J Syst Sci Complex (2020) 33: 1785–1803

Global Exponential Convergence of Neutral Type Competitive Neural Networks with D Operator and Mixed Delay

AOUITI Chaouki · ASSALI El Abed · BEN GHARBIA Imen

DOI: 10.1007/s11424-020-8225-x
Received: 18 August 2018 / Revised: 22 February 2019

Abstract The models of competitive neural network (CNN) was in recent past proposed to describe the dynamics of cortical cognitive maps with unsupervised synaptic modifications, where there are two types of memories: Long-term memories (LTM) and short-term memories (STM), LTM presents unsupervised and slow synaptic modifications and STM characterize the fast neural activity. This paper is concerned with a class of neutral type CNN's with mixed delay and D operator. By employing the appropriate differential inequality theory, some sufficient conditions are given to ensure that all solutions of the model converge exponentially to zero vector. Finally, an illustrative example is also given at the end of this paper to show the effectiveness of the proposed results.

Keywords Competitive neural networks, D operator, exponential convergence, neutral type delay.

1 Introduction

During the last few decades, some famous neural networks (NN's) models, such as Cohen Grossberg neural networks (CGNN's), Hopfield neural networks (HNN's), bidirectional associative memory neural networks (BAM), shunting inhibitory cellular neural networks (SICNN's) and competitive neural networks (CNN's) have received attention due to their wide range of applications including static image processing, combinatorial optimization, fault diagnosis, associative memory, pattern recognition, signal processing and several other scientific areas[1–12]. Especially, CNN's as one of the popular artificial NNs were investigated by many researchers[13–25]. As is well known, in this model, there are two-types of state variables: The short-term memory variables (STM) and long-term memory (LTM). Thus, there are two time scales in these NNs, one corresponds to the fast changes of the NN's states and the other corresponds to the slow changes of the synapses by external stimuli[13].

AOUITI Chaouki · ASSALI El Abed · BEN GHARBIA Imen
University of Carthage, Faculty of Sciences of Bizerta, Department of Mathematics, Research Units of Mathematics and Applications UR13ES47, 7021 Zarzouna, Bizerta, Tunisia.
Email: chaouki.aouiti@fsb.rnu.tn; elabed.assali@fsb.rnu.tn; imen.bengharbia@fsb.rnu.tn.
◇ *This paper was recommended for publication by Editor SUN Jian.*

On other hand, time delays can be encountered in the implementations of NNs, and the existence of time delays occurs in the response and communication time of neurons, and its existence may lead to instability (or) bad performance of systems. So, it is important to consider the dynamics of NNs with delay[8]. In particular, a type of time delay, namely neutral type time delays is one of many delay types and is objectively existent. Up to now, neutral type NNs can be classified as non-operator-based neutral functional differential equations[8–12, 26, 27, 43] or D-operator-based neutral functional differential equations[28, 30]. In general, neutral type NNs with D operator have more realistic significance than non-operator-based ones in most applications of NNs dynamics. This is to say, it is necessary to consider the effect of neutral type NNs with D operator when studying the stability of state estimation of NNs. Up to date, a few researchers have introduced the neutral type NNs with D operator[29–34]. For example, in [29] Aiping Zhang investigated the almost periodic solutions of SICNNs with neutral type proportional delays and D operators by using Lyapunov-Krasovskii functional method and differential inequality techniques. In [30], the authors investigated the existence and global exponential stability of pseudo almost periodic solutions of a class of neutral type cellular neural networks with D operator by using Lyapunov functional method and differential inequality techniques. In [31], the authors studied the global exponential convergence of neutral type SICNNs with D operator by using differential inequality techniques. Further, in [32] by using the Banach fixed point theorem and applying inequality techniques, some new sufficient conditions are obtained for the existence and exponential stability of the unique anti-periodic solution. In [33], the global exponential convergence of solutions for cellular neural networks with neutral type delays and D operator had been studied by differential inequality techniques. In [34] Zhang focused on the existence and global exponential stability of pseudo almost periodic solutions for neutral type shunting inhibitory cellular neural networks with D operator.

In addition, the global exponential convergence behavior of solutions plays a fundamental role in characterizing the behavior of NNs since the exponential convergence rate can be estimated (we refer the reader to [35, 36]). Therefore, it is worthwhile to investigate the global exponential convergence of neutral type CNN's with mixed delay and D operator. However, until now, the results on global exponential convergence of addressed system have not been obtained. The main contributions of this paper can be summarized in the following:

1) It is the first time to focus on global exponential convergence of neutral-type CNN's with mixed delay and D operator;

2) the neutral type competitive neural networks with D operator and mixed delay in this paper are more general than those of numerous previous works[13–25, 37];

3) we establish some new sufficient conditions which guarantee the global exponential convergence of neutral type CNN's with mixed delay and D operator;

4) the achieved results in this paper are original and complement to the ones obtained previously in [38].

The approach used in these papers consists of two steps.

♣ Description of the Model and Preliminaries.

♣ Global exponential convergence results.

The rest of the paper is organized as follows. In Section 2, models and preliminaries are presented that will be used later. In Section 3 applying the differential inequality theory and global exponential convergence theory, we will obtain some sufficient conditions which ensure the global exponential convergence of neutral-type CNNs with mixed delay and D operator. In Section 4, numerical simulations are given to illustrate the effectiveness of our theoretical results. In Section 5, we give a brief conclusion.

2 Description of the Model and Preliminaries

Consider the following competitive with neutral type delays and D operator:

$$\begin{cases} \text{STM}: \ \varepsilon\big[x_i(t)-p_i(t)x_i(t-\alpha_i(t))\big]' &= -c_i(t)x_i(t)+\sum\limits_{j=1}^{n}a_{ij}(t)f_j(x_j(t)) \\ & \quad +\sum\limits_{j=1}^{n}b_{ij}(t)f_j(x_j(t-\tau_{ij}(t)) \\ & \quad +\sum\limits_{j=1}^{n}d_{ij}(t)\displaystyle\int_0^{+\infty}N_{ij}(u)f_j(x_j(t-u))du \\ & \quad +B_i(t)\sum\limits_{j=1}^{n}m_{ij}(t)y_j+I_i(t), \\ \text{LTM}: \ \big[m_{ij}(t)-q_i(t)m_{ij}(t-\beta_i(t))\big]' &= -e_i(t)m_{ij}(t)+f_i(x_i(t))+J_i(t), \end{cases} \tag{1}$$

where $n \geq 2$, $t \geq t_0$ $i,j=1,2\cdots,n$, $x_i(\cdot)$ is the neuron currrent activity level; $m_{ij}(\cdot)$ is the synaptic efficiency; $c_i(\cdot)$, $e_i(\cdot)>0$ are the rate of decay; $p_i(\cdot)$, $q_i(\cdot)$, $a_{ij}(\cdot)$ and $b_{ij}(\cdot)$ represent the connection weights; $d_{ij}(\cdot)$ is the distributed delayed connection weight of the unit j on the unit i; $N_{ij}(t)$ is the delay kernel at time t; $y_j(\cdot)$ is the constant external stimulus; $f_j(x_j(\cdot))$ is the output of neurons; $I_i(t)$, $J_i(t)$ denote the external inputs on the ith neuron at time t; $B_i(\cdot)>0$ is the strength of the external stimulus; $\tau_{ij}(\cdot)\geq 0$, $\alpha_i(\cdot)\geq 0$ and $\beta_i(\cdot)\geq 0$ correspond to the transmission delays; ε is a fast time scale decided by STM and $\varepsilon>0$.

