J Syst Sci Complex (2021) 34: 1345–1363

Robust Control for Discrete-Time T-S Fuzzy Singular Systems*

CHEN Jian · YU Jinpeng

DOI: 10.1007/s11424-020-0059-z
Received: 25 March 2020 / Revised: 19 May 2020
©The Editorial Office of JSSC & Springer-Verlag GmbH Germany 2020

Abstract This paper deals with the robust admissibility and state feedback stabilization problems for discrete-time T-S fuzzy singular systems with norm-bounded uncertainties. By introducing a new approximation technique, the initial membership functions are conveniently expressed in piecewise-linear functions with the consideration of the approximation errors. By utilizing the piecewise-linear membership functions, the fuzzy weighting-based Lyapunov function and the use of auxiliary matrices, the admissibility of the systems is determined by examining the conditions at some sample points. The conditions can be reduced into the normal parallel distributed compensation ones by choosing special values of some slack matrices. Furthermore, the authors design the robust state feedback controller to guarantee the closed-loop system to be admissible. Two examples are provided to illustrate the advantage and effectiveness of the proposed method.

Keywords Admissible, discrete-time singular systems, piecewise-linear membership functions, state feedback, T-S fuzzy models.

1 Introduction

Most complex physical plants and industrial processes can be modeled as nonlinear systems which are difficult to analysis and synthesis[1–5]. T-S fuzzy model[1] can successfully approximate a wide class of nonlinear systems and attracts considerable attention in recent years[6–11]. The quadratic Lyapunov method is popular to obtain the stability and stabilization conditions via a set of linear matrix inequalities (LMIs)[12–17]. However, this method may lead to conservatism because a single positive definite matrix is required in the Lyaponov function

CHEN Jian
School of Information and Control Engineering, Qingdao University of Technology, Qingdao 266520*, China.* Email: janec@163.com.
YU Jinpeng (Corresponding author)
School of Automation, Qingdao University, Qingdao 266071*, China.* Email: yjp1109@hotmail.com.
*This paper was supported in part by the National Natural Science Foundation of China under Grant Nos. 61973179 and 61803220, and in part by the Taishan scholar Special Project Fund under Grant No. TSQN20161026.
◇*This paper was recommended for publication by Editor LI Hongyi.*

for all rules. Scholars have made great efforts on the aspect of reducing the conservatism of the stability criteria. Recently, many results have been reported in terms of all kinds of methods. For continuous-time systems, a fuzzy Lyapunov function-based approach[18], a fuzzy weighting-dependent approach[19], and a fuzzy Lyapunov function combined with a line-integral function[20, 21] are used for analysis and design. Other work tries to exploit the characteristics of the membership functions. A membership function dependent method is proposed in [22]. The imperfect premise matching method[23] and piecewise-linear membership function method[24] are given and extended to the tracking control, sampled-data output feedback control for continuous systems[25–27]. For discrete-time systems, a piecewise Lyapunov function method[28], a Kronecker-product approach[29] are given. In the fuzzy controller design, the controller shares the same membership functions so that the implementation cost is increased. The membership functions or the fuzzy controller rules are not necessarily the same, which can reduce the complexity of the control systems. However, it makes the controller design more complicated. In [23, 25, 26], the local or global information of membership functions was employed to relax the stability criteria.

The singular system is used to describe many types of complex systems, such as biological systems, robotic systems, or chemical systems[30–32]. A review of the latest literatures, more interests are concentrated on the admissibility and robust control of singular systems[33]. In [34, 35], the authors have studied the H_∞ control problems via dynamic feedback controller. The methods in [36] and [37] give the bounded real lemma to deal with the controller design. In [38], the dissipative control and filtering are studied. A sufficient and necessary condition of H_∞ control[29] is put forward for discrete-time singular systems. In [39, 40], fuzzy singular models are introduced, and the robust stability are studied. The H_∞ filter design for fuzzy discrete-time singular systems are studied in [41]. The output feedback control for nonlinear discrete-time systems is investigated in [42]. The sliding mode control[43] and finite-time control via output feedback[44] are studied recently. From this, we can find that many achievements in the research of singular fuzzy systