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New Results on Interval General Cohen-Grossberg BAM Neural Networks

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Abstract This paper is concerned with an interval general Cohen-Grossberg bidirectional associative memory neural networks with mixed delays. Under proper conditions, the authors studied the existence, the uniqueness and the global exponential stability of almost automorphic solutions for the suggested system. The proposed method was mainly based on the exponential dichotomy of linear differential equation, the Banach's fixed point principle and the differential inequality techniques. The authors illustrate with an example to demonstrate the effectiveness of the proposed findings.

Keywords Almost automorphic solution, bi-almost automorphic function, global exponential stability, interval general CGBAM neural networks.

1 Introduction

Artificial neural networks (ANNs) has been widely investigated (see [1–18]). In 1983, Cohen and Grossberg proposed one of the most popular ANNs called Cohen-Grossberg neural network (see [19]). The study of dynamic behaviors of Cohen-Grossberg neural networks (CGNNs) has quickly attracted many attention and new interesting results have been obtained: In [20], Yang studied the existence and the global exponential stability of periodic solution for Cohen-Grossberg shunting inhibitory cellular neural networks with delays and impulses. In [21], Xu, et al. investigated the existence and the uniqueness of almost automorphic solutions of CGNNs with delays. Paper [22], dealt with the existence and the exponential stability of pseudo almost automorphic solutions for Cohen-Grossberg neural networks with mixed delays. In [3], the asymptotic almost automorphic solution for impulsive Cohen-Grossberg neural networks with mixed delays is analysed. In 1988, Kosko proposed bidirectional associative memories (BAMs) as typical models of ANNs (see [23]). BAMs has been proved to have widespread applications in various fields such medical image edge detection, medical event detection in electronic health

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records, diagnosis prediction in health care, pattern recognition and robotics (see [24–26]). These applications heavily depend on the dynamic behaviors of BAMs that is why extensive results have been proved: In [27], Xu and Zhang studied the existence and the global exponential stability of anti-periodic solutions for BAM neural networks with inertial term and delay. Then, in [28], they studied the existence and the exponentially stability of anti-periodic solutions for neutral BAM neural networks with time-varying delays in the leakage terms. In [29], Yang, et al. investigated the almost automorphic solution for neutral type high-order Hopfield BAM neural networks with time-varying leakage delays on time scales. Paper [5], dealt with the existence and the global exponential stability of pseudo almost periodic solution for neutral delay BAM neural networks with time-varying delay in leakage terms. Paper [4] focused on the analyse of the (μ, ν)-pseudo-almost automorphic solutions for high-order Hopfield bidirectional associative memory neural networks. The reader can also see papers [30–32], and so on. The combination of the two previous models gives Cohen-Grossberg Bidirectional Associative Memory neural networks (CGBAMs) which is our central model in this work.

In fact, time delay exists in practical dynamical systems, including ANNs, because neurons cannot respond instantaneously (see [33, 34]). It can changes their dynamical behavior (see [1–11, 35]). In this work, we look at two different types of delays (time-varying delays, distributed delays) in the analysis of our main model.

Indeed, several real phenomena can be more or less periodic. Therefore, studying these phenomena requires concepts that go beyond the concept of periodicity. From a mathematical point of view, many mathematicians have proposed more appropriate classes of functions to explain complex behaviors such as the class of Almost Automorphic functions (AA). This class was introduced in the literature by Bochner, et al. (see [36, 37]). AA functions became an attractive topic in the qualitative theory of differential equations because of their applications in physics, mathematical biology, control theory, and other related areas. In neural network theory, an important question can be asked: What will be the nature of output when all the parameters are almost automorphic? A lot of research work has been published to answer this question. However, interval general CGBAMs with almost automorphic connection weights have never been investigated. This is a very challenging problem. Motivated by the aforementioned discussion, in this paper we try to establish the dynamics of the system defined by the following equations:

$$\begin{cases} \dot{x}_i(t) = -\alpha_i^1(x_i(t))\Big\{ a_i^1(x_i(t)) - \sum_{j=1}^{p} b_{ji}^1(t) f_j^1\big[x_j(t-\tau_{ji}), y_j(t-\upsilon_{ji})\big] \\ \qquad - \sum_{j=1}^{p} d_{ji}^1(t) \int_{-\infty}^{t} K_{ji}(t-s) g_j^1\big[x_j(s), y_j(s)\big] ds - I_i(t) \Big\}, \quad 1 \le i \le n, \\ \dot{y}_j(t) = -\alpha_j^2(x_i(t))\Big\{ a_j^2(y_j(t)) - \sum_{i=1}^{n} b_{ij}^2(t) f_i^2\big[x_i(t-\zeta_{ij}), y_i(t-\vartheta_{ij})\big] \\ \qquad - \sum_{i=1}^{n} d_{ij}^2(t) \int_{-\infty}^{t} G_{ij}(t-s) g_i^2\big[x_i(s), y_i(s)\big] ds - J_j(t) \Big\}, \quad 1 \le j \le p, \end{cases} \tag{1}$$

where $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, p$ (n and p are the number of neurons in layers); $x_i(\cdot), y_j(\cdot)$

are the activations of the i^{th} and j^{th} neurons; $\alpha_i^1(\cdot), \alpha_j^2(\cdot)$ represent the amplification functions; $a_i^1(\cdot)$, $a_j^2(\cdot)$ represent the rate with which the i^{th} and j^{th} neuron will reset their potential to the resting state in isolation when they are disconnected from the network and the external inputs; $b_{ji}^1(\cdot)$, $b_{ji}^2(\cdot)$, $d_{ji}^1(\cdot)$, $d_{ij}^1(\cdot)$ are the connection weights, which denote the strengths of connectivity between the cells j and i; $\tau_{ji}, \upsilon_{ji}, \zeta_{ij}, \vartheta_{ij} > 0$ are the constant time delay; $f_j^1(\cdot,\cdot)$, $f_i^2(\cdot,\cdot)$, $g_j^1(\cdot,\cdot)$, $g_j^1(\cdot,\cdot)$ are the activation functions; $K_{ji}(\cdot), G_{ij}(\cdot)$ are the transmission delay kernels; $I_i(\cdot)$, $J_j(\cdot)$ denote the i^{th} and j^{th} component of an external input source introduced from outside the network to the cell i and j respectively.

The system (1) is supplement with initial value given by:

$$\begin{cases} x_i(s) = \phi_i(s), & s \in (-\infty, 0], \\ y_j(s) = \psi_j(s), & s \in (-\infty, 0], \end{cases} \tag{2}$$

where $\phi_i(\cdot), \psi_i(\cdot)$ are continuous functions on $(-\infty, 0]$.

Remark 1.1 Our motivation for this letter stems from the fact that the system in Equation (1) can exist in many applications of science or engineering. The success of these applications relies on understanding the underlying dynamical behavior of the model.

Our main purpose is to present new criteria concerning the existence, the uniqueness and the global exponential stability of almost automorphic solutions for System (1) by using the exponential dichotomy theory, the Banach fixed point and the differential inequality technique.

Remark 1.2 Our principal contributions are:

- The choice of model in Equation (1) is significant since it includes Hophold neural networks, BAM neural networks, cellular neural networks and Lotka-Volterra competition models as special cases. We generalize the results of papers [1–5, 11, 27, 38].
- The class of bi-almost automorphic functions is never used in the theory of neural networks.
- The study of the existence, the uniqueness and the global exponential stability of the almost automorphic solutions of system in Equation (1) is firstly put forward.
- The research for the almost automorphic solutions of dynamic systems are complicated. The fundamental property of uniform continuity is not verified. Our findings improve many results reported in the literature (see [20, 21, 27, 31, 38–42]).

The rest of this paper is organized as follows: In Section 2, we give useful definitions, assumptions and lemmas. Section 3 is devoted to establish new criteria for the existence, the uniqueness and the global exponential stability of almost automorphic solution of system in Equation (1). In Section 4, a numerical example is given to illustrate the feasibility of the obtained results. We conclude with remarks.

2 Preliminaries

Throughout this paper, we will use the following concepts and notations. $BC(\mathbb{R}, \mathbb{R}^n)$ denotes the set of bounded continued functions from $\mathbb{R}$ to $\mathbb{R}^n$.

Note that $(BC(\mathbb{R},\mathbb{R}^n), \|\cdot\|_\infty)$ is a Banach space where $\|\cdot\|_\infty$ denotes the sup norm

$$\| f \|_\infty := \sup_{t\in\mathbb{R}} \|f(t)\|.$$

For the sake of simplicity, we adapt the following notation: For $f \in BC(\mathbb{R},\mathbb{R})$, let

$$f^* = \sup_{t\in\mathbb{R}} |f(t)|, \quad f_* = \inf_{t\in\mathbb{R}} |f(t)|.$$

Definition 2.1 (see [38, 43]) A continuous function $f : \mathbb{R} \to \mathbb{R}^n$ is called almost automorphic if for every real sequence $(s'_n)_{n\in\mathbb{N}}$, there exists a subsequence $(s_n)_{n\in\mathbb{N}}$ such that $g(t) = \lim_{n\to\infty} f(t+s_n)$ is well defined for each $t \in \mathbb{R}$ and $\lim_{n\to\infty} g(t-s_n) = f(t)$ for each $t \in \mathbb{R}$. The collection of all almost automorphic functions which go from $\mathbb{R}$ to $\mathbb{R}^n$ is denoted by $AA(\mathbb{R},\mathbb{R}^n)$.

Definition 2.2 (see [44]) A continuous function $F(t,s) : \mathbb{R}\times\mathbb{R} \to \mathbb{R}^n$ is called bi-almost automorphic if for every real sequence $(s'_n)_{n\in\mathbb{N}}$, there exists a subsequence $(s_n)_{n\in\mathbb{N}}$ such that $G(t,s) = \lim_{n\to\infty} F(t+s_n, s+s_n)$ is well defined for each $t,s \in \mathbb{R}$ and $\lim_{n\to\infty} G(t-s_n, s-s_n) = F(t,s)$ for each $t,s \in \mathbb{R}$. The collection of such functions is denoted by $bAA(\mathbb{R}\times\mathbb{R},\mathbb{R}^n)$.

Remark 2.3 (see [44])

- If $f \in C(\mathbb{R}\times\mathbb{R},\mathbb{R})$ and $f(t,s) = g(t-s)$ for some $g \in C(\mathbb{R},\mathbb{R}^n)$, then $f \in bAA(\mathbb{R}\times\mathbb{R},\mathbb{R}^n)$.
- The concept of bi-almost automorphic function is a natural generalization of the function $f(t,s)$ having the same period in the two arguments, that is $f(t+T,s+T) = f(s,t)$ for all $t,s \in \mathbb{R}$ for some $T \in \mathbb{R}\backslash\{0\}$.