In this paper, taking $\varepsilon=1$ for convenience. After setting

$$S_i(t)=\sum_{j=1}^{n}y_jm_{ij}(t)=m_i(t)^{\mathrm{T}}y,$$

where $y=(y_1,y_2,\cdots,y_n)^{\mathrm{T}}$, $m_i(t)=(m_{i1}(t),m_{i2}(t),\cdots,m_{in}(t))^{\mathrm{T}}$, then (1) can be written as

$$\begin{cases} \mathrm{STM}: \ \big[x_i(t)-p_i(t)x_i(t-\alpha_i(t))\big]' = -c_i(t)x_i(t)+\sum\limits_{j=1}^{n}a_{ij}(t)f_j(x_j(t)) \\ \qquad\qquad +\sum\limits_{j=1}^{n}b_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) \\ \qquad\qquad +\sum\limits_{j=1}^{n}d_{ij}(t)\int_0^{+\infty}N_{ij}(u)f_j(x_j(t-u))du \\ \qquad\qquad +B_i(t)S_i(t)+I_i(t), \\ \mathrm{LTM}: \ \big[S_i(t)-q_i(t)S_i(t-\beta_i(t))\big]' = -e_i(t)S_i(t)+|y|^2f_i(x_i(t))+J_i(t), \end{cases} \tag{2}$$

where $i,j=1,2\cdots,n$, $k\in\mathbb{Z}_+$, $|y|^2=y_1^2+y_2^2+\cdots+y_n^2$ is a constant without loss of generality, the input stimulus y is assumed to be a normalized vector of unit magnitude $|y|^2=1$, then (2) are simplified as follows:

$$\begin{cases} \mathrm{STM}: \ \big[x_i(t)-p_i(t)x_i(t-\alpha_i(t))\big]' = -c_i(t)x_i(t)+\sum\limits_{j=1}^{n}a_{ij}(t)f_j(x_j(t)) \\ \qquad\qquad +\sum\limits_{j=1}^{n}b_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) \\ \qquad\qquad +\sum\limits_{j=1}^{n}d_{ij}(t)\int_0^{+\infty}N_{ij}(u)f_j(x_j(t-u))du \\ \qquad\qquad +B_i(t)S_i(t)+I_i(t), \\ \mathrm{LTM}: \ \big[S_i(t)-q_i(t)S_i(t-\beta_i(t))\big]' = -e_i(t)S_i(t)+f_i(x_i(t))+J_i(t). \end{cases} \tag{3}$$

The initial conditions of the system (3) are given by

$$\begin{aligned} &x_i(s)=\varphi_i(s), \quad s\in]-\infty,0], \\ &S_i(s)=\phi_i(s), \quad s\in]-\infty,0], \quad i=1,2,\cdots,n, \end{aligned} \tag{4}$$

where $\varphi_i(\cdot)$, $\phi_i(\cdot)$ are the real-valued continuous function defined on $]-\infty,0]$.

For convenience, we introduce the following notations:

$$g^+=\sup_{t\in[0,+\infty)}|g(t)|, \quad g^-=\inf_{t\in[0,+\infty)}|g(t)|, \quad \|x\|=\max_{1\le i\le n}|x_i|,$$

where g is a bounded and continuous function defined on $[0,+\infty)$. We denote $\mathbb{R}^n$ the set of all n-dimensional real vector, $\mathbb{R}=\mathbb{R}^1$ the set of real numbers.

Let $x=(x_1,x_2,\cdots,x_n)^{\mathrm{T}}\in\mathbb{R}^n$, $S=(S_1,S_2,\cdots,S_n)^{\mathrm{T}}\in\mathbb{R}^n$, $|x|$, $|S|$ are the absolute-value vector given by $|x|=(|x_1|,|x_2|,\cdots,|x_n|)^{\mathrm{T}}$, $|S|=(|S_1|,|S_2|,\cdots,|S_n|)^{\mathrm{T}}$.

Throughout the remainder of this paper, we also assume that the following conditions hold:

(H1) For each $i=1,2,\cdots,n$, there exist two bounded and continuous functions:

$$c_i^*, \ e_i^*:\mathbb{R}\to(0,+\infty),$$

and two positive constants k_i, $\overline{k}_i$, such that

$$\mathrm{e}^{-\int_s^t c_i(\theta)d\theta} \le k_i \mathrm{e}^{-\int_s^t c_i^*(\theta)d\theta}, \quad \mathrm{e}^{-\int_s^t e_i(\theta)d\theta} \le \overline{k}_i \mathrm{e}^{-\int_s^t \mathrm{e}_i^*(\theta)d\theta},$$

for all $t, s \in \mathbb{R}$ and $t - s \ge 0$.

(H2) For each $j = 1, 2, \cdots, n$, there exists a positive constant L_j such that

$$|f_j(u)| \le L_j |u|.$$

(H3) The delay kernel $N_{ij} : [0, +\infty) \to \mathbb{R}$ is continuous, and $|N_{ij}(t)|\mathrm{e}^{\omega t}$ is integrable on $[0, +\infty)$ for $\omega > 0$, and $i,\ j = 1, 2, \cdots, n$.

(H4) For $i,\ j = 1, 2, \cdots, n$, there exist positive constants $\xi_1, \xi_2, \cdots, \xi_n$ and λ_0 such that

- $$\sup_{t\ge 0}\Big\{ - c_i^*(t) + k_i \Big[\frac{1}{1-p_i^+}|c_i(t)p_i(t)| + \xi_i^{-1}\sum_{j=1}^n |a_{ij}(t)|L_j\xi_j \frac{1}{1-p_j^+} + \xi_i^{-1}\sum_{j=1}^n |b_{ij}(t)|L_j\xi_j \frac{1}{1-p_j^+} + \xi_i^{-1}\sum_{j=1}^n |d_{ij}(t)| \int_0^{+\infty} |N_{ij}(u)|du L_j \xi_j \frac{1}{1-p_j^+} + |B_i(s)|\frac{1}{1-q_i^+}\Big]\Big\} < 0,$$
- $$\sup_{t\ge 0}\Big\{ - \mathrm{e}_i^*(t) + \overline{k}_i \Big[\frac{1}{1-q_i^+}|e_i(s)q_i(s)| + L_i \frac{1}{1-p_i^+}\Big]\Big\} < 0,$$
- $I_i(t) = O(\mathrm{e}^{-\lambda_0 t})$, as $t \to +\infty$,
- $J_i(t) = O(\mathrm{e}^{-\lambda_0 t})$ as $t \to +\infty$.

Remark 2.1 It is well known that the global exponential convergence plays a key role in characterizing the dynamical behavior of neutral type CNNs (3) since the exponential convergence rate can be unveiled. In particular, assume that the leakage term coefficient function $c_i(\cdot)$ and $e_i(\cdot)$ are not oscillating, i.e.,

$$\begin{cases} \inf\limits_{t\in\mathbb{R}} c_i(t) > 0, \\ \inf\limits_{t\in\mathbb{R}} e_i(t) > 0, \quad i = 1, 2, \cdots, n, \end{cases} \tag{5}$$

some sufficient conditions ensuring the global exponential convergence for all solutions of NNs have been established in [39, 40].

On the other hand, as pointed out in [41], equations with oscillating coefficients appear in linearizations of population dynamics models with seasonal fluctuations, where during some seasons the death or harvesting rates may be greater than the birth rate. Clearly (5) is a special case of (H1) with $\inf_{t\in\mathbb{R}} c_i(t) = c_i^*$ and $\inf_{t\in\mathbb{R}} e_i(t) = e_i^*$, which means that our results are essentially new and more meaningful.

3 Global Exponential Convergence Results

In this section, we will establish a sufficient condition which ensures that all solutions of system (3) converge exponentially to zero vector.

Theorem 3.1 *If Assumptions* (H1)–(H4) *hold. Then, for each solution* $x(t)$, $S(t)$ *of the system* (3), *there exists a constant* $\lambda \in (0, \lambda_0)$ *such that*

$$x_i(t) = O(\mathrm{e}^{-\lambda t}), \quad S_i(t) = O(\mathrm{e}^{-\lambda t}) \quad as \quad t \to +\infty,$$

where $i \in \{1, 2, \cdots, n\}$ *and* $x_i(t)$, $S_i(t)$ *are an arbitrary state vector of* (3), (4).

Proof Let $(x_1(t), x_2(t), \cdots, x_n(t), S_1(t), S_2(t), \cdots, S_n(t))^{\mathrm{T}}$ be an arbitrary solution of the system (3) associated with the initial condition (4). Set, for all $i \in \{1, 2, \cdots, n\}$,

$$\begin{aligned} v_i(t) &= \xi_i^{-1} x_i(t), \quad V_i(t) = v_i(t) - p_i(t) v_i(t - \alpha_i(t)), \\ u_i(t) &= \xi_i^{-1} S_i(t), \quad U_i(t) = u_i(t) - q_i(t) u_i(t - \beta_i(t)). \end{aligned} \tag{6}$$

Then the system (3) can be rewritten as the following:

$$\begin{cases} \mathrm{STM}: \ V_i'(t) = [v_i(t) - p_i(t) v_i(t - \alpha_i(t))]' \\ \qquad = -c_i(t) V_i(t) - c_i(t) p_i(t) v_i(t - \alpha_i(t)) + \xi_i^{-1} \displaystyle\sum_{j=1}^n a_{ij}(t) f_j(x_j(t)) \\ \qquad + \xi_i^{-1} \displaystyle\sum_{j=1}^n b_{ij}(t) f_j(x_j(t - \tau_{ij}(t))) \\ \qquad + \xi_i^{-1} \displaystyle\sum_{j=1}^n d_{ij}(t) \int_0^{+\infty} N_{ij}(u) f_j(x_j(t-u)) du + \xi_i^{-1} B_i(t) S_i(t) + \xi_i^{-1} I_i(t), \\ \mathrm{LTM}: \ U_i'(t) = [u_i(t) - q_i(t) u_i(t - \beta_i(t))]' \\ \qquad = -e_i(t) U_i(t) - e_i(t) q_i(t) u_i(t - \beta_i(t)) + \xi_i^{-1} f_i(x_i(t)) + \xi_i^{-1} J_i(t). \end{cases} \tag{7}$$

