J Syst Sci Complex (2020) 33: 706–724

Event-Triggered Consensus for Discrete-Time Multi-agent Systems with Parameter Uncertainties Based on a Predictive Control Scheme

LIU Xiaoyu · XUAN Yongbo · ZHANG Zhongyu · DIAO Zhaoshi · MU Zhenxing · LI Zhitao

DOI: 10.1007/s11424-020-8318-6
Received: 12 November 2018 / Revised: 9 April 2019

Abstract In this paper, the event-triggered consensus for linear discrete-time multi-agent systems with parameter uncertainties is investigated. The parameter uncertainty is assumed to be norm-bounded. An event-triggered consensus protocol based on the predictive control method is proposed to make the multi-agent system achieve consensus. And for the design of the consensus protocol, the problem of estimating the control input is transformed into the problem of estimating state differences between agents. Furthermore, the event-triggered consensus protocol proposed in this paper only demands each agent to mornitor its state to determine its event-triggered instants. A sufficient existence condition for the consensus protocol is proposed based on the linear matrix inequality. And a sufficient condition for the nonexistence of the Zeno-like behaviour is also derived. Finally, a numerical example is given to illustrate that the event-triggered consensus protocol proposed in this paper can make the multi-agent system with parameter uncertainties achieve consensus effectively.

Keywords Consensus, event-triggered strategy, multi-agent systems, predictive scheme.

1 Introduction

In the 1970s, the agent was proposed in the field of intelligence firstly[1]. Then more and more researchers began to pay their attention to the multi-agent systems. Olfati-Saber and Murray investigated the consensus for multi-agent systems with directed topologies and one-order integrator dynamics. The effects of switching communication topologies and communication delays

LIU Xiaoyu (Corresponding author)
China North Industries Group Corporation Limited China Ordnance Industry Navigation and Control Technology Institute, Beijing 100821*, China.* Email: liuxiaoyu9426@163.com.
XUAN Yongbo
Beijing Aeronuatical Technology Research Center, Beijing 100076*, China.*
ZHANG Zhongyu · DIAO Zhaoshi · MU Zhenxing · LI Zhitao
China North Industries Group Corporation Limited China Ordnance Industry Navigation and Control Technology Institute, Beijing 100821*, China.*
◇ *This paper was recommended for publication by Editor SUN Jian.*

were also taken into account[2]. Ren, et al. proposed necessary and sufficient consensus conditions for multi-agent systems based on the eigenvalues of Laplacian matrix[3–5]. He and Cao considered the consensus problem of high-order multi-agent systems and provided a sufficient and necessary condition for the consensus[6]. It was worth noting that there was a common assumption in the aforementioned works, i.e., all the dynamics of agents were assumed to be certain. But in some practical situations, such an assumption was too restrictive[7–10]. So the consensus problem of multi-agent systems with uncertainties received increasing attention in recent years. Zhang, et al. investigated the distributed robust consensus for multi-agent systems with uncertain double-order integrators and directed communication topologies[11]. Song, et al. studied the second-order leader-following consensus problem of nonlinear multi-agent systems without assuming that the interaction diagraph was strongly connected or contained a directed spanning tree[12]. Li, et al. considered the distributed robust consensus for linear multi-agent systems with parameter uncertainties. And the distributed consensus protocols were designed for both continuous-time and discrete-time multi-agent systems[13]. Su and Huang studied the cooperative global output regulation problem of the heterogeneous second order nonlinear uncertain multi-agent systems[14]. Hu, et al. considered the consensus for heterogeneous multi-agent systems which are governed by the Euler-Lagrange system and the double-order integrator system respectively. And the parameters of the Euler-Lagrange system are assumed to be uncertain[15]. Hu and Cao investigated the consensus problem of nonlinear multi-agent systems by introducing the event-triggered mechanism into intermittent control[16].

It should be noted that in all the above results agents should broadcast its information to their neighbors periodically (discrete-time) or continuously (continuous-time). It is clear that periodic or continuous communication in multi-agent systems would leads to inefficient implementations with respect to energy consumption, communication resources, and processor usage[17–21]. Therefore, more and more researchers began to investigate the consensus for multi-agent systems without periodic or continuous communication. Much attention had been paid to the event-triggered strategy in order to save the communication resources. Johansson and Dimarogonas investigated the consensus for linear continuous-time multi-agent systems based on the event-triggered strategy[22]. Then Seyboth, et al. designed the consensus protocols based on the event-triggered strategy for multi-agent systems with one-order/second-order integrator dynamics respectively[23]. For the multi-agent systems with the general linear dynamics, Zhang, et al. proposed a consensus protocol based on the event-triggered strategy to make them achieve consensus at last[24]. But it had been assumed that the communication topology among agents was undirected in [24]. Motivated by this point, Yang, et al. investigated the consensus for multi-agent systems with the general linear dynamics and the directed communication topology, and the continuous-time and discrete-time cases were considered in [25] and [26] where the Zeno behaviour and Zeno-like behaviour were also proved nonexistent for the continuous-time and discrete-time cases, respectively.

But to the best of my knowledge, there are few works which consider the event-triggered consensus problem of discrete-time multi-agent systems with parameter uncertainties, which motivates this study. In this paper, an event-triggered consensus protocol is proposed for discrete-time multi-agent systems with parameter uncertainties based on a predictive control scheme. The main challenge of designing the event-triggered consensus protocol is to estimate the control input. The problem of estimating the control input is transformed into the problem of estimating the state differences between agents. Furthermore, the event-triggered consensus protocol proposed in this paper only requires each agent to monitor its state to determine its event-triggered instants. When the event is triggered, the agent will update its consensus protocol. At the same time, its state and state difference information will be sent to its out-neighbor agents. And the agent will update its consensus protocol and estimation of state difference when it receives the state and state difference information from its in-neighbor agents. In addition, a sufficient existence condition for the feedback gain matrix K is proposed in terms of the linear matrix inequality. And a sufficient condition for the nonexistence of the Zeno-like behaviour is also derived. From the simulation, it can be seen that the method in this paper can make the discrete-time multi-agent systems with parameter uncertainties achieve consensus and save much communication resources.

The rest of this paper is organized as follow. Some useful notations and the graph theory are introduced in Section 2. The design of the event-triggered consensus protocol based on the predictive scheme is given in Section 3. In Sections 4 and 5, the analysis of consensus and Zeno-like behaviour are presented. A numerical example is given in Section 6 to illustrate the efficiency of the event-triggered consensus protocol presented in this paper. At last, Section 7 concludes the paper.

2 Design of the Event-Triggered Consensus Protocol

2.1 Notation and Graph Theory

The notations and the graph theory used in this paper are introduced in this section. Let $\mathcal{R}^{m\times n}$ denote the set of $m \times n$ real matrices. $0_{m\times n}$ denotes the $m \times n$ matrix with all zeros. I_n denotes the $n \times n$ identity matrix. 1_n denotes the $n \times 1$ column vector of all ones. A diagonal matrix with x_i $(i = 1, 2, \cdots, n)$ is denoted by $\text{diag}(x_1, x_2, \cdots, x_n)$. $A \otimes B$ denotes the Kronecker product of matrices A and B. Let $\| * \|$ denote the Euclidean norm for vectors and the induced 2-norm for matrices respectively. $\text{Re}(*)$ denotes the real part of a complex number and $\lambda_i(*)$ denotes the ith eigenvalue of a matrix.