Example 2.4 (see [44]) $f(t,s) = \sin(t)\cos(s)$ is a bi-almost automorphic function from $\mathbb{R}\times\mathbb{R}$ to $\mathbb{R}$.

Definition 2.5 Let $x \in \mathbb{R}^p$ and $Q(t)$ be a $p\times p$ continuous matrix defined on $\mathbb{R}$. The linear system

$$x'(t) = Q(t)x(t) \tag{3}$$

is said to admit an exponential dichotomy on $\mathbb{R}$ if there exist positive constants b,δ and projection P and the fundamental solution matrix $X(t)$ of (3) satisfy

$$\|X(t)PX^{-1}(s)\| \le b\mathrm{e}^{-\delta(t-s)}, \quad \text{for } t \ge s,$$
$$\|X(t)(I-P)X^{-1}(s)\| \le b\mathrm{e}^{-\delta(t-s)}, \quad \text{for } t \le s,$$

where I is the identity matrix.

Lemma 2.6 *Let $c_i(\cdot)$ be an almost automorphic function on $\mathbb{R}$. For each $1 \le i \le n$,*

$$M[c_i] = \lim_{T\to+\infty} \frac{1}{T}\int_t^{t+T} c_i(s)ds > 0,$$

then the linear system

$$x'(t) = \mathrm{diag}(-c_1(t), -c_2(t), \cdots, -c_n(t))x(t) \tag{4}$$

admits an exponential dichotomy on $\mathbb{R}$.

Lemma 2.7 *The inhomogeneous linear system*

$$x'(t) = -c(t)x(t) + f(t)$$

has a unique bounded solution for a vector $f \in C(\mathbb{R}, \mathbb{R}^n)$ *if and only if the inhomogeneous linear system* (4) *has exponential dichotomy.*

Throughout this paper, it will be assume that:

For all $1 \leq i \leq n$, $1 \leq j \leq p$, $b^1_{ji}(\cdot)$, $d^1_{ji}(\cdot)$, $b^2_{ij}(\cdot)$, $d^2_{ij}(\cdot)$, $I_i(\cdot)$, $J_i(\cdot) \in AA(\mathbb{R}, \mathbb{R})$ and we introduce the following fundamental assumptions.

Assumption 1 $\alpha^1_i(u)$ are uniformly continuous functions and there are positive constants α^{1*}_i, α^1_{i*} such that $0 < \alpha^1_{i*} \leq \alpha^1_i(u) \leq \alpha^{1*}_i$, $\forall u \in \mathbb{R}$, $i = 1, 2, \cdots, n$. $\alpha^2_j(u)$ are uniformly continuous functions and there are positive constants α^{2*}_j, α^2_{j*} such that $0 < \alpha^2_{j*} \leq \alpha^2_j(u) \leq \alpha^{2*}_j$, $\forall u \in \mathbb{R}$, $j = 1, 2, \cdots, p$.

Assumption 2 $a^1_i(u)$, $i = 1, 2, \cdots, n$, are uniformly continuous functions and there exist positive constants a^{1*}_i, a^1_{i*} such that $a^1_{i*} \leq \frac{a^1_i(u) - a^1_i(v)}{u-v} \leq a^{1*}_i$, $\forall u, v \in \mathbb{R}$, $u \neq v$, $a^1_i(0) = 0$. $a^2_j(u)$, $j = 1, 2, \cdots, p$, are uniformly continuous functions and there exist positive constants a^{2*}_j, a^1_{j*} such that $a^2_{j*} \leq \frac{a^2_j(u) - a^2_j(v)}{u-v} \leq a^{2*}_j$, $\forall u, v \in \mathbb{R}, u \neq v, a^2_j(0) = 0$.

Assumption 3 For all $1 \leq i \leq n$, $1 \leq j \leq p$, there exist a nonnegative constants numbers L^f_j, M^f_j, L^f_i, M^f_i, L^g_j, M^g_j, L^g_i, M^g_i, for all x, y, u, $v \in \mathbb{R}$, such that

$$\begin{cases} |f^1_j(x,y) - f^1_j(u,v)| \leq L^f_j \mid x-u \mid + M^f_j \mid y-v \mid, \\ |f^2_i(x,y) - f^2_i(u,v)| \leq L^f_i \mid x-u \mid + M^f_i \mid y-v \mid, \\ |g^1_j(x,y) - g^1_j(u,v)| \leq L^g_j \mid x-u \mid + M^g_j \mid y-v \mid, \\ |g^2_i(x,y) - g^2_i(u,v)| \leq L^g_i \mid x-u \mid + M^g_i \mid y-v \mid. \end{cases}$$

Assumption 4 For all $i \in \{1, 2, \cdots, n\}, j \in \{1, 2, \cdots, p\}$, the delay kernels K_{ji}, G_{ij} : $[0, +\infty) \longrightarrow \mathbb{R}$ are almost automorphic, integrable and there exists a real number λ such that

$$\int_0^{+\infty} K_{ji}(m)dm = \int_0^{+\infty} G_{ij}(m)dm = 1,$$
$$\int_0^{+\infty} \mathrm{e}^{\lambda m} K_{ji}(m)dm < \infty, \quad \int_0^{+\infty} \mathrm{e}^{\lambda m} G_{ij}(m)dm < \infty.$$

Assumption 5

$$r = \max\left\{ \max_{1 \leq i \leq n} \left\{ \frac{1}{a^1_{i*}\alpha^1_{i*}} \sum_{j=1}^p \left[b^{1*}_{ji}(L^f_j + M^f_j) + d^{1*}_{ji}(L^g_j + M^g_j) \right] \alpha^{1*}_j \right\} \right.$$
$$\left. \max_{1 \leq i \leq n} \left\{ \frac{1}{a^2_{j*}\alpha^2_{j*}} \sum_{i=1}^n \left[b^{2*}_{ij}(L^f_i + M^f_i) + d^{2*}_{ij}(L^g_i + M^g_i) \right] \alpha^{2*}_i \right\} \right\} < 1.$$

Now, we need the following lemmas.

Lemma 2.8 (see [43]) *Let $\phi(\cdot) \in AA(\mathbb{R}, \mathbb{R})$, $a \in \mathbb{R}$ be a constant. Then $\phi(\cdot - a) \in AA(\mathbb{R}, \mathbb{R})$.*

Lemma 2.9 (see [43]) *If $\varphi, \psi \in AA(\mathbb{R}, \mathbb{R})$, then we have*

$$\begin{cases} \varphi + \psi \in AA(\mathbb{R}, \mathbb{R}), \\ \varphi \times \psi \in AA(\mathbb{R}, \mathbb{R}). \end{cases}$$

Lemma 2.10 *If $f_j^1(\cdot, \cdot) \in C(\mathbb{R} \times \mathbb{R}, \mathbb{R})$ satisfies Assumption 3, $x(\cdot), y(\cdot) \in AA(\mathbb{R}, \mathbb{R})$, τ, υ are nonnegative constants then $f_j^1\left[x(\cdot - \tau), y(\cdot - \upsilon)\right] \in bAA(\mathbb{R} \times \mathbb{R}, \mathbb{R}^n)$.*

Proof $x(\cdot), y(\cdot) \in AA(\mathbb{R}, \mathbb{R}^n)$. Let $(s_n')_{n \in \mathbb{N}}$ be a sequence of real numbers. By hypothesis we can extract a subsequence $(s_n)_{n \in \mathbb{N}}$ of $(s_n')_{n \in \mathbb{N}}$ such that:

$$\lim_{n \to +\infty} x\left(t - \tau + s_n\right) = x^1(t - \tau), \quad \forall t \in \mathbb{R}, \quad \lim_{n \to +\infty} x^1\left(t - \tau - s_n\right) = x(t - \tau), \quad \forall t \in \mathbb{R}$$

and

$$\lim_{n \to +\infty} y\left(t - \upsilon + s_n\right) = y^1(t - \upsilon), \quad \forall t \in \mathbb{R}, \quad \lim_{n \to +\infty} y^1\left(t - \upsilon - s_n\right) = y(t - \upsilon), \quad \forall t \in \mathbb{R}.$$

Obviously,

$$\begin{aligned} &\left| f_j^1\left[x(t - \tau + s_n), y(t - \upsilon + s_n)\right] - f_j^1\left[x^1(t - \tau), y^1(t - \upsilon)\right] \right| \\ &\le L_j^f \left|x(t - \tau + s_n) - x^1(t - \tau)\right| + M_j^f \left|y(t - \upsilon + s_n) - y^1(t - \upsilon)\right| \to 0 \text{ when } n \to +\infty. \end{aligned}$$

Therefore, $\lim_{t \to \infty} f_j^1\left[x(t - \tau + s_n), y(t - \upsilon + s_n)\right] = f_j^1\left[x^1(t - \tau), y^1(t - \upsilon)\right]$. By the same way, we have

$$\lim_{t \longrightarrow \infty} f_j^1\left[x^1(t - \tau - s_n), y^1(t - \upsilon - s_n)\right] = f_j^1\left[x(t - \tau), y(t - \upsilon)\right].$$

Then, $f_j^1\left[x(\cdot - \tau), y(\cdot - \upsilon)\right] \in bAA(\mathbb{R} \times \mathbb{R}, \mathbb{R}^n)$. ∎

Lemma 2.11 *Assume that Assumption 4 holds. For all $1 \le i \le n$, $1 \le j \le p$, if $x_j(\cdot), y_j(\cdot) \in AA(\mathbb{R}, \mathbb{R}^n)$ then the function*

$$t \longmapsto \int_{-\infty}^t K_{ji}(t - s) g_j^1\left[x_j(s), y_j(s)\right] ds = \int_0^\infty K_{ji}(s) g_j^1\left[x_j(t - s), y_j(t - s)\right] ds \in AA(\mathbb{R}, \mathbb{R}^n).$$

Proof Suppose that

$$\Phi_{ji}(t) = \int_{-\infty}^t K_{ji}(t - s) g_j\left[x_j(s), y_j(s)\right] ds. \tag{5}$$

Our goal is to show that $\Phi_{ij}(\cdot) \in AA(\mathbb{R}, \mathbb{R}^n)$.