Set

$$\begin{aligned} \varGamma_i(\sigma) = \sup_{t>0} \Bigg\{ & \sigma - c_i^*(t) + k_i \Bigg[\frac{\mathrm{e}^{\sigma \alpha_i^+}}{1 - p_i^+ \mathrm{e}^{\sigma \alpha_i^+}} |c_i(s) p_i(s)| + \xi_i^{-1} \sum_{j=1}^n |a_{ij}(s)| L_j \xi_j \frac{1}{1 - p_j^+ \mathrm{e}^{\sigma \alpha_j^+}} \\ & + \xi_i^{-1} \sum_{j=1}^n |b_{ij}(s)| L_j \xi_j \frac{\mathrm{e}^{\sigma \tau_{ij}^+}}{1 - p_j^+ \mathrm{e}^{\sigma \alpha_j^+}} + \xi_i^{-1} \sum_{j=1}^n |d_{ij}(s)| \int_0^{+\infty} |N_{ij}(u)| \mathrm{e}^{\sigma u} du \frac{L_j \xi_j}{1 - p_j^+ \mathrm{e}^{\sigma \alpha_j^+}} \\ & + |B_i(s)| \frac{1}{1 - q_i^+ \mathrm{e}^{\sigma \beta_i^+}} + \sigma \Bigg] \Bigg\}, \\ \varUpsilon_i(\sigma) = \sup_{t>0} \Bigg\{ & \sigma - \mathrm{e}_i^*(t) + \overline{k}_i \Bigg[\frac{\mathrm{e}^{\sigma \beta_i^+}}{1 - q_i^+ \mathrm{e}^{\sigma \beta_i^+}} |e_i(s) q_i(s) + L_i \frac{1}{1 - p_i^+ \mathrm{e}^{\sigma \alpha_i^+}} + \sigma \Bigg] \Bigg\}. \end{aligned}$$

Then by (H4), and the continuity of $\Gamma_i(\sigma)$, $\Upsilon_i(\sigma)$, we obtain

$$\begin{aligned}\Gamma_i(0)=\sup_{t>0}\Big\{&-c_i^*(t)+k_i\Big[\frac{1}{1-p_i^+}|c_i(s)p_i(s)|+\xi_i^{-1}\sum_{j=1}^n|a_{ij}(s)|L_j\xi_j\frac{1}{1-p_j^+}\\&+\xi_i^{-1}\sum_{j=1}^n|b_{ij}(s)|L_j\xi_j\frac{1}{1-p_j^+}+\xi_i^{-1}\sum_{j=1}^n|d_{ij}(s)|\int_0^{+\infty}|N_{ij}(u)|duL_j\xi_j\frac{1}{1-p_j^+}\\&+|B_i(s)|\frac{1}{1-q_i^+}\Big]\Big\}<0,\\\Upsilon_i(0)=\sup_{t>0}\Big\{&-\mathrm{e}_i^*(t)+\overline{k}_i\Big[\frac{1}{1-q_i^+}|e_i(s)q_i(s)|+L_i\frac{1}{1-p_i^+}\Big]\Big\}<0,\end{aligned}$$

and we can choose a constant

$$0<\lambda\in\min\Big\{\omega,\lambda_0,\min_{1\le i\le n}\inf_{t\ge0}c_i^*(t),\min_{1\le i\le n}\inf_{t\ge0}e_i^*(t)\Big\}$$

such that $1-p_j^+\mathrm{e}^{\lambda\alpha_j^+}>0$, $1-q_j^+\mathrm{e}^{\lambda\beta_j^+}>0$ and

$$\begin{aligned}\Gamma_i(\lambda)=\sup_{t>0}\Big\{&\lambda-c_i^*(t)+k_i\Big[\frac{\mathrm{e}^{\lambda\alpha_i^+}}{1-p_i^+\mathrm{e}^{\lambda\alpha_i^+}}|c_i(s)p_i(s)|+\xi_i^{-1}\sum_{j=1}^n|a_{ij}(s)|L_j\xi_j\frac{1}{1-p_j^+\mathrm{e}^{\lambda\alpha_j^+}}\\&+\xi_i^{-1}\sum_{j=1}^n|b_{ij}(s)|L_j\xi_j\frac{\mathrm{e}^{\lambda\tau_{ij}^+}}{1-p_j^+\mathrm{e}^{\lambda\alpha_j^+}}+|B_i(s)|\frac{1}{1-q_i^+\mathrm{e}^{\lambda\beta_i^+}}\\&+\xi_i^{-1}\sum_{j=1}^n|d_{ij}(s)|\int_0^{+\infty}|N_{ij}(u)|\mathrm{e}^{\lambda u}duL_j\xi_j\frac{1}{1-p_j^+\mathrm{e}^{\lambda\alpha_j^+}}+\lambda\Big]\Big\}<0,\end{aligned}\tag{8}$$

$$\Upsilon_i(\lambda)=\sup_{t>0}\Big\{\lambda-\mathrm{e}_i^*(t)+\overline{k}_i\Big[\frac{\mathrm{e}^{\lambda\beta_i^+}}{1-q_i^+\mathrm{e}^{\lambda\beta_i^+}}|e_i(s)q_i(s)|+L_i\frac{1}{1-p_i^+\mathrm{e}^{\lambda\alpha_i^+}}+\lambda\Big]\Big\}<0.\tag{9}$$

Let $V(t)=(V_1(t),V_2(t),\cdots,V_n(t))$, $U(t)=(U_1(t),U_2(t),\cdots,U_n(t))$,

$$\|\Phi\|_1=\max\{\|\varphi\|_\xi,\|\phi\|_\xi\},$$

where

$$\|\varphi\|_\xi=\sup_{t\le0}\max_{1\le i\le n}\xi_i^{-1}|\varphi_i(t)-p_i(t)\varphi_i(t-\alpha_i(t))|,\tag{10}$$

$$\|\phi\|_\xi=\sup_{t\le0}\max_{1\le i\le n}\xi_i^{-1}|\phi_i(t)-q_i(t)\phi_i(t-\beta_i(t))|.\tag{11}$$

For all $\varepsilon>0$, we have

$$\|V(0)\|<(\|\Phi\|_1+\varepsilon),\tag{12}$$

$$\|U(0)\|<(\|\Phi\|_1+\varepsilon),\tag{13}$$

and

$$\|V(t)\|<(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t}<K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t},\tag{14}$$

$$\|U(t)\|<(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t}<K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t},\tag{15}$$

with K is a sufficiently great constant such that

$$|\xi_i^{-1}I_i(t)| < \lambda K(\|\varPhi\|_1+\varepsilon)\mathrm{e}^{-\lambda(t-t_0)}, \tag{16}$$

$$|\xi_i^{-1}J_i(t)| < \lambda K(\|\varPhi\|_1+\varepsilon)\mathrm{e}^{-\lambda(t-t_0)}, \quad \text{for all } t\geq 0. \tag{17}$$

In what follows, we will show

$$\|V(t)\| < K(\|\varPhi\|_1+\varepsilon)\mathrm{e}^{-\lambda t}, \tag{18}$$

$$\|U(t)\| < K(\|\varPhi\|_1+\varepsilon)\mathrm{e}^{-\lambda t}, \quad \text{for all } \ t>0. \tag{19}$$

If Equations (18) and (19) are not true, without loss of generality, there must exist $i\in\{1,2,\cdots,n\}$ and $t_1>0$ such that

$$|V_i(t_1)| = \|V(t_1)\| = K(\|\varPhi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1}, \tag{20}$$

$$|U_i(t_1)| = \|U(t_1)\| = K(\|\varPhi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1}, \tag{21}$$

and

$$\|V(t)\| < K(\|\varPhi\|_1+\varepsilon)\mathrm{e}^{-\lambda t}, \tag{22}$$

$$\|U(t)\| < K(\|\varPhi\|_1+\varepsilon)\mathrm{e}^{-\lambda t}, \tag{23}$$

for all $t\in(-\infty,t_1]$.