The communication topology among the N agents can be represented with a weighted graph $\mathcal{G} = (\mathcal{V}, \varepsilon, \mathcal{A})$. The N agents in a multi-agent system can be regarded as the nodes $\mathcal{V} = 1, 2, \cdots, N$ of the graph $\mathcal{G}$. A directed graph contains a directed spanning tree if there are directed paths from one node to other ones. The adjacency matrix can be defined as $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{N\times N}$ associated with the directed graph $\mathcal{G}$. Assume that for all $i \in \mathcal{V}$, $a_{ii} = 0$, $a_{ij} > 0$ if $e_{ij} \in \varepsilon$ and $a_{ij} = 0$ otherwise. The directed edge $e_{ij} \in \varepsilon$ denotes that agent j can receive information from agent i. So agent i can be called as agent j's in-neighbor agent and agent j can be called as agent i's out-neighbor agent. $\mathcal{L} = [l_{ij}] \in \mathcal{R}^{N\times N}$ denotes the Laplacian matrix of the directed graph $\mathcal{G}$, where $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ $(i \neq j)$.

2.2 Preliminaries

A discrete-time multi-agent system with the parameter uncertainty is consisted of N agents, where the dynamics of agent i is characterized as follow:

$$x_i(k+1) = (A + \Delta A)x_i(k) + Bu_i(k), \tag{1}$$

where $x_i(k) \in \mathcal{R}^{n\times 1}$ and $u_i(k) \in \mathcal{R}^{m\times 1}$ are the state and the control input respectively. $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$ are constant matrices. $i = 1, 2, \cdots, N$. $\Delta A \in \mathcal{R}^{n\times n}$ is an unknown matrix which represents the uncertainty and assumed to be in the form of $\Delta A = DFE$, where F is the uncertainty with the compatible dimension satisfying $F^{\mathrm{T}}F \le \sigma^2 I$. F is Lebesgue measurable and $\sigma > 0$ is a constant. D and E are known constant matrices which characterise the structure of the uncertainty. The communication topology among the N agents can be described as a directed weighted graph $\mathcal{G}$. And the following assumption is necessary to obtain the main result.

Assumption 1 The matrix pair (A, B) in (1) is stabilizable and the graph $\mathcal{G}$ contains a directed spanning tree.

2.3 Design of the Protocol

The well-known consensus protocol for general linear discrete-time multi-agent systems is

$$u_i(k) = K\sum_{j=1}^{N} a_{ij}\big[x_i(k) - x_j(k)\big]. \tag{2}$$

In consensus protocol (2), it can be seen that the periodic communication is necessary because each agent should monitor the states of its neighbor agents at each sampling instant. In this paper, an event-triggered strategy is designed. An agent will send its current state information to its out-neighbor agents only when its event is triggered. The basic idea is that an agent predicts its future states on the basis of its normal model and its state on the last triggered instant, and an event will be triggered when the estimation error is large than than a prescribed threshold function.

Let $k^i_{s_i}$ denote the most recent triggering instant of agent i, $s_i = 1, 2, \cdots$ represent the sequence number of the triggering instants of the agent i, and $\widehat{u}_i$ represent the control input estimation of agent i, and an estimation of $x_i(k)$ can be obtained as

$$\widehat{x}_i(k) = A^{(k-k^i_{s_i})}x_i(k^i_{s_i}) + \sum_{h=k^i_{s_i}}^{k-1} A^{(k-h-1)}B\widehat{u}_i(h). \tag{3}$$

Define the estimation error as

$$e_i(k) = \widehat{x}_i(k) - x_i(k). \tag{4}$$

Then the triggering function is defined as

$$f_i(k) = \big\|e_i(k)\big\| - c\alpha^k, \tag{5}$$

where $c > 0$, $\max|\text{Re}\{\lambda(A+\lambda_i(\mathcal{L})BK)\}| < \alpha < 1$, $\lambda_i(\mathcal{L}) \neq 0$, $i = 2, 3, \cdots, N$.

For the triggering function (5), when $f_i(k) \geq 0$, the event of agent i is triggered. Then agent i sends its current information (including the state x_i and the estimated state difference vector $\widehat{\theta}_i$ defined below) to its out-neighbor agents and updates its own consensus protocol. At the same time, the estimation error $e_i(k)$ is reset to 0. And if the triggering function $f_i(k) < 0$, it means that the communication between agent i and its out-neighbor agents is unnecessary until the next event is triggered. On the other hand, agent i will receive the latest information from its in-neighbor agents when the events of its in-neighbor agents are triggered.

An event-triggered consensus protocol similar to (2) is proposed as follows

$$u_i(k) = K\sum_{j=1}^{N} a_{ij}\big[\widehat{x}_i(k) - \widehat{x}_j(k)\big], \tag{6}$$

where $K \in \mathcal{R}^{m\times n}$ is the feedback gain matrix to be determined and $\widehat{x}_j(k)$ is the estimation of $x_j(k)$.

Similar to (3), $\widehat{x}_j(k)$ can be obtained by

$$\widehat{x}_j(k) = A^{(k-k^j_{s_j})}x_j(k^j_{s_j}) + \sum_{h=k^j_{s_j}}^{k-1} A^{(k-h-1)}B\widehat{u}_j(h). \tag{7}$$

From (3) and (7), it can be seen that the main challenge to implement the protocol (6) is how to estimate the control input $\widehat{u}_i$ and $\widehat{u}_j$. A method of estimating $\widehat{u}_i$ and $\widehat{u}_j$ is presented as follows.

The state difference between agent i and other agents can be defined as $\theta_{ij}(k) = x_i(k) - x_j(k)$, and the state difference vector

$$\theta_i(k) = [\theta^{\rm T}_{i1}(k), \theta^{\rm T}_{i2}(k), \cdots, \theta^{\rm T}_{i(i-1)}(k), \theta^{\rm T}_{i(i+1)}(k), \cdots, \theta^{\rm T}_{iN}(k)]^{\rm T},$$

so the traditional consensus protocol (2) can be rewritten as

$$u_i(k) = K(a^*_i \otimes I_n)\theta_i(k), \tag{8}$$

where $a^*_i = [a_{i1}, a_{i2}, \cdots, a_{i(i-1)}, a_{i(i+1)}, \cdots, a_{iN}]$.

From (8), we can see that the problem of estimating u_i can be transformed into the problem of estimating θ_i. Then the following equation can be obtained by substituting (8) into (1) if $\Delta A = 0$,

$$\theta_i(k+1) = [I_{N-1} \otimes A + (d_i + 1_{N-1}a^*_i - \mathcal{A}^*_i) \otimes BK]\theta_i(k), \tag{9}$$

where

$$d_i = \mathrm{diag}(l_{11}, l_{22} \cdots, l_{(i-1)(i-1)}, l_{(i+1)(i+1)}, \cdots, l_{NN}),$$

$$\mathcal{A}_i^* = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1(i-1)} & a_{1(i+1)} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2(i-1)} & a_{2(i+1)} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{(i-1)1} & a_{(i-1)2} & \cdots & a_{(i-1)(i-1)} & a_{(i-1)(i+1)} & \cdots & a_{(i-1)N} \\ a_{(i+1)1} & a_{(i+1)2} & \cdots & a_{(i+1)(i-1)} & a_{(i+1)(i+1)} & \cdots & a_{(i+1)N} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{N(i-1)} & a_{N(i+1)} & \cdots & a_{NN} \end{bmatrix}.$$

From (9), the following predictor can be designed to estimate $\theta_i(k)$

$$\widehat{\theta}_i(k) = \mathit{\Omega}_i^{(k-k_{s_i}^i)} \widehat{\theta}_i(k_{s_i}^i), \tag{10}$$

where $\widehat{\theta}_i(k)$ is the estimation of $\theta_i(k)$, $k_{s_i}^i$ is the most recent triggering instant of agent i, $\mathit{\Omega}_i = I_{N-1} \otimes A + (d_i + 1_{N-1} a_i^* - \mathcal{A}_i^*) \otimes BK$.