Let $x_j(\cdot), y_j(\cdot) \in AA(\mathbb{R}, \mathbb{R}^n)$, by using Lemma 2.10 we have $s \mapsto g_j\left[x_j(s), y_j(s)\right]$ belongs on $bAA(\mathbb{R} \times \mathbb{R}, \mathbb{R}^n)$. Now, let $(s_n')_{n \in \mathbb{N}}$ be a sequence of real numbers. By hypothesis we can extract a subsequence $(s_n)_{n \in \mathbb{N}}$ of $(s_n')_{n \in \mathbb{N}}$ such that for all $t, s \in \mathbb{R}$:

$$\lim_{n \to +\infty} K_{ji}(t - s + s_n) = K_{ji}^1(t - s), \quad \lim_{n \to +\infty} K_{ji}^1(t - s - s_n) = K_{ji}(t - s)$$

and

$$\lim_{n\to+\infty} g_j\big[x_j(s+s_n), y_j(s+s_n)\big] = g_j^1\big[x_j(s), y_j(s)\big],$$
$$\lim_{n\to+\infty} g_j^1\big[x_j(s-s_n), y_j(s-s_n)\big] = g_j\big[x_j(s), y_j(s)\big].$$

Pose

$$\Phi_{ji}^1(t) = \int_{-\infty}^{t} K_{ji}(t-s) g_j^1\big[x_j(s), y_j(s)\big] ds.$$

Obviously,

$$\begin{aligned}
\big|\Phi_{ji}(t+s_n) - \Phi_{ji}^1(t)\big| &= \bigg| \int_{-\infty}^{t+s_n} K_{ji}(t-s+s_n) g_j\big[x_j(s+s_n), y_j(s+s_n)\big] ds \\
&\quad - \int_{-\infty}^{t} K_{ji}(t-s) g_j^1\big[x_j(s), y_j(s)\big] ds \bigg| \\
&= \bigg| \int_{-\infty}^{t} K_{ji}(t-u) g_j\big[x_j(u+s_n), y_j(u+s_n)\big] du \\
&\quad - \int_{-\infty}^{t} K_{ji}(t-s) g_j^1\big[x_j(s), y_j(s)\big] ds \bigg| \\
&\le \int_{-\infty}^{t} K_{ji}(t-u) \big| g_j\big[x_j(u+s_n), y_j(u+s_n)\big] - g_j^1\big[x_j(u), y_j(u)\big]\big| du.
\end{aligned}$$

Using the Lebesgue dominated convergence theorem, we obtain

$$\lim_{n\longrightarrow\infty} \Phi_{ji}(t+s_n) = \Phi_{ji}^1(t).$$

By the same way, we have

$$\lim_{n\longrightarrow\infty} \Phi_{ji}^1(t-s_n) = \Phi_{ji}(t).$$

Then, $\Phi_{ij}(\cdot) \in AA(\mathbb{R}, \mathbb{R}^n)$. ∎

3 Existence, Uniqueness and Global Exponential Stability of Almost Automorphic Solutions

In this section, we start by studying the existence and the uniqueness of almost automorphic solution of system in Equation (1).

By Assumption 1, the antiderivatives of $\frac{1}{\alpha_i^1(x_i(t))}$ and $\frac{1}{\alpha_j^2(y_j(t))}$ exist.

Then we choose an antiderivatives $F_i^1(x_i)$ of $\frac{1}{\alpha_i^1(x_i(t))}$ and $F_j^2(y_j)$ of $\frac{1}{\alpha_j^2(y_j(t))}$ with $F_i^1(0) = F_j^2(0) = 0$. Obviously, $(F_i^1)'(x_i) = \frac{1}{\alpha_i^1(x_i(t))}$ and $(F_j^2)'(y_j) = \frac{1}{\alpha_j^2(y_j(t))}$.

By $\alpha_i^1(x_i(t)) > 0,\ \alpha_j^2(y_j(t)) > 0$, we see that $F_i^1(x_i), F_j^2(y_j)$ are strictly monotone increasing respectively on x_i and y_j.

By derivative theorem for inverse function, there exist an inverse functions $(F_i^1)^{-1}(x_i)$ of $F_i^1(x_i)$ and $(F_j^2)^{-1}(y_j)$ of $F_j^2(y_j)$ which are continuous and differential.

Moreover, we have $((F_i^1)^{-1}(x_i))' = \alpha_i^1(x_i(t))$ and $((F_j^2)^{-1}(y_j))' = \alpha_j^2(y_j(t))$. Denoting $(F_i^1)'(x_i)x_i'(t) = \frac{x_i'(t)}{\alpha_i^1(x_i(t))} = u_i'(t)$, $(F_j^2)'(y_j)y_j'(t) = \frac{y_j'(t)}{\alpha_j^2(y_j(t))} = v_j'(t)$, we get $x_i(t) = (F_i^1)^{-1}(u_i(t))$ and $y_j(t) = (F_j^2)^{-1}(v_j(t))$. Then, we get:

$$\begin{cases} \dot{u}_i(t) = -a_i^1((F_i^1)^{-1}(u_i(t))) + \displaystyle\sum_{j=1}^{p} b_{ji}^1(t) f_j^1\left[(F_i^1)^{-1}(u_i(t-\tau_{ji})), (F_j^2)^{-1}(v_j(t-\upsilon_{ji}))\right] \\ \qquad + \displaystyle\sum_{j=1}^{p} d_{ji}^1(t) \int_{-\infty}^{t} K_{ji}(t-s) g_j^1\left[(F_i^1)^{-1}(u_i(s)), (F_j^2)^{-1}(v_j(s))\right] ds + I_i(t), \quad 1 \le i \le n, \\ \dot{y}_j(t) = -a_j^2((F_j^2)^{-1}(v_j(t)) + \displaystyle\sum_{i=1}^{n} b_{ij}^2(t) f_i^2\left[(F_i^1)^{-1}(u_i(t-\zeta_{ij})), (F_j^2)^{-1}(v_j(t-\vartheta_{ij}))\right] \\ \qquad + \displaystyle\sum_{i=1}^{n} d_{ij}^2(t) \int_{-\infty}^{t} G_{ij}(t-s) g_i^2\left[(F_i^1)^{-1}(u_i(s)), (F_j^2)^{-1}(v_j(s))\right] ds + J_j(t), \quad 1 \le j \le p, \end{cases} \tag{6}$$

By using Assumption 2 and the mean value theorem, we have

$$a_i^1((F_i^1)^{-1}(u_i(t))) = \left[a_i^1((F_i^1)^{-1}(\theta_i u_i(t)))\right]' u_i(t) = \widetilde{a}_i^1(u_i(t))u_i(t),$$

where θ_i is a constant such that $0 \le \theta_i \le 1$.

$$a_j^2((F_j^2)^{-1}(v_j(t))) = \left[a_j^2((F_j^2)^{-1}(\theta_j v_j(t)))\right]' v_j(t) = \widetilde{a}_j^2(v_j(t))v_j(t),$$

where θ_j is a constant such that $0 \le \theta_j \le 1$.

Substituting this into (6) yields,

$$\begin{cases} \dot{u}_i(t) = -\widetilde{a}_i^1(u_i(t))u_i(t) + \displaystyle\sum_{j=1}^{p} b_{ji}^1(t) f_j^1\left[(F_i^1)^{-1}(u_i(t-\tau_{ji})), (F_j^2)^{-1}(v_j(t-\upsilon_{ji}))\right] \\ \qquad + \displaystyle\sum_{j=1}^{p} d_{ji}^1(t) \int_{-\infty}^{t} K_{ji}(t-s) g_j^1\left[(F_i^1)^{-1}(u_i(s)), (F_j^2)^{-1}(v_j(s))\right] ds + I_i(t), \quad 1 \le i \le n, \\ \dot{y}_j(t) = -\widetilde{a}_j^2(v_j(t))v_j(t) + \sum_{i=1}^{n} b_{ij}^2(t) f_i^2\left[(F_i^1)^{-1}(u_i(t-\zeta_{ij})), (F_j^2)^{-1}(v_j(t-\vartheta_{ij}))\right] \\ \qquad + \displaystyle\sum_{i=1}^{n} d_{ij}^2(t) \int_{-\infty}^{t} G_{ij}(t-s) g_i^2\left[(F_i^1)^{-1}(u_i(s)), (F_j^2)^{-1}(v_j(s))\right] ds + J_j(t), \quad 1 \le j \le p. \end{cases} \tag{7}$$

Remark 3.1 Evidently, System (1) has a unique almost automorphic solution if and only if System (7) has a unique almost automorphic solution. Then we only need to consider the almost automorphic solution of System (7).

By the Lagrange theorem we have

$$\begin{aligned} \left|(F_i^1)^{-1}(u) - (F_i^1)^{-1}(v)\right| &= \left|\left[(F_i^1)^{-1}(v + \theta_i(u-v))\right]'(u-v)\right| \\ &= \left|\alpha_i^1(v + \theta_i(u-v))\right|\left|u-v\right| \end{aligned}$$

and

$$\begin{aligned} \left|(F_j^2)^{-1}(u) - (F_j^2)^{-1}(v)\right| &= \left|\left[(F_j^2)^{-1}(v + \theta_j(u-v))\right]'(u-v)\right| \\ &= \left|\alpha_j^2(v + \theta_j(u-v))\right|\left|u-v\right|. \end{aligned}$$

By Assumption 1, we get

$$\alpha_{i*}^1|u-v| \le |(F_i^1)^{-1}(u)-(F_i^1)^{-1}(v)| \le \alpha_i^{1*}|u-v|,$$

$$\alpha_{j*}^2|u-v| \le |(F_j^2)^{-1}(u)-(F_j^2)^{-1}(v)| \le \alpha_j^{2*}|u-v|.$$

Combined with Assumption 2, we obtain

$$a_{i*}^1\alpha_{i*}^1 \le \left[a_i^1((F_i^1)^{-1}(\cdot))\right] \le a_i^{1*}\alpha_i^{1*},$$

$$a_{j*}^2\alpha_{j*}^2 \le \left[a_j^2((F_j^2)^{-1}(\cdot))\right] \le a_j^{2*}\alpha_j^{2*}.$$

For any arbitrary vector $Z(t)=(x_1(t),x_2(t),\cdots,x_n(t),y_1(t),y_2(t),\cdots,y_p(t))^{\mathrm{T}}$, we define the norm

$$\|Z(t)\| = \max\left\{\max_{1\le i\le n}\{|x_i(t)|\};\max_{1\le j\le p}\{|y_j(t)|\}\right\}.$$

Theorem 3.2 *Under Assumptions* 1–5*, System* (7) *has a unique almost automorphic solution in the region*

$$\Delta = \left\{Z\in AA(\mathbb{R},\mathbb{R}^{n+p}), \|Z-Z_0\|_\infty \le \frac{rM}{1-r}\right\},$$

where

$$Z_0(t) = \begin{pmatrix} \int_{-\infty}^t \mathrm{e}^{-\int_s^t \widetilde{a}_1^1(\varphi_1(u))du} I_1(s)ds \\ \vdots \\ \int_{-\infty}^t \mathrm{e}^{-\int_s^t \widetilde{a}_n^1(\varphi_n(u))du} I_n(s)ds \\ \int_{-\infty}^t \mathrm{e}^{-\int_s^t \widetilde{a}_1^2(\psi_1(u))du} J_1(s)ds \\ \vdots \\ \int_{-\infty}^t \mathrm{e}^{-\int_s^t \widetilde{a}_p^2(\psi_p(u))du} J_p(s)ds \end{pmatrix}.$$