Notice that

$$\begin{aligned}\mathrm{e}^{\lambda\theta}|y_j(\theta)| &\leq \mathrm{e}^{\lambda\theta}|v_j(\theta)-p_j(\theta)v_j(\theta-\alpha_j(\theta))| + \mathrm{e}^{\lambda\theta}|p_j(\theta)v_j(\theta-\alpha_j(\theta))|\\ &\leq \mathrm{e}^{\lambda\theta}|V_j(\theta)| + p_j^+\mathrm{e}^{\lambda\alpha_j^+}\mathrm{e}^{\lambda(\theta-\alpha_j(\theta))}|v_j(\theta-\alpha_j(\theta))|\\ &\leq K(\|\varPhi\|_1+\varepsilon)+p_j^+\mathrm{e}^{\lambda\alpha_j^+}\sup_{s\in(-\infty,t]}\mathrm{e}^{\lambda s}|v_j(s)|, \end{aligned}\tag{24}$$

$$\begin{aligned}\mathrm{e}^{\lambda\theta}|u_j(\theta)| &\leq \mathrm{e}^{\lambda\theta}|u_j(\theta)-q_j(\theta)u_j(\theta-\beta_j(\theta))| + \mathrm{e}^{\lambda\theta}|q_j(\theta)u_j(\theta-\beta_j(\theta))|\\ &\leq K(\|\varPhi\|_1+\varepsilon)+q_j^+\mathrm{e}^{\lambda\beta_j^+}\sup_{s\in(-\infty,t]}\mathrm{e}^{\lambda s}|u_j(s)|, \end{aligned}\tag{25}$$

and

$$\mathrm{e}^{\lambda t}|v_j(t)| \leq \sup_{s\in(-\infty,t]}\mathrm{e}^{\lambda s}|v_j(s)| \leq \frac{K((\|\varPhi\|_1+\varepsilon))}{1-p_j^+\mathrm{e}^{\lambda\alpha_j^+}}, \tag{26}$$

$$\mathrm{e}^{\lambda t}|u_j(t)| \leq \sup_{s\in(-\infty,t]}\mathrm{e}^{\lambda s}|u_j(s)| \leq \frac{K((\|\varPhi\|_1+\varepsilon))}{1-q_j^+\mathrm{e}^{\lambda\beta_j^+}}, \tag{27}$$

where $\theta\in(-\infty,t)$, $t\in(-\infty,t_1)$, $j=1,2,\cdots,n$.

Furthermore,

$$\begin{aligned}V_i'(s)+c_i(s)V_i(s) &= -c_i(s)p_i(s)v_i(s-\alpha_i(s))+\xi_i^{-1}\sum_{j=1}^n a_{ij}(s)f_j(x_j(s))\\ &\quad +\xi_i^{-1}\sum_{j=1}^n b_{ij}(s)f_j(x_j(s-\tau_{ij}(s)))+\xi_i^{-1}\sum_{j=1}^n d_{ij}(s)\int_0^{+\infty}N_{ij}(u)\\ &\quad \times f_j(x_j(s-u))du+\xi_i^{-1}B_i(s)S_i(s)+\xi_i^{-1}I_i(s), \end{aligned}\tag{28}$$

$$U_i'(s)+e_i(s)U_i(s) = -e_i(s)q_i(s)u_i(s-\beta_i(s))+\xi_i^{-1}f_i(x_i(s))+\xi_i^{-1}J_i(s), \tag{29}$$

with $s \in [0,t]$, $t \in [0,t_1]$, $i \in \{1,2,\cdots,n\}$.

Multiplying both sides of (28) (resp (29)) by $\mathrm{e}^{\int_0^s c_i(u)du}$ (resp $\mathrm{e}^{\int_0^s e_i(u)du}$) and integrating it on $[0,t]$, we have

$$\begin{aligned} V_i(t) = {} & V_i(0)\mathrm{e}^{-\int_0^t c_i(\theta)dv} + \int_0^t \mathrm{e}^{-\int_s^t c_i(u)du}\Big[- c_i(s)p_i(s)v_i(s-\alpha_i(s)) \\ & +\xi_i^{-1}\sum_{j=1}^n a_{ij}(s)f_j(x_j(s)) + \xi_i^{-1}\sum_{j=1}^n b_{ij}(s)f_j(x_j(s-\tau_{ij}(s))) \\ & +\xi_i^{-1}\sum_{j=1}^n d_{ij}(s)\int_0^{+\infty} N_{ij}(u)f_j(x_j(s-u))du + \xi_i^{-1}B_i(s)S_i(s) + \xi_i^{-1}I_i(s)\Big]ds, \end{aligned} \tag{30}$$

$$\begin{aligned} U_i(t) = {} & U_i(0)\mathrm{e}^{-\int_0^t e_i(u)du} \\ & + \int_0^t \mathrm{e}^{-\int_s^t e_i(u)du}\Big[-e_i(s)q_i(s)u_i(s-\beta_i(s)) + \xi_i^{-1}f_i(x_i(s)) + \xi_i^{-1}J_i(s)\Big]ds,\ t\in[0,t_1]. \end{aligned} \tag{31}$$

Thus, combining (8), (12), (16), (22) and (30), we have

$$\begin{aligned} |V_i(t_1)| = {} & \Big| V_i(0)\mathrm{e}^{-\int_{t_0}^{t_1} c_i(u)du} + \int_0^{t_1} \mathrm{e}^{-\int_s^{t_1} c_i(u)du}\Big[- c_i(s)p_i(s)v_i(s-\alpha_i(s)) \\ & +\xi_i^{-1}\sum_{j=1}^n a_{ij}(s)f_j(x_j(s)) + \xi_i^{-1}\sum_{j=1}^n b_{ij}(s)f_j(x_j(s-\tau_{ij}(s))) \\ & +\xi_i^{-1}\sum_{j=1}^n d_{ij}(s)\int_0^{+\infty} N_{ij}(u)f_j(x_j(s-u))du + \xi_i^{-1}B_i(s)S_i(s) + \xi_i^{-1}I_i(s)\Big]ds\Big| \\ \le {} & |V_i(0)|k_i\mathrm{e}^{-\int_0^{t_1} c_i^*(u)du} + \int_0^{t_1} \mathrm{e}^{-\int_s^{t_1} c_i^*(u)du}k_i\Big| - c_i(s)p_i(s)v_i(s-\alpha_i(s)) \\ & +\xi_i^{-1}\sum_{j=1}^n a_{ij}(s)f_j(x_j(s)) + \xi_i^{-1}\sum_{j=1}^n b_{ij}(s)f_j(x_j(s-\tau_{ij}(s))) \\ & +\xi_i^{-1}\sum_{j=1}^n d_{ij}(s)\int_0^{+\infty} N_{ij}(u)f_j(x_j(s-u))du + \xi_i^{-1}B_i(s)S_i(s) + \xi_i^{-1}I_i(s)ds\Big| \\ \le {} & (\|\varPhi\|_1 + \varepsilon)k_i\mathrm{e}^{-\int_0^{t_1} c_i^*(u)du} \\ & + \int_0^{t_1} \mathrm{e}^{-\int_s^{t_1} c_i^*(u)du}k_i\Big[|c_i(s)||p_i(s)y_i(s-\alpha_i(s))| + \xi_i^{-1}\sum_{j=1}^n |a_{ij}(s)|L_j\xi_j|y_j(s)| \\ & \qquad +\xi_i^{-1}\sum_{j=1}^n |b_{ij}(s)|L_j\xi_j|y_j(s-\tau_{ij}(s))| \\ & \qquad +\xi_i^{-1}\sum_{j=1}^n |d_{ij}(s)|\int_0^{+\infty} |N_{ij}(u)|L_j\xi_j|y_j(s-u)|du \\ & \qquad +\xi_i^{-1}|B_i(s)|\xi_i|u_i(s)| + \xi_i^{-1}|I_i(s)|ds\Big] \end{aligned}$$

$$
\begin{aligned}
&\leq (\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1}k_i\mathrm{e}^{-\int_0^{t_1}[c_i^*(u)-\lambda]du}+\int_0^{t_1}\mathrm{e}^{-\int_s^{t_1}[c_i^*(u)-\lambda]du}k_i\Bigg[\frac{\mathrm{e}^{\lambda\alpha_i^+}|c_i(s)p_i(s)|}{1-p_i^+\mathrm{e}^{\lambda\alpha_i^+}}\\
&\quad+\xi_i^{-1}\sum_{j=1}^n|a_{ij}(s)|L_j\xi_j\frac{1}{1-p_j^+\mathrm{e}^{\lambda\alpha_j^+}}+\xi_i^{-1}\sum_{j=1}^n|b_{ij}(s)|L_j\xi_j\frac{\mathrm{e}^{\lambda\tau_{ij}^+}}{1-p_j^+\mathrm{e}^{\lambda\alpha_j^+}}\\
&\quad+\xi_i^{-1}\sum_{j=1}^n|d_{ij}(s)|\int_0^{+\infty}|N_{ij}(u)|\mathrm{e}^{\lambda u}du L_j\xi_j\frac{1}{1-p_j^+\mathrm{e}^{\lambda\alpha_j^+}}\\
&\quad+|B_i(s)|\frac{1}{1-q_i^+\mathrm{e}^{\lambda\beta_i^+}}+\lambda\Bigg]dsK(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1}\\
&\leq (\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1}k_i\mathrm{e}^{-\int_0^{t_1}[c_i^*(u)-\lambda]du}\\
&\quad+\int_0^{t_1}\mathrm{e}^{-\int_s^{t_1}[c_i^*(u)-\lambda]du}\big[c_i^*(s)-\lambda\big]dsK(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1}\\
&=K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1}\left[\left(\frac{k_i}{K}-1\right)\mathrm{e}^{-\int_0^{t_1}[c_i^*(u)-\lambda]du}+1\right]<K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1},
\end{aligned}
\tag{32}
$$

which contradicts (20).

Similarly, combining (9), (13), (17), (23) and (31) we get

$$
\begin{aligned}
|U_i(t_1)|&=\Bigg|U_i(0)\mathrm{e}^{-\int_0^t e_i(u)du}+\int_0^t\mathrm{e}^{-\int_s^t e_i(u)du}\\
&\quad\times\big[-e_i(s)q_i(s)u_i(s-\beta_i(s))+\xi_i^{-1}f_i(x_i(s))+\xi_i^{-1}J_i(s)\big]ds\Bigg|\\
&\leq |U_i(0)\overline{k}_i|\mathrm{e}^{-\int_0^t e_i^*(u)du}+\int_0^t\mathrm{e}^{-\int_s^t e_i^*(u)du}\overline{k}_i\\
&\quad\times\Bigg[|e_i(s)||q_i(s)||u_i(s-\beta_i(s))|+\xi_i^{-1}L_i\xi_i|y_i(s)|+\xi_i^{-1}|J_i(s)|\Bigg]ds\Bigg|\\
&\leq (\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1}\overline{k}_i\mathrm{e}^{-\int_0^t[e_i^*(u)-\lambda]du}\\
&\quad+\int_0^t\mathrm{e}^{-\int_s^t[e_i^*(u)-\lambda]du}\overline{k}_i\Bigg[\frac{\mathrm{e}^{\lambda\beta_i^+}}{1-p_i^+\mathrm{e}^{\lambda\beta_i^+}}|e_i(s)q_i(s)|+L_i\frac{1}{1-p_i^+\mathrm{e}^{\lambda\alpha_i^+}}+\lambda\Bigg]ds\\
&\quad\times K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1}\\
&\leq (\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1}\overline{k}_i\mathrm{e}^{-\int_0^t[e_i^*(u)-\lambda]du}+\int_0^t\mathrm{e}^{-\int_s^t[e_i^*(u)-\lambda]du}\big[e_i^*(s)-\lambda\big]ds\\
&\quad\times K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1}\\
&=(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1}\left[\left(\frac{\overline{k}_i}{K}-1\right)\mathrm{e}^{-\int_0^t[e_i^*(u)-\lambda]du}+1\right]\\
&<K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1},
\end{aligned}
\tag{33}
$$

which contradicts (21).

Therefore, (18) and (19) hold. Letting $\varepsilon\to 0^+$, one has from (18) and (19) that

$$
\|V(t)\|\leq K\|\Phi\|_1\mathrm{e}^{-\lambda t}, \tag{34}
$$

$$
\|U(t)\|\leq K\|\Phi\|_1\mathrm{e}^{-\lambda t},\quad \text{for all}\quad t>0. \tag{35}
$$

Then, arguing as in the proof of (24)–(27), in view of (34) and (35), one can show

$$\mathrm{e}^{\lambda t}|v_j(t)| \le \sup_{s\in(-\infty,t]} \mathrm{e}^{\lambda s}|v_j(s)| \le \frac{K\|\Phi\|_1}{1-p_j^+\mathrm{e}^{\lambda\alpha_j^+}},$$
$$\mathrm{e}^{\lambda t}|u_j(t)| \le \sup_{s\in(-\infty,t]} \mathrm{e}^{\lambda s}|u_j(s)| \le \frac{K\|\Phi\|_1}{1-q_j^+\mathrm{e}^{\lambda\beta_j^+}},$$

and

$$|v_j(t)| \le \frac{K\|\Phi\|_1}{1-p_j^+\mathrm{e}^{\lambda\alpha_j^+}}\mathrm{e}^{-\lambda t},$$
$$|u_j(t)| \le \frac{K\|\Phi\|_1}{1-q_j^+\mathrm{e}^{\lambda\beta_j^+}}\mathrm{e}^{-\lambda t}, \quad \forall t>0, \quad j=1,2,\cdots,n.$$

Thus, we have $x_i(t)=O(\mathrm{e}^{-\lambda t})$, $S_i(t)=O(\mathrm{e}^{-\lambda t})$ as $t\to+\infty$. The proof of Theorem 3.1 is completed. ∎

Remark 3.2 In the following, we summarize the process steps for obtaining the global exponential convergence of the solution of the system (3). For establishing some sufficient conditions of global exponential stability in this paper, we only employ the method of proof by contradiction rather than traditional Lyapunov functional approach.

Process steps for obtaining main results:

Step 1 By using calculus basic formula,

$$v_i(t)=\xi_i^{-1}x_i(t), \quad V_i(t)=v_i(t)-p_i(t)v_i(t-\alpha_i(t)),$$
$$u_i(t)=\xi_i^{-1}S_i(t), \quad U_i(t)=u_i(t)-q_i(t)u_i(t-\beta_i(t)).$$

Step 2 We define the norm

$$\|\Phi\|_1=\max\{\|\varphi\|_\xi, \|\phi\|_\xi\}.$$

Step 3 We will show for all $t>0$

$$\|V(t)\|<K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t}, \quad \|U(t)\|<K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t}.$$

Step 4 We assume that Step 3 is not true, i.e., there must exist $i\in\{1,2,\cdots,n\}$ and $t_1>0$ such that

$$|V_i(t_1)|=\|V(t_1)\|=K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1},$$
$$|U_i(t_1)|=\|U(t_1)\|=K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1},$$

and

$$\|V(t)\|<K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t}, \quad \|U(t)\|<K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t},$$

for all $t\in(-\infty,t_1]$.

Step 5 Using (H1)–(H4) we have

$$|V(t_1)|<K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1}, \quad |U(t_1)|<K(\|\Phi\|_1+\varepsilon)\mathrm{e}^{-\lambda t_1}.$$

We find a contradiction with Step 4.

Step 6 We have $x_i(t)=O(\mathrm{e}^{-\lambda t})$, $S_i(t)=O(\mathrm{e}^{-\lambda t})$ as $t\to+\infty$.

4 Example and Remarks

In this section, we give an example to illustrate the results obtained in the previous sections.

Considering the following neutral-type competitive neural networks with multi-proportional delays and leakage delays

$$\begin{cases} \text{STM}: \ [x_1(t) - p_1(t)x_1(t-\alpha_1(t))]' = -c_1(t)x_1(t) + \sum_{j=1}^{2} a_{1j}(t)f_j(x_j(t)) \\ \qquad + \sum_{j=1}^{2} b_{1j}(t)f_j(x_j(t-\tau_{1j}(t))) \\ \qquad + \sum_{j=1}^{n} d_{1j}(t)\int_0^{+\infty} N_{1j}(u)f_j(x_j(t-u))du \\ \qquad + B_1(t)S_1(t) + I_1(t), \\ \text{STM}: \ [x_2(t) - p_2(t)x_2(t-\alpha_2(t))]' = -c_2(t)x_2(t) + \sum_{j=1}^{2} a_{2j}(t)f_j(x_j(t)) \\ \qquad + \sum_{j=1}^{2} b_{2j}(t)f_j(x_j(t-\tau_{2j}(t))) \\ \qquad + \sum_{j=1}^{n} d_{2j}(t)\int_0^{+\infty} N_{2j}(u)f_j(x_j(t-u))du \\ \qquad + B_2(t)S_2(t) + I_2(t), \\ \text{LTM}: \ [S_1(t) - q_1(t)S_1(t-\beta_1(t))]' = -e_1(t)S_1(t) + f_1(x_1(t)) + J_1(t), \\ \text{LTM}: \ [S_2(t) - q_2(t)S_2(t-\beta_2(t))]' = -e_2(t)S_1(t) + f_2(x_2(t)) + J_2(t), \end{cases} \tag{36}$$

where $f_i(u) = \frac{1}{20}\arctan(u) \Rightarrow L_i = \frac{1}{20}$, $\alpha_1(u) = \beta_1(u) = \frac{1}{2}|\cos(u)|$, $\alpha_2(u) = \beta_2(u) = \frac{1}{2}|\sin(u)|$, $p_1(u) = q_1(u) = \frac{1}{100}\sin(u)$, $p_2(u) = q_2(u) = \frac{1}{100}\cos(u)$, $c_1(t) = e_1(t) = \frac{1}{10}(1+\frac{3}{2}\sin(t))$, $c_2(t) = e_2(t) = \frac{1}{10}(1+\frac{3}{2}\cos(t))$, $\tau_{ij}(t) = \frac{1}{2}|\sin(t)|$, $N_{ij}(t) = \frac{1}{10}\mathrm{e}^{-2t}$, $B_1(t) = 0.01\sin(t)$, $B_2(t) = 0.02\cos(t)$,

$$(a_{ij}(t))_{1\le i,j\le 2} = \begin{pmatrix} 0.02\sin(t) & 0.02\cos(t) \\ 0.01\cos(t) & 0.01\sin(t) \end{pmatrix}, \quad (b_{ij}(t))_{1\le i,j\le 2} = \begin{pmatrix} 0.01\cos(t) & 0.015\sin(t) \\ 0.02\sin(t) & 0.01\cos(t) \end{pmatrix},$$

$$(d_{ij}(t))_{1\le i,j\le 2} = \begin{pmatrix} 0.01\sin(t) & 0.01\cos(t) \\ 0.01\cos(t) & 0.03\sin(t) \end{pmatrix}, \quad (I_i(t))_{1\le i\le 2} = \begin{pmatrix} \mathrm{e}^{-2t}\sin^4(t) \\ \mathrm{e}^{-2t}\cos^4(t) \end{pmatrix},$$

$$(J_i(t))_{1\le i\le 2} = \begin{pmatrix} \mathrm{e}^{-2t}\sin^4(t) \\ \mathrm{e}^{-2t}\cos^4(t) \end{pmatrix}.$$

Let $\omega = 1$, $\xi_1 = \xi_2 = 1$, $c_i^*(t) = \mathrm{e}_i^*(t) = \frac{1}{10}$, $k_i = \overline{k}_i = \mathrm{e}^{\frac{3}{10}}$, $\mathrm{e}^{-\int_s^t c_i(u)du} \le \mathrm{e}^{\frac{3}{10}}\mathrm{e}^{-(t-s)}$,

$$\mathrm{e}^{-\int_s^t e_i(u)du} \le \mathrm{e}^{\frac{3}{10}}\mathrm{e}^{-(t-s)}, \quad i = 1,2, \quad t \ge s.$$

Then, for $\lambda_0 = 2$, we have

$$\begin{aligned}\varGamma_1(0) = \sup_{t\geq 0}\Big\{ &- c_1^*(t)\\ &+k_1\Big[\frac{1}{1-p_1^+}|c_1(t)p_1(t)| + \xi_1^{-1}\sum_{j=1}^{2}|a_{1j}(t)|L_j\xi_j\frac{1}{1-p_j^+}\\ &+\xi_1^{-1}\sum_{j=1}^{2}|b_{ij}(t)|L_j\xi_j\frac{1}{1-p_j^+} + \xi_1^{-1}\sum_{j=1}^{2}|d_{1j}(t)|\int_0^{+\infty}|N_{1j}(u)|duL_j\xi_j\frac{1}{1-p_j^+}\\ &+|B_1(t)|\frac{1}{1-q_1^+}\Big]\Big\} = -0.0827 < 0,\end{aligned}$$

$$\begin{aligned}\varGamma_2(0) = \sup_{t\geq t_0}\Big\{ &- c_2^*(t)\\ &+k_2\Big[\frac{1}{1-p_2^+}|c_2(t)p_2(t)| + \xi_2^{-1}\sum_{j=1}^{2}|a_{2j}(t)|L_j\xi_j\frac{1}{1-p_j^+}\\ &+\xi_2^{-1}\sum_{j=1}^{2}|b_{2j}(t)|L_j\xi_j\frac{1}{1-p_j^+} + \xi_i^{-1}\sum_{j=1}^{2}|d_{2j}(t)|\int_0^{+\infty}|N_{2j}(u)|duL_j\xi_j\frac{1}{1-p_j^+}\\ &+|B_2(t)|\frac{1}{1-q_2^+}\Big]\Big\} = -0.0690 < 0,\end{aligned}$$

$$\varUpsilon_1(0) = \sup_{t\geq 0}\Big\{ - \mathrm{e}_1^*(t) + \overline{k}_1\Big[\frac{1}{1-p_1^+}|e_1(s)q_1(s)| + L_1\frac{1}{1-p_1^+}\Big]\Big\} = -0.0284 < 0,$$

$$\varUpsilon_1(0) = \sup_{t\geq t_0}\Big\{ - \mathrm{e}_2^*(t) + \overline{k}_2\Big[\frac{1}{1-p_2^+}|e_2(s)q_2(s)| + L_2\frac{1}{1-p_2^+}\Big]\Big\} = -0.0284 < 0.$$

This implies that all the conditions in Theorem 3.1 are satisfied, then all solution of the system (3) converge exponentially to the zero vector $(0,0)^{\mathrm{T}}$. The state trajectories of the system (3) with $t \in [0,150]$ and $t \in [0,1000]$ are illustrated in Figure 1 and Figure 2, respectively. Figure 3 and Figure 4 depict the time responses of state variables $x_1(t)$, $x_2(t)$, $S_1(t)$ and $S_2(t)$ with different initial conditions.

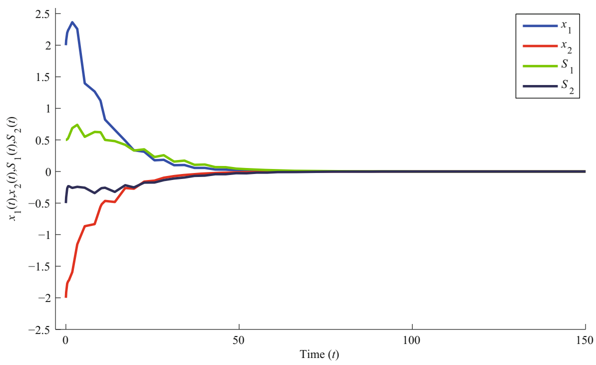

Figure 1 Transient responses of the state x_1, x_2, S_1 and S_2 of the system (36) for $t \in [0,150]$

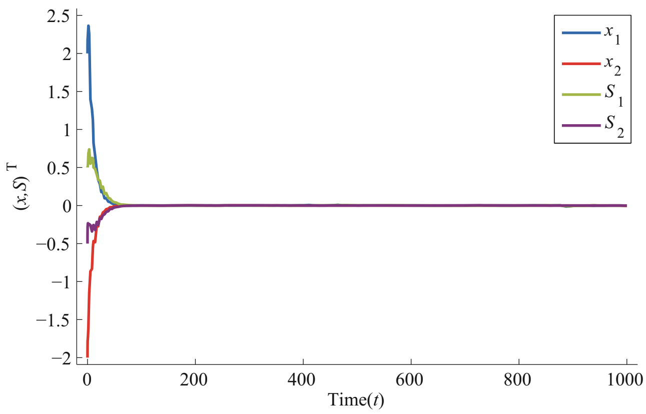

Figure 2 Transient responses of the state x_1, x_2, S_1 and S_2 of the system (36) for $t \in [0, 1000]$