Therefore, $\widehat{u}_i(k)$ can be designed as

$$\widehat{u}_i(k) = K(a_i^* \otimes I_n) \mathit{\Omega}_i^{(k-k_{s_i}^i)} \widehat{\theta}_i(k_{s_i}^i), \quad k \geq k_{s_i}^i. \tag{11}$$

$\widehat{u}_j(k)$ can be designed as

$$\widehat{u}_j(k) = K(a_j^* \otimes I_n) \mathit{\Omega}_j^{(k-k_{s_j}^j)} \widehat{\theta}_j(k_{s_j}^j), \quad k \geq k_{s_j}^j, \tag{12}$$

where $k_{s_j}^{ij}$ is the most recent triggering instant of agent j.

Remark 2.1 It should be noted that (10) utilizes the artificial closed-loop system (9) to predict the future state. This prediction is not always precise, however, simulation results show that this method has a good performance.

Remark 2.2 It should be noted that if the event of agent j is triggered, then agent j will send its current state $x_j(k_{s_j}^j)$ and the estimated state difference vector $\widehat{\theta}_j(k_{s_j}^j)$ to its neighbor agent i at the triggering instant $k_{s_j}^j$. At the same time, agent i will update the state difference vector $\widehat{\theta}_i$ using the received information. Then $\widehat{\theta}_i(k)$, $k \geq k_{s_j}^j$ will be estimated based on the updated $\widehat{\theta}_i$.

Remark 2.3 Without loss of generality, all the agents are assumed to be triggered at $k = 0$. Therefore, $\widehat{\theta}_p(0) = \theta_p(0)$, $p = 1, 2, \cdots, N$.

From (11)–(12), the event-triggered consensus protocol (6) can be rewritten as

$$\begin{aligned} u_i(k) &= K\sum_{j=1}^{N} a_{ij}\left[\widehat{x}_i(k)-\widehat{x}_j(k)\right] \\ &= K\sum_{j=1}^{N} a_{ij}\Bigg[\Big(A^{(k-k_{s_i}^i)}x_i(k_{s_i}^i)+\sum_{h=k_{s_i}^i}^{k-1}A^{(k-h-1)}BK \\ &\qquad \times(a_i^*\otimes I_n)\Omega_i^{(h-k_{s_i}^i)}\widehat{\theta}_i(k_{s_i}^i)\Big)-\Big(A^{(k-k_{s_j}^j)}x_j(k_{s_j}^j) \\ &\qquad +\sum_{h=k_{s_j}^j}^{k-1}A^{(k-h-1)}BK(a_j^*\otimes I_n)\Omega_j^{(h-k_{s_j}^j)}\widehat{\theta}_j(k_{s_j}^j)\Big)\Bigg], \end{aligned} \tag{13}$$

where $K\in\mathcal{R}^{m\times n}$ is the feedback gain matrix to be determined.

Definition 2.4 For the discrete-time multi-agent system (1), if $\lim_{k\to\infty}\|x_i(k)-x_j(k)\|=0$ holds, it can be said that the protocol (13) solves the consensus problem of the multi-agent system (1) or the multi-agent system (1) achieves consensus under the protocol (13).

Lemma 2.5 (see [27]) *If the graph $\mathcal{G}$ contains a directed spanning tree, zero is the simple eigenvalue of the Laplacian matrix $\mathcal{L}$ and all the other eigenvalues have positive real parts. Otherwise, 1_N is a right eigenvector associated with the zero eigenvalue.*

Lemma 2.6 (see [13]) *For the parameter uncertainty $\Delta A = DFE$ which satisfies that $F^{\rm T}F\leq\sigma^2 I$, the discrete-time system $x_i(k+1)=(A+\Delta A)x_i(k)$ is quadratically stable if and only if A is Schur stable and $\|E(zI-A)^{-1}D\|_\infty<\frac{1}{\sigma}$ holds, where $\sigma>0$ is a constant.*

Lemma 2.7 (see [28]) *There exists a positive-definite matrix P such that*

$$P(A+H_1F_1E_1)^{\rm T}+(A+H_1F_1E_1)P<0$$

for all admissible uncertainty F_1 satisfying $F_1^{\rm T}F_1\leq\varrho^2 I$ if and only if there exists a scalar $\varepsilon>0$ such that

$$PA^{\rm T}+AP+\frac{1}{\varepsilon}PE_1^{\rm T}E_1P+\varepsilon\varrho^2H_1H_1^{\rm T}<0.$$

Lemma 2.8 *For the discrete-time multi-agent system* (1) *with the event-triggered consensus protocol* (13) *and the triggering function* (5)*, if all the matrices $A+\lambda_s(\mathcal{L})BK$ $(s=2,3,\cdots,N)$ are Schur, then all the matrices $\Omega_i=I_{N-1}\otimes A+(d_i+1_{N-1}a_i^*-\mathcal{A}_i^*)\otimes BK$ $(i=1,2,\cdots,N)$ are also Schur.*

Proof See the appendix. ∎

3 Analysis of the Consensus for Multi-agent Systems with Parameter Uncertainties

Following the aforementioned sections, the following Theorem can be obtained, which gives a necessary and sufficient condition for the solvability of the consensus problem under the proposed protocol.

Theorem 3.1 *For the linear discrete-time multi-agent system* (1) *under the event-triggered consensus protocol* (13) *and the triggering function* (5)*, the consensus problem of the multi-agent system* (1) *can be solved if and only if all the matrices* $A+\lambda_i(\mathcal{L})BK$ *are Schur, where* $\lambda_i(\mathcal{L}) \neq 0$ *and* $\|E(zI - A - \lambda_i(\mathcal{L})BK)^{-1}D\|_\infty < \frac{1}{\sigma}$ $(i = 2, 3, \cdots, N)$ *holds. Furthermore, if there exist matrices* $Q > 0$*,* R *and a scalar* $\gamma > 0$ *such that*

$$\begin{bmatrix} -Q & QA^{\mathrm{T}} & QE^{\mathrm{T}} & R^{\mathrm{T}} \\ AQ & Q - \sigma^2 DD^{\mathrm{T}} + \gamma\kappa^2 BB^{\mathrm{T}} & 0 & 0 \\ EQ & 0 & -I & 0 \\ R & 0 & 0 & -\gamma I \end{bmatrix} < 0, \tag{14}$$

where $\kappa = \max|\lambda_i(\mathcal{L})|$ $(i = 2, 3, \cdots, N)$*. The feedback gain matrix is given by* $K = RQ^{-1}$*.*

Proof From the estimation error $e_i(k)$, it is clear that

$$A^{(k-k^i_{s_i})}x_i(k^i_{s_i}) + \sum_{h=k^i_{s_i}}^{k-1} A^{(k-h-1)}BK(a_i^* \otimes I_n)\varOmega_i^{(h-k^i_{s_i})}\widehat{\theta}_i(k^i_{s_i}) = e_i(k) + x_i(k). \tag{15}$$

Then, substituting (15) into (13) yields

$$u_i(k) = K\sum_{j=1}^{N} a_{ij}[x_i(k) + e_i(k) - x_j(k) - e_j(k)] = K[l_i x(k) + l_i e(k)], \tag{16}$$

where $x(k) = [x_1^{\mathrm{T}}(k), x_2^{\mathrm{T}}(k), \cdots, x_N^{\mathrm{T}}(k)]^{\mathrm{T}}$, $e(k) = [e_1^{\mathrm{T}}(k), e_2^{\mathrm{T}}(k), \cdots, e_N^{\mathrm{T}}(k)]^{\mathrm{T}}$, and $l_i = [l_{i1}, l_{i2}, \cdots, l_{iN}]$ represents the ith row of the Laplacian matrix $\mathcal{L}$.

Substituting (16) into (1) yields

$$x_i(k+1) = (A + \Delta A)x_i(k) + BK[l_i x(k) + l_i e(k)]. \tag{17}$$

Define $\delta_i(k) = x_i(k) - x_1(k)$, so it can be known that the multi-agent system (1) can achieve consensus if $\lim_{k\to\infty}\|\delta_i(k)\| = 0$ holds. Then it can be obtained based on (17) that

$$\delta_i(k+1) = x_i(k+1) - x_1(k+1) = (A + \Delta A)\delta_i(k) + BK(l_i - l_1)[x(k) - e(k)]. \tag{18}$$

Then (18) can be transformed into the following form.

$$\delta(k+1) = \varPi\delta(k) + [(\mathcal{A}_{22} + 1_{N-1}a_1 + \mathcal{M}) \otimes BK]e(k), \tag{19}$$

where

$$\varPi = I_{N-1} \otimes (A + \Delta A) + (\mathcal{L}_{22} + 1_{N-1}a_1^*) \otimes BK, \quad \delta(k) = [\delta_2^{\mathrm{T}}(k), \delta_3^{\mathrm{T}}(k), \cdots, \delta_N^{\mathrm{T}}(k)]^{\mathrm{T}},$$
$$a_1^* = [a_{12}, a_{13}, \cdots, a_{1N}], \quad a_i = [a_{i1}, a_{i2}, \cdots, a_{iN}], \quad \alpha_i = a_{i1} + a_{i2} + \cdots + a_{iN},$$

$$\mathcal{M} = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 \\ \alpha_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1 & 0 & \cdots & 0 \end{bmatrix} \in \mathcal{R}^{(N-1)\times N}, \quad \mathcal{L}_{22} = \begin{bmatrix} \alpha_2 & -a_{23} & \cdots & -a_{2N} \\ -a_{32} & \alpha_3 & \cdots & -a_{3N} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{N2} & -a_{N3} & \cdots & \alpha_N \end{bmatrix},$$

$$\mathcal{A}_{22} = \begin{bmatrix} -a_{21} & -a_{22} & \cdots & -a_{2N} \\ -a_{31} & -a_{32} & \cdots & -a_{3N} \\ \vdots & \vdots & & \vdots \\ -a_{N1} & -a_{N2} & \cdots & -a_{NN} \end{bmatrix}.$$

For agent i, if its event is triggered, i.e., $f_i(k) \geq 0$, then its estimation error $e_i(k)$ will be reset to 0. Therefore, before the event is triggered, $f_i(k)$ will not cross 0 and the estimation error $e_i(k)$ satisfies $\|e_i(k)\| \leq c\alpha^k$. And α satisfies $0 < \alpha < 1$, hence, $\|e(k)\| \leq \sqrt{N}c\alpha^k$ and $\lim_{k\to\infty} \|e(k)\| = 0$ holds. Therefore, the stability problem of the system (19) can be transformed into the stability problem of the following system.

$$\delta(k+1) = [I_{N-1} \otimes (A + \Delta A) + (\mathcal{L}_{22} + 1_{N-1}a_1^*) \otimes BK]\delta(k). \tag{20}$$

Following Lemma 2.8, the invertible matrix can be taken as

$$S^{-1} = \begin{bmatrix} 1 & 0 \\ -1_{N-1} & I_{N-1} \end{bmatrix}.$$

By the definition of the Laplacian matrix $\mathcal{L}$, the following equation can be derived.

$$S^{-1}\mathcal{L}S = \begin{bmatrix} 0 & -a_1^* \\ 0 & \mathcal{L}_{22} + 1_{N-1}a_1^* \end{bmatrix}. \tag{21}$$

It can be proved similar to Lemma 2.8 that the eigenvalues of the matrix Π are the same as the ones of $(A + \Delta A) + \lambda_i(\mathcal{L})BK$, $i = 2, 3, \cdots, N$. Therefore, the stability of the system (20) is equivalent to the stability of the following system.

$$\delta_i(k+1) = [A + \Delta A + \lambda_i(\mathcal{L})BK]\delta_i(k), \quad i = 2, 3, \cdots, N. \tag{22}$$

Following Lemma 2.6, it can be seen that the system (22) with the parameter uncertainty $\Delta A = DFE$ is quadratically stable for the admissible uncertainty F which satisfies $F^{\mathrm{T}}F \leq \sigma^2 I$ if and only if $A + \lambda_i(\mathcal{L})BK$ $(i = 2, 3, \cdots, N)$ are Schur stable and $\|E(zI - A - \lambda_i(\mathcal{L})BK)^{-1}D\|_\infty < \frac{1}{\sigma}$ holds, where $\sigma > 0$ is a constant. Therefore, the multi-agent system (1) can achieve consensus under the event-triggered consensus protocol (13) and the triggering function (5) if and only if all the matrices $A + \lambda_i(\mathcal{L})BK$, $i = 2, 3, \cdots, N$ are Schur and $\|E(zI - A - \lambda_i(\mathcal{L})BK)^{-1}D\|_\infty < \frac{1}{\sigma}$ holds.

Next, it will be proved that the method of choosing the feedback gain matrix K in this paper can guarantee the quadratic stability of the system (22) with the admissible parameter uncertainty ΔA. In virtue of the discrete-time bounded real lemma in [29], it can be known that the matrices $A+\lambda_i(\mathcal{L})BK$ are Schur and $\|E(zI-A-\lambda_i(\mathcal{L})BK)^{-1}D\|_\infty < \frac{1}{\sigma}$ holds if and only if there exist matrices $P_i > 0$ $(i = 2, 3, \cdots, N)$ such that

$$\Xi_i^{\mathrm{T}} P_i \Xi_i - P_i + E^{\mathrm{T}} E + \sigma^2 \Xi_i^{\mathrm{T}} P_i D(I - \sigma^2 D P_i D^{\mathrm{T}})^{-1} D P_i \Xi_i < 0, \tag{23}$$

where $\Xi_i = A + \lambda_i(\mathcal{L})BK$.

Assumed that $|\lambda_i(\mathcal{L})| \leq \kappa$, $i = 2, 3, \cdots, N$, if there exists a matrix $P > 0$ such that the following inequality holds for all $|\lambda| \leq \kappa$, then (23) surely holds.

$$\Xi^{\mathrm{T}} P \Xi - P + E^{\mathrm{T}} E + \sigma^2 \Xi^{\mathrm{T}} P D(I - \sigma^2 D P D^{\mathrm{T}})^{-1} D P \Xi < 0, \tag{24}$$

where $\Xi = A + \lambda BK$.

It is clear that (24) can be rewritten as

$$\Xi^{\mathrm{T}}(P^{-1} - \sigma^2 D D^{\mathrm{T}})^{-1} \Xi - P + E^{\mathrm{T}} E < 0. \tag{25}$$

Define $Q = P^{-1}$, then (25) can be rewritten into the following form based on the Schur Complement Lemma in [30].

$$\begin{bmatrix} -Q^{-1} & \Xi^{\mathrm{T}} & E^{\mathrm{T}} \\ \Xi & Q - \sigma^2 D D^{\mathrm{T}} & 0 \\ E & 0 & -I \end{bmatrix} < 0. \tag{26}$$

Then (26) can be decomposed into the following form by substituting $\Xi = A + \lambda BK$.

$$\begin{bmatrix} -Q^{-1} & A^{\mathrm{T}} & E^{\mathrm{T}} \\ A & Q - \sigma^2 D D^{\mathrm{T}} & 0 \\ E & 0 & -I \end{bmatrix} + \begin{bmatrix} 0 \\ B \\ 0 \end{bmatrix} \lambda \begin{bmatrix} K & 0 & 0 \end{bmatrix} + \begin{bmatrix} K^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix} \lambda \begin{bmatrix} 0 & B^{\mathrm{T}} & 0 \end{bmatrix} < 0. \tag{27}$$

Following Lemma 2.7, it can be known that (27) holds for all $|\lambda| \leq \kappa$ if and only if there exists a scalar γ can make the following inequality hold.

$$\begin{bmatrix} -Q^{-1} & A^{\mathrm{T}} & E^{\mathrm{T}} & K^{\mathrm{T}} \\ A & Q - \sigma^2 D D^{\mathrm{T}} + \gamma \kappa^2 B B^{\mathrm{T}} & 0 & 0 \\ E & 0 & -I & 0 \\ K & 0 & 0 & -\gamma I \end{bmatrix} < 0. \tag{28}$$

Define $R = KQ$ and $\Phi = \mathrm{diag}(Q, I, I, I) > 0$, then the (14) in Theorem 3.1 can be derived by multiplying both sides of (28) by Φ. The proof is completed. ▮

4 Analysis of the Zeno-Like Behaviour

Zeno behaviour is a definition in hybrid systems which means that infinite discrete transitions occur in a finite time interval. For the event-triggered consensus problem of continuous-time multi-agent systems, Zeno behaviour means that there is no a positive lower bound on the inter-event times[23]. However, Zeno-like behaviour is defined for the event-triggered consensus problem of discrete-time multi-agent systems in the sense of sampling[26]. And Zeno-like behaviour is said to be nonexistent when all the inter-event times are ensured to be greater than at least one sampling interval. Similar to [31], a sufficient condition for the nonexistence of the Zeno-like behaviour is given in this paper.

Theorem 4.1 *For the discrete-time multi-agent system* (1) *under the event-triggered consensus protocol* (13) *and the triggering function* (5)*, the Zeno-like behaviour is nonexistent if the function* (29) *has a solution* $\tau_1^i > 1$ *for* $0 < a_A + \beta < 1$ *or the function* (30) *has a solution* $\tau_2^i > 1$ *for* $a_A + \beta > 1$ *for all the event-triggered instants* $k_{s_i}^i$*, where* $i = 1, 2, \cdots, N$.

$$(\mu_i + \varphi_1 + \phi_i + \omega_i \|\widehat{\theta}_i(k_1^i)\|)\tau_1^i = c\alpha^{k_1^i + \tau_1^i}, \tag{29}$$

$$(\mu_i + \varphi_1 (a_A + \beta)^{k_1^i + \tau_2^i} + \phi_i + \omega_i \|\widehat{\theta}_i(k_1^i)\|)\tau_2^i = c\alpha^{k_1^i + \tau_2^i}. \tag{30}$$

Proof It is clear that

$$\begin{aligned} e_i(k+1) - e_i(k) &= A^{(k+1-k_{s_i}^i)} x_i(k_{s_i}^i) + \sum_{h=k_{s_i}^i}^{k} A^{(k-h)} B\widehat{u}_i(h) - x_i(k+1) \\ &\quad - A^{(k-k_{s_i}^i)} x_i(k_{s_i}^i) - \sum_{h=k_{s_i}^i}^{k-1} A^{(k-h-1)} B\widehat{u}_i(h) + x_i(k) \\ &= (A - I_n)\left[A^{(k-k_{s_i}^i)} x_i(k_{s_i}^i) + \sum_{h=k_{s_i}^i}^{k-1} A^{(k-h-1)} B\widehat{u}_i(h) - x_i(k)\right] \\ &\quad + B\widehat{u}_i(k) - Bu_i(k) - \Delta A x_i(k) \\ &= (A - I_n) e_i(k) + B\widehat{u}_i(k) - Bu_i(k) - \Delta A x_i(k). \end{aligned} \tag{31}$$

Then (16) can be rewritten as

$$u_i(k) = K(l_i^* \otimes I_n)\delta(k) + K(l_i \otimes I_n) e(k), \tag{32}$$

where $l_i^* = [l_{i2}, l_{i3}, \cdots, l_{iN}]$, $l_i = [l_{i1}, l_{i2}, \cdots, l_{iN}]$.

Substituting (32) into (31) yields

$$\begin{aligned} e_i(k+1) - e_i(k) =& (A - I_n) e_i(k) + B\widehat{u}_i(k) \\ & - BK(l_i^* \otimes I_n)\delta(k) - BK(l_i \otimes I_n) e(k) - \Delta A x_i(k). \end{aligned} \tag{33}$$

From (19), it can be derived that

$$\begin{aligned}\delta(k) &= \Pi\delta(k-1) + We(k-1)\\ &= \Pi^2\delta(k-2) + \Pi We(k-2) + We(k-1)\\ &= \Pi^k\delta(0) + \sum_{s=0}^{k-1}\Pi^{k-s-1}We(s),\end{aligned} \tag{34}$$

where $\Pi = I_{N-1}\otimes(A+\Delta A) + (\mathcal{L}_{22} + 1_{N-1}a_1^*)\otimes BK$, $W = (\mathcal{A}_{22} + 1_{N-1}a_1 + \mathcal{M})\otimes BK$.

Then it has that

$$\begin{aligned}\|\delta(k)\| &= \|\Pi^k\delta(0) + \sum_{s=0}^{k-1}\Pi^{k-s-1}We(s)\|\\ &\leq \|\Pi^k\|\|\delta(0)\| + \sum_{s=0}^{k-1}\|\Pi^{k-s-1}\|\|W\|\|e(s)\|\\ &\leq \beta_1\|\Pi\|^k + \sum_{s=0}^{k-1}\beta_2\|\Pi\|^{k-s-1}\alpha^s\\ &= \beta_1 a_\Pi^k + \beta_2(a_\Pi^{k-1} + a_\Pi^{k-2}\alpha + \cdots + \alpha^{k-1})\\ &\leq \beta_1 a_\Pi^k + \beta_2\frac{a_\Pi^k + \alpha^k}{\alpha - a_\Pi}\\ &= \left(\beta_1 + \frac{\beta_2}{\alpha - a_\Pi}\right)a_\Pi^k + \frac{\beta_2}{\alpha - a_\Pi}\alpha^k,\end{aligned} \tag{35}$$

where $\beta_1 = \|\delta(0)\|$, $\beta_2 = \sqrt{N}c\|W\|$, $a_\Pi = \|I_{N-1}\otimes A + (\mathcal{L}_{22} + 1_{N-1}a_1^*)\otimes BK\| + \sigma\|D\|\|E\|$.

Following (12), the following equation can be obtained.

$$\|B\widehat{u}_i(k)\| = \|BK(a_i^*\otimes I_n)\Omega_i^{(k-k_{s_i}^i)}\widehat{\theta}_i(k_{s_i}^i)\| \leq \|BK\|\|a_i^*\otimes I_n\|a_{\Omega_i}^{k-k_{s_i}^i}\|\widehat{\theta}_i(k_{s_i}^i)\|, \tag{36}$$

where $a_{\Omega_i} = \|\Omega_i\|$.

Then (1) can be rewritten as

$$\begin{aligned}x_i(k) &= (A+\Delta A)x_i(k-1) + Bu_i(k-1)\\ &= (A+\Delta A)^k x_i(0) + \sum_{s=0}^{k-1}(A+\Delta A)^{k-s-1}Bu_i(s).\end{aligned} \tag{37}$$

From $\Delta A = DFE$ and $F^{\mathrm{T}}F \leq \sigma^2 I$, it can be known that $\|F\| \leq \sigma$, where $\sigma > 0$ is a constant. Then it can be obtained based on (32) and (35).

$$\begin{aligned}&\|\Delta A x_i(k)\|\\ \leq&\|\Delta A\|\|x_i(k)\|\\ \leq&\|D\|\|F\|\|E\|\left(\|(A+\Delta A)^k x_i(0)\| + \sum_{s=0}^{k-1}\|(A+\Delta A)^{k-s-1}Bu_i(s)\|\right)\end{aligned}$$

$$
\begin{aligned}
&\le\sigma\|D\|\|E\|(\|A\|+\|\Delta A\|)^k\|x_i(0)\|+\sum_{s=0}^{k-1}(\|A\|+\|\Delta A\|)^{k-s-1}\|Bu_i(s)\|\\
&\le\beta(a_A+\beta)^k\|x_i(0)\|+\sum_{s=0}^{k-1}(a_A+\beta)^{k-s-1}\|Bu_i(s)\|\\
&\le\beta\|x_i(0)\|(a_A+\beta)^k+\sum_{s=0}^{k-1}(a_A+\beta)^{k-s-1}\|BK\|\|l_i^*\otimes I_n\|\|\delta(s)\|\\
&\quad+\sum_{s=0}^{k-1}(a_A+\beta)^{k-s-1}\|BK\|\|l_i\otimes I_n\|\|e(s)\|\\
&\le\eta_1(a_A+\beta)^k+\sum_{s=0}^{k-1}(a_A+\beta)^{k-s-1}\eta_2 a_\Pi^s\\
&\quad+\sum_{s=0}^{k-1}(a_A+\beta)^{k-s-1}\eta_3\alpha^s+\sum_{s=0}^{k-1}(a_A+\beta)^{k-s-1}\eta_4\alpha^s\\
&\le\eta_2\frac{(a_A+\beta)^k+a_\Pi^k}{a_\Pi-(a_A+\beta)}+(\eta_3+\eta_4)\frac{(a_A+\beta)^k+\alpha^k}{\alpha-(a_A+\beta)}+\eta_1(a_A+\beta)^k\\
&=\varphi_1(a_A+\beta)^k+\varphi_2 a_\Pi^k+\varphi_3\alpha^k,
\end{aligned}
\tag{38}
$$

where

$$
\begin{aligned}
&a_A=\|A\|,\quad \beta=\sigma\|D\|\|E\|,\quad \eta_1=\beta\|x_i(0)\|,\quad \eta_2=\|BK\|\|l_i^*\otimes I_n\|\left(\beta_1+\frac{\beta_2}{\alpha-a_\Pi}\right),\\
&\eta_3=\|BK\|\|l_i^*\otimes I_n\|\frac{\beta_2}{\alpha-a_\Pi},\quad \eta_4=\|BK\|\|l_i\otimes I_n\|\sqrt{N}c,\\
&\varphi_1=\eta_1+\frac{\eta_2}{a_\Pi-a_A-\beta}+\frac{\eta_3+\eta_4}{\alpha-a_A-\beta},\quad \varphi_2=\frac{\eta_2}{a_\Pi-a_A-\beta},\quad \varphi_3=\frac{\eta_3+\eta_4}{\alpha-a_A-\beta}.
\end{aligned}
$$

Then substituting (35), (36) and (38) into (33) yields

$$
\begin{aligned}
&\|e_i(k+1)-e_i(k)\|\\
&=\|(A-I_n)e_i(k)+B\widehat{u}_i(k)-Bu_i(k)-\Delta Ax_i(k)\|\\
&\le\|A-I_n\|\|e_i(k)\|+\|B\widehat{u}_i(k)\|+\|Bu_i(k)\|+\|\Delta Ax_i(k)\|\\
&\le\mu_i\alpha^k+\varphi_1(a_A+\beta)^k+\phi_i a_\Pi^k+\omega_i a_{\Omega_i}^{k-k_{s_i}^i}\|\widehat{\theta}_i(k_{s_i}^i)\|.
\end{aligned}
\tag{39}
$$

Define k_1^i and k_2^i as the two consecutive triggering instants of the agent i, and they satisfy that $0<k_1^i<k_2^i$ and $e_i(k_1^i)=e_i(k_2^i)=0$. Therefore, it can be seen that $e_i(k)=\sum_{s=k_1^i}^{k-1}[e(s+1)-e(s)]$, then it has that

$$
\|e_i(k)\|\le\sum_{s=k_1^i}^{k-1}\|e(s+1)-e(s)\|. \tag{40}
$$

Then substituting (39) into (40) yields

$$
\|e_i(k)\|\le\sum_{s=k_1^i}^{k-1}\left[\mu_i\alpha^s+\varphi_1(a_A+\beta)^s+\phi_i a_\Pi^s+\omega_i a_{\Omega_i}^{s-k_1^i}\|\widehat{\theta}_i(k_1^i)\|\right]. \tag{41}
$$

From the triggering function (5), it can be seen that if the triggering function $f_i(k) \geq 0$, i.e., $\|e_i(k)\| \geq c\alpha^k$, the event of agent i is triggered. Therefore, the events will not be triggered before the following equality holds.

$$\sum_{s=k_1^i}^{k-1} \left[\mu_i\alpha^s + \varphi_1(a_A+\beta)^s + \phi_i a_{\Pi}^s + \omega_i a_{\Omega_i}^{s-k_1^i}\|\widehat{\theta}_i(k_1^i)\|\right] = c\alpha^k. \tag{42}$$

From the previous sections, it can be known that the multi-agent system (1) can achieve consensus if $\lim_{k\to\infty}\|\delta(k)\| = 0$ holds, so the matrix Π is Schur based on (19). Furthermore, it can be seen that Ω_i $(i = 1, 2, \cdots, N)$ are also Schur from Lemma 2.8. Hence, $0 < a_{\Pi}, a_{\Omega_i} < 1$. And it has been defined that $\max|\mathrm{Re}(\lambda_i(\Pi))| < \alpha < 1$ in the triggering function (5). Define $\tau^i = k_2^i - k_1^i$ and consider the different possible values of $a_A + \beta$, it can be sure that the value of τ^i must be greater than or equal to the solutions of the function, i.e., $\tau^i \geq \tau_1^i$ or $\tau^i \geq \tau_2^i$. Therefore, the Zeno-like behaviour can be proved nonexistent if the function (29) has a solution $\tau_1^i > 1$ when $0 < a_A + \beta < 1$ or the function (29) has a solution $\tau_2^i > 1$ when $a_A + \beta > 1$ for all the event-triggered instants $k_{s_i}^i$, where $i = 1, 2, \cdots, N$. The proof is completed. ∎

5 Simulation

In this section, a numerical example is given to illustrate the effectiveness of the method proposed in this paper. Consider the discrete-time multi-agent system consists of six agents. The dynamics model of agent i is described as the system (1) with

$$A = \begin{bmatrix} 0.6375 & 0.3625 \\ -0.1813 & 1.1813 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1625 \\ -0.0187 \end{bmatrix}, \quad D = \begin{bmatrix} 0.2 & 0.1 \\ -0.2 & 0.1 \end{bmatrix},$$

$$F = 0.05, \quad E = \begin{bmatrix} 0.2 & -0.1 \\ 0 & 0.4 \end{bmatrix}, \quad \Delta A = DFE = \begin{bmatrix} 0.002 & 0.001 \\ -0.002 & 0.003 \end{bmatrix}.$$

The communication topology among the six agents is described by a weighted graph as in Figure 1. And the Laplacian matrix of the graph is

$$L = \begin{bmatrix} 3 & 0 & 0 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

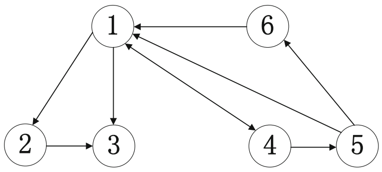

Figure 1 Communication topology

Let the initial state of the system (1) be $x_1(0) = [1.4 \ \ 0.3]^{\mathrm{T}}$, $x_2(0) = [-0.5 \ \ 1.2]^{\mathrm{T}}$, $x_3(0) = [0 \ \ 1]^{\mathrm{T}}$, $x_4(0) = [0.7 \ \ 0]^{\mathrm{T}}$, $x_5(0) = [0.8 \ \ -1.5]^{\mathrm{T}}$, $x_6(0) = [1.4 \ \ -0.2]^{\mathrm{T}}$. The feedback gain matrix K is choosen as $[-1 \ \ 2]$. And the other parameters are chosen as $c = 0.5$ and $\alpha = 0.92$.

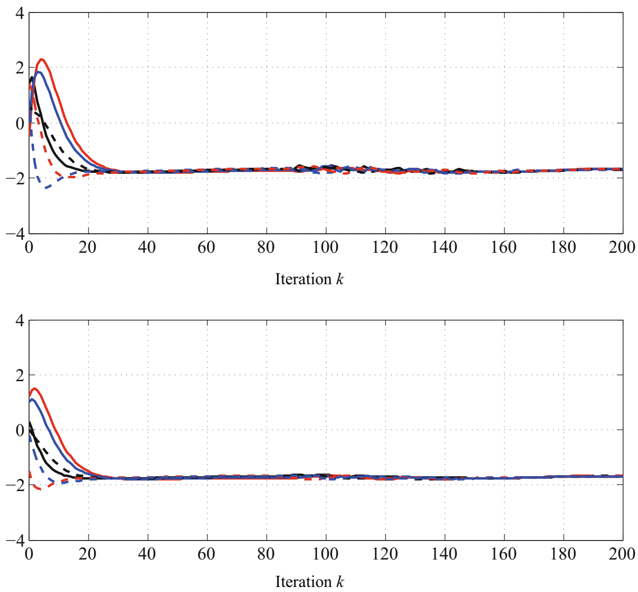

Figure 2 The state trajectories of all the agents

Figure 2 shows the state trajectories of six agents. It can be seen that the discrete-time multi-agent system with the parameter uncertainty can achieve consensus. Therefore, the event-triggered consensus protocol based on the predictive scheme proposed in this paper can solve the consensus problem of multi-agent systems effectively. And in Figure 3, the estimation error of each agent and the threshold of errors are presented. It can be seen that when the estimation error reaches the threshold, the event is triggered, then the estimation error is reset to zero.

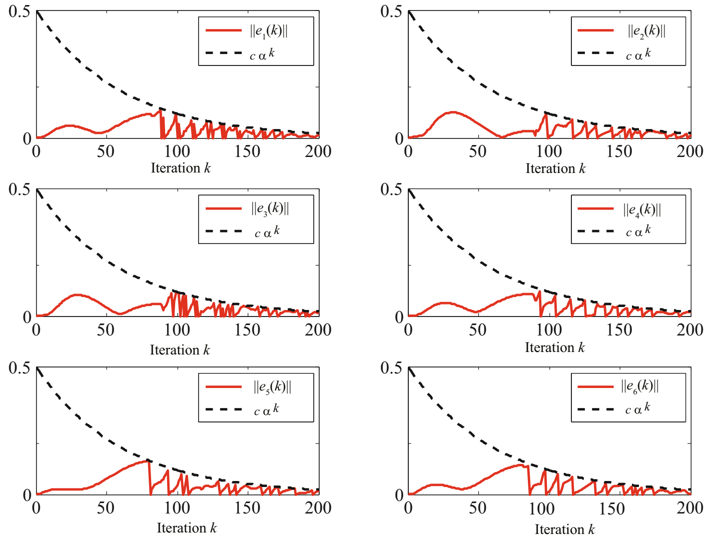

Figure 3 The measurement errors and the threshold of errors

6 Conclusion

This paper has investigated the event-triggered consensus for discrete-time multi-agent systems with the parameter uncertainty based on a predictive control scheme. An event-triggered protocol has been designed for the discrete-time multi-agent systems with the parameter uncertainty to achieve consensus. The consensus protocol proposed in this paper only requires each agent to observe its own state to determine the event-triggered instants. And a necessary and sufficient condition for the solvability of the consensus problem has been proposed and a method to design the feedback gain matrix has been presented in terms of the linear matrix inequality. And a sufficient condition for the nonexistence of the Zeno-like behaviour has been also derived. At last, a numerical example has been given to illustrate that the event-triggered consensus protocol proposed in this paper can make the multi-agent system with the parameter uncertainty achieve consensus.

References

[1] Maes P, *Desiging Autonomous Agents: Theory and Practice from Biology to Engineering and Back*, Elsevier Science, Amsterdam, Netherlands, 1990.

[2] Olfatisaber R and Murray R M, Consensus problems in networks of agents with switching topology and time-delays, *IEEE Transactions on Automatic Control*, 2004, **49**(9): 1520–1533.

[3] Ren W and Beard R W, Consensus seeking in multiagent systems under dynamically changing interaction topologies, *IEEE Transactions on Automatic Control*, 2005, **50**(5): 655–661.

[4] Yu W, Chen G, and Cao M, Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems, *Automatica*, 2010, **46**(6): 1089–1095.

[5] Yang X R and Liu G P, Necessary and sufficient consensus conditions of descriptor multi-agent systems, *IEEE Transactions on Circuits & Systems I: Regular Papers*, 2012, **59**(11): 2669–2677.

[6] He W and Cao J, Consensus control for high-order multi-agent systems, *IET Control Theory & Applications*, 2011, **5**(1): 231–238.

[7] Song Q, Liu F, Cao J, et al., M-matrix strategies for pinning-controlled leader-following consensus in multiagent systems with nonlinear dynamics, *IEEE Transactions on Cybernetics*, 2013, **43**(6): 1688–1697.

[8] Li S, Feng G, Luo X, et al., Output consensus of heterogeneous linear discrete-time multiagent systems with structural uncertainties, *IEEE Transactions on Cybernetics*, 2017, **45**(12): 2868–2879.

[9] Huang W, Zeng J, and Sun H, Robust consensus for linear multi-agent systems with mixed uncertainties, *Systems & Control Letters*, 2015, **76**: 56–65.

[10] Wu X, Tang Y, Cao J, et al., Distributed consensus of stochastic delayed multi-agent systems under asynchronous switching, *IEEE Trans Cybernetics*, 2016, **46**(8): 1817–1827.

[11] Zhang W, Wang Z, and Guo Y, Robust consensus for uncertain multi-agent systems on directed communication topologies, *Proceedings of 49th IEEE Conference on Decision and Control*, Atlanta, GA, USA, 2010, 6317–6322.

[12] Song Q, Cao J, and Yu W, Second-order leader-following consensus of nonlinear multi-agent systems via pinning control, *Systems & Control Letters*, 2010, **59**(9): 553–562.

[13] Li Z, Duan Z, Xie L, et al., Distributed robust control of linear multi-agent systems with parameter uncertainties, *International Journal of Control*, 2012, **85**(8): 1039–1050.

[14] Su Y and Huang J, Cooperative global output regulation of heterogeneous second-order nonlinear uncertain multi-agent systems, *Automatica*, 2013, **49**(11): 3345–3350.

[15] Hu H X, Yu W, Xuan Q, et al., Group consensus for heterogeneous multi-agent systems with parametric uncertainties, *Neurocomputing*, 2014, **142**(1): 383–392.

[16] Hu A and Cao J, Consensus of multi-agent systems via intermittent event-triggered control, *International Journal of Systems Science*, 2017, **48**(2): 280–287.

[17] Zou L, Wang Z, Gao H, et al., Event-triggered state estimation for complex networks with mixed time delays via sampled data information: The continuous-time case, *IEEE Transactions on Cybernetics*, 2015, **45**(12): 2804–2815.

[18] Hu A, Cao J, Hu M, et al., Event-triggered consensus of multi-agent systems with noises, *Journal of the Franklin Institute*, 2015, **352**(9): 3489–3503.

[19] Wu D, Sun X M, Tan Y, et al., On designing event-triggered schemes for networked control systems subject to one-step packet dropout, *IEEE Transactions on Industrial Informatics*, 2016, **12**(3): 902–910.

[20] Wu D, Sun X M, Wen C, et al., Redesigned predictive event-triggered controller for networked control system with delays, *IEEE Transactions on Cybernetics*, 2016, **46**(10): 2195–2206.

[21] Nowzari C and Cortés J, Distributed event-triggered coordination for average consensus on weight-balanced digraphs, *Automatica*, 2016, **68**: 237–244.

[22] Johansson K H and Dimarogonas D V, Event-triggered control for multi-agent systems, *Proceddings of 48th IEEE Conference Decision Control and 28th Chinese Control Conference*, 2009, 7131–7136.

[23] Seyboth G S, Dimarogonas D V, and Johansson K H, *Event-Based Broadcasting for Multi-Agent Average Consensus*, Pergamon Press, Inc., 2013.

[24] Zhang Z, Hao F, Zhang L, et al., Consensus of linear multi-agent systems via event-triggered control, *International Journal of Control*, 2014, **87**(6): 1243–1251.

[25] Yang D, Ren W, and Liu X, Decentralized consensus for linear multi-agent systems under general directed graphs based on event-triggered/self-triggered strategy, *Proc. 54th IEEE Conference Decision and Control*, 2015, 1983–1988.

[26] Yang D, Liu X, and Chen W, Periodic event/self-triggered consensus for general continuous-time linear multi-agent systems under general directed graphs, *IET Control Theory & Applications*, 2015, **9**(3): 428–440.

[27] Ren W and Cao Y, *Distributed Coordination of Multi-agent Networks: Emergent Problems, Models, and Issues*, Springer-Verlag, London, 2011.

[28] Xie L, Fu M, and de Souza C E, H_∞ control and quadratic stabilization of systems with parameter uncertainty via output feedback, *IEEE Transactions on Automatic Control*, 1992, **37**(8): 1253–1256.

[29] Zhou K and Doyle J, *Essentials of Robust Control*, Upper Saddle River, NJ, Prentice Hall, 1998.

[30] Boyd S P, Linear matrix inequalities in system and control theory, *Procedings of IEEE International Conference Robotics Automation*, 1994, **85**(5): 798–799.

[31] Eqtami A, Dimarogonas D V, and Kyriakopoulos K J, Event-triggered control for discrete-time systems, *Proceedings of American Control Conference*, 2010, 4719–4724.

[32] Brewer J, Kronecker products and matrix calculus in system theory, *IEEE Transactions on Circuits and Systems*, 1978, **25**(9): 772–781.

Appendix

Proof of Lemma 2.8

Proof The invertible matrix can be taken as $S_i^{-1} = \begin{bmatrix} P_i^{\mathrm{T}} & Q_i^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, where $P_i = [1, 0, 0, \cdots, 0] \in \mathcal{R}^{1\times N}$, $Q_i \in \mathcal{R}^{(N-1)\times N}$ is the matrice which is derived by inserting -1_{N-1} before the ith column of the identity matrix I_{N-1}, i.e.,

$$Q_i = \begin{bmatrix} 1 & 0 & \cdots & -1 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -1 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & -1 & \cdots & 0 & 1 \end{bmatrix}.$$

Following the definition of the Laplacian matrix $\mathcal{L}$, it can be derived that

$$S_i^{-1}\mathcal{L}S_i = \begin{bmatrix} 0 & l_1^i \\ 0 & d_i + 1_{N-1}a_i^* - \mathcal{A}_i^* \end{bmatrix}, \tag{43}$$

where $l_1^i = [l_{11}, l_{12}, \cdots, l_{1(i-1)}, l_{1(i+1)}, \cdots, l_{1N}]$.

Assumed that $\lambda_1(\mathcal{L}) = 0, \lambda_2(\mathcal{L}), \cdots, \lambda_N(\mathcal{L})$ are the eigenvalues of the Laplacian matrix $\mathcal{L}$. From (42), it can be seen that $\lambda_s(\mathcal{L}),\ s = 2, 3, \cdots, N$ are the eigenvalues of $d_i + 1_{N-1}a_i^* - \mathcal{A}_i^*$. Therefore, there exists an invertible matrix T_i such that $d_i + 1_{N-1}a_i^* - \mathcal{A}_i^*$ is similar to a Jordan canonical matrix.

$$T_i^{-1}(d_i + 1_{N-1}a_i^* - \mathcal{A}_i^*)T_i = J_i = \mathrm{diag}(J_1^i, J_2^i, \cdots, J_{m_i}^i), \tag{44}$$

where $J_n^i,\ n = 1, 2, \cdots, m_i$ are upper triangular Jordan blocks. And the principal diagonal elements of J_n^i are $\lambda_s(\mathcal{L})$, $s = 2, 3, \cdots, N$.

Therefore, it has that

$$(T_i \otimes I_n)^{-1} \varUpsilon (T_i \otimes I_n) = I_{N-1} \otimes A + J_i \otimes BK, \tag{45}$$

where $\varUpsilon = I_{N-1} \otimes A + (d_i + 1_{N-1}a_i^* - \mathcal{A}_i^*) \otimes BK$, and $I_{N-1} \otimes A + J_i \otimes BK$ is an upper triangular block matrix.

According to the properties of Kronecker product[32], it can be known that the eigenvalues of $I_{N-1} \otimes A + J_i \otimes BK$ are given by the eigenvalues of $A + \lambda_s(\mathcal{L})BK$, $s = 2, 3, \cdots, N$, i.e., the eigenvalues of the matrix $\varOmega_i$ are the same as the ones of $A + \lambda_s(\mathcal{L})BK$, $s = 2, 3, \cdots, N$. As a result, if all the matrices $A + \lambda_s(\mathcal{L})BK$, $s = 2, 3, \cdots, N)$ are Schur, the matrix $\varOmega_i$ is surely Schur. The proof is completed. ∎