Proof Define the nonlinear operator $\varTheta_Z : AA(\mathbb{R},\mathbb{R}^{n+p}) \mapsto AA(\mathbb{R},\mathbb{R}^{n+p})$ as follows: For all $Z_{(\varphi,\psi)^{\mathrm{T}}}=(\varphi_1,\varphi_2,\cdots,\varphi_n,\psi_1,\psi_2,\cdots,\psi_p)\in AA(\mathbb{R},\mathbb{R}^{n+p})$,

$$\varTheta_Z(t) = \begin{pmatrix} \int_{-\infty}^t \mathrm{e}^{-\int_s^t \widetilde{a}_1^1(\varphi_1(u))du} \varGamma_1^1(s)ds \\ \vdots \\ \int_{-\infty}^t \mathrm{e}^{-\int_s^t \widetilde{a}_n^1(\varphi_n(u))du} \varGamma_n^1(s)ds \\ \int_{-\infty}^t \mathrm{e}^{-\int_s^t \widetilde{a}_1^2(\psi_1(u))du} \varGamma_1^2(s)ds \\ \vdots \\ \int_{-\infty}^t \mathrm{e}^{-\int_s^t \widetilde{a}_p^2(\psi_p(u))du} \varGamma_p^2(s)ds \end{pmatrix},$$

$$
\begin{aligned}
\Gamma_i^1(s,\varphi,\psi) &= \sum_{j=1}^{p} b_{ji}^1(s) f_j^1\big[(F_i^1)^{-1}(\varphi_i(s-\tau_{ji})),(F_j^2)^{-1}(\psi_j(s-\upsilon_{ji}))\big] \\
&\quad + \sum_{j=1}^{p} d_{ji}^1(s)\int_{-\infty}^{s} K_{ji}(s-u) g_j^1\big[(F_i^1)^{-1}(\varphi_i(u)),(F_j^2)^{-1}(\psi_j(u))\big]du + I_i(s), \\
&\quad 1 \le i \le n, \\
\Gamma_j^2(s,\varphi,\psi) &= \sum_{i=1}^{n} b_{ij}^2(s) f_i^2\big[(F_i^1)^{-1}(u_i(s-\zeta_{ij})),(F_j^2)^{-1}(v_j(s-\vartheta_{ij}))\big] \\
&\quad + \sum_{i=1}^{n} d_{ij}^2(s)\int_{-\infty}^{s} G_{ij}(s-u) g_i^2\big[(F_i^1)^{-1}(u_i(u)),(F_j^2)^{-1}(v_j(u))\big]du + J_j(s), \\
&\quad 1 \le j \le p.
\end{aligned}
$$

For all $1 \le i \le n$, $1 \le j \le p$, by using Lemmas 2.8–2.11, the functions Γ_i^1 and Γ_j^2 are almost automorphic.

Suppose that Assumptions 1–5 hold. Because $M[\alpha_i^1] > 0, M[\alpha_j^2] > 0$, the linear system

$$
\begin{cases}
\dot{x}_i(t) = -\widetilde{a}_i^1(u_i(t))u_i(t), \quad 1 \le i \le n, \\
\dot{y}_j(t) = -\widetilde{a}_j^2(t)(v_j(t))v_j(t), \quad 1 \le j \le p,
\end{cases}
$$

admits an exponential dichotomy on $\mathbb{R}$. By Lemma 2.7, System (7) has a unique almost automorphic solution $Z_{(\varphi,\psi)^{\mathrm{T}}}$ which can be expressed as follows

$$
\begin{aligned}
Z_{(\varphi,\psi)^{\mathrm{T}}}(t) = \Bigg(&\int_{-\infty}^{t} \mathrm{e}^{-\int_s^t \widetilde{a}_1^1(u(m))dm}\Bigg[\sum_{j=1}^{p} b_{j1}^1(s) f_j^1\big[(F_i^1)^{-1}(\varphi_j(s-\tau_{j1})),(F_j^2)^{-1}(\psi_j(s-\upsilon_{j1}))\big] \\
&+ \sum_{j=1}^{p} d_{j1}^1(s)\int_{-\infty}^{s} K_{j1}(s-m) g_j^1\big[(F_i^1)^{-1}(\varphi_j(m)),(F_j^2)^{-1}(\psi_j(m))\big]dm + I_1(s)\Bigg]ds, \cdots, \\
&\int_{-\infty}^{t} \mathrm{e}^{-\int_s^t \widetilde{a}_n^1(u(m))dm}\Bigg[\sum_{j=1}^{p} b_{jn}^1(s) f_j^1\big[(F_i^1)^{-1}(\varphi_j(s-\tau_{jn})),(F_j^2)^{-1}(\psi_j(s-\upsilon_{jn}))\big] \\
&+ \sum_{j=1}^{p} d_{jn}^1(s)\int_{-\infty}^{s} K_{jn}(s-m) g_j^1\big[(F_i^1)^{-1}(\varphi_j(m)),(F_j^2)^{-1}(\psi_j(m))\big]dm + I_n(s)\Bigg]ds, \cdots, \\
&\int_{-\infty}^{s} \mathrm{e}^{-\int_s^t \widetilde{a}_1^2(v(m))dm}\Bigg[\sum_{i=1}^{n} b_{i1}^2(s) f_i^2\big[(F_i^1)^{-1}(\varphi_i(s-\zeta_{i1})),(F_j^2)^{-1}(\psi_i(s-\vartheta_{i1}))\big] \\
&+ \sum_{i=1}^{n} d_{i1}^2(s)\int_{-\infty}^{s} G_{i1}(s-m) g_i^2\big[(F_i^1)^{-1}(\varphi_i(m)),(F_j^2)^{-1}(\psi_i(m))\big]dm + J_1(s)\Bigg]ds, \cdots, \\
&\int_{-\infty}^{s} \mathrm{e}^{-\int_s^t \widetilde{a}_p^2(v(m))dm}\Bigg[\sum_{i=1}^{n} b_{ip}^2(s) f_i^2\big[(F_i^1)^{-1}(\varphi_i(s-\zeta_{ip})),(F_j^2)^{-1}(\psi_i(s-\vartheta_{ip}))\big] \\
&+ \sum_{i=1}^{n} d_{ip}^2(s)\int_{-\infty}^{s} G_{ip}(s-m) g_i^2\big[(F_i^1)^{-1}(\varphi_i(m)),(F_j^2)^{-1}(\psi_i(m))\big]dm + J_p(s)\Bigg]ds\Bigg).
\end{aligned}
$$

One has

$$
\begin{aligned}
M &= \|Z_0\| \\
&= \max\Bigg\{ \sup_{t\in\mathbb{R}} \max_{1\le i\le n} \Bigg\{ \Bigg| \int_{-\infty}^{t} \mathrm{e}^{-\int_s^t \tilde{a}_i^1(u(m))dm} I_i(s)ds \Bigg| \Bigg\}; \\
&\qquad \sup_{t\in\mathbb{R}} \max_{1\le j\le p} \Bigg\{ \Bigg| \int_{-\infty}^{t} \mathrm{e}^{-\int_s^t \tilde{a}_j^2(v(m))dm} J_j(s)ds \Bigg| \Bigg\} \Bigg\} \\
&\le \max\Bigg\{ \sup_{t\in\mathbb{R}} \max_{1\le i\le n} \Bigg\{ \Bigg| \int_{-\infty}^{t} \mathrm{e}^{-a_{i*}^1\alpha_{i*}^1(t-s)} I_i(s)ds \Bigg| \Bigg\}; \\
&\qquad \sup_{t\in\mathbb{R}} \max_{1\le j\le p} \Bigg\{ \Bigg| \int_{-\infty}^{t} \mathrm{e}^{-a_{j*}^2\alpha_{j*}^2(t-s)} J_j(s)ds \Bigg| \Bigg\} \Bigg\} \\
&\le \max\Bigg\{ \max_{1\le i\le n} \bigg(\frac{I_i^*}{a_{i*}^1\alpha_{i*}^1} \bigg); \max_{1\le j\le p} \bigg(\frac{J_j^*}{a_{j*}^2\alpha_{j*}^2} \bigg) \Bigg\}.
\end{aligned}
$$

After

$$\|Z\| \le \|Z - Z_0\| + \|Z_0\| \le \frac{rM}{1-r} + M.$$

Set $\Delta = \big\{Z \in AA(\mathbb{R},\mathbb{R}^{n+p}) : \|Z - Z_0\| \le \frac{rM}{1-r}\big\}$. Clearly, Δ is a closed convex subset of $AA(\mathbb{R},\mathbb{R}^{n+p})$. We have, for $1\le i\le n,\ 1\le j\le p$,

$$
\begin{aligned}
&\big\|\Theta_Z(t) - Z_0(t)\big\| \\
&= \max\Bigg\{ \sup_{t\in\mathbb{R}} \max_{1\le i\le n} \Bigg\{ \Bigg| \int_{-\infty}^{t} \mathrm{e}^{-\int_s^t \tilde{a}_i^1(u_i(m))dm} \Bigg[\sum_{j=1}^{p} b_{ji}^1(s) f_j^1\big[(F_i^1)^{-1}(\varphi_i(s-\tau_{ji})), (F_j^2)^{-1}(\psi_j(s-\upsilon_{ji}))\big] \\
&\quad + \sum_{j=1}^{p} d_{ji}^1(s) \int_{-\infty}^{s} K_{ji}(s-m) g_j^1\big[(F_i^1)^{-1}(\varphi_i(m)), (F_j^2)^{-1}(\psi_j(m))\big] dm \Bigg] ds \Bigg| \Bigg\}; \\
&\quad \sup_{t\in\mathbb{R}} \max_{1\le j\le p} \Bigg\{ \Bigg| \int_{-\infty}^{t} \mathrm{e}^{-\int_s^t \tilde{a}_j^2(v_j(m))dm} \Bigg[\sum_{i=1}^{n} b_{ij}^2(s) f_i^2\big[(F_i^1)^{-1}(u_i(s-\zeta_{ij})), (F_j^2)^{-1}(v_j(s-\vartheta_{ij}))\big] \\
&\quad + \sum_{i=1}^{n} d_{ij}^2(s) \int_{-\infty}^{s} G_{ij}(s-m) g_i^2\big[(F_i^1)^{-1}(u_i(m)), (F_j^2)^{-1}(v_j(m))\big] dm \Bigg] ds \Bigg| \Bigg\} \Bigg\} \\
&\le \max\Bigg\{ \sup_{t\in\mathbb{R}} \max_{1\le i\le n} \Bigg\{ \int_{-\infty}^{t} \mathrm{e}^{-\int_s^t \tilde{a}_i^1(u_i(m))dm} \Bigg[\sum_{j=1}^{p} \big|b_{ji}^1(s)\big| \big|f_j^1\big[(F_i^1)^{-1}(\varphi_i(s-\tau_{ji})), (F_j^2)^{-1}(\psi_j(s-\upsilon_{ji}))\big]\big| \\
&\quad + \sum_{j=1}^{p} \big|d_{ji}^1(s)\big| \int_{-\infty}^{s} K_{ji}(s-m) \big|g_j^1\big[(F_i^1)^{-1}(\varphi_i(m)), (F_j^2)^{-1}(\psi_j(m))\big]\big| dm \Bigg] ds \Bigg\};
\end{aligned}
$$

$$
\begin{aligned}
&\sup_{t\in\mathbb{R}}\max_{1\le j\le p}\bigg\{\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t \widetilde{a}_j^2(v_j(m))dm}\bigg[\sum_{i=1}^{n}\big|b_{ij}^2(s)\big|\big|f_i^2\big[(F_i^1)^{-1}(u_i(s-\zeta_{ij})),(F_j^2)^{-1}(v_j(s-\vartheta_{ij}))\big]\big|\\
&+\sum_{i=1}^{n}\big|d_{ij}^2(s)\big|\int_{-\infty}^{s}G_{ij}(s-m)\big|g_i^2\big[(F_i^1)^{-1}(u_i(m)),(F_j^2)^{-1}(v_j(m))\big]\big|dm\bigg]ds\bigg\}\bigg\}\\
&\le\max\bigg\{\max_{1\le i\le n}\bigg\{\frac{1}{a_{i*}^1\alpha_{i*}^1}\sum_{j=1}^{p}\big[b_{ji}^{1*}(L_j^f+M_j^f)+d_{ji}^{1*}(L_j^g+M_j^g)\big]\alpha_j^{1*}\bigg\};\\
&\max_{1\le i\le n}\bigg\{\frac{1}{a_{j*}^2\alpha_{j*}^2}\sum_{i=1}^{n}\big[b_{ij}^{2*}(L_i^f+M_i^f)+d_{ij}^{2*}(L_i^g+M_i^g)\big]\alpha_i^{2*}\bigg\}\bigg\}\|Z\|=r\|Z\|,
\end{aligned}
$$

then $\Theta_Z\in\Delta$.

We next prove that the mapping Θ is a contraction mapping of the Δ. Let Z, $\widetilde{Z}\in\Delta$,

$$
\begin{aligned}
&\big\|\Theta_Z(t)-\Theta_{\widetilde{Z}}(t)\big\|\\
&=\max\bigg\{\sup_{t\in\mathbb{R}}\max_{1\le i\le n}\bigg\{\bigg|\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t \widetilde{a}_i^1(u_i(m))dm}\bigg[\sum_{j=1}^{p}b_{ji}^1(s)\bigg\{f_j^1\big[(F_i^1)^{-1}(\varphi_j(s-\tau_{ji})),(F_j^2)^{-1}(\psi_j(s-\upsilon_{ji}))\big]\\
&-f_j^1\big[(F_i^1)^{-1}(\widetilde{\varphi}_j(s-\tau_{ji})),(F_j^2)^{-1}(\widetilde{\psi}_j(s-\upsilon_{ji}))\big]\bigg\}\\
&+\sum_{j=1}^{p}d_{ji}^1(s)\int_{-\infty}^{s}K_{ji}(s-m)\bigg\{g_j^1\big[(F_i^1)^{-1}(\varphi_j(m)),(F_j^2)^{-1}(\psi_j(m))\big]\\
&-g_j^1\big[(F_i^1)^{-1}(\widetilde{\varphi}_j(m)),(F_j^2)^{-1}(\widetilde{\psi}_j(m))\big]dm\bigg\}ds\bigg|\bigg\};\\
&\sup_{t\in\mathbb{R}}\max_{1\le j\le p}\bigg\{\bigg|\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t \widetilde{a}_j^2(v_j(m))dm}\bigg[\sum_{i=1}^{n}b_{ij}^2(s)\bigg\{f_i^2\bigg[(F_i^1)^{-1}(\varphi_i(s-\zeta_{ij})),(F_j^2)^{-1}(\psi_i(s-\vartheta_{ij}))\bigg]\\
&-f_i^2\big[(F_i^1)^{-1}(\widetilde{\varphi}_i(s-\zeta_{ij})),(F_j^2)^{-1}(\widetilde{\psi}_i(s-\vartheta_{ij}))\big]\bigg\}\\
&+\sum_{i=1}^{n}d_{ij}^2(s)\int_{-\infty}^{t}G_{ij}(s-m)\bigg\{g_i^2\big[(F_i^1)^{-1}(\varphi_i(m)),(F_j^2)^{-1}(\psi_i(m))\big]\\
&-g_i^2\big[(F_i^1)^{-1}(\widetilde{\varphi}_i(m)),(F_j^2)^{-1}(\widetilde{\psi}_i(m))\big]dm\bigg\}ds\bigg|\bigg\}\bigg\}\\
&\le\max\bigg\{\max_{1\le i\le n}\bigg\{\frac{1}{a_{i*}^1\alpha_{i*}^1}\sum_{j=1}^{p}\big[b_{ji}^{1*}(L_j^f+M_j^f)+d_{ji}^{1*}(L_j^g+M_j^g)\big]\alpha_j^{1*}\bigg\};\\
&\max_{1\le i\le n}\bigg\{\frac{1}{a_{j*}^2\alpha_{j*}^2}\sum_{i=1}^{n}\big[b_{ij}^{2*}(L_i^f+M_i^f)+d_{ij}^{2*}(L_i^g+M_i^g)\big]\alpha_i^{2*}\bigg\}\bigg\}\|Z-\widetilde{Z}\|=r\|Z-\widetilde{Z}\|,
\end{aligned}
$$

which prove that Θ is a contraction mapping. Then, by virtue of the Banach fixed point theorem, Θ has a unique fixed point which corresponds to the solution of system in Equation (7). The proof is completed. ∎

Remark 3.3 To the best of our knowledge, there have been no results on the almost automorphic solutions for interval general Cohen-Grossberg BAM neural networks with time-varying coefficients and mixed time-varying delays until now. Hence, the obtained results are essentially new.

Now, we establish new results for the global exponential stability of almost automorphic solution of system in Equation (1).

Definition 3.4 Let $Z^*(t) = (x_1^*(t), x_2^*(t), \cdots, x_n^*(t), y_1^*(t), y_2^*(t), \cdots, y_p^*(t))^{\mathrm{T}}$ an almost automorphic solution of system in Equation (1) with initial value

$$\phi^*(s) = (\varphi_1^*(s), \varphi_2^*(s), \cdots, \varphi_n^*(s), \psi_1^*(s), \psi_2^*(s), \cdots, \psi_p^*(s))^{\mathrm{T}}.$$

If there exist a positive constant λ and $M > 1$ such that for every solution $Z(t) = (x_1(t), x_2(t), \cdots, x_n(t), y_1(t), y_2(t), \cdots, y_p(t))^{\mathrm{T}}$ of system in Equation (6) with any initial value $\phi(t) = (\varphi_1(s), \cdots, \varphi_n(s), \psi_1(s), \cdots, \psi_p(s))^{\mathrm{T}}$ satisfies

$$\| x - x^* \| \leq M \| \phi - \phi^* \| \mathrm{e}^{-\lambda t}, \quad \| y - y^* \| \leq M \| \phi - \phi^* \| \mathrm{e}^{-\lambda t}, \quad \forall t \geq 0,$$

then, Z^* is said to be globally exponentially stable.

Theorem 3.5 *Under Assumptions* 1–5 *and Theorem* 3.2*, the unique almost automorphic solution of system in Equation* (7) *is globally exponentially stable.*

Proof Suppose that $Z(t) = (x_1(t), x_2(t), \cdots, x_n(t), y_1(t), y_2(t), \cdots, y_p(t))^{\mathrm{T}}$ be an arbitrary solution of system in Equation (6) with initial value $\phi(t) = (\varphi_1(t), \varphi_2(t), \cdots, \varphi_n(t), \psi_1(t), \psi_2(t), \cdots, \psi_p(t))^{\mathrm{T}}$. It follows from Theorem 3.2 that system in Equation (1) has one and only one almost automorphic solution $Z^*(t) = (x_1^*(t), x_2^*(t), \cdots, x_n^*(t), y_1^*(t), y_2^*(t), \cdots, y_p^*(t))^{\mathrm{T}} \in \Delta$, with initial value $\phi^*(t) = (\varphi_1^*(t), \varphi_2^*(t), \cdots, \varphi_n^*(t), \psi_1^*(t), \psi_2^*(t), \cdots, \psi_p^*(t))^{\mathrm{T}}$. Let $U_i(t) = u_i(t) - u_i^*(t)$ and $V_j(t) = v_j(t) - v_j^*(t)$, for $i = 1, 2, \cdots, n,\ j = 1, 2, \cdots, p$.

Similarly to 7, we have

$$\begin{aligned}
&a_i^1((F_i^1)^{-1}(u_i(t))) - a_i^1((F_i^1)^{-1}(u_i^*(t))) \\
&= a_i^1((F_i^1)^{-1}(U_i(t) + u_i^*(t))) - a_i^1((F_i^1)^{-1}(u_i^*(t))) \\
&= \left[a_i^1((F_i^1)^{-1}(u_i^*(t) + \theta U_i(t)))\right]' U_i(t) \\
&= \beta_i^1(U_i(t)) U_i(t)
\end{aligned}$$

such that $\beta_i^1(U_i(t)) = \left[a_i^1((F_i^1)^{-1}(u_i^*(t) + \theta U_i(t)))\right]' U_i(t)$, $0 \leq \theta \leq 1$ and

$$\begin{aligned}
&a_j^2((F_j^2)^{-1}(v_j(t))) - a_j^2((F_j^2)^{-1}(v_j^*(t))) \\
&= a_j^2((F_j^2)^{-1}(V_j(t) + v_j^*(t))) - a_j^2((F_j^2)^{-1}(v_j^*(t))) \\
&= \left[a_j^2((F_j^2)^{-1}(v_j^*(t) + \theta V_j(t)))\right]' V_j(t) \\
&= \beta_j^2(V_j(t)) V_j(t)
\end{aligned}$$

such that $\beta_j^2(V_j(t)) = \big[a_j^2((F_j^2)^{-1}(v_j^*(t)+\theta V_j(t)))\big]$, $0\le\theta\le 1$. Then, we have

$$\begin{cases} \dot{U}_i(t) = -\beta_i^1(U_i(t))U_i(t) + \displaystyle\sum_{j=1}^{p} b_{ji}^1(t)\big\{f_j^1\big[(F_i^1)^{-1}(u_j(t-\tau_{ji})),(F_j^2)^{-1}(v_j(t-\upsilon_{ji}))\big] \\ \qquad -f_j^1\big[(F_i^1)^{-1}(u_j^*(t-\tau_{ji})),(F_j^2)^{-1}(v_j^*(t-\upsilon_{ji}))\big]\big\} \\ \qquad +\displaystyle\sum_{j=1}^{p} d_{ji}^1(t)\int_{-\infty}^{t} K_{ji}(t-m)\big\{g_j^1\big[(F_i^1)^{-1}(u_j(m)),(F_j^2)^{-1}(v_j(m))\big] \\ \qquad -g_j^1\big[(F_i^1)^{-1}(u_j^*(m)),(F_j^2)^{-1}(v_j^*(m))\big]\big\}dm, \quad 1\le i\le n, \\ \dot{V}_j(t) = -\beta_j^2(V_j(t))V_j(t) + \displaystyle\sum_{i=1}^{n} b_{ij}^2(t)\big\{f_i^2\big[(F_i^1)^{-1}(u_i(t-\zeta_{ij})),(F_j^2)^{-1}(v_i(t-\vartheta_{ij}))\big] \\ \qquad -f_i^2\big[(F_i^1)^{-1}(u_i^*(t-\zeta_{ij})),(F_j^2)^{-1}(v_i^*(t-\vartheta_{ij}))\big]\big\} \\ \qquad +\displaystyle\sum_{i=1}^{n} d_{ij}^2(t)\int_{-\infty}^{t} G_{ij}(t-m)\big\{g_i^2\big[(F_i^1)^{-1}(u_i(m)),(F_j^2)^{-1}(v_i(m))\big] \\ \qquad -g_i^2\big[(F_i^1)^{-1}(u_i^*(m)),(F_j^2)^{-1}(v_i^*(m))\big]\big\}dm, \quad 1\le j\le p. \end{cases} \tag{8}$$

The initial conditions of System (8) are

$$\begin{cases} \dot{U}_i(t) = \varphi_i(s)-\varphi_i^*(s), \quad s\in(-\infty,0], \quad 1\le i\le n, \\ \dot{V}_j(t) = \psi(s)_j-\psi_j^*(s), \quad s\in(-\infty,0], \quad 1\le j\le p. \end{cases} \tag{9}$$

For $i=1,2,\cdots,n$, $j=1,2,\cdots,p$, $w\in[0,+\infty[$, let Γ_i^1,Γ_j^1, be defined by

$$\widetilde{\Gamma}_i^1(w) = a_{i*}^1\alpha_{i*}^1 - w - \sum_{j=1}^{p}\left[b_{ji}^{1*}(L_j^f e^{\tau_{ji}w}+M_j^f e^{\upsilon_{ji}w}) + d_{ji}^{1*}\int_{-\infty}^{t} K_{ji}(t-u)(L_j^g+M_j^g)e^{wu}du\right]\alpha_j^1$$

and

$$\widetilde{\Gamma}_j^2(w) = a_{j*}^2\alpha_{j*}^2 - w - \sum_{i=1}^{n}\left[b_{ij}^{2*}(L_i^f e^{\zeta_{ij}w}+M_i^f e^{\vartheta_{ij}w}) + d_{ij}^{2*}\int_{-\infty}^{t} G_{ij}(t-u)(L_i^g+M_i^g)e^{wu}du\right]\alpha_j^2.$$

In view of Assumption 4, for $i=1,2,\cdots,n$, $j=1,2,\cdots,p$, we obtain

$$\begin{cases} \widetilde{\Gamma}_i^1(0) = a_{i*}^1\alpha_{i*}^1 - \displaystyle\sum_{j=1}^{p}\left[b_{ji}^{1*}(L_j^f+M_j^f) + d_{ji}^{1*}\int_{-\infty}^{t} K_{ji}(t-u)(L_j^g+M_j^g)du\right]\alpha_j^1 > 0, \\ \widetilde{\Gamma}_j^2(0) = a_{j*}^2\alpha_{j*}^2 - \displaystyle\sum_{i=1}^{n}\left[b_{ij}^{2*}(L_i^f+M_i^f) + d_{ij}^{2*}\int_{-\infty}^{t} G_{ij}(t-u)(L_i^g+M_i^g)du\right]\alpha_i^2 > 0. \end{cases}$$

Both, $\widetilde{\Gamma}_i^1(.)$ and $\widetilde{\Gamma}_j^2(.)$ are continuous on $[0,\infty[$ such that $\widetilde{\Gamma}_i^1(w)\longrightarrow-\infty$ when $w\longmapsto+\infty$, $\exists\varepsilon_i^*>0$ such that $\widetilde{\Gamma}_i^1(\varepsilon_i^*)=0$ and $\widetilde{\Gamma}_i^1(\varepsilon_i)>0$ for $\varepsilon_i\in(0,\varepsilon_i^*)$ and $\widetilde{\Gamma}_j^2(w)\longrightarrow-\infty$ when $w\longmapsto+\infty$, $\exists\zeta_j^*>0$ such that $\Gamma_j^2(\zeta_j^*)=0$ and $\widetilde{\Gamma}_j^2(\zeta_j)>0$ for $\zeta_j\in(0,\zeta_j^*)$.

By choosing $\eta=\min\{\varepsilon_0,\varepsilon_1^*,\cdots,\varepsilon_n^*,\zeta_1^*,\zeta_2^*,\cdots,\zeta_p^*\}$, we obtain

$$\begin{cases} \widetilde{\Gamma}_i^1(\eta)\ge 0, \quad i=1,2,\cdots,n, \\ \widetilde{\Gamma}_j^2(\eta)\ge 0, \quad j=1,2,\cdots,p. \end{cases}$$

We can choose a positive constant $\lambda \in (0,\eta)$ such that

$$\begin{cases} \dfrac{1}{a^1_{i*}\alpha^1_{i*}-\lambda}\displaystyle\sum_{j=1}^{p}\Big\{b^{1*}_{ji}(L^f_j \mathrm{e}^{\lambda\tau_{ji}}+M^f_j \mathrm{e}^{\lambda\upsilon_{ji}})+d^{1*}_{ji}\int_{-\infty}^{t} G_{ij}(t-m)(L^g_i+M^g_i)\mathrm{e}^{\lambda m}dm\Big\}\alpha^1_j<1,\\ \dfrac{1}{a^2_{j*}\alpha^2_{j*}-\lambda}\displaystyle\sum_{i=1}^{n}\Big\{b^{2*}_{ij}(L^f_i \mathrm{e}^{\lambda\zeta_{ij}}+M^f_i \mathrm{e}^{\lambda\vartheta_{ij}})+d^{2*}_{ij}\int_{-\infty}^{t} K_{ji}(s-u)(L^g_j+M^g_j)\mathrm{e}^{\lambda m}dm\Big\}\alpha^2_i<1. \end{cases}$$

Multiplying Equation (8) by $\mathrm{e}^{-\int_0^s \beta^1_i(U_i(m))dm}$ and $\mathrm{e}^{-\int_0^s \beta^2_j(V_j(m))dm}$ and integrating on $[0,t]$, we get

$$\begin{cases} \dot U_i(t)=U_i(0)\mathrm{e}^{-\int_0^t \beta^1_i(U_i(m))dm}\\ \qquad +\displaystyle\int_0^t \mathrm{e}^{-\int_0^t \beta^1_i(U_i(m))dm}\Big\{\Big[\sum_{j=1}^{p} b^1_{ji}(s)\big\{f^1_j\big[(F^1_i)^{-1}(u_j(s-\tau_{ji})),(F^2_j)^{-1}(v_j(s-\upsilon_{ji}))\big]\\ \qquad -f^1_j\big[(F^1_i)^{-1}(u^*_j(s-\tau_{ji})),(F^2_j)^{-1}(v^*_j(s-\upsilon_{ji}))\big]\big\}\\ \qquad +\displaystyle\sum_{j=1}^{p} d^1_{ji}(t)\int_{-\infty}^{s} K_{ji}(s-m)\Big\{g^1_j\big[(F^1_i)^{-1}(u_j(m)),(F^2_j)^{-1}(v_j(m))\big]\\ \qquad -g^1_j\big[(F^1_i)^{-1}(u^*_j(m)),(F^2_j)^{-1}(v^*_j(m))\big]dm\Big\}ds,\quad 1\le i\le n,\\ \dot V_j(t)=V_j(0)\mathrm{e}^{-\int_0^t \beta^2_j(V_j(m))dm}\\ \qquad +\displaystyle\int_0^t \mathrm{e}^{-\int_0^t \beta^2_j(V_j(m))dm}\Big\{\Big[\sum_{i=1}^{n} b^2_{ij}(s)\Big\{f^2_i\big[(F^1_i)^{-1}(u_i(s-\zeta_{ij})),(F^2_j)^{-1}(v_i(s-\vartheta_{ij}))\big]\\ \qquad -f^2_i\big[(F^1_i)^{-1}(u^*_i(s-\zeta_{ij})),(F^2_j)^{-1}(v^*_i(s-\vartheta_{ij}))\big]\Big\}\\ \qquad +\displaystyle\sum_{i=1}^{n} d^2_{ij}(s)\int_{-\infty}^{s} G_{ij}(s-m)\Big\{g^2_i\big[(F^1_i)^{-1}(u_i(m)),(F^2_j)^{-1}(v_i(m))\big]\\ \qquad -g^2_i\big[(F^1_i)^{-1}(u^*_i(m)),(F^2_j)^{-1}(v^*_i(m))\big]dm\Big\}ds,\quad 1\le j\le p. \end{cases}$$

Let

$$N=\max\Big\{\max_{1\le i\le n}\frac{a^1_{i*}\alpha^1_{i*}}{\sum_{j=1}^{p}\big[b^{1*}_{ji}(L^f_j+M^f_j)+d^{1*}_{ji}(L^g_j+M^g_j)\big]\alpha^{1*}_j};\\ \max_{1\le j\le p}\frac{a^2_{j*}\alpha^2_{j*}}{\sum_{i=1}^{n}\big[b^{2*}_{ij}(L^f_i+M^f_i)+d^{2*}_{ij}(L^g_i+M^g_i)\big]\alpha^{2*}_i}\Big\}.$$

Besides, $\forall\, t\in(-\infty,0]$,

$$\begin{cases} \parallel U\parallel\le N\mathrm{e}^{-\lambda t}\parallel\phi-\phi^*\parallel,\\ \parallel V\parallel\le N\mathrm{e}^{-\lambda t}\parallel\phi-\phi^*\parallel. \end{cases}\tag{10}$$

We claim that, for $t>0$,

$$\begin{cases} \parallel U\parallel\le N\mathrm{e}^{-\lambda t}\parallel\phi-\phi^*\parallel,\\ \parallel V\parallel\le N\mathrm{e}^{-\lambda t}\parallel\phi-\phi^*\parallel, \end{cases}\tag{11}$$

If (10) is false, then there must be some $t_1 > 0$ some $i \in \{1, 2, \cdots, n\}$, $j \in \{1, 2, \cdots, p\}$, for any $p > 1$ and some k such that

$$\begin{cases} \| U(t_1) \| = pN \| \phi - \phi^* \| \mathrm{e}^{-\lambda t_1}, \\ \| V(t_1) \| = pN \| \phi - \phi^* \| \mathrm{e}^{-\lambda t_1}, \end{cases} \tag{12}$$

and

$$\begin{cases} \| U(t) \| < pN \| \phi - \phi^* \| \mathrm{e}^{-\lambda t}, \quad \forall\ t \in (-\infty, t_1], \\ \| V(t) \| < pN \| \phi - \phi^* \| \mathrm{e}^{-\lambda t}, \quad \forall\ t \in (-\infty, t_1]. \end{cases} \tag{13}$$

Now, we have the following:

$$\begin{aligned}
\big|U_i(t_1)\big| &= \Bigg| U_i(0)\mathrm{e}^{-\int_0^{t_1} \beta_i^1(U_i(m))dm} \\
&\quad + \int_0^{t_1} \mathrm{e}^{-\int_0^{t_1} \beta_i^1(U_i(m))dm} \Bigg\{ \Bigg[\sum_{j=1}^{p} b_{ji}^1(s) \Bigg\{ f_j^1 \Big[(F_i^1)^{-1}(u_j(s-\tau_{ji})), (F_j^2)^{-1}(v_j(s-\upsilon_{ji})) \Big] \\
&\quad - f_j^1 \Big[(F_i^1)^{-1}(u_j^*(s-\tau_{ji})), (F_j^2)^{-1}(v_j^*(s-\upsilon_{ji})) \Big] \Bigg\} \\
&\quad + \sum_{j=1}^{p} d_{ji}^1(t) \int_0^{\infty} K_{ji}(m) \Bigg\{ g_j^1 \Big[(F_i^1)^{-1}(u_j(s-m)), (F_j^2)^{-1}(v_j(s-m)) \Big] \\
&\quad - g_j^1 \Big[(F_i^1)^{-1}(u_j^*(s-m)), (F_j^2)^{-1}(v_j^*(s-m)) \Big] dm \Bigg\} ds \Bigg| \\
&\leq \| \phi - \phi^* \| \mathrm{e}^{-t_1 a_{i*}^1 \alpha_{i*}^1} + \int_0^{t_1} \mathrm{e}^{-(t_1-s) a_{i*}^1 \alpha_{i*}^1} \Bigg\{ \sum_{j=1}^{p} b_{ji}^{1*} (L_j^f + M_j^f) \| \phi - \phi^* \| \\
&\quad + \sum_{j=1}^{p} d_{ji}^{1*} \int_0^{\infty} K_{ji}(m)(L_j^g + M_j^g) \| \phi - \phi^* \| dm \Bigg\} ds \\
&\leq \| \phi - \phi^* \| \mathrm{e}^{-t_1 a_{i*}^1 \alpha_{i*}^1} \\
&\quad + \int_0^{t_1} \mathrm{e}^{-(t_1-s) a_{i*}^1 \alpha_{i*}^1} \Bigg\{ \sum_{j=1}^{p} b_{ji}^{1*} (L_j^f \mathrm{e}^{(s-\tau_{ji})\lambda} + M_j^f \mathrm{e}^{(s-\upsilon_{ji})\lambda}) pN \| \phi - \phi^* \| \\
&\quad + \sum_{j=1}^{p} d_{ji}^{1*} \int_0^{\infty} K_{ji}(m)(L_j^g + M_j^g) \mathrm{e}^{-\lambda(s-m)} dm pN \| \phi - \phi^* \| \Bigg\} ds \\
&\leq \| \phi - \phi^* \| \mathrm{e}^{-t_1 a_{i*}^1 \alpha_{i*}^1} + \int_0^{t_1} \mathrm{e}^{-(t_1-s) a_{i*}^1 \alpha_{i*}^1} pN \| \phi - \phi^* \| \\
&\quad \mathrm{e}^{-\lambda s} \Bigg\{ \sum_{j=1}^{p} b_{ji}^{1*} (L_j^f \mathrm{e}^{\lambda \tau_{ji}} + M_j^f \mathrm{e}^{\lambda \upsilon_{ji}}) + \sum_{j=1}^{p} d_{ji}^{1*} \int_0^{\infty} K_{ji}(m)(L_j^g + M_j^g) \mathrm{e}^{\lambda m} dm \Bigg\} ds
\end{aligned}$$

$$
\begin{aligned}
&\le pN \parallel \phi-\phi^* \parallel \mathrm{e}^{-\lambda t_1}\Big\{\mathrm{e}^{(\lambda-a^1_{i*}\alpha^1_{i*})t_1}\Big[\frac{1}{N}-\frac{1}{a^1_{i*}\alpha^1_{i*}-\lambda}\sum_{j=1}^{p}\Big\{b^{1*}_{ji}(L^f_j\mathrm{e}^{\lambda\tau_{ji}}+M^f_j\mathrm{e}^{\lambda\upsilon_{ji}})\\
&+d^{1*}_{ji}\int_0^\infty K_{ji}(m)(L^g_j+M^g_j)\mathrm{e}^{\lambda m}dm\Big\}\alpha^1_j\Big]+\frac{1}{a^1_{i*}\alpha^1_{i*}-\lambda}\sum_{j=1}^{p}\Big\{b^{1*}_{ji}(L^f_j\mathrm{e}^{\lambda\tau_{ji}}+M^f_j\mathrm{e}^{\lambda\upsilon_{ji}})\\
&+d^{1*}_{ji}\int_0^\infty K_{ji}(m)(L^g_j+M^g_j)\mathrm{e}^{\lambda m}dm\Big\}\alpha^1_j\Big\}\\
&\le pN \parallel \phi-\phi^* \parallel \mathrm{e}^{-\lambda t_1}\Big\{\frac{1}{a^1_{i*}\alpha^1_{i*}-\lambda}\sum_{j=1}^{p}\Big\{b^{1*}_{ji}(L^f_j\mathrm{e}^{\lambda\tau_{ji}}+M^f_j\mathrm{e}^{\lambda\upsilon_{ji}})\\
&+d^{1*}_{ji}\int_0^\infty K_{ji}(m)(L^g_j+M^g_j)\mathrm{e}^{\lambda m}dm\Big\}\alpha^1_j\Big\}\\
&< pN \parallel \phi-\phi^* \parallel \mathrm{e}^{-\lambda t_1}. \qquad (14)
\end{aligned}
$$

We can easy obtain some upper bound of $\big|V_j(t_1)\big|$ as follows:

$$
|V_j(t_1)| < pN \parallel \phi-\phi^* \parallel \mathrm{e}^{-\lambda t_1}. \qquad (15)
$$

(14) and (15) contradict (11), then (10) holds. Letting $p \longrightarrow 1$, then (11) holds.
Hence, the almost automorphic solution Z of System (7) is globally exponentially stable. ∎

4 Numerical Example and Comparisons

In this section, to illustrate the feasibility of our theoretical findings obtained in previous sections, we give a numerical example. Consider the following interval general CGBAM neural networks with mixed delays:

$$
\begin{cases}
\dot{x}_i(t) = -\alpha^1_i(x_i(t))\Big\{a^1_i(x_i(t)) - \sum_{j=1}^{2} b^1_{ji}(t)f^1_j\big[x_j(t-\tau_{ji}), y_j(t-\upsilon_{ji})\big]\\
\qquad -\sum_{j=1}^{2} d^1_{ji}(t)\int_{-\infty}^{t} K_{ji}(t-s)g^1_j\big[x_j(s), y_j(s)\big]ds - I_i(t)\Big\}, \quad 1\le i\le 2,\\
\dot{y}_j(t) = -\alpha^2_j(x_i(t))\Big\{a^2_j(y_j(t)) - \sum_{i=1}^{2} b^2_{ij}(t)f^2_i\big[x_i(t-\zeta_{ij}), y_i(t-\vartheta_{ij})\big]\\
\qquad -\sum_{i=1}^{2} d^2_{ij}(t)\int_{-\infty}^{t} G_{ij}(t-s)g^2_i\big[x_i(s), y_i(s)\big]ds - J_j(t)\Big\}, \quad 1\le j\le 2.
\end{cases} \qquad (16)
$$

For all $x, y \in \mathbb{R}$, $i, j = 1, 2$, we have:

$$
(\alpha^1_i(x_i(t)))_{1\le i\le 2} = \begin{pmatrix} 0.4+0.1\cos(x_i(t)) \\ 0.4-0.1\cos(x_i(t)) \end{pmatrix},
$$

$$
(\alpha^2_j(y_j(t)))_{1\le j\le 2} = \begin{pmatrix} 0.4+0.1\sin(y_j(t)) \\ 0.4-0.1\sin(y_j(t)) \end{pmatrix},
$$

$$
\begin{aligned}
&(a_i^1(x_i(t)))_{1\le i\le 2} = \begin{pmatrix} 0.2x_i(t) \\ 0.2x_i(t) \end{pmatrix},\\
&(a_j^2(y_j(t)))_{1\le j\le 2} = \begin{pmatrix} 0.4y_j(t) \\ 0.4y_j(t) \end{pmatrix},\\
&\tau_{ji} = \upsilon_{ji} = \zeta_{ij} = \vartheta_{ij} = L_i^f = M_i^f = L_i^g = M_i^g = L_j^f = M_j^f = L_j^g = M_j^g = 0.5,\\
&K_{ji}(t) = G_{ij}(t) = \mathrm{e}^{-t}, \quad f_j^1(x,y) = g_j^1(x,y) = f_i^1(x,y) = g_i^1(x,y) = \sin x + \sin y.\\
&b_{ji}^1(t) = \begin{pmatrix} 0.04\cos\frac{1}{2+\sin t+\sin\sqrt{2}t} & 0 \\ 0 & 0.04\sin\frac{1}{2+\cos t+\cos\sqrt{2}t} \end{pmatrix},\\
&d_{ji}^1(t) = \begin{pmatrix} 0.04\cos\frac{1}{2+\sin t+\sin\sqrt{2}t} & 0 \\ 0 & 0.04\sin\frac{1}{2+\cos t+\cos\sqrt{3}t} \end{pmatrix},\\
&I_i(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix},\\
&b_{ij}^2(t) = \begin{pmatrix} 0.04\sin\frac{1}{2+\cos t+\cos\sqrt{3}t} & 0 \\ 0 & 0.04\sin\frac{1}{2+\cos t+\cos\sqrt{2}t} \end{pmatrix},\\
&d_{ij}^2(t) = \begin{pmatrix} 0.04\sin\frac{\pi}{2+\sin t+\sin\sqrt{3}t} & 0 \\ 0 & 0.04\sin\frac{1}{2+\cos t+\cos\sqrt{5}t} \end{pmatrix},\\
&J_j(t) = \begin{pmatrix} \sin t \\ \cos t \end{pmatrix},\\
&r = \max\left\{ \max_{1\le i\le 2}\left\{ \frac{1}{a_{i*}^1\alpha_{i*}^1}\sum_{j=1}^{2}\left[b_{ji}^{1*} + d_{ji}^{1*}\right]\alpha_j^{1*} \right\}; \max_{1\le i\le 2}\left\{ \frac{1}{a_{j*}^2\alpha_{j*}^2}\sum_{i=1}^{2}\left[b_{ij}^{2*} + d_{ij}^{2*}\right]\alpha_i^{2*} \right\}\right\}\\
&\quad = 0.4 < 1.
\end{aligned}
$$

According to Theorem 3.2 and Theorem 3.5, the system in Equation (1) has a unique almost automorphic solution, which is globally exponentially stable.