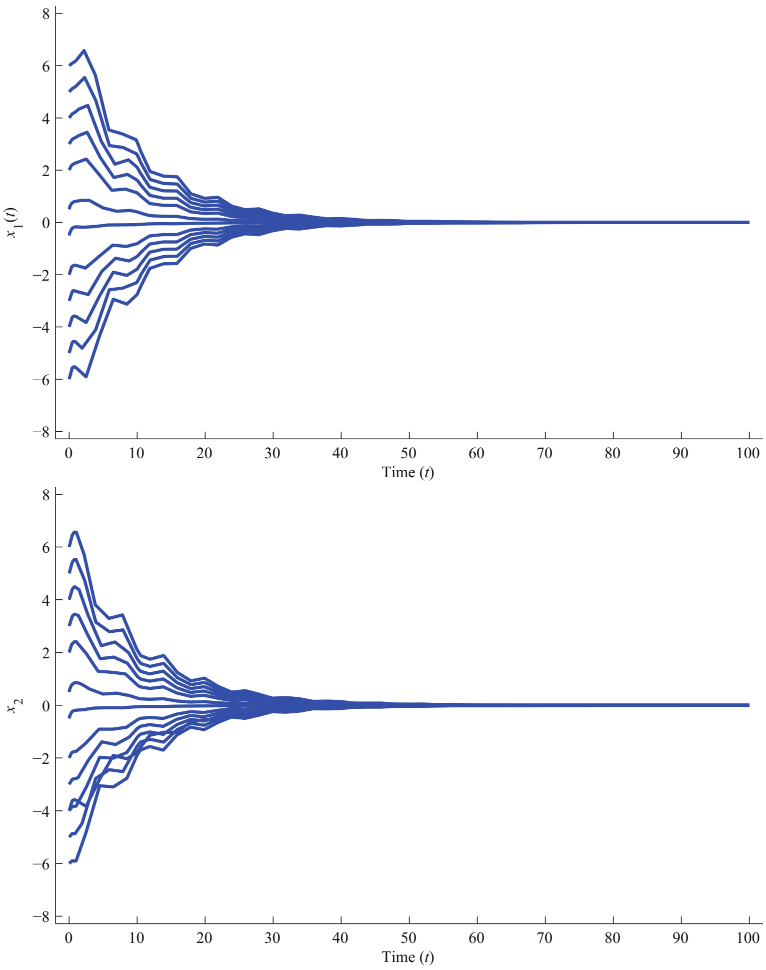

Figure 3 Transient response of state variables $x_1(t)$ and $x_2(t)$

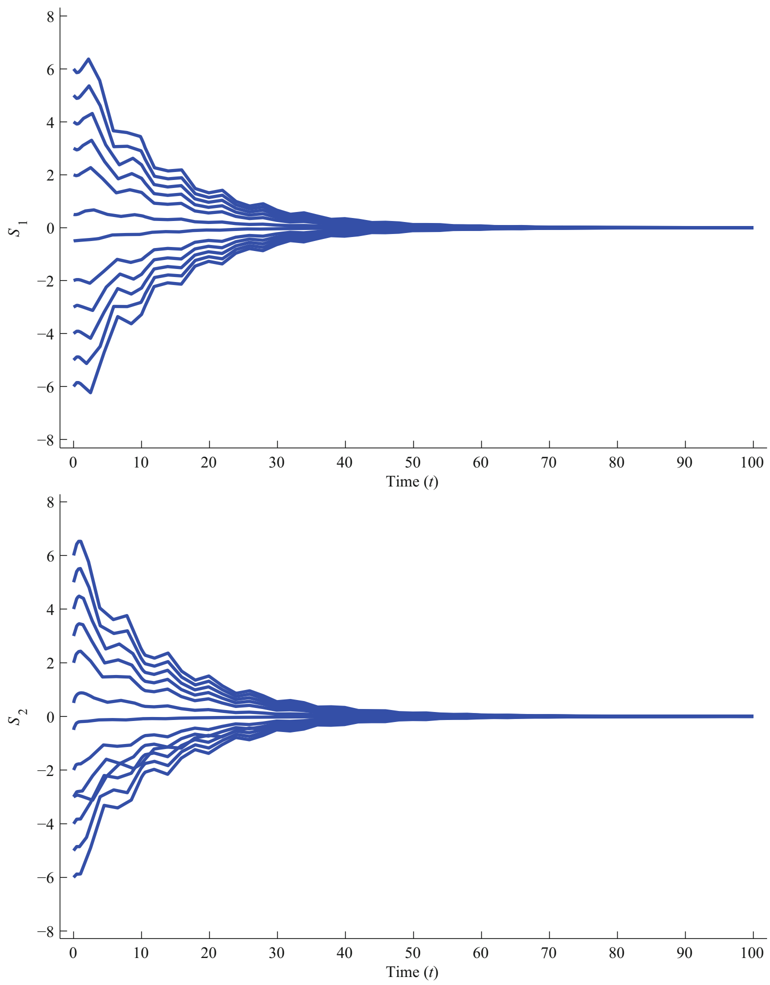

Figure 4 Transient response of state variables $S_1(t)$ and $S_2(t)$

Remark 4.1 To the best of our knowledge, the exponential convergence for the neutral-type (CNNs) with D operator has not been considered before. Thus, all the results in the references in [13–25, 37] cannot be applicable to prove that all the solutions of (1) converge exponentially to the zero vector $(0,0)^{\mathrm{T}}$. This implies that the results of this paper are essentially new and have wider application range of the proposed approach.

Remark 4.2 In example above, replacing $c_1(t)=e_1(t)=\frac{1}{10}(1+\frac{3}{2}\sin(t))$, $c_2(t)=e_2(t)=\frac{1}{10}(1+\frac{3}{2}\cos(t))$ with $c_1(t)=e_1(t)=-4$, $c_2(t)=e_2(t)=-5$, respectively, it is easily to see that (H1)–(H4) are not satisfied. Some numerical simulations in Figure 5 illustrate that the exponential convergence does not exist. This demonstrates the validity of the theoretical result of this paper.

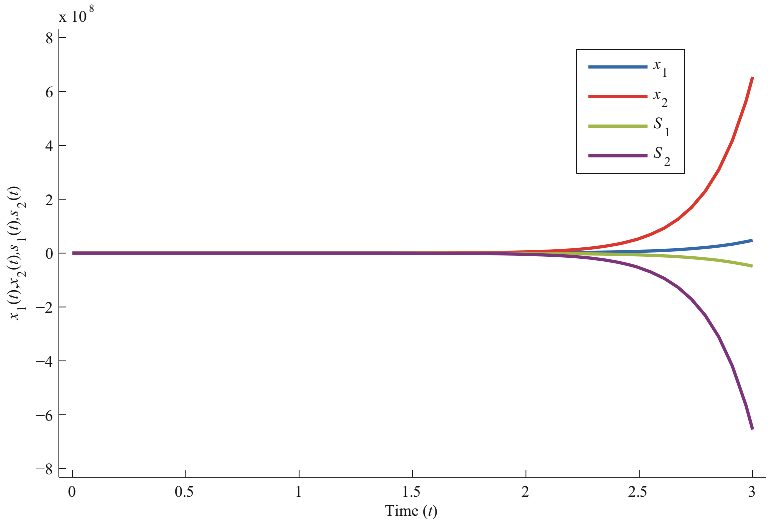

Figure 5 Transient responses of the state x_1, x_2, S_1 and S_2 of the system (36) for $(x(0), S(0))^{\mathrm{T}} = (300, 200, -300, -200)^{\mathrm{T}}$ with $c_1(t) = e_1(t) = -4,\ c_2(t) = e_2(t) = -5$

Remark 4.3 In this paper, a new exponential function inequality has been added to deal with the case that leakage term coefficient function is oscillating (see (H1)), which is different from the results in [42]. In addition, some numerical simulations are introduced to illustrate that the exponential convergence of addressed system does not exist when the condition is not verified, the validity of the conclusion of this article is demonstrated from the other point of view.

Remark 4.4 In [13], the authors developed the stability problem for competitive neural networks with multi-proportional delays. Further, the authors in [25] investigated the existence and global exponential stability analysis of almost periodic solution for delayed competitive neural networks with discontinuous activations. In [42], the authors studied the existence and the global exponential stability of almost periodic solutions of a class of neutral type competitive neural networks with mixed time-varying delays and leakage delays on time scales. However, in this paper, we provide new criteria concerning the global exponential convergence of neutral type competitive neural networks with mixed delay and D operator. The method of this paper is to construct a delay differential inequality rather than a Lyapunov functional, whose results can be easily checked. Until now, there are no results on the global exponential convergence CNNs with mixed delay and D operator, which means that our results are essentially new and more meaningful.

5 Conclusion

The global exponential convergence plays an important role in designing neural networks to serve mankind. Thus, many researchers have extensively focused on this question recently. In this paper, neutral-type (CNNs) with D operator have been studied. By employing the differential inequality theory, some sufficient conditions for the global exponential convergence have been established. The method affords a possible method to analyze the global exponential stability of anti-periodic and pseudo almost periodic solutions for neutral-type (CNNs) with D operator. The corresponding results will appear in the near future.

References

[1] Li X and Song S, Impulsive control for existence, uniqueness, and global stability of periodic solutions of recurrent neural networks with discrete and continuously distributed delays, *IEEE Transactions on Neural Networks and Learning Systems*, 2013, **24**(6): 868–877.

[2] Li X and Fu X, Lag synchronization of chaotic delayed neural networks via impulsive control, *IMA Journal of Mathematical Control and Information*, 2011, **29**(1): 133–145.

