheme is designed via backstepping method, and the problem caused by unknown backlash-like hysteresis and stochastic disturbance can be solved. In addition, we have guaranteed the boundedness of all signals of the closed-loop system almost surely, and the tracking error z_1 can asymptotically converge to zero in probability. At last, a simulation result is given to confirm the effectiveness of the proposed method. The results of this study are based on that the signs of unknown virtual control coefficients are known. When the virtual control coefficient is completely unknown, how to construct an appropriate mechanism to achieve our control objective is still a challenging task. Thus, this issue is what we need to pay attention to in our future study.

References

[1] Zou A M, Hou Z G, and Tan M, Adaptive control of a class of nonlinear pure-feedback systems using fuzzy backstepping approach, *IEEE Transaction on Fuzzy Systems*, 2008, **16**(4): 886–897.

[2] Li Y X, Finite time command filtered adaptive fault tolerant control for a class of uncertain nonlinear systems, *Automatic*, 2019, **106**: 117–123.

[3] Zhao K, Song Y D, Chen C L P, et al., Control of nonlinear systems under dynamic constraints: A unified barrier function-based approach, *Automatic*, DOI: 10.1016/j.automatica.2020.109102.

[4] Yang H Y, Jiang Y C, and Yin S, Adaptive fuzzy fault-tolerant control for markov jump systems with additive and multiplicative actuator faults, *IEEE Transaction on Fuzzy Systems*, 2021, **29**(4): 772–785.

[5] Tong S C, Ming X, and Li Y X, Observer-based adaptive fuzzy tracking control for strict-feedback nonlinear systems with unknown control gain functions, *IEEE Transactions on Cybernetics*, 2020, **50**(9): 3903–3913.

[6] Wang L J, Basin M V, Li H Y, et al., Observer-based composite adaptive fuzzy control for nonstrict-feedback systems with actuator failures, *IEEE Transactions on Fuzzy Systems*, 2018, **26**(4): 2336–2347.

[7] Li R C and Zhang X F, Adaptive sliding mode observer design for a class of T-S fuzzy descriptor fractional order systems, *IEEE Transactions on Fuzzy Systems*, 2020, **28**(9): 1951–1960.

[8] Liu C X, Wen G L, Zhao Z J, et al., Neural-network-based lliding-mode control of an uncertain robot using dynamic model approximated switching gain, *IEEE Transactions on Cybernetics*, 2021, **51**(5): 2339–2346.

[9] Zhang Y Y and Li S, Adaptive near-optimal consensus of high-order nonlinear multi-agent systems with heterogeneity, *Automatic*, 2017, **85**: 426–432.

[10] Xie X P, Zhou Q, Yue D, et al., Relaxed control design of discrete-time takagi-sugeno fuzzy systems: An event-triggered real-time scheduling approach, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2018, **48**(12): 2251–2262.

[11] Lai G Y, Liu Z, Zhang Y, et al., Adaptive backstepping-based tracking control of a class of uncertain switched nonlinear systems, *Automatic*, 2019, **91**: 301–310.

[12] Xu N, Zao X D, Zong G D, et al., Adaptive control design for uncertain switched nonstrict-feedback nonlinear systems to achieve asymptotic tracking performance, *Applied Mathematics and Computation*, DOI: 10.1016/j.amc.2021.126344.

[13] Guo J and Diao J D, Prediction-based event-triggered identification of quantized input FIR systems with quantized output observations, *Science China-Information Sciences*, 2020, **63**(1): 136–147.

[14] Diao S Z, Sun W, Su S F, et al., Adaptive fuzzy event-triggered control for single-link flexible-joint robots with actuator failures, *IEEE Transactions on Cybernetics*, DOI: 10.1109/TCYB.2021.3049536.

[15] Chen Z Y, Nu B, Zhao X D, et al., Model-based adaptive event-triggered control of nonlinear continuous-time systems, *Applied Mathematics and Computation*, DOI: 10.1016/j.amc.2021.126330.

[16] Huo X, Karimi R H, Zhao X D, et al., Adaptive-critic design for decentralized event-triggered control of constrained nonlinear interconnected systems within an identifier-critic framework, *IEEE*

Transactions on Cybernetics, DOI: 10.1109/TCYB.2020.3037321.

[17] Wang T, Qiu J B, Yin S, et al., Performance-based adaptive fuzzy tracking control for networked industrial processes, *IEEE Transactions on Cybernetics*, 2016, **46**(8): 1760–1770.

[18] Wu Z J, Yang J, and Shi P, Adaptive tracking for stochastic nonlinear systems with markovian switching, *IEEE Transactions on Automatic Control*, 2010, **55**(9): 2135–2141.

[19] Sun K K, Mou S S, Qiu J B, et al., Adaptive fuzzy control for nontriangular structural stochastic switched nonlinear systems with full state constraints, *IEEE Transactions on Fuzzy Systems*, 2019, **27**(8): 1587–1601.

[20] Yang Y, Miao S T, Yue D, et al., Adaptive neural containment seeking of stochastic nonlinear strict-feedback multi-agent systems, *Neurocomputing*, 2020, **400**: 393–400.

[21] Wang F, Wang J M, Wang K, et al., Adaptive backstepping sliding mode control of uncertain semi-strict nonlinear systems and application to permanent magnet synchronous motor, *Journal of Systems Science and Complexity*, 2021, **34**(2): 552–571.

[22] Fu X X, Kang Y, Li P F, et al., Control for a class of stochastic mechanical systems based on the discrete-time approximate observer, *Journal of Systems Science and Complexity*, 2019, **32**(2): 526–541.

[23] Chen C, Liu Z, Zhang Y, et al., Asymptotic fuzzy tracking control for a class of stochastic strict-feedback systems, *IEEE Transactions on Fuzzy Systems*, 2017, **25**(3): 556–568.

[24] Sui S, Chen C L P, and Tong S C, Fuzzy adaptive finite-time control design for nontriangular stochastic nonlinear systems, *IEEE Transactions on Fuzzy Systems*, 2019, **27**(11): 172–184.

[25] Li T S, Li Z F, Wang D, et al., Output-feedback adaptive neural control for stochastic nonlinear time-varying delay systems with unknown control directions, *IEEE Transactions on Neural Networks and Learning Systems*, 2015, **26**(6): 1188–1201.

[26] Wu L B and Yang G H, Adaptive output neural network control for a class of stochastic nonlinear systems with dead-zone nonlinearities, *IEEE Transactions on Neural Networks and Learning Systems*, 2017, **28**(3): 726–739.

[27] Li P P and Shen Y J, Adaptive sampled-data observer design for a class of nonlinear systems with unknown hysteresis, *Journal of Systems Science and Complexity*, 2020, **52**(1): 561–579.

[28] Shen F, Wang X J, and Yin X H, Adaptive output-feedback control for a class of stochastic nonlinear systems with unknown control directions and hysteresis input, *International Journal of Systems Science*, 2020, **52**(3): 657–670.

[29] Huang L T, Li Y M, and Tong S C, Fuzzy adaptive output feedback control for MIMO switched nontriangular structure nonlinear systems with unknown control directions, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2020, **50**(2): 550–564.

[30] Qiu J B, Sun K K, Rudas I J, et al., Command filter-based adaptive NN control for MIMO nonlinear systems with full-state constraints and actuator hysteresis, *IEEE Transactions on Cybernetics*, 2020, **50**(7): 2905–2915.

[31] Lu K X, Liu Z, Chen C L P, et al., Event-triggered neural control of nonlinear systems with rate-dependent hysteresis input based on a new filter, *IEEE Transactions on Neural Networks and Learning Systems*, 2020, **31**(4): 1270–1284.

[32] Li Y M, Tong S C, and Li T S, Adaptive fuzzy output feedback control of MIMO nonlinear uncertain systems with time-varying delays and unknown backlash-like hysteresis, *Neurocomputing*, 2012, **93**: 56–66.

[33] Wang J H, Liu Z, Zhang Y, et al., Neural adaptive event-triggered control for nonlinear uncertain

stochastic systems with unknown hysteresis, *IEEE Transactions on Neural Networks and Learning Systems*, 2019, **30**(11): 3300–3312.

[34] Wang H Q, Chen B, Liu K F, et al., Adaptive neural tracking control for a class of nonstrict-feedback stochastic nonlinear systems with unknown backlash-like hysteresis, *IEEE Transactions on Neural Networks and Learning Systems*, 2014, **25**(5): 947–958.

[35] Li Y X and Tong S C, A bound estimation approach for adaptive fuzzy asymptotic tracking of uncertain stochastic nonlinear systems, *IEEE Transactions on Cybernetics*, DOI: 10.1109/TCYB.2020.3030276.

[36] Su C Y, Stepanenko Y, Svoboda J, et al., Robust adaptive control of a class of nonlinear systems with unknown backlash-like hysteresis, *IEEE Transactions on Automatic Control*, 2000, **45**(12): 2427–2432.

[37] Li H Y, Zhao S Y, He W, et al., Adaptive finite-time tracking control of full state constrained nonlinear systems with dead-zone, *Automatica*, 2019, **100**: 99–107.

[38] Liu Y J, Lu S, Tong S C, et al., Adaptive control-based barrier Lyapunov functions for a class of stochastic nonlinear systems with full state constraints, *Automatica*, 2018, **87**: 83–93.

[39] Jin X and Xu J X, Iterative learning control for output-constrained systems with both parametric and nonparametric uncertainties, *Automatica*, 2013, **49**(8): 2508–2516.

[40] Sun W, Su S F, Wu Y Q, et al., Adaptive fuzzy control with high-order barrier Lyapunov functions for high-order uncertain nonlinear systems with full-state constraints, *IEEE Transactions on Cybernetics*, 2020, **50**(8): 3424–3432.

[41] Yu X and Xie X J, Output feedback regulation of stochastic nonlinear systems with stochastic iISS inverse dynamics, *IEEE Transactions on Automatic Control*, 2010, **55**(2): 304–320.