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Interval Type-2 Fuzzy Passive Filtering for Nonlinear Singularly Perturbed PDT-Switched Systems and Its Application*

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Abstract The problem of designing a passive filter for nonlinear switched singularly perturbed systems with parameter uncertainties is explored in this paper. Firstly, the multiple-time-scale phenomenon is settled effectively by introducing a singular perturbation parameter in the plant. Secondly, the interval type-2 fuzzy set theory is employed where parameter uncertainties are expressed in membership functions rather than the system matrices. It is worth noting that interval type-2 fuzzy sets of the devised filter are different from the plant, which makes the design of the filter more flexible. Thirdly, the persistent dwell-time switching rule, as a kind of time-dependent switching rules, is used to manage the switchings among nonlinear singularly perturbed subsystems, and this rule is more general than dwell-time and average dwell-time switching rules. Next, sufficient conditions are provided for guaranteeing that the filtering error system is globally uniformly exponentially stable with a passive performance. Furthermore, on the basis of the linear matrix inequalities, the explicit expression of the designed filter can be obtained. Finally, a tunnel diode electronic circuit is rendered as an example to confirm the correctness and the validity of the developed filter.

Keywords Interval type-2 fuzzy model, passive filter, persistent dwell-time switching rule, singularly perturbed nonlinear systems.

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1 Introduction

Substantial theoretical achievements and abundant practical applications about hybrid systems have sprung up in the past few years, such as [1–9]. When concentrating on the properties of continuous dynamics and discrete dynamics, such hybrid systems are assigned as switched systems and discrete dynamics can be abstracted as the switching rule[10–12]. Served as the manager of the running time and the sequence of subsystems, the switching rule could characterize various systems. For example, in [13], the asynchronous filtering issue was studied for the switched systems. By employing the hidden Markov model-based technique, the authors in [14] proposed a non-fragile state estimation method for the switched neural networks with probabilistic quantized outputs. Moreover, in [15], the fault-tolerant controller that can achieve multiple objectives was given for persistent dwell-time (PDT) switched systems. Among time-dependent switching rules applied in the published literature, dwell-time (DT), average dwell-time (ADT) and PDT switching rules have played vital roles and received extensive attention[16–21]. In DT switching rule, only a single element, slow switching, occurs, which is the main factor affecting its generalization. In this regard, it is worth highlighting that ADT switching rule contains not only slow switching but also fast switching. This feature allows ADT switching rule to express more situations than DT switching rule. Additionally, PDT switching rule can remove the restriction on the frequency of fast switching in ADT switching rule to some degree. Furthermore, the activated subsystem in slow switching of PDT switching rule could sustain a shorter running time than ADT switching rule. To sum up, PDT switching rule is more general than DT and ADT switching rules[22]. Thereupon, PDT switching rule is adopted and excavated in this paper.

On the other hand, it is well known that modeling complex hybrid systems, such as power systems and nuclear reactors, frequently encounters the situation that there are some extremely smaller parameters[23, 24]. These small parasitic parameters may result in the numerical ill-conditioning problem, which means that some state vectors are exceedingly susceptible to the perturbations and change quickly. If this actual situation is not considered deliberately, the accuracy of the model will be unsatisfactory, which will bring some difficulties to analyze and synthesize this system[25–31]. Accordingly, singularly perturbed systems (SPSs) are employed to remedy this trouble by employing a small singular perturbation parameter ε to reflect the separation of the fast and the slow states[24]. Since the concept of SPSs was put forward, the number of relevant researches has grown, which has produced many achievements on this theme. For example, the authors in [32] used an ε-dependent Lyapunov function for fast sampling SPSs to obtain two theorems: One was ε-independent to deduce the satisfactory controller, and the other was used to estimate the upper bound of ε. As noted in [33], the non-fragile control issue of nonlinear SPSs with semi-Markov jump parameters was investigated.

Furthermore, SPSs under PDT switching rule may be difficult to handle directly when the nonlinearity exists, which is an interesting topic that gains much attention[34]. The nonlinearity is usually caused by the physical traits of devices or environmental factors, like the hysteresis curve of transformer and high temperature. A brilliant strategy to resolve nonlinearity is the type-1 Takagi-Sugeno (T-S) fuzzy set theory, whose essence is a weighted sum of linear subsystems[6, 35, 36]. The parameter uncertainties, as a vital issue, add extra complexity to the T-S fuzzy model[37]. Commonly, some restrictive assumptions on the uncertain characteristic of systems are exploited in most papers when settling this problem for fuzzy systems, such as the sector bound. But the process of calculating these special structures of uncertainty may increase the dimension of the resulting linear matrix inequalities (LMIs), which increases the complexity of the calculation. Hence, this fact arouses a fascinating study of searching for an alternative approach to tackle the parameter uncertainties under the special pattern of the T-S fuzzy modeling paradigm. Fortunately, Lam and Seneviratne resolved the system stability issue with parameter uncertainties by an interval type-2 (IT2) T-S fuzzy model based on the type-2 fuzzy set theory in [38]. Compared to the traditional approach, this new way describes the uncertainty more accurately by using the firing strength of grades of memberships (GFs), which comprises the lower and upper GFs and the corresponding nonlinear weighting functions[39]. This merit stimulates the enthusiasm of scholars to extend the scope of applying the IT2 T-S fuzzy model. In [40], the design of the switched filter for IT2 fuzzy systems was investigated, where the filter has the same fuzzy sets of the plant. In the sense of slow and fast switchings, the stability of IT2 fuzzy systems was guaranteed by means of the mode-dependent ADT method[41]. The filter with different fuzzy rules for IT2 fuzzy systems was devised in [42], which makes the structure of the filter more flexible. As far as the authors' knowledge, there are scarce results on the issue of designing a filter for IT2 T-S fuzzy SPSs with PDT switching rule. In order to narrow this blank, this work is carried out.

In summary, this paper focuses on how to get the trace of a certain signal of the switched IT2 T-S fuzzy SPSs. The main work is to design a filter to estimate this signal. And the filtering error system (FES) needs to be globally uniformly exponentially stable (GUES) with a given passive performance index. In short, the main contributions of this paper can be divided into the following three aspects:

(i) A novel system model reflecting the phenomena of nonlinearities, multiple-time-scale and parameter uncertainties is proposed by employing PDT switching rule, the singularly perturbed and the IT2 T-S fuzzy set theories;

(ii) Based on the establishment of ε-dependent multiple Lyapunov-like functions (MLFs), sufficient conditions for FES to be GUES with a passive performance index can be obtained by confining the changes of MLFs at the switching instants and during the persistent times of each acting system;

(iii) The adopted technology based on the strength of the IT2 T-S fuzzy model, which enables the plant and the filter to have different premise variables and membership functions (MFs), can improve the flexibility of the filter and reduce the conservatism of the obtained results to some extent.

The notations used are standard and one can refer to [39].

2 Problem Formulation

An applicable mathematical model is formed and interpreted in this section and a corresponding filter system is also established, then FES can be obtained. Finally, some necessary definitions and lemmas are given.

2.1 System Construction

Considering the nonlinear SPSs with PDT switching rule, the mth rule of a discrete-time IT2 T-S fuzzy model is established as follows:

Plant Rule m: IF $g_1(x(k))$ is Q_{m1}, $g_2(x(k))$ is Q_{m2}, $\cdots$, and $g_\xi(x(k))$ is $Q_{m\xi}$, THEN

$$\begin{cases} x(k+1) = A^{\varepsilon}_{m\sigma(k)}x(k) + D_{1m\sigma(k)}w(k), \\ z(k) = B_{m\sigma(k)}x(k) + D_{2m\sigma(k)}w(k), \\ y(k) = C_{m\sigma(k)}x(k) + D_{3m\sigma(k)}w(k), \end{cases} \tag{1}$$

with

$$A^{\varepsilon}_{m\sigma(k)} \triangleq \begin{bmatrix} A_{1m\sigma(k)} & \varepsilon A_{2m\sigma(k)} \\ A_{3m\sigma(k)} & \varepsilon A_{4m\sigma(k)} \end{bmatrix}, \quad D_{1m\sigma(k)} \triangleq \begin{bmatrix} D_{1sm\sigma(k)} \\ D_{1fm\sigma(k)} \end{bmatrix}, \quad D_{3m\sigma(k)} \triangleq \begin{bmatrix} D_{3sm\sigma(k)} \\ D_{3fm\sigma(k)} \end{bmatrix},$$
$$B_{m\sigma(k)} \triangleq \begin{bmatrix} B_{sm\sigma(k)} & B_{fm\sigma(k)} \end{bmatrix}, \quad C_{m\sigma(k)} \triangleq \mathrm{diag}\left\{C_{sm\sigma(k)}, C_{fm\sigma(k)}\right\},$$
$$x(k) \triangleq \begin{bmatrix} x_s^{\mathrm{T}}(k) & x_f^{\mathrm{T}}(k) \end{bmatrix}^{\mathrm{T}},$$

in which, ε is a positive scalar and stands for the singular perturbation parameter; for the time $k \in \mathbb{N}$, $x(k) \in \mathbb{R}^{\varkappa_x}$ expresses the system state vector, which includes the slow state vector $x_s(k) \in \mathbb{R}^{\varkappa_{x_s}}$ and the fast state vector $x_f(k) \in \mathbb{R}^{\varkappa_{x_f}}$ satisfying $\varkappa_{x_s} + \varkappa_{x_f} = \varkappa_x$; $y(k) \in \mathbb{R}^{\varkappa_y}$, $z(k) \in \mathbb{R}^{\varkappa_z}$ and $w(k) \in l_2[0,\infty)$ denote the measurement output, the signal to be estimated and the outside disturbance signal, respectively; $\sigma(k)$ indicates PDT switching signal valued in a finite set $\mathcal{S} \triangleq \{1, 2, \cdots, \wp\}$, where $\wp$ symbolizes the total number of subsystems; for $\sigma(k) \triangleq i$, A^{ε}_{mi}, B_{mi}, C_{mi}, D_{1mi}, D_{2mi} and D_{3mi} are the system matrices that have appropriate dimensions. $g_1(x(k))$, $g_2(x(k))$, $\cdots$, and $g_\xi(x(k))$ imply the premise variables of the plant. For $m \in \mathcal{M} \triangleq \{1, 2, \cdots, \theta\}$ and $\ell \in \mathcal{L} \triangleq \{1, 2, \cdots, \xi\}$, $Q_{m\ell}$ signifies the IT2 fuzzy set of the mth plant rule to the function $g_\ell(x(k))$, where the positive scalars θ and ξ indicate the number of the IF-THEN rules and the number of the fuzzy sets to the plant, respectively. Through the IT2 T-S fuzzy reasoning[43], the overall switched nonlinear SPSs ($\mathit{\Gamma}$) are presented as follows:

$$\begin{cases} x(k+1) = \sum\limits_{m=1}^{\theta} f_m(x(k))\left[A^{\varepsilon}_{mi}x(k) + D_{1mi}w(k)\right], \\ z(k) = \sum\limits_{m=1}^{\theta} f_m(x(k))\left[B_{mi}x(k) + D_{2mi}w(k)\right], \\ y(k) = \sum\limits_{m=1}^{\theta} f_m(x(k))\left[C_{mi}x(k) + D_{3mi}w(k)\right], \end{cases} \tag{2}$$

where $f_m(x(k))$ are the grades of membership to the plant and they are assumed to be made by blending the lower grade of membership (LGM) $\underline{f}_m(x(k))$ and the upper grade of membership (UGM) $\overline{f}_m(x(k))$ with two nonlinear weighting functions, $\underline{\alpha}_m(x(k))$ and $\overline{\alpha}_m(x(k))$, as shown follows:

$$f_m(x(k)) \triangleq \frac{\widetilde{f}_m(x(k))}{\sum_{\widetilde{m}=1}^{\theta} \widetilde{f}_{\widetilde{m}}(x(k))}, \quad \widetilde{f}_m(x(k)) \triangleq \underline{\alpha}_m(x(k))\underline{f}_m(x(k)) + \overline{\alpha}_m(x(k))\overline{f}_m(x(k)),$$

with

$$\sum_{m=1}^{\theta} f_m(x(k)) = 1, \quad 0 \le f_m(x(k)) \le 1,$$
$$\underline{\alpha}_m(x(k)) + \overline{\alpha}_m(x(k)) = 1, \quad 0 \le \underline{\alpha}_m(x(k)) \le 1, \quad 0 \le \overline{\alpha}_m(x(k)) \le 1.$$

For the mth rule of the plant, the firing strength is represented by an interval set which is defined and confined by

$$F_m(x(k)) \triangleq \left[\underline{f}_m(x(k))\ \overline{f}_m(x(k))\right],$$

where

$$\underline{f}_m(x(k)) \triangleq \prod_{\ell=1}^{\xi} \underline{\mu}_{Q_{m\ell}}(g_\ell(x(k))), \quad \overline{f}_m(x(k)) \triangleq \prod_{\ell=1}^{\xi} \overline{\mu}_{Q_{m\ell}}(g_\ell(x(k))),$$

with

$$0 \le \underline{f}_m(x(k)) \le \overline{f}_m(x(k)) \le 1, \quad 0 \le \underline{\mu}_{Q_{m\ell}}(g_\ell(x(k))) \le \overline{\mu}_{Q_{m\ell}}(g_\ell(x(k))) \le 1,$$

in which $\underline{\mu}_{Q_{m\ell}}(g_\ell(x(k)))$ and $\overline{\mu}_{Q_{m\ell}}(g_\ell(x(k)))$ express the lower membership function (LMF) and the upper membership function (UMF), respectively.

Remark 2.1 In the IT2 T-S fuzzy theory, the parameter uncertainties are embedded in MFs instead of system matrices, which is advantageous to the derivation and computation. An interval set including LMF and UMF is used to describe this kind of MFs with uncertainties. And the mixture of LGM and UGM with nonlinear weighting functions represents the actual GFs of systems.

Remark 2.2 A piecewise function, $\sigma(k)$, which is continuous on the right, can be employed as a switching signal. In order to stabilize systems and describe more practical systems, several switching rules are defined by scholars, among which PDT switching rule is the one adopted in this paper. Consequently, a brief introduction to PDT switching rule is given. The time sequence generated by PDT switching rule is composed of T-portion and τ-portion. Those two portions take place in turn. In each τ-portion, only a single subsystem is allowed to act, whose running time should be no less than the positive scalar τ_{PDT}. At the same time, quite a few subsystems with running times less than τ_{PDT} can act in the T-portion and the length of each T-portion should be less than the positive scalar T_{PDT}. In addition, a τ-portion and a successive T-portion make up a stage.

Remark 2.3 Figure 1 facilitates the comprehension of the generation and the rule of PDT switching sequence. Along the abscissa axis denoting the time, the minimum intervals, such as $\left(k_{l_g}, k_{l_g}+1\right)$, are the sampling intervals; the intervals similar to $\left(k_{l_g-1}, k_{l_g}\right)$ mean the running intervals of acting subsystems; otherwise, $\tau^{(g)} \triangleq k_{l_g+1}-k_{l_g}$ and $T^{(g)} \triangleq k_{l_{g+1}}-k_{l_g+1}$ represent running times of τ-portion and T-portion in the gth stage, respectively. And the vertical axis signifies the value of the energy function. Some special symbols are employed for the following process, and they are stated here as well: $Tim(\Delta_t)$ shows the sustained running time of the subsystem Δ_t; $Num\left(k_{l_{g-1}}, k_{l_g}\right)$ denotes the total number of acting subsystems in the interval $\left[k_{l_{g-1}}, k_{l_g}\right)$. Furthermore, we can derive the following inequality[44]:

$$0 \leq Num\,(t,k) \leq \left(\frac{k-t}{\tau_{PDT}+T_{PDT}}+1\right)(T_{PDT}+1)\,. \tag{3}$$

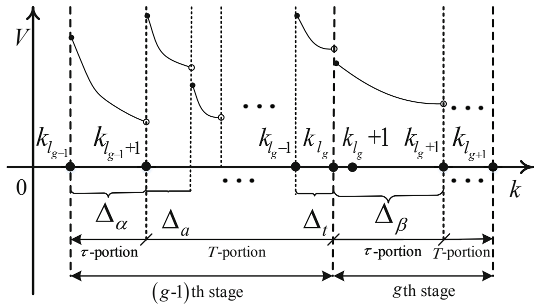

Figure 1 The sketch map of the energy function variation subject to PDT switching rule

Definition 2.4 (see [44]) PDT switching rule has the following features with two positive constants τ_{PDT} and T_{PDT}:

(i) There is a T-portion between every two neighbouring τ-portions;

(ii) The duration of τ-portion is no less than τ_{PDT} and the duration of T-portion is no more than T_{PDT};

(iii) A single subsystem is activated in a τ-portion, and in T-portion, a set of subsystems whose running times are shorter than τ_{PDT} can switch among each other.

2.2 Filter Form

With the aid of defining the filter state vector $\widehat{x}\,(k)$, the output signal $y\,(k)$ from the plant, as well as the assumption that the mode of the filter shares the same one of the plant, one can gain the output signal of the filter $\widehat{z}\,(k)$ as the estimate signal to $z\,(k)$. The IT2 T-S fuzzy format of the filter is described by:

Filter Rule n: IF $\widehat{g}_1\,(x\,(k))$ is $\widehat{Q}_{n1}$, $\widehat{g}_2\,(x\,(k))$ is $\widehat{Q}_{n2}$, $\cdots$, and $\widehat{g}_{\widehat{\xi}}\,(x\,(k))$ is $\widehat{Q}_{n\widehat{\xi}}$, THEN

$$\begin{cases} \widehat{x}\,(k+1)=\widehat{A}_{ni}\widehat{x}\,(k)+\widehat{E}_{ni}y\,(k)\,, \\ \widehat{z}\,(k)=\widehat{B}_{ni}\widehat{x}\,(k)\,, \end{cases} \tag{4}$$

with

$$\widehat{x}(k) \triangleq \begin{bmatrix} \widehat{x}_s(k) \\ \widehat{x}_f(k) \end{bmatrix}, \quad \widehat{A}_{ni} \triangleq \begin{bmatrix} \widehat{A}_{1ni} & \widehat{A}_{2ni} \\ \widehat{A}_{3ni} & \widehat{A}_{4ni} \end{bmatrix}, \quad \widehat{E}_{ni} \triangleq \begin{bmatrix} \widehat{E}_{1ni} & \widehat{E}_{2ni} \\ \widehat{E}_{3ni} & \widehat{E}_{4ni} \end{bmatrix}, \quad \widehat{B}_{ni} \triangleq \begin{bmatrix} \widehat{B}_{sni} & \widehat{B}_{fni} \end{bmatrix},$$

in which, $\widehat{A}_{ni}$, $\widehat{B}_{ni}$ and $\widehat{E}_{ni}$ are the filter gains to be determined. $\widehat{g}_1(x(k))$, $\widehat{g}_2(x(k))$, $\cdots$, and $\widehat{g}_{\widehat{\xi}}(x(k))$ imply the premise variables of the filter. For $n \in \mathcal{N} \triangleq \{1, 2, \cdots, \chi\}$ and $\widehat{\ell} \in \widehat{\mathcal{L}} \triangleq \left\{1, 2, \cdots, \widehat{\xi}\right\}$, $\widehat{Q}_{n\widehat{\ell}}$ signifies the IT2 fuzzy set of nth filter rule to the function $\widehat{g}_{\widehat{\ell}}(x(k))$, where the positive scalars χ and $\widehat{\xi}$ indicate the number of the IF-THEN rules and the number of the fuzzy sets to the filter, separately. Through the IT2 T-S fuzzy reasoning, the overall switched IT2 T-S fuzzy filter ($\widehat{\Gamma}$) is described as follows:

$$\begin{cases} \widehat{x}(k+1) = \sum_{n=1}^{\chi} h_n(x(k)) \left[\widehat{A}_{ni}\widehat{x}(k) + \widehat{E}_{ni} y(k)\right], \\ \widehat{z}(k) = \sum_{n=1}^{\chi} h_n(x(k)) \widehat{B}_{ni}\widehat{x}(k), \end{cases} \tag{5}$$

in which $h_n(x(k))$ are the grades of membership to the filter.

The IT2 T-S fuzzy grades of membership towards the filter, $h_n(x(k))$, are assumed to be composed by $\widetilde{h}_n(x(k))$, which is obtained by blending LGM $\underline{h}_n(x(k))$ and UGM $\overline{h}_n(x(k))$ of the filter with two nonlinear weighting functions, $\underline{\beta}_n(x(k))$ and $\overline{\beta}_n(x(k))$, as exhibited below:

$$h_n(x(k)) \triangleq \frac{\widetilde{h}_n(x(k))}{\sum_{\widetilde{n}=1}^{\chi} \widetilde{h}_{\widetilde{n}}(x(k))}, \quad \widetilde{h}_n(x(k)) \triangleq \underline{\beta}_n(x(k))\,\underline{h}_n(x(k)) + \overline{\beta}_n(x(k))\,\overline{h}_n(x(k)),$$

with

$$\sum_{n=1}^{\chi} h_n(x(k)) = 1, \quad 0 \leq h_n(x(k)) \leq 1,$$
$$\underline{\beta}_n(x(k)) + \overline{\beta}_n(x(k)) = 1, \quad 0 \leq \underline{\beta}_n(x(k)) \leq 1, \quad 0 \leq \overline{\beta}_n(x(k)) \leq 1.$$

For the nth rule of the filter, the firing strength is defined by an interval set satisfying:

$$H_n(x(k)) \triangleq \left[\underline{h}_n(x(k))\ \overline{h}_n(x(k))\right],$$

where

$$\underline{h}_n(x(k)) \triangleq \prod_{\widehat{\ell}=1}^{\widehat{\xi}} \underline{\mu}_{\widehat{Q}_{n\widehat{\ell}}}\left(\widehat{g}_{\widehat{\ell}}(x(k))\right), \quad \overline{h}_n(x(k)) \triangleq \prod_{\widehat{\ell}=1}^{\widehat{\xi}} \overline{\mu}_{\widehat{Q}_{n\widehat{\ell}}}\left(\widehat{g}_{\widehat{\ell}}(x(k))\right),$$

with

$$0 \leq \underline{h}_n(x(k)) \leq \overline{h}_n(x(k)) \leq 1, \quad 0 \leq \underline{\mu}_{\widehat{Q}_{n\widehat{\ell}}}\left(\widehat{g}_{\widehat{\ell}}(x(k))\right) \leq \overline{\mu}_{\widehat{Q}_{n\widehat{\ell}}}\left(\widehat{g}_{\widehat{\ell}}(x(k))\right) \leq 1,$$

in which $\underline{\mu}_{\widehat{Q}_{n\widehat{\ell}}}\left(\widehat{g}_{\widehat{\ell}}(x(k))\right)$ and $\overline{\mu}_{\widehat{Q}_{n\widehat{\ell}}}\left(\widehat{g}_{\widehat{\ell}}(x(k))\right)$ express LMF and UMF of the filter, respectively.

2.3 Filtering Error System

Consider the plant (Γ) as well as the filter $(\widehat{\Gamma})$ and define the augmented state vector $\widetilde{x}(k) \triangleq \left[x^{\mathrm{T}}(k)\ \widehat{x}^{\mathrm{T}}(k)\right]^{\mathrm{T}}$, the error signal $\widetilde{z}(k) \triangleq z(k) - \widehat{z}(k)$ as well as the grades of membership to FES $\varpi_{mn}(x(k)) \triangleq f_m(x(k))\, h_n(x(k))$ for $\forall m \in \mathcal{M}, n \in \mathcal{N}$ such that the following conditions hold:

$$1 = \sum_{m=1}^{\theta} f_m(x(k)) = \sum_{n=1}^{\chi} h_n(x(k)) = \sum_{m=1}^{\theta}\sum_{n=1}^{\chi} f_m(x(k))\, h_n(x(k)) = \sum_{m=1}^{\theta}\sum_{n=1}^{\chi} \varpi_{mn}(x(k)).$$

From the above discussion, FES $(\widetilde{\Gamma})$ can be deduced as:

$$\begin{cases} \widetilde{x}(k+1) = \sum_{m=1}^{\theta}\sum_{n=1}^{\chi} \varpi_{mn}(x(k)) \left[\widetilde{A}^{\varepsilon}_{mni}\widetilde{x}(k) + \widetilde{D}_{1mni} w(k)\right], \\ \widetilde{z}(k) = \sum_{m=1}^{\theta}\sum_{n=1}^{\chi} \varpi_{mn}(x(k)) \left[\widetilde{B}_{mni}\widetilde{x}(k) + \widetilde{D}_{2mi} w(k)\right], \end{cases} \tag{6}$$

with

$$\widetilde{A}^{\varepsilon}_{mni} \triangleq \begin{bmatrix} A^{\varepsilon}_{mi} & 0 \\ \widehat{E}_{ni}C_{mi} & \widehat{A}_{ni} \end{bmatrix}, \quad \widetilde{D}_{1mni} \triangleq \begin{bmatrix} D_{1mi} \\ \widehat{E}_{ni}D_{3mi} \end{bmatrix}, \quad \widetilde{B}_{mni} \triangleq \begin{bmatrix} B_{mi} & -\widehat{B}_{ni} \end{bmatrix}, \quad \widetilde{D}_{2mi} \triangleq D_{2mi}.$$

The method in [45] is adopted for further analyzing FES in this note, which is recalled as follows for $\varphi \in \mathbb{N}$, $\psi \in \mathbb{N}$:

(i) The interest state space Ω is separated equally into φ state subspaces Ω_ϑ with $\Omega = \cup_{\vartheta=1}^{\varphi}\Omega_\vartheta$;

(ii) The footprint of uncertainty (FOU) Λ is divided equally into $\psi+1$ subspaces Λ_ρ with $\Lambda = \cup_{\rho=1}^{\psi+1}\Lambda_\rho$.

With the aid of the different LGM and UGM in Ω_ϑ and Λ_ρ, the grades of membership to FES have another expression for $\forall \rho \in \mathcal{R} \triangleq \{1, 2, \cdots, \psi+1\}$:

$$\varpi_{mn}(x(k)) \triangleq \sum_{\rho=1}^{\psi+1} \tau_{mn\rho}(x(k)) \left[\underline{\gamma}_{mn\rho}(x(k))\, \underline{\varpi}_{mn\rho}(x(k)) + \overline{\gamma}_{mn\rho} x(k)\, \overline{\varpi}_{mn\rho}(x(k))\right], \tag{7}$$

in which

$$\tau_{mn\rho}(x(k)) \triangleq \begin{cases} 1, & \varpi_{mn}(x(k)) \in \text{the FOU subspace } \rho, \\ 0, & \text{otherwise}, \end{cases} \tag{8}$$

with

$$\sum_{m=1}^{\theta}\sum_{n=1}^{\chi} \varpi_{mn}(x(k)) = 1, \quad \underline{\gamma}_{mn\rho}(x(k)) + \overline{\gamma}_{mn\rho}(x(k)) = 1,$$
$$0 \le \varpi_{mn}(x(k)), \quad 0 \le \underline{\gamma}_{mn\rho}(x(k)) \le 1, \quad 0 \le \overline{\gamma}_{mn\rho}(x(k)) \le 1.$$

On the other hand, the variable scalar $\varepsilon_{\varrho\nu_\varrho\vartheta\rho}(x_\varrho(k))$ is introduced to format LGM and UGM in different FOU, and is defined by the below three limitations:

(i) When ϱ takes values as $1, 2, \cdots, \varkappa_x$, $\nu_\varrho \in \{1, 2\}$, and for $x(k) \in \Omega_\vartheta$, $\vartheta \in \widetilde{\mathcal{R}} \triangleq \{1, 2, \cdots, \varphi\}$, the inequalities $0 \le \varepsilon_{\varrho\nu_\varrho\vartheta\rho}(x_\varrho(k)) \le 1$ as well as the equality $\varepsilon_{\varrho 1\vartheta\rho}(x_\varrho(k)) + \varepsilon_{\varrho 2\vartheta\rho}(x_\varrho(k)) = 1$ hold;

(ii) The equation $\varepsilon_{\varrho\nu_\varrho\vartheta\rho}(x_\varrho(k)) = 0$ holds when $x(k) \notin \Omega_\vartheta$;

(iii) $\sum_{\vartheta=1}^{\varphi} \sum_{\nu_1=1}^{2} \cdots \sum_{\nu_{\varkappa_x}=1}^{2} \prod_{\varrho=1}^{\varkappa_x} \varepsilon_{\varrho\nu_\varrho\vartheta\rho}(x_\varrho(k)) = 1$.

In this way, LMF and UMF in different FOU are expressed as follows for $\forall m \in \mathcal{M}$, $\forall n \in \mathcal{N}$, $\forall \rho \in \mathcal{R}$:

$$\underline{\varpi}_{mn\rho}(x(k)) \triangleq \sum_{\vartheta=1}^{\varphi} \sum_{\nu_1=1}^{2} \cdots \sum_{\nu_{\varkappa_x}=1}^{2} \prod_{\varrho=1}^{\varkappa_x} \varepsilon_{\varrho\nu_\varrho\vartheta\rho}(x_\varrho(k)) \underline{\omega}_{mn\nu_1\cdots\nu_{\varkappa_x}\vartheta\rho},$$

$$\overline{\varpi}_{mn\rho}(x(k)) \triangleq \sum_{\vartheta=1}^{\varphi} \sum_{\nu_1=1}^{2} \cdots \sum_{\nu_{\varkappa_x}=1}^{2} \prod_{\varrho=1}^{\varkappa_x} \varepsilon_{\varrho\nu_\varrho\vartheta\rho}(x_\varrho(k)) \overline{\omega}_{mn\nu_1\cdots\nu_{\varkappa_x}\vartheta\rho},$$

satisfying

$$0 \le \underline{\omega}_{mn\nu_1\cdots\nu_\iota\vartheta\rho} \le \overline{\omega}_{mn\nu_1\cdots\nu_\iota\vartheta\rho} \le 1.$$

The target of this paper is to design a filter such that FES is GUES with a passive performance. Some necessary definitions and lemmas are given below.

Definition 2.5 (see [15]) The FES $(\widetilde{\Gamma})$ is GUES, if there exist scalars $0 < \widetilde{\phi} < 1$, $\delta > 0$ and the below inequality is satisfied under the condition of zero disturbance:

$$\|\widetilde{x}(k)\|^2 \le \delta\widetilde{\phi}^{k-k_0} \|\widetilde{x}(k_0)\|^2, \quad k_0 \le k.$$

Definition 2.6 (see [46]) The FES $(\widetilde{\Gamma})$ is passive, if the below inequality is achieved under the condition of zero-initial state for $0 < r$:

$$0 \le \sum_{k=0}^{\infty} \left[\mathrm{sym}\left\{\widetilde{z}^{\mathrm{T}}(k) w(k)\right\} + r^2 w^{\mathrm{T}}(k) w(k)\right]. \tag{9}$$

Lemma 2.7 (see [47]) *For two scalars $0 < \varepsilon \le \overline{\varepsilon}$, and two symmetric matrices $\mathcal{H}_1$, $\mathcal{H}_2$ that have the same dimension, the below necessary and sufficient condition exists:*

$$\mathcal{H}_1 + \varepsilon\mathcal{H}_2 < 0 \iff \begin{cases} \mathcal{H}_1 \le 0, \\ \mathcal{H}_1 + \overline{\varepsilon}\mathcal{H}_2 < 0. \end{cases}$$

Lemma 2.8 (see [48]) *For three real matrices $\mathcal{A}_{mni}$, $\mathcal{B} > 0$, $\mathcal{C}_{mni}$, and a scalar $\varpi_{mn} \in [0, 1]$, the following inequality holds:*

$$\left[\sum_{m=1}^{\theta} \sum_{n=1}^{\chi} \varpi_{mn}\mathcal{A}_{mni}\right]^{\mathrm{T}} \mathcal{B}_i \left[\sum_{m=1}^{\theta} \sum_{n=1}^{\chi} \varpi_{mn}\mathcal{C}_{mni}\right] \le \frac{1}{2} \sum_{m=1}^{\theta} \sum_{n=1}^{\chi} \varpi_{mn} \left[\mathcal{A}_{mni}^{\mathrm{T}}\mathcal{B}_i\mathcal{C}_{mni} + \mathcal{C}_{mni}^{\mathrm{T}}\mathcal{B}_i\mathcal{A}_{mni}\right].$$

Remark 2.9 In every state subspace, the value of uncertain GMs is a linear sum of LGM and UGM, and is partly determined by some nonlinear functions, such as $\underline{\alpha}_m(x(k))$ and $\overline{\alpha}_m(x(k))$. The designed filter can work under different forms of these nonlinear functions. In this sense, such uncertainties can illustrate the universality of our methods.

3 Main Results

In this section, GUES and passive performance issues of FES are studied. And then, via some decoupling and matrix transformation techniques, the explicit format of the filter will be obtained. Moreover, in the rest of the paper, $\underline{\widetilde{f}}_m(x(k))$, $\overline{\widetilde{f}}_m(x(k))$, $\underline{\widetilde{h}}_n(x(k))$, $\overline{\widetilde{h}}_n(x(k))$, $\tau_{mn\rho}(x(k))$, $\varpi_{mn}(x(k))$, $\underline{\varpi}_{mn\rho}(x(k))$, $\overline{\varpi}_{mn\rho}(x(k))$, $\underline{\gamma}_{mn\rho}(x(k))$, $\overline{\gamma}_{mn\rho}(x(k))$ are separately rewritten as $\underline{\widetilde{f}}_m$, $\overline{\widetilde{f}}_m$, $\underline{\widetilde{h}}_n$, $\overline{\widetilde{h}}_n$, $\tau_{mn\rho}$, ϖ_{mn}, $\underline{\varpi}_{mn\rho}$, $\overline{\varpi}_{mn\rho}$, $\underline{\gamma}_{mn\rho}$, $\overline{\gamma}_{mn\rho}$ for simplicity.

Theorem 3.1 *Considering that the state space and FOU are partitioned into $\varphi(\in \mathbb{N}^+)$ and $\psi+1(\in \mathbb{N}^+)$ subspaces separately, for the admissible switching sequence obeying PDT switching rule and given scalars $0<\theta$, $0<\chi$, $0<\varepsilon$, $0<r$, $0<T_{PDT}$, $0<\tau_{PDT}$, $0<\lambda<1$, $1<\mu$, $\lambda\mu\neq 1$, FES $(\widetilde{\Gamma})$ is GUES with a passive performance level r, if for $\forall m\in\mathcal{M}$, $\forall n\in\mathcal{N}$, $\forall\rho\in\mathcal{R}$, $\forall i_\varrho\in\{1,2\}$, $i\neq j$, and i, $j\in\mathcal{S}$, there exist symmetric matrices P_i^ε, $\mathcal{X}_i$, $0\leq\mathcal{W}_{mn\rho i}$, $0\leq\mathcal{Y}_{mn\rho i}$, $0\leq\mathcal{V}_{mn\rho i}$, $0\leq\mathcal{U}_{mn\rho i}$, $0\leq\mathcal{R}_{mn\rho i}$ such that the following conditions hold:*

$$(T_{PDT}+1)\ln(\mu)+(T_{PDT}+\tau_{PDT})\ln(\lambda)<0, \tag{10}$$

$$P_i^\varepsilon>0, \tag{11}$$

$$P_i^\varepsilon-\mu P_j^\varepsilon<0, \tag{12}$$

$$\overline{\omega}\Xi_{1mn\rho i}^\varepsilon+\omega_2\mathcal{X}_i+\omega_3\mathcal{Y}_{mn\rho i}<0, \tag{13}$$

$$\Xi_{2mn\rho i}^\varepsilon+\mathcal{X}_i+\mathcal{Y}_{mn\rho i}>0, \tag{14}$$

$$\widetilde{r}>0, \tag{15}$$

where

$$\overline{\omega}\triangleq\overline{\varpi}_{mn\nu_1\cdots\nu_{\varkappa_x}\vartheta\rho},\quad \omega_2\triangleq\overline{\varpi}_{mn\nu_1\cdots\nu_{\varkappa_x}\vartheta\rho}-\frac{1}{\theta\chi},\quad \omega_3\triangleq\overline{\varpi}_{mn\nu_1\cdots\nu_{\varkappa_x}\vartheta\rho}-\underline{\omega}_{mn\nu_1\cdots\nu_{\varkappa_x}\vartheta\rho},$$

$$r_1\triangleq\mu^{T_{PDT}+1+\frac{T_{PDT}+1}{\tau_{PDT}+T_{PDT}}},\quad r_2\triangleq\lambda\mu^{\frac{T_{PDT}+1}{\tau_{PDT}+T_{PDT}}},\quad \widetilde{r}\triangleq\frac{r_1(1-\lambda)}{r^2(1-r_2)}-\frac{1}{r^2},$$

$$\Xi_{1mn\rho i}^\varepsilon\triangleq\begin{bmatrix}\Xi_{1mn\rho i}^{\varepsilon(1,1)} & \Xi_{1mn\rho i}^{\varepsilon(1,2)}\\ * & \Xi_{1mn\rho i}^{\varepsilon(2,2)}\end{bmatrix},\quad \Xi_{2mn\rho i}^\varepsilon\triangleq\begin{bmatrix}\Xi_{2mn\rho i}^{\varepsilon(1,1)} & \Xi_{2mn\rho i}^{\varepsilon(1,2)}\\ * & \Xi_{2mn\rho i}^{\varepsilon(2,2)}\end{bmatrix},$$

with

$$\Xi_{1mn\rho i}^{\varepsilon(1,1)}\triangleq\widetilde{A}_{mni}^{\varepsilon T}(P_i^\varepsilon+\mathcal{V}_{mn\rho i})\widetilde{A}_{mni}^\varepsilon-\lambda P_i^\varepsilon+\widetilde{B}_{mni}^{\mathrm{T}}(\widetilde{r}I+\mathcal{U}_{mn\rho i})\widetilde{B}_{mni},$$

$$\Xi_{1mn\rho i}^{\varepsilon(1,2)}\triangleq\widetilde{A}_{mni}^{\varepsilon T}(P_i^\varepsilon+\mathcal{V}_{mn\rho i})\widetilde{D}_{1mni}-\widetilde{B}_{mni}^{\mathrm{T}}+\widetilde{B}_{mni}^{\mathrm{T}}(\widetilde{r}I+\mathcal{U}_{mn\rho i})\widetilde{D}_{2mni},$$

$$\Xi_{1mn\rho i}^{\varepsilon(2,2)}\triangleq\widetilde{D}_{1mni}^{\mathrm{T}}(P_i^\varepsilon+\mathcal{V}_{mn\rho i})\widetilde{D}_{1mni}-r^2I-\mathrm{sym}\left\{\widetilde{D}_{2mi}\right\}+\widetilde{D}_{2mni}^{\mathrm{T}}(\widetilde{r}I+\mathcal{U}_{mn\rho i})\widetilde{D}_{2mni},$$

$$\Xi_{2mn\rho i}^{\varepsilon(1,1)}\triangleq\widetilde{A}_{mni}^{\varepsilon T}(P_i^\varepsilon-\mathcal{W}_{mn\rho i})\widetilde{A}_{mni}^\varepsilon-\lambda P_i^\varepsilon+\widetilde{B}_{mni}^{\mathrm{T}}(\widetilde{r}I-\mathcal{R}_{mn\rho i})\widetilde{B}_{mni},$$

$$\Xi_{2mn\rho i}^{\varepsilon(1,2)}\triangleq\widetilde{A}_{mni}^{\varepsilon T}(P_i^\varepsilon-\mathcal{W}_{mn\rho i})\widetilde{D}_{1mni}-\widetilde{B}_{mni}^{\mathrm{T}}+\widetilde{B}_{mni}^{\mathrm{T}}(\widetilde{r}I-\mathcal{R}_{mn\rho i})\widetilde{D}_{2mni},$$

$$\Xi_{2mn\rho i}^{\varepsilon(2,2)}\triangleq\widetilde{D}_{1mni}^{\mathrm{T}}(P_i^\varepsilon-\mathcal{W}_{mn\rho i})\widetilde{D}_{1mni}-r^2I-\mathrm{sym}\left\{\widetilde{D}_{2mi}\right\}+\widetilde{D}_{2mni}^{\mathrm{T}}(\widetilde{r}I-\mathcal{R}_{mn\rho i})\widetilde{D}_{2mni}.$$

Proof Choose the MLF candidate for the FES as

$$V_{\sigma(k)}(\widetilde{x}(k))\triangleq\widetilde{x}^{\mathrm{T}}(k)P_{\sigma(k)}^\varepsilon\widetilde{x}(k). \tag{16}$$

The target function is exhibited as follows when the subsystem i is running

$$\begin{aligned}\mathcal{J}_i(k) \triangleq\ & V_i(\widetilde{x}(k+1)) - \lambda V_i(\widetilde{x}(k)) \\ & - \left[\operatorname{sym}\left\{\widetilde{z}^{\mathrm{T}}(k)\, w(k)\right\} + r^2 w^{\mathrm{T}}(k)\, w(k) - \widetilde{r}\widetilde{z}^{\mathrm{T}}(k)\, \widetilde{z}(k)\right]. \end{aligned} \tag{17}$$

The introduction of some slack matrices satisfying the below inequalities can be beneficial to the rest steps for finding the desired filter[49]:

$$\begin{aligned}
&\left[\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\sum_{\rho=1}^{\psi+1}\tau_{mn\rho}\left(\underline{\gamma}_{mn\rho}\underline{\varpi}_{mn\rho} + \overline{\gamma}_{mn\rho}\overline{\varpi}_{mn\rho}\right) - 1\right]\mathcal{X}_i = 0,\\
&\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\sum_{\rho=1}^{\psi+1}\tau_{mn\rho}\overline{\gamma}_{mn\rho}\left(\overline{\varpi}_{mn\rho} - \underline{\varpi}_{mn\rho}\right)\mathcal{Y}_{mn\rho i} \geq 0,\\
&\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\sum_{\rho=1}^{\psi+1}\tau_{mn\rho}\overline{\varpi}_{mn\rho}\mathcal{V}_{mn\rho i} \geq 0,\\
&\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\sum_{\rho=1}^{\psi+1}\tau_{mn\rho}\underline{\gamma}_{mn\rho}\left(\overline{\varpi}_{mn\rho} - \underline{\varpi}_{mn\rho}\right)\mathcal{W}_{mn\rho i} \geq 0,\\
&\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\sum_{\rho=1}^{\psi+1}\tau_{mn\rho}\overline{\varpi}_{mn\rho}\mathcal{U}_{mn\rho i} \geq 0,\\
&\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\sum_{\rho=1}^{\psi+1}\tau_{mn\rho}\underline{\gamma}_{mn\rho}\left(\overline{\varpi}_{mn\rho} - \underline{\varpi}_{mn\rho}\right)\mathcal{R}_{mn\rho i} \geq 0.
\end{aligned}$$

By Lemma 2.8 and the aforesaid six conditions, a straightforward result is demonstrated as follows:

$$\begin{aligned}
\mathcal{J}_i(k) =\ & \widetilde{x}^{\mathrm{T}}(k+1)\, P_i^{\varepsilon}\widetilde{x}(k+1) - \lambda\widetilde{x}^{\mathrm{T}}(k)\, P_i^{\varepsilon}\widetilde{x}(k) - \operatorname{sym}\left\{\widetilde{z}^{\mathrm{T}}(k)\, w(k)\right\}\\
& - r^2 w^{\mathrm{T}}(k)\, w(k) + \widetilde{r}\widetilde{z}^{\mathrm{T}}(k)\, \widetilde{z}(k)\\
=\ & \left\{\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\varpi_{mn}\left[\widetilde{A}_{mni}^{\varepsilon}\widetilde{x}(k) + \widetilde{D}_{1mni}w(k)\right]\right\}^{\mathrm{T}}\\
& \times P_i^{\varepsilon}\left\{\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\varpi_{mn}\left[\widetilde{A}_{mni}^{\varepsilon}\widetilde{x}(k) + \widetilde{D}_{1mni}w(k)\right]\right\}\\
& + \widetilde{r}\left\{\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\varpi_{mn}\left[\widetilde{B}_{mni}\widetilde{x}(k) + \widetilde{D}_{2mi}w(k)\right]\right\}^{\mathrm{T}}\\
& \times\left\{\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\varpi_{mn}\left[\widetilde{B}_{mni}\widetilde{x}(k) + \widetilde{D}_{2mi}w(k)\right]\right\}\\
& - \operatorname{sym}\left\{\left[\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\varpi_{mn}\left[\widetilde{B}_{mni}\widetilde{x}(k) + \widetilde{D}_{2mi}w(k)\right]\right]^{\mathrm{T}} w(k)\right\}\\
& - \lambda\widetilde{x}^{\mathrm{T}}(k)\, P_i^{\varepsilon}\widetilde{x}(k) - r^2 w^{\mathrm{T}}(k)\, w(k)
\end{aligned}$$

$$
\begin{aligned}
&\le \sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\varpi_{mn}\left[\widetilde{A}_{mni}^{\varepsilon}\widetilde{x}\left(k\right)+\widetilde{D}_{1mni}w\left(k\right)\right]^{\mathrm{T}}P_{i}^{\varepsilon}\left[\widetilde{A}_{mni}^{\varepsilon}\widetilde{x}\left(k\right)+\widetilde{D}_{1mni}w\left(k\right)\right]\\
&\quad -\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\varpi_{mn}\mathrm{sym}\left\{\left[\widetilde{B}_{mni}\widetilde{x}\left(k\right)+\widetilde{D}_{2mi}w\left(k\right)\right]^{\mathrm{T}}w\left(k\right)\right\}\\
&\quad +\widetilde{r}\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\varpi_{mn}\left[\widetilde{B}_{mni}\widetilde{x}\left(k\right)+\widetilde{D}_{2mi}w\left(k\right)\right]^{\mathrm{T}}\left[\widetilde{B}_{mni}\widetilde{x}\left(k\right)+\widetilde{D}_{2mi}w\left(k\right)\right]\\
&\quad +\eta^{\mathrm{T}}\left(k\right)\left\{\left[\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\sum_{\rho=1}^{\psi+1}\tau_{mn\rho}\left(\underline{\gamma}_{mn\rho}\underline{\varpi}_{mn\rho}+\overline{\gamma}_{mn\rho}\overline{\varpi}_{mn\rho}\right)-1\right]\mathcal{X}_{i}\right\}\eta\left(k\right)\\
&\quad +\eta^{\mathrm{T}}\left(k\right)\left[\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\sum_{\rho=1}^{\psi+1}\tau_{mn\rho}\left(1-\underline{\gamma}_{mn\rho}\right)\left(\overline{\varpi}_{mn\rho}-\underline{\varpi}_{mn\rho}\right)\mathcal{Y}_{mn\rho i}\right]\eta\left(k\right)\\
&\quad +\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\sum_{\rho=1}^{\psi+1}\tau_{mn\rho}\overline{\varpi}_{mn\rho}\left[\widetilde{A}_{mni}^{\varepsilon}\widetilde{x}\left(k\right)+\widetilde{D}_{1mni}w\left(k\right)\right]^{\mathrm{T}}\mathcal{V}_{mn\rho i}\\
&\quad \times\left[\widetilde{A}_{mni}^{\varepsilon}\widetilde{x}\left(k\right)+\widetilde{D}_{1mni}w\left(k\right)\right]+\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\sum_{\rho=1}^{\psi+1}\tau_{mn\rho}\underline{\gamma}_{mn\rho}\left(\overline{\varpi}_{mn\rho}-\underline{\varpi}_{mn\rho}\right)\\
&\quad \times\left[\widetilde{A}_{mni}^{\varepsilon}\widetilde{x}\left(k\right)+\widetilde{D}_{1mni}w\left(k\right)\right]^{\mathrm{T}}\mathcal{W}_{mn\rho i}\left[\widetilde{A}_{mni}^{\varepsilon}\widetilde{x}\left(k\right)+\widetilde{D}_{1mni}w\left(k\right)\right]\\
&\quad +\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\sum_{\rho=1}^{\psi+1}\tau_{mn\rho}\overline{\varpi}_{mn\rho}\left[\widetilde{B}_{mni}\widetilde{x}\left(k\right)+\widetilde{D}_{2mi}w\left(k\right)\right]^{\mathrm{T}}\mathcal{U}_{mn\rho i}\\
&\quad \times\left[\widetilde{B}_{mni}\widetilde{x}\left(k\right)+\widetilde{D}_{2mi}w\left(k\right)\right]+\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\sum_{\rho=1}^{\psi+1}\tau_{mn\rho}\underline{\gamma}_{mn\rho}\left(\overline{\varpi}_{mn\rho}-\underline{\varpi}_{mn\rho}\right)\\
&\quad \times\left[\widetilde{B}_{mni}\widetilde{x}\left(k\right)+\widetilde{D}_{2mi}w\left(k\right)\right]^{\mathrm{T}}\mathcal{R}_{mn\rho i}\left[\widetilde{B}_{mni}\widetilde{x}\left(k\right)+\widetilde{D}_{2mi}w\left(k\right)\right]\\
&\quad -\lambda\widetilde{x}^{\mathrm{T}}\left(k\right)P_{i}^{\varepsilon}\widetilde{x}\left(k\right)-r^{2}w^{\mathrm{T}}\left(k\right)w\left(k\right)\\
&=\eta^{\mathrm{T}}\left(k\right)\left\{\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\sum_{\rho=1}^{\psi+1}\tau_{mn\rho}\left[\overline{\varpi}_{mn\rho}\varXi_{1mn\rho i}^{\varepsilon}+\overline{\varpi}_{mn\rho}\mathcal{X}_{i}+\left(\overline{\varpi}_{mn\rho}-\underline{\varpi}_{mn\rho}\right)\mathcal{Y}_{mn\rho i}\right]-\mathcal{X}_{i}\right\}\eta\left(k\right)\\
&\quad -\eta^{\mathrm{T}}\left(k\right)\left[\sum_{m=1}^{\theta}\sum_{n=1}^{\chi}\sum_{\rho=1}^{\psi+1}\tau_{mn\rho}\underline{\gamma}_{mn\rho}\left(\overline{\varpi}_{mn\rho}-\underline{\varpi}_{mn\rho}\right)\left(\varXi_{2mn\rho i}^{\varepsilon}+\mathcal{Y}_{mn\rho i}+\mathcal{X}_{i}\right)\right]\eta\left(k\right),
\end{aligned}
$$

with

$$\eta\left(k\right)\triangleq\left[\widetilde{x}^{\mathrm{T}}\left(k\right)\ w^{\mathrm{T}}\left(k\right)\right]^{\mathrm{T}}.$$

From (13), the following inequality holds:

$$\sum_{\vartheta=1}^{\varphi}\sum_{\nu_{1}=1}^{2}\cdots\sum_{\nu_{\varkappa_{x}}=1}^{2}\prod_{\varrho=1}^{\varkappa_{x}}\varepsilon_{\varrho\nu_{\varrho}\vartheta_{\rho}}\left(x_{\varrho}\left(k\right)\right)\left(\overline{\varpi}\varXi_{1mn\rho i}^{\varepsilon}+\omega_{2}\mathcal{X}_{i}+\omega_{3}\mathcal{Y}_{mn\rho i}\right)<0,$$

which indicates that

$$\overline{\varpi}_{mn\rho}\varXi_{1mn\rho i}^{\varepsilon}+\left(\overline{\varpi}_{mn\rho}-\frac{1}{\theta\chi}\right)\mathcal{X}_{i}+\left(\overline{\varpi}_{mn\rho}-\underline{\varpi}_{mn\rho}\right)\mathcal{Y}_{mn\rho i}<0. \tag{18}$$

Combining (18) and (8), we can get that

$$\sum_{\rho=1}^{\psi+1} \tau_{mn\rho} \left[\overline{\varpi}_{mn\rho} \Xi^{\varepsilon}_{1mn\rho i} + \left(\overline{\varpi}_{mn\rho} - \frac{1}{\theta\chi} \right) \mathcal{X}_i + \left(\overline{\varpi}_{mn\rho} - \underline{\varpi}_{mn\rho} \right) \mathcal{Y}_{mn\rho i} \right] < 0.$$

Therefore, it follows that

$$\sum_{m=1}^{\theta} \sum_{n=1}^{\chi} \sum_{\rho=1}^{\psi+1} \tau_{mn\rho} \left[\overline{\varpi}_{mn\rho} \Xi^{\varepsilon}_{1mn\rho i} + \overline{\varpi}_{mn\rho} \mathcal{X}_i + \left(\overline{\varpi}_{mn\rho} - \underline{\varpi}_{mn\rho} \right) \mathcal{Y}_{mn\rho i} \right] - \mathcal{X}_i < 0.$$

Via the above discussion, the inequality $\mathcal{J}_i(k) \leq 0$ can be obtained from (13) and (14).

When $w(k) \equiv 0$, from $\mathcal{J}_i(k) \leq 0$ and condition (12), ones can know that $V_i(\widetilde{x}(k+1)) \leq \lambda V_i(\widetilde{x}(k))$ and $0 \leq V_i(\widetilde{x}(k)) \leq \mu V_j(\widetilde{x}(k))$. Based on Figure 1 and $0 < \lambda < 1$, $\tau^{(g)} \geq \tau_{PDT}$, for $k \in \left[k_{l_{g-1}}, k_{l_g}\right)$, it can be inferred that

$$\begin{aligned} V_{\sigma(k)}(\widetilde{x}(k)) &\leq \mu^{Num\left(k_{l_{g-1}},k\right)} \lambda^{k-k_{l_{g-1}}} V_{\sigma\left(k_{l_{g-1}}\right)}\left(\widetilde{x}\left(k_{l_{g-1}}\right)\right) \\ &\leq (\mu\lambda)^{T^{(g)}} \mu\lambda^{\tau^{(g)}} V_{\sigma\left(k_{l_{g-1}}\right)}\left(\widetilde{x}\left(k_{l_{g-1}}\right)\right) \leq \phi_g V_{\sigma\left(k_{l_{g-1}}\right)}\left(\widetilde{x}\left(k_{l_{g-1}}\right)\right), \end{aligned}$$

with

$$\phi_g \triangleq (\mu\lambda)^{T^{(g)}} \mu\lambda^{\tau_{PDT}}.$$

Thereafter, consider the following two cases:

(i) When $\mu\lambda < 1$, it can be observed that $\phi_g < 1$;

(ii) When $\mu\lambda > 1$, $\phi_g \leq (\mu\lambda)^{T_{PDT}} \mu\lambda^{\tau_{PDT}} < 1$ can be achieved under the condition (10) and $T^{(g)} \leq T_{PDT}$.

Then, denoting k_0 and $V_0(\widetilde{x}(k_0))$ as the initial time and the initial value of Lyapunov function, respectively, it follows that

$$V_{\sigma(k)}(\widetilde{x}(k)) \leq \phi^g V_{\sigma(k_0)}(\widetilde{x}(k_0)),$$

where

$$\phi \triangleq \begin{cases} \max\limits_{\widetilde{g}\in[0,g]} \phi_{\widetilde{g}}, & \mu\lambda < 1, \\ (\mu\lambda)^{T_{PDT}} \mu\lambda^{\tau_{PDT}}, & \mu\lambda > 1, \end{cases}$$

satisfying

$$\phi < 1,$$

which means

$$\|\widetilde{x}(k)\|^2 \leq \frac{\phi^g \max_{\sigma(k)\in\mathcal{S}} \lambda_{\max}\left(P^{\varepsilon}_{\sigma(k)}\right)}{\min_{\sigma(k)\in\mathcal{S}} \lambda_{\min}\left(P^{\varepsilon}_{\sigma(k)}\right)} \|\widetilde{x}(k_0)\|^2 \leq \delta \widetilde{\phi}^{k-k_0} \|\widetilde{x}(k_0)\|^2,$$

with

$$\widetilde{\phi} \triangleq \max_{k_0 \leq k} \left(\phi^{\frac{g}{k-k_0+1}} \right), \quad \delta \triangleq \frac{\widetilde{\phi} \max_{\sigma(k)\in\mathcal{S}} \lambda_{\max}\left(P^{\varepsilon}_{\sigma(k)}\right)}{\min_{\sigma(k)\in\mathcal{S}} \lambda_{\min}\left(P^{\varepsilon}_{\sigma(k)}\right)}.$$

By Definition 2.5, the conditions from Theorem 3.1 can guarantee that FES is GUES.

On the other hand, from $\mathcal{J}_i(k) \leq 0$, ones can know that

$$V_i(\widetilde{x}(k+1)) \leq \lambda V_i(\widetilde{x}(k)) + \mathrm{sym}\left\{\widetilde{z}^{\mathrm{T}}(k)\, w(k)\right\} + r^2 w^{\mathrm{T}}(k)\, w(k) - \widetilde{r}\widetilde{z}^{\mathrm{T}}(k)\,\widetilde{z}(k). \tag{19}$$

When $k \in [k_{l_{g-1}}, k_{l_g})$ and $V_0(\widetilde{x}(k_0)) = 0$, it can be inferred from (12) and the above inequality that

$$\begin{aligned}
&V_{\sigma(k)}(\widetilde{x}(k)) \\
\leq &\sum_{t=k_0}^{k-1} \mu^{Num(t,k)} \lambda^{k-t-1} \left[\mathrm{sym}\left\{\widetilde{z}^{\mathrm{T}}(t)\, w(t)\right\} + r^2 w^{\mathrm{T}}(t)\, w(t) - \widetilde{r}\widetilde{z}^{\mathrm{T}}(t)\,\widetilde{z}(t)\right] \\
= &\sum_{t=k_0}^{k-1} \mu^{Num(t,k)} \lambda^{k-t-1} \left[\left(rw(t) + \frac{1}{r}\widetilde{z}(t)\right)^{\mathrm{T}} \left(rw(t) + \frac{1}{r}\widetilde{z}(t)\right) - \frac{r_1(1-\lambda)}{r^2(1-r_2)}\widetilde{z}^{\mathrm{T}}(t)\,\widetilde{z}(t)\right].
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
&\sum_{t=k_0}^{k-1} \mu^{Num(t,k)} \lambda^{k-t-1} \left(rw(t) + \frac{1}{r}\widetilde{z}(t)\right)^{\mathrm{T}} \left(rw(t) + \frac{1}{r}\widetilde{z}(t)\right) \\
\geq &\frac{r_1(1-\lambda)}{r^2(1-r_2)} \sum_{t=k_0}^{k-1} \mu^{Num(t,k)} \lambda^{k-t-1} \widetilde{z}^{\mathrm{T}}(t)\,\widetilde{z}(t).
\end{aligned}$$

Considering $\mu > 1$, $0 < \lambda < 1$ and inequality (3), we can get that

$$\begin{aligned}
&\sum_{t=k_0}^{k-1} r_2^{k-t-1} \left(rw(t) + \frac{1}{r}\widetilde{z}(t)\right)^{\mathrm{T}} \left(rw(t) + \frac{1}{r}\widetilde{z}(t)\right) \\
\geq &\frac{(1-\lambda)}{r^2(1-r_2)} \sum_{t=k_0}^{k-1} \lambda^{k-t-1} \widetilde{z}^{\mathrm{T}}(t)\,\widetilde{z}(t).
\end{aligned}$$

Then, we can deduce that

$$\begin{aligned}
&\sum_{k=k_0+1}^{\infty} \sum_{t=k_0}^{k-1} r_2^{k-t-1} \left(rw(t) + \frac{1}{r}\widetilde{z}(t)\right)^{\mathrm{T}} \left(rw(t) + \frac{1}{r}\widetilde{z}(t)\right) \\
\geq &\frac{(1-\lambda)}{r^2(1-r_2)} \sum_{k=k_0+1}^{\infty} \sum_{t=k_0}^{k-1} \lambda^{k-t-1} \widetilde{z}^{\mathrm{T}}(t)\,\widetilde{z}(t),
\end{aligned}$$

from which, we can obtain

$$\sum_{t=k_0}^{\infty} \left(rw(t) + \frac{1}{r}\widetilde{z}(t)\right)^{\mathrm{T}} \left(rw(t) + \frac{1}{r}\widetilde{z}(t)\right) \geq \frac{1}{r^2} \sum_{t=k_0}^{\infty} \widetilde{z}^{\mathrm{T}}(t)\,\widetilde{z}(t).$$

Concluding the above discussion, the inequality (9) holds, which implies that FES meets a given passive performance index under the criteria in Theorem 3.1. This ends this proof. ∎

Remark 3.2 In the process of settling the fuzzy summation, two scalars with the explicit relationship of size, $\overline{\omega}_{mn\nu_1\cdots\nu_{\varkappa_x}\vartheta\rho}$ and $\underline{\omega}_{mn\nu_1\cdots\nu_{\varkappa_x}\vartheta\rho}$, are obtained across the dividing processes of FOU and the state space. It is easily acknowledged that the utilization of some LMIs stemmed from this relationship can reduce the conservatism of stability conditions by large. Besides, the results in Theorem 3.1 also take the information of MFs fully into consideration by these two scalars. By contrast, the conclusions based on the type-1 T-S fuzzy model in most references are not related to the message of MFs. This point also displays the advancement and superiority of the type-2 T-S fuzzy model compared to the type-1 T-S fuzzy model.

Theorem 3.3 *Considering that the state space and FOU are divided into* $\varphi(\in \mathbb{N}^+)$ *and* $\psi+1(\in \mathbb{N}^+)$ *subspaces separately, for the admissible switching sequence obeying PDT switching rule and given scalars* ζ_1, ζ_2, $\widetilde{\zeta}_1$, $\widetilde{\zeta}_2$, $0<\theta$, $0<\chi$, $0<\overline{\varepsilon}$, $0<r$, $0<T_{PDT}$, $0<\tau_{PDT}$, $0<\lambda<1$, $1<\mu$, $\lambda\mu\neq 1$ *and symmetric matrices* $\mathcal{G}\triangleq \operatorname{diag}\{\mathcal{G}_1,\mathcal{G}_2\}$, $\widetilde{\mathcal{G}}\triangleq \operatorname{diag}\{\widetilde{\mathcal{G}}_1,\widetilde{\mathcal{G}}_2\}$, *FES* $(\widetilde{\Gamma})$ *is GUES with a passive performance level* r *for* $\varepsilon\in(0,\overline{\varepsilon}]$, *if for* $\forall m\in\mathcal{M}$, $\forall n\in\mathcal{N}$, $\forall\rho\in\mathcal{R}$, $\forall i_\varrho\in\{1,2\}$, $i\neq j$, *and* i, $j\in\mathcal{S}$, *there exist symmetric matrices* $\mathcal{P}_{1i}\in\mathbb{R}^{2\varkappa_x}$, $\mathcal{P}_{2i}\in\mathbb{R}^{2\varkappa_x}$, $\mathcal{W}_{mn\rho i}\in\mathbb{R}^{\varkappa_x}$, $\mathcal{X}_i\triangleq\begin{bmatrix}\mathcal{X}_{1i} & \mathcal{X}_{2i}\\ * & \mathcal{X}_{3i}\end{bmatrix}\in\mathbb{R}^{2\varkappa_x+1}$, $0\le\mathcal{Y}_{mn\rho i}\triangleq\begin{bmatrix}\mathcal{Y}_{1mn\rho i} & \mathcal{Y}_{2mn\rho i}\\ * & \mathcal{Y}_{3mn\rho i}\end{bmatrix}\in\mathbb{R}^{2\varkappa_x+1}$, $0\le\mathcal{V}_{mn\rho i}\in\mathbb{R}^{\varkappa_x}$, $0<\widetilde{\mathcal{U}}_{mn\rho i}\le(\widetilde{r}I)^{-1}$, $\widetilde{\mathcal{R}}_{mn\rho i}$, *and matrices* $\mathring{A}_{ni}$, $\mathring{B}_{ni}$, $\mathring{E}_{ni}$, $\mathcal{I}_{mn\rho i}\triangleq\begin{bmatrix}\mathcal{I}_{1mn\rho i} & \zeta_1\mathcal{I}_{3ni}\\ \mathcal{I}_{2mn\rho i} & \zeta_2\mathcal{I}_{3ni}\end{bmatrix}\in\mathbb{R}^{2\varkappa_x}$, $\widetilde{\mathcal{I}}_{mn\rho i}\triangleq\begin{bmatrix}\widetilde{\mathcal{I}}_{1mn\rho i} & \widetilde{\zeta}_1\mathcal{I}_{3ni}\\ \widetilde{\mathcal{I}}_{2mn\rho i} & \widetilde{\zeta}_2\mathcal{I}_{3ni}\end{bmatrix}\in\mathbb{R}^{2\varkappa_x}$, *such that* (10), (15) *and the following conditions hold:*

$$\mathcal{P}_{1i}-\mathcal{W}_{mn\rho i}<0,\mathcal{P}_i^{\overline{\varepsilon}}-\mathcal{W}_{mn\rho i}<0 \tag{20}$$
$$\mathcal{P}_{1i}>0, \tag{21}$$
$$\mathcal{P}_i^{\overline{\varepsilon}}>0, \tag{22}$$
$$\mathcal{P}_{1i}-\mu\mathcal{P}_{1j}<0, \tag{23}$$
$$\mathcal{P}_i^{\overline{\varepsilon}}-\mu\mathcal{P}_j^{\overline{\varepsilon}}<0, \tag{24}$$
$$\widetilde{\Xi}_{1mn\rho i}>0, \tag{25}$$
$$\Xi_{1mn\rho i}^{\overline{\varepsilon}}>0, \tag{26}$$
$$\widetilde{\Xi}_{2mn\rho i}<0, \tag{27}$$
$$\Xi_{2mn\rho i}^{\overline{\varepsilon}}<0, \tag{28}$$

where

$$\widetilde{\Xi}_{1mn\rho i}\triangleq\begin{bmatrix}\widetilde{\Xi}^{(1,1)}_{1mn\rho i} & 0 & \widetilde{\Xi}^{(1,3)}_{1mn\rho i} & \widetilde{\Xi}^{(1,4)}_{1mn\rho i}\\ * & -\widetilde{\mathcal{R}}_{mn\rho i} & \widetilde{B}_{mni} & \widetilde{D}_{2mi}\\ * & * & \widetilde{\Xi}^{(3,3)}_{1mn\rho i} & \widetilde{\Xi}^{(3,4)}_{1mn\rho i}\\ * & * & * & \widetilde{\Xi}^{(4,4)}_{1mn\rho i}\end{bmatrix},$$

$$\Xi^{\overline{\varepsilon}}_{1mn\rho i}\triangleq\begin{bmatrix}\widetilde{\Xi}^{\overline{\varepsilon}(1,1)}_{1mn\rho i} & 0 & \widetilde{\Xi}^{\overline{\varepsilon}(1,3)}_{1mn\rho i} & \widetilde{\Xi}^{(1,4)}_{1mn\rho i}\\ * & -\widetilde{\mathcal{R}}_{mn\rho i} & \widetilde{B}_{mni} & \widetilde{D}_{2mi}\\ * & * & \widetilde{\Xi}^{\overline{\varepsilon}(3,3)}_{1mn\rho i} & \widetilde{\Xi}^{(3,4)}_{1mn\rho i}\\ * & * & * & \widetilde{\Xi}^{(4,4)}_{1mn\rho i}\end{bmatrix},$$

$$\widetilde{\Xi}_{2mn\rho i} \triangleq \begin{bmatrix} \widetilde{\Xi}_{2mn\rho i}^{(1,1)} & 0 & \widetilde{\Xi}_{2mn\rho i}^{(1,3)} & \widetilde{\Xi}_{2mn\rho i}^{(1,4)} \\ * & -\widetilde{\mathcal{U}}_{mn\rho i} & \omega_1 \widetilde{B}_{mni} & \omega_1 \widetilde{D}_{2mi} \\ * & * & \widetilde{\Xi}_{2mn\rho i}^{(3,3)} & \widetilde{\Xi}_{2mn\rho i}^{(3,4)} \\ * & * & * & \widetilde{\Xi}_{2mn\rho i}^{(4,4)} \end{bmatrix},$$

$$\Xi_{2mn\rho i}^{\overline{\varepsilon}} \triangleq \begin{bmatrix} \widetilde{\Xi}_{2mn\rho i}^{\overline{\varepsilon}(1,1)} & 0 & \widetilde{\Xi}_{2mn\rho i}^{\overline{\varepsilon}(1,3)} & \widetilde{\Xi}_{2mn\rho i}^{(1,4)} \\ * & -\widetilde{\mathcal{U}}_{mn\rho i} & \omega_1 \widetilde{B}_{mni} & \omega_1 \widetilde{D}_{2mi} \\ * & * & \widetilde{\Xi}_{2mn\rho i}^{\overline{\varepsilon}(3,3)} & \widetilde{\Xi}_{2mn\rho i}^{(3,4)} \\ * & * & * & \widetilde{\Xi}_{2mn\rho i}^{(4,4)} \end{bmatrix},$$

with

$$A_{mi} \triangleq \begin{bmatrix} A_{1mi} & 0 \\ A_{3mi} & 0 \end{bmatrix}, \quad A_{mi}^{\overline{\varepsilon}} \triangleq \begin{bmatrix} 0 & A_{2mi} \\ 0 & A_{4mi} \end{bmatrix}, \quad \mathcal{X}_{2i} \triangleq \begin{bmatrix} \mathcal{X}_{12i} \\ \mathcal{X}_{22i} \end{bmatrix}, \quad \mathcal{Y}_{2mn\rho i} \triangleq \begin{bmatrix} \mathcal{Y}_{12mn\rho i} \\ \mathcal{Y}_{12mn\rho i} \end{bmatrix},$$

$$\overline{\Xi}_{1mn\rho i}^{(1,3)} \triangleq \begin{bmatrix} \mathcal{I}_{1mn\rho i} A_{mi}^{\overline{\varepsilon}} & 0 \\ \mathcal{I}_{2mn\rho i} A_{mi}^{\overline{\varepsilon}} & 0 \end{bmatrix}, \quad \widetilde{\Xi}_{1mn\rho i}^{(1,3)} \triangleq \begin{bmatrix} \mathcal{I}_{1mn\rho i} A_{mi} + \zeta_1 \mathring{E}_{ni} C_{mi} & \zeta_1 \mathring{A}_{ni} \\ \mathcal{I}_{2mn\rho i} A_{mi} + \zeta_2 \mathring{E}_{ni} C_{mi} & \zeta_2 \mathring{A}_{ni} \end{bmatrix},$$

$$\overline{\Xi}_{2mn\rho i}^{(1,3)} \triangleq \begin{bmatrix} \widetilde{\mathcal{I}}_{1mn\rho i} A_{mi}^{\overline{\varepsilon}} & 0 \\ \widetilde{\mathcal{I}}_{2mn\rho i} A_{mi}^{\overline{\varepsilon}} & 0 \end{bmatrix}, \quad \widetilde{\Xi}_{2mn\rho i}^{(1,3)} \triangleq \begin{bmatrix} \omega_1 \left(\widetilde{\mathcal{I}}_{1mn\rho i} A_{mi} + \zeta_1 \mathring{E}_{ni} C_{mi} \right) & \omega_1 \widetilde{\zeta}_1 \mathring{A}_{ni} \\ \omega_1 \left(\widetilde{\mathcal{I}}_{2mn\rho i} A_{mi} + \zeta_2 \mathring{E}_{ni} C_{mi} \right) & \omega_1 \widetilde{\zeta}_2 \mathring{A}_{ni} \end{bmatrix},$$

$$\widetilde{\Xi}_{1mn\rho i}^{(1,4)} \triangleq \begin{bmatrix} \mathcal{I}_{1mn\rho i} D_{1mi} + \zeta_1 \mathring{E}_{ni} D_{3mi} \\ \mathcal{I}_{2mn\rho i} D_{1mi} + \zeta_2 \mathring{E}_{ni} D_{3mi} \end{bmatrix}, \quad \widetilde{\Xi}_{2mn\rho i}^{(1,4)} \triangleq \begin{bmatrix} \omega_1 \left(\widetilde{\mathcal{I}}_{1mn\rho i} D_{1mi} + \widetilde{\zeta}_1 \mathring{E}_{ni} D_{3mi} \right) \\ \omega_1 \left(\widetilde{\mathcal{I}}_{2mn\rho i} D_{1mi} + \widetilde{\zeta}_2 \mathring{E}_{ni} D_{3mi} \right) \end{bmatrix},$$

$$\widetilde{\Xi}_{1mn\rho i}^{(3,4)} \triangleq \begin{bmatrix} -B_{mi}^{\mathrm{T}} + \mathcal{X}_{12i} + \mathcal{Y}_{12mn\rho i} \\ \mathring{B}_{ni}^{\mathrm{T}} + \mathcal{X}_{22i} + \mathcal{Y}_{22mn\rho i} \end{bmatrix}, \quad \widetilde{\Xi}_{2mn\rho i}^{(3,4)} \triangleq \begin{bmatrix} -\overline{\varpi} B_{mi}^{\mathrm{T}} + \omega_2 \mathcal{X}_{12i} + \omega_3 \mathcal{Y}_{12mn\rho i} \\ \overline{\varpi} \mathring{B}_{ni}^{\mathrm{T}} + \omega_2 \mathcal{X}_{22i} + \omega_3 \mathcal{Y}_{22mn\rho i} \end{bmatrix},$$

$$\widetilde{\Xi}_{1mn\rho i}^{(1,1)} \triangleq \mathcal{G} \left(\mathcal{P}_{1i} - \mathcal{W}_{mn\rho i} \right) \mathcal{G}^{\mathrm{T}} + \mathrm{sym} \left\{ \mathcal{I}_{mn\rho i} \mathcal{G}^{\mathrm{T}} \right\},$$

$$\widetilde{\Xi}_{2mn\rho i}^{(1,1)} \triangleq \widetilde{\mathcal{G}} \left(\mathcal{P}_{1i} + \mathcal{V}_{mn\rho i} \right) \widetilde{\mathcal{G}}^{\mathrm{T}} + \mathrm{sym} \left\{ \widetilde{\mathcal{I}}_{mn\rho i} \widetilde{\mathcal{G}}^{\mathrm{T}} \right\},$$

$$\widetilde{\Xi}_{1mn\rho i}^{\overline{\varepsilon}(1,1)} \triangleq \widetilde{\Xi}_{1mn\rho i}^{(1,1)} + \overline{\varepsilon} \mathcal{G} \mathcal{P}_{2i} \mathcal{G}^{\mathrm{T}}, \quad \widetilde{\Xi}_{2mn\rho i}^{\overline{\varepsilon}(1,1)} \triangleq \widetilde{\Xi}_{2mn\rho i}^{(1,1)} + \overline{\varepsilon} \widetilde{\mathcal{G}} \mathcal{P}_{2i} \widetilde{\mathcal{G}}^{\mathrm{T}}, \widetilde{\Xi}_{1mn\rho i}^{\overline{\varepsilon}(1,3)} \triangleq \widetilde{\Xi}_{1mn\rho i}^{(1,3)} + \overline{\varepsilon} \overline{\Xi}_{1mn\rho i}^{(1,3)},$$

$$\widetilde{\Xi}_{2mn\rho i}^{\overline{\varepsilon}(1,3)} \triangleq \widetilde{\Xi}_{2mn\rho i}^{(1,3)} + \overline{\varepsilon} \omega_1 \overline{\Xi}_{2mn\rho i}^{(1,3)}, \quad \widetilde{\Xi}_{1mn\rho i}^{(3,3)} \triangleq -\lambda \mathcal{P}_{1i} + \mathcal{X}_{1i} + \mathcal{Y}_{1mn\rho i}, \quad \widetilde{\Xi}_{1mn\rho i}^{\overline{\varepsilon}(3,3)} \triangleq \widetilde{\Xi}_{1mn\rho i}^{(3,3)} - \overline{\varepsilon} \lambda \mathcal{P}_{2i},$$

$$\widetilde{\Xi}_{2mn\rho i}^{(3,3)} \triangleq -\lambda \overline{\varpi} \mathcal{P}_{1i} + \omega_2 \mathcal{X}_{1i} + \omega_3 \mathcal{Y}_{1mn\rho i}, \quad \widetilde{\Xi}_{2mn\rho i}^{\overline{\varepsilon}(3,3)} \triangleq \widetilde{\Xi}_{2mn\rho i}^{(3,3)} - \overline{\varepsilon} \lambda \overline{\varpi} \mathcal{P}_{2i},$$

$$\widetilde{\Xi}_{1mn\rho i}^{(4,4)} \triangleq -\mathrm{sym} \left\{ \widetilde{D}_{2mi} \right\} - r^2 I + \mathcal{X}_{3i} + \mathcal{Y}_{3mn\rho i}, \quad \omega_1 \triangleq \sqrt{\overline{\varpi}}, \quad \mathcal{P}_i^{\overline{\varepsilon}} \triangleq \mathcal{P}_{1i} + \overline{\varepsilon} \mathcal{P}_{2i},$$

$$\widetilde{\Xi}_{2mn\rho i}^{(4,4)} \triangleq -\mathrm{sym} \left\{ \overline{\varpi} \widetilde{D}_{2mi} \right\} - \overline{\varpi} r^2 I + \omega_2 \mathcal{X}_{3i} + \omega_3 \mathcal{Y}_{3mn\rho i}.$$

In this case, the gains of filter are presented as follows:

$$\left[\widehat{A}_{ni} \ \widehat{B}_{ni} \ \widehat{E}_{ni} \right] = \left[\mathcal{I}_{3ni}^{-1} \mathring{A}_{ni} \ \mathring{B}_{ni} \ \mathcal{I}_{3ni}^{-1} \mathring{E}_{ni} \right].$$

Proof Combining Lemma 2.7 with (21)–(28), it can be deduced that for $\varepsilon \in (0, \overline{\varepsilon}]$:

$$P_i^{\varepsilon} > 0, \tag{29}$$
$$P_i^{\varepsilon} - \mu P_j^{\varepsilon} < 0, \tag{30}$$
$$\Xi_{1mn\rho i}^{\varepsilon} > 0, \tag{31}$$
$$\Xi_{2mn\rho i}^{\varepsilon} < 0, \tag{32}$$

where

$$P_i^{\varepsilon} \triangleq \mathcal{P}_{1i} + \varepsilon \mathcal{P}_{2i},$$
$$\Xi_{1mn\rho i}^{\varepsilon} \triangleq \begin{bmatrix} \Xi_{1mn\rho i}^{\varepsilon(1,1)} & 0 & \Xi_{1mn\rho i}^{\varepsilon(1,3)} & \widetilde{\Xi}_{1mn\rho i}^{(1.4)} \\ * & -\widetilde{\mathcal{R}}_{mn\rho i} & \widetilde{B}_{mni} & \widetilde{D}_{2mi} \\ * & * & \Xi_{1mn\rho i}^{\varepsilon(3,3)} & \widetilde{\Xi}_{1mn\rho i}^{(3.4)} \\ * & * & * & \widetilde{\Xi}_{1mn\rho i}^{(4.4)} \end{bmatrix},$$
$$\Xi_{2mn\rho i}^{\varepsilon} \triangleq \begin{bmatrix} \Xi_{2mn\rho i}^{\varepsilon(1,1)} & 0 & \Xi_{2mn\rho i}^{\varepsilon(1,3)} & \widetilde{\Xi}_{2mn\rho i}^{(1.4)} \\ * & -\widetilde{\mathcal{U}}_{mn\rho i} & \omega_1 \widetilde{B}_{mni} & \omega_1 \widetilde{D}_{2mi} \\ * & * & \Xi_{2mn\rho i}^{\varepsilon(3,3)} & \widetilde{\Xi}_{2mn\rho i}^{(3.4)} \\ * & * & * & \widetilde{\Xi}_{2mn\rho i}^{(4.4)} \end{bmatrix},$$

with

$$\Xi_{1mn\rho i}^{\varepsilon(1,1)} \triangleq \widetilde{\Xi}_{1mn\rho i}^{(1,1)} + \varepsilon \mathcal{G} \mathcal{P}_{2i} \mathcal{G}^{\mathrm{T}}, \quad \Xi_{1mn\rho i}^{\varepsilon(1,3)} \triangleq \widetilde{\Xi}_{1mn\rho i}^{(1,3)} + \varepsilon \overline{\Xi}_{1mn\rho i}^{(1,3)}, \quad \Xi_{1mn\rho i}^{\varepsilon(3,3)} \triangleq \widetilde{\Xi}_{1mn\rho i}^{(3,3)} - \varepsilon \lambda \mathcal{P}_{2i},$$
$$\Xi_{2mn\rho i}^{\varepsilon(1,1)} \triangleq \widetilde{\Xi}_{2mn\rho i}^{(1,1)} + \varepsilon \widetilde{\mathcal{G}} \mathcal{P}_{2i} \widetilde{\mathcal{G}}^{\mathrm{T}}, \quad \Xi_{2mn\rho i}^{\varepsilon(1,3)} \triangleq \widetilde{\Xi}_{2mn\rho i}^{(1,3)} + \varepsilon \omega_1 \overline{\Xi}_{2mn\rho i}^{(1,3)}, \quad \Xi_{2mn\rho i}^{\varepsilon(3,3)} \triangleq \widetilde{\Xi}_{2mn\rho i}^{(3,3)} - \varepsilon \lambda \overline{\omega} \mathcal{P}_{2i}.$$

It is easily observed that (29) and (30) are equivalent to (11) and (12), respectively. Considering (20), the following inequality holds:

$$\left[\mathcal{G}\left(P_i^{\varepsilon} - \mathcal{W}_{mn\rho i}\right) + \mathcal{I}_{mn\rho i}\right]\left(P_i^{\varepsilon} - \mathcal{W}_{mn\rho i}\right)^{-1}\left[\mathcal{G}\left(P_i^{\varepsilon} - \mathcal{W}_{mn\rho i}\right) + \mathcal{I}_{mn\rho i}\right]^{\mathrm{T}} < 0,$$

which follows that

$$\mathcal{G}\left(P_i^{\varepsilon} - \mathcal{W}_{mn\rho i}\right)\mathcal{G}^{\mathrm{T}} + \mathcal{I}_{mn\rho i}\mathcal{G}^{\mathrm{T}} + \mathcal{G}\mathcal{I}_{mn\rho i}^{\mathrm{T}} < -\mathcal{I}_{mn\rho i}\left(P_i^{\varepsilon} - \mathcal{W}_{mn\rho i}\right)^{-1}\mathcal{I}_{mn\rho i}^{\mathrm{T}}. \tag{33}$$

Applying inequality (33) to (31), ones have

$$0 < \Xi_{1mn\rho i}^{\varepsilon} < \mathring{\Xi}_{1mn\rho i}^{\varepsilon}, \tag{34}$$

where

$$\mathring{\Xi}_{1mn\rho i}^{\varepsilon} \triangleq \begin{bmatrix} \mathring{\Xi}_{1mn\rho i}^{\varepsilon(1,1)} & 0 & \Xi_{1mn\rho i}^{\varepsilon(1,3)} & \widetilde{\Xi}_{1mn\rho i}^{(1.4)} \\ * & -\widetilde{\mathcal{R}}_{mn\rho i} & \widetilde{B}_{mni} & \widetilde{D}_{2mi} \\ * & * & \Xi_{1mn\rho i}^{\varepsilon(3,3)} & \widetilde{\Xi}_{1mn\rho i}^{(3.4)} \\ * & * & * & \widetilde{\Xi}_{1mn\rho i}^{(4.4)} \end{bmatrix},$$

$$\mathring{\Xi}_{1mn\rho i}^{\varepsilon(1,1)} \triangleq -\mathcal{I}_{mn\rho i} \left(P_i^{\varepsilon} - \mathcal{W}_{mn\rho i}\right)^{-1} \mathcal{I}_{mn\rho i}^{\mathrm{T}}.$$

Perform a congruent transformation with $\operatorname{diag}\{\mathcal{I}_{mn\rho i}^{-T}, I, I, I\}$ and apply Schur complement to (34). Defining $\widetilde{\mathcal{R}}_{mn\rho i} \triangleq (\widetilde{r}I - \mathcal{R}_{mn\rho i})^{-1}$, (14) can be obtained. Furthermore, setting $\widetilde{\mathcal{U}}_{mn\rho i} \triangleq (\widetilde{r}I + \mathcal{U}_{mn\rho i})^{-1}$ and using the same method to deal with (32), it can be inferred that (13) holds. This ends the proof. ∎

4 Simulation

As an example to prove the powerful application of the designed filter, a tunnel diode circuit is exhibited in Figure 2. The volt-ampere characteristic of the tunnel diode is as follows [50]:

$$i_d(\kappa) = 0.002u_d(\kappa) + \hbar u_d^3(\kappa),$$

where, for the continuous time κ, $i_d(\kappa)$ is the current through this tunnel diode; $u_d(\kappa) \in [-3, 3]$ denotes the voltage across this diode; $\hbar \in [0.01, 0.03]$ represents the parameter uncertainties.

Setting $x_s(\kappa) \triangleq u_d(\kappa)$, $x_f(\kappa) \triangleq i_L(\kappa)$, $g_1(x_s(\kappa), \hbar) \triangleq 0.002 + \hbar x_s^2(\kappa)$ and considering the Kirchoff voltage and current law, ones have

$$\begin{cases} C\dot{x}_s(\kappa) = -g_1(x_s(\kappa), \hbar)\, x_s(\kappa) + x_f(\kappa), \\ L\dot{x}_f(\kappa) = -x_s(\kappa) - Rx_f(\kappa), \end{cases}$$

where C and L symbolize the values of the capacitor and the inductance, respectively; $R \in \{R_1, R_2\}$ is the value of the resistance, where the two different values correspond to two subsystems.

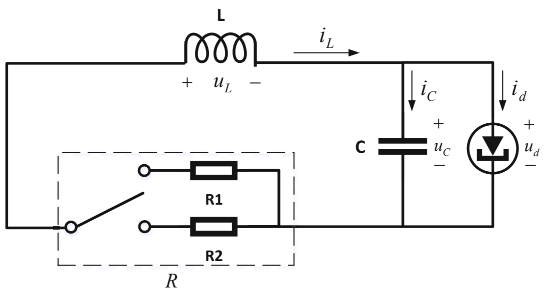

Figure 2 The tunnel diode circuit

Supposing that $|x_s(\kappa)| \leq 3$, the system matrices of $\mathit{\Gamma}$ are expressed by

$$A_{11}^{\varepsilon} = \begin{bmatrix} -\frac{g_{1\min}}{C} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_1}{L} \end{bmatrix}, \quad A_{12}^{\varepsilon} = \begin{bmatrix} -\frac{g_{1\min}}{C} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_2}{L} \end{bmatrix},$$

$$A_{21}^{\varepsilon} = \begin{bmatrix} -\frac{g_{1\max}}{C} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_1}{L} \end{bmatrix}, \quad A_{22}^{\varepsilon} = \begin{bmatrix} -\frac{g_{1\max}}{C} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_2}{L} \end{bmatrix},$$

$$D_{1mi} = \begin{bmatrix} -0.1 \; 0.1 \end{bmatrix}^{\mathrm{T}}, \quad m \in \{1, 2\}, \quad i \in \{1, 2\}, \quad g_{1\max} = 0.272, \quad g_{1\min} = 0.002.$$

For $\theta = 2$, $\xi = 1$, $\chi = 2$ and $\widehat{\xi} = 1$, LMF and UMF of the plant and the filter are given as follows, and the grades of membership of the plant and the filter are shown in Figure 3 and Figure 4, respectively[42]:

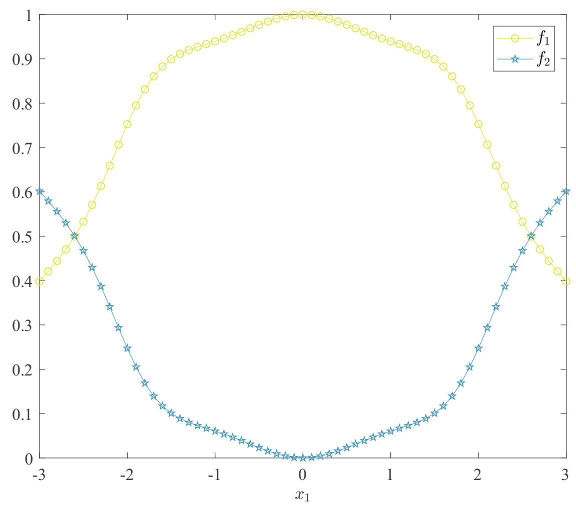

Figure 3 Membership functions of the plant

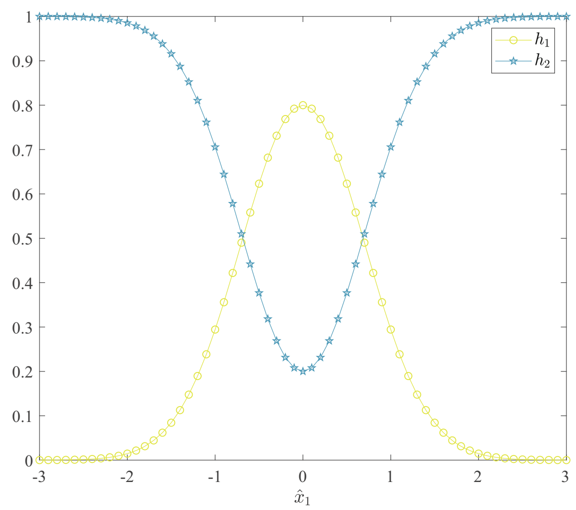

Figure 4 Membership functions of the filter

$$\underline{\mu}_{Q_{11}}(x_s(k)) = \frac{g_{1\max} - g_1(x_s(k), 0.03)}{g_{1\max} - g_{1\min}}, \quad \overline{\mu}_{Q_{11}}(x_s(k)) = \frac{g_{1\max} - g_1(x_s(k), 0.01)}{g_{1\max} - g_{1\min}},$$
$$\underline{\mu}_{Q_{21}}(x_s(k)) = \frac{g_1(x_s(k), 0.01) - g_{1\min}}{g_{1\max} - g_{1\min}}, \quad \overline{\mu}_{Q_{21}}(x_s(k)) = \frac{g_1(x_s(k), 0.03) - g_{1\min}}{g_{1\max} - g_{1\min}},$$

and

$$\underline{\mu}_{\widehat{Q}_{11}}(x_s(k)) = 0.8e^{-x_s^2(k)}, \quad \underline{\mu}_{\widehat{Q}_{21}}(x_s(k)) = 1 - \underline{\mu}_{\widehat{Q}_{11}}(x_s(k)),$$
$$\overline{\mu}_{\widehat{Q}_{11}}(x_s(k)) = \underline{\mu}_{\widehat{Q}_{11}}(x_s(k)), \quad \overline{\mu}_{\widehat{Q}_{21}}(x_s(k)) = \underline{\mu}_{\widehat{Q}_{21}}(x_s(k)).$$

LGM and UGM are independent of the state space Ω and FOU. Therefore, Ω is equally divided into 10 regions, and FOU is not divided for reducing the complexity of computing. Then the minimum value of state variate is $\underline{x}_{s\vartheta\rho} = 0.6 \times (\vartheta - 6)$ and the maximum value of the state variate is $\overline{x}_{s\vartheta\rho} = 0.6 \times (\vartheta - 5)$. Other functions relative to MFs are listed as follows:

$$\underline{\alpha}_m(x_s(k)) = \sin^2(x_s(k)), \quad \overline{\alpha}_m(x_s(k)) = 1-\underline{\alpha}_m(x_s(k)), \quad \varepsilon_{11\vartheta\rho}(x_s(k)) = 1 - \frac{x_s(k)-\underline{x}_{s\vartheta\rho}}{\overline{x}_{s\vartheta\rho}-\underline{x}_{s\vartheta\rho}},$$
$$\underline{\beta}_n(x_s(k)) = \cos^2(x_s(k)), \quad \overline{\beta}_n(x_s(k)) = 1 - \underline{\beta}_n(x_s(k)), \quad \varepsilon_{12\vartheta\rho}(x_s(k)) = 1 - \varepsilon_{11\vartheta\rho}(x_s(k)),$$
$$\underline{\omega}_{mn1\vartheta\rho} = \underline{f}_m(\underline{x}_{s\vartheta\rho})\underline{h}_n(\underline{x}_{s\vartheta\rho}), \quad \underline{\omega}_{mn2\vartheta\rho} = \underline{f}_m(\overline{x}_{s\vartheta\rho})\underline{h}_n(\overline{x}_{s\vartheta\rho}),$$
$$\overline{\omega}_{mn1\vartheta\rho} = \overline{f}_m(\underline{x}_{s\vartheta\rho})\overline{h}_n(\underline{x}_{s\vartheta\rho}), \quad \overline{\omega}_{mn2\vartheta\rho} = \overline{f}_m(\overline{x}_{s\vartheta\rho})\overline{h}_n(\overline{x}_{s\vartheta\rho}).$$

The parameters of the circuit are chosen as $C = 0.02$ F, $L = 0.0001$ H, $R_1 = 10\ \Omega$, $R_2 = 13\ \Omega$. Setting the sampling time as $T = 0.1585$ s, the discrete-time model is obtained. Other parameters used are shown below for $m \in \{1, 2\}$, $i \in \{1, 2\}$:

$$r = 1.55,\ \tau_{PDT} = 3,\ T_{PDT} = 4,\ \zeta_1 = -0.01,\ \zeta_2 = 0.01,\ \widetilde{\zeta}_1 = -0.1,\ \widetilde{\zeta}_2 = 0.1,\ \lambda = 0.95,\ \mu = 1.05,$$
$$\mathcal{G}_1 = \mathrm{diag}\{-0.01, -0.01\},\ \mathcal{G}_2 = \mathrm{diag}\{0.01, -0.01\},\ \widetilde{\mathcal{G}}_1 = \mathrm{diag}\{-0.01, -0.01\},\ D_{2mi} = -0.1,$$
$$\widetilde{\mathcal{G}}_2 = \mathrm{diag}\{-0.01, 0.01\}, \quad B_{mi} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \quad C_{mi} = \mathrm{diag}\{2, 2\}, \quad D_{3mi} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^{\mathrm{T}}.$$

And then by calculating the conditions in Theorem 3.3, the filter gains are shown as follows:

$$\widehat{A}_{11} = \begin{bmatrix} 0.0685 & 0.0055 \\ 0.0069 & 0.0005 \end{bmatrix}, \quad \widehat{A}_{12} = \begin{bmatrix} 0.1131 & 0.0087 \\ 0.0087 & 0.0007 \end{bmatrix}, \quad \widehat{A}_{21} = \begin{bmatrix} 0.0639 & 0.0051 \\ 0.0064 & 0.0005 \end{bmatrix},$$
$$\widehat{A}_{22} = \begin{bmatrix} 0.0960 & 0.0074 \\ 0.0074 & 0.0006 \end{bmatrix}, \quad \widehat{E}_{11} = \begin{bmatrix} 2.3427 & -2.4500 \\ 0.2343 & -0.2450 \end{bmatrix}, \quad \widehat{E}_{12} = \begin{bmatrix} 3.2495 & -3.3944 \\ 0.2500 & -0.2611 \end{bmatrix},$$
$$\widehat{E}_{21} = \begin{bmatrix} 1.9451 & -2.0438 \\ 0.1945 & -0.2044 \end{bmatrix}, \quad \widehat{E}_{22} = \begin{bmatrix} 2.9156 & -3.0229 \\ 0.2243 & -0.2325 \end{bmatrix}, \quad \widehat{B}_{11} = \begin{bmatrix} 0.2944 \\ 0.0236 \end{bmatrix}^{\mathrm{T}},$$
$$\widehat{B}_{12} = \begin{bmatrix} 0.5046 & 0.0391 \end{bmatrix}, \quad \widehat{B}_{21} = \begin{bmatrix} 0.1535 & 0.0123 \end{bmatrix}, \quad \widehat{B}_{22} = \begin{bmatrix} 0.2882 & 0.0223 \end{bmatrix}.$$

Set the initial states $x(0) = \begin{bmatrix} -1 & -0.5 \end{bmatrix}^{\mathrm{T}}$, $\widehat{x}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}}$, and the outside disturbance $w(k) = 5\sin(0.05k)\exp(-0.009k)$. The responses of the actual signal $z(k)$ and the estimation signal $\widehat{z}(k)$ with PDT sequence are displayed in Figure 5. From Figure 5, it can be observed that $\widehat{z}(k)$ can follow $z(k)$, which further reveals the effectiveness of the designed filter in Theorem 3.3.

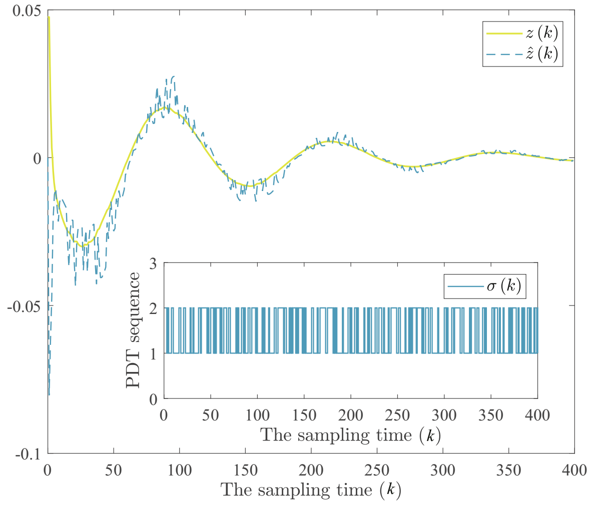

Figure 5 Responses of the actual signal $z(k)$ and the estimation signal $\widehat{z}(k)$ with PDT sequence

5 Conclusion

This study has presented a new approach to devise a filter for the nonlinear switched SPSs with parameter uncertainties by employing the IT2 T-S fuzzy theory. Through the stability theory of the switched systems, sufficient conditions have been obtained to make sure FES is GUES with a prescribed passive performance index. The expected filter gains have been achieved by solving a set of LMIs. Finally, an application example has been adopted to show the validity of the proposed method. In addition, for making this method more practical, our future work will focus on the switched IT2 T-S fuzzy SPSs under network-induced phenomena and some new matrix transformation techniques that can reduce the computational complexity.

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