[3] Li X and Rakkiyappan R, Stability results for TakagiSugeno fuzzy uncertain BAM neural networks with time delays in the leakage term, *Neural Computing and Applications*, 2013, **22**(1): 203–219.

[4] Zhang X, Lü X, and Li X, Sampled-data-based lag synchronization of chaotic delayed neural networks with impulsive control, *Nonlinear Dynamics*, 2017, **90**(3): 2199–2207.

[5] Zhang H, Ye R, Cao J, et al., Lyapunov functional approach to stability analysis of RiemannLiouville fractional neural networks with time varying delays, *Asian Journal of Control*, 2018, **20**(5): 1938–1951.

[6] Alzahrani E A, Akca H, and Li X, New synchronization schemes for delayed chaotic neural networks with impulses, *Neural Computing and Applications*, 2017, **28**(9): 2823–2837.

[7] Rakkiyappan R, Velmurugan G, Li X, et al., Global dissipativity of memristor-based complex-valued neural networks with time-varying delays, *Neural Computing and Applications*, 2016, **27**(3): 629–649.

[8] Aouiti C, M'hamdi M S, and Touati A, Pseudo almost automorphic solutions of recurrent neural networks with time-varying coefficients and mixed delays, *Neural Processing Letters*, 2017, **45**(1): 121–140.

[9] Li X and Wu J, Stability of nonlinear differential systems with state-dependent delayed impulses, *Automatica*, 2016, **64**: 63–69.

[10] Li X and Cao J, An impulsive delay inequality involving unbounded time-varying delay and applications, *IEEE Transactions on Automatic Control*, 2017, **62**(7): 3618–3625.

[11] Aouiti C, M'hamdi M S, Chérif F, et al., Impulsive generalised high-order recurrent neural networks with mixed delays: Stability and periodicity, *Neurocomputing*, 2018, **321**: 296–307

[12] Aouiti C, Neutral impulsive shunting inhibitory cellular neural networks with time-varying coefficients and leakage delay, *Cognitive Neurodynamics*, 2016, **10**(6): 573–591.

[13] Zhou L and Zhao Z, Exponential stability of a class of competitive neural networks with multi-proportional delays, *Neural Processing Letters*, 2016, **44**(3): 651–663.

[14] Lu H and He Z, Global exponential stability of delayed competitive neural networks with different time scales, *Neural Networks*, 2005, **18**(3): 243–250.

[15] Cui B T, Chen J, and Lou X Y, New results on global exponential stability of competitive

neural networks with different time scales and time-varying delays, *Chinese Physics B*, 2008, **17**: 1670–1677.

[16] Gu H, Jiang H, and Teng Z, Existence and global exponential stability of equilibrium of competitive neural networks with different time scales and multiple delays, *Journal of the Franklin Institute*, 2010, **347**(5): 719–731.

[17] Nie X and Cao J, Multistability of competitive neural networks with time-varying and distributed delays, *Nonlinear Analysis: Real World Applications*, 2009, **10**(2): 928–942.

[18] Yang X, Huang C, and Cao J, An LMI approach for exponential synchronization of switched stochastic competitive neural networks with mixed delays, *Neural Computing and Applications*, 2012, **21**(8): 2033–2047.

[19] Nie X and Cao J, Existence and global stability of equilibrium point for delayed competitive neural networks with discontinuous activation functions, *International Journal of Systems Science*, 2012, **43**(3): 459–474.

[20] Nie X and Cao J, Multistability of second-order competitive neural networks with nondecreasing saturated activation functions, *IEEE Transactions on Neural Networks*, 2011, **22**(11): 1694–1708.

[21] Meyer-Base A and Thummler V, Local and global stability analysis of an unsupervised competitive neural network, *IEEE Transactions on Neural Networks*, 2008, **19**(2): 346–351.

[22] Meyer-Baese A, Pilyugin S S, and Chen Y, Global exponential stability of competitive neural networks with different time scales, *IEEE Transactions on Neural Networks*, 2003, **14**(3): 716–719.

[23] Meyer-Bse A, Pilyugin S, Wismller A, et al., Local exponential stability of competitive neural networks with different time scales, *Engineering Applications of Artificial Intelligence*, 2004, **17**(3): 227–232.

[24] Nie X and Cao J, Exponential stability of competitive neural networks with time-varying and distributed delays, *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, 2008, **222**(6): 583–594.

[25] Tan Y and Jing K, Existence and global exponential stability of almost periodic solution for delayed competitive neural networks with discontinuous activations, *Mathematical Methods in the Applied Sciences*, 2016, **39**(11): 2821–2839.

[26] Li X and Wu J, Sufficient stability conditions of nonlinear differential systems under impulsive control with state-dependent delay, *IEEE Transactions on Automatic Control*, 2018, **63**(1): 306–311.

[27] Cao J and Wang J, Global asymptotic and robust stability of recurrent neural networks with time delays, *IEEE Transactions on Circuits and Systems I: Regular Papers*, 2005, **52**(2): 417–426.

[28] Candan T, Existence of positive periodic solutions of first order neutral differential equations with variable coefficients, *Applied Mathematics Letters*, 2006, **52**: 142–148.

[29] Zhang A, Almost periodic solutions for SICNNs with neutral type proportional delays and D operators, *Neural Processing Letters*, 2017, **47**(1): 57–70.

[30] Xu Y, Exponential stability of pseudo almost periodic solutions for neutral type cellular neural networks with D operator, *Neural Processing Letters*, 2017, **46**(1): 329–342.

[31] Yao L, Global exponential convergence of neutral type shunting inhibitory cellular neural networks with D operator, *Neural Processing Letters*, 2017, **45**(2): 401–409.

[32] Chen Z, Global exponential stability of anti-periodic solutions for neutral type CNNs with D operator, *International Journal of Machine Learning and Cybernetics*, 2018, **9**(7): 1109–1115.

[33] Yao L, Global convergence of CNNs with neutral type delays and D operator, *Neural Computing and Applications*, 2016, **29**(1): 105–109.

[34] Zhang A, Pseudo almost periodic solutions for neutral type SICNNs with D operator, *Journal of Experimental & Theoretical Artificial Intelligence*, 2017, **29**(4): 795–807.

[35] Yu Y, Global exponential convergence for a class of neutral functional differential equations with proportional delays, *Mathematical Methods in the Applied Sciences*, 2016, **39**(15): 4520–4525.

[36] Liu X, Improved convergence criteria for HCNNs with delays and oscillating coefficients in leakage terms, *Neural Computing and Applications*, 2016, **27**(4): 917–925.

[37] Wang Y and Huang L, Global stability analysis of competitive neural networks with mixed time-varying delays and discontinuous neuron activations, *Neurocomputing*, 2015, **152**: 85–96.

[38] Aouiti C, abed Assali E, Cao J, et al., Global exponential convergence of neutral-type competitive neural networks with multi-proportional delays, distributed delays and time-varying delay in leakage delays, *International Journal of Systems Science*, 2018, **49**(10): 2202–2214.

[39] Yi X, Shao J, Yu Y, et al., New convergence behavior of high-order Hopfield neural networks with time-varying coefficients, *Journal of Computational and Applied Mathematics*, 2008, **219**(1): 216–222.

[40] Xiong W, New result on convergence for HCNNs with time-varying leakage delays, *Neural Computing and Applications*, 2015, **26**(2): 485–491.

[41] Berezansky L and Braverman E, On exponential stability of a linear delay differential equation with an oscillating coefficient, *Applied Mathematics Letters*, 2009, **22**(12): 1833–1837.

[42] Li B, Li Y, and Meng X, The existence and global exponential stability of almost periodic solutions for neutral-type CNNs on time scales, *Mathematics*, 2019, **7**(4): 321–350.

[43] Li X and Song S, Stabilization of delay systems: Delay-dependent impulsive control, *IEEE Transactions on Automatic Control*, 2017, **62**(1): 406–411.

[44] Aouiti C, Gharbia I B, Cao J, et al., Existence and global exponential stability of pseudo almost periodic solution for neutral delay BAM neural networks with time-varying delay in leakage terms, *Chaos, Solitons & Fractals*, 2018, **107**: 111–127.

[45] Aouiti C, Coirault P, Miaadi F, et al., Finite time boundedness of neutral high-order Hopfield neural networks with time delay in the leakage term and mixed time delays, *Neurocomputing*, 2017, **260**: 378–392.

[46] Liu B and Huang L, Existence and uniqueness of periodic solutions for a kind of first order neutral functional differential equations, *Journal of Mathematical Analysis and Applications*, 2006, **322**(1): 121–132.