The simulation results can be seen in the following figures: Figure 1 depicts the numerical simulation of (x_1, x_2, y_1, y_2) for system in Equation (1), Figure 2 represents the orbit of (x_1, x_2) for system in Equation (16), Figure 3 represents the orbit of (y_1, y_2) for system in Equation (16), Figure 4 represents the orbit of (x_1, x_2, y_1) for system in Equation (16), and Figure 5 represents the orbit of (x_2, y_1, y_2) for system in Equation (16).

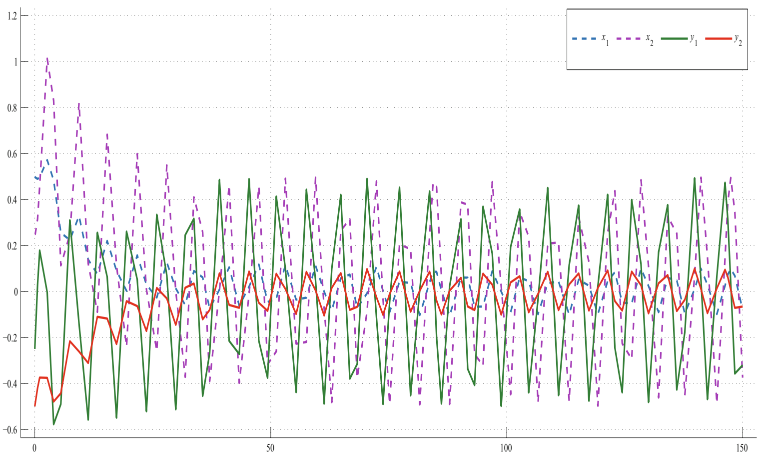

Figure 1 Solutions (x_1, x_2, y_1, y_2) of system in Equation (16)

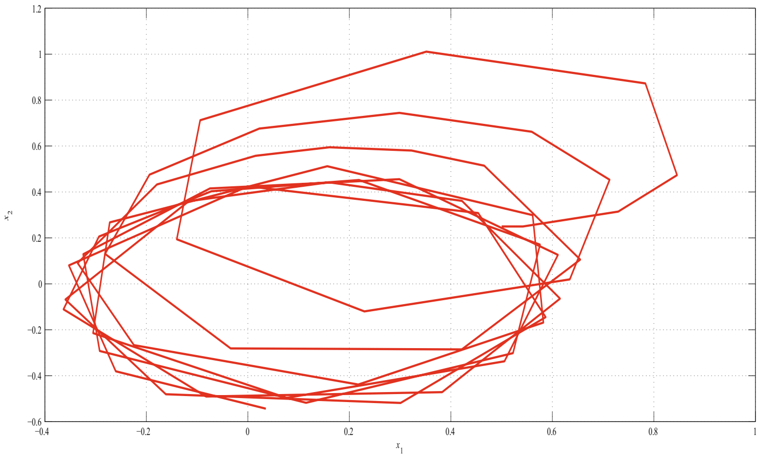

Figure 2 Orbit of (x_1, x_2) of system in Equation (16)

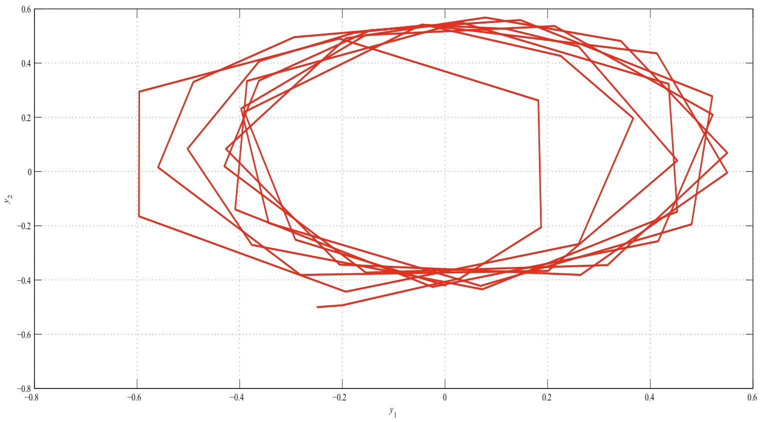

Figure 3 Orbit of (y_1, y_2) of system in Equation (16)

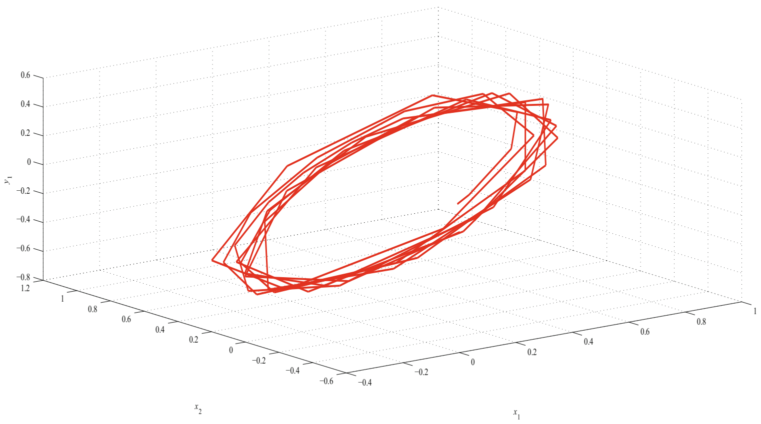

Figure 4 Orbit of (x_1, x_2, y_1) of system in Equation (16)

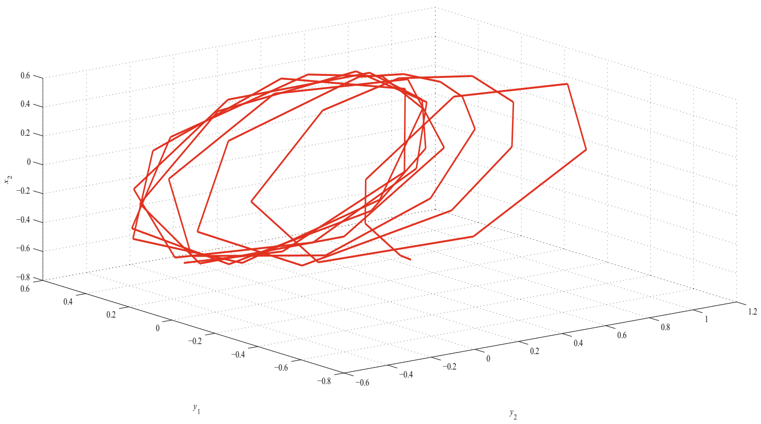

Figure 5 Orbit of (x_2, y_1, y_2) of system in Equation (16)

Remark 4.1 1) Figures 1–5 confirm that the proposed conditions in our theoretical results are effective for this example.

2) The global exponential stability meaning the study of the behaviors of trajectories (x_1, x_2, y_1, y_2) with initial conditions respectively $(0.5, 0.1, -0.1, -0.5)$.

5 Comparison with Previous Results

In [32], the authors investigated the dynamics behavior of a class of interval general BAM neural networks with multiple delays. Based on the fundamental solution matrix of coefficients, inequality technique and Lyapunov method, they derived sufficient conditions to ensure the existence and the exponential stability of anti-periodic solutions of the suggested system. The model studied in [32] is without distributed delay. a_i, b_j are constants (not time dependent). In our work, we dealt with general Cohen-Grossberg BAM neural networks with mixed delays (transmission delay and distributed delays) so, our model is the most general. The analysis methods are totally different than the methods used for paper [32]. In [31], the author studied a class of general BAM neural networks with multiple delays. Employing the exponential dichotomy theory, fixed-point theorem, and constructing suitable Lyapunov functionals, some criteria are established to ensure the existence and the global exponential stability of pseudo almost periodic solutions. However, the class of almost automorphic functions covers the class of periodic, almost periodic and pseudo almost periodic functions. Our outcomes are essentially new and generalize previously results in [31, 32]. In [39], a class of general Cohen-Grossberg BAM neural networks has been investigated. The existence of periodic solution for the suggested systems have been obtained. By observing Figures 3–10 of paper [39], we see that the dynamic behavior of the solutions (x_1, x_2, y_1, y_2) of Systems (4.1) and (4.2) are perfectly periodic. However, in this paper, we deal with almost automorphic solutions, Figures 1–5, affirm our main results, they show an almost automorphic behavior and not a periodic behavior. Roughly, our results generalize enormously many previous works in the aforementioned references ([31, 32, 39]) and are very significant.

Remark 5.1 In light of Theorems 3.2–3.5, the existence, the uniqueness and the global exponential stability of almost automorphic solution of system in Equation (1) are obtained, indicating that the sufficient conditions in both theorems can be used to solve optimization problem by converting object function into energy function. Our results are important because system in Equation (1) have significant applications in pattern completion, classification, feature detection, data compression, approximation, control, and so on (see [45–47]).

6 Conclusions

In this paper, a class of interval general CGBAM neural networks with mixed delays have been dealt with. By using the exponential dichotomy of linear differential equation, the Banach fixed point principle and the differential inequality techniques new sufficient conditions for the existence, the uniqueness and the global exponential stability of the almost automorphic solutions have been established. Finally, in the numerical example section, one can easily see that the simulation support our theoretical findings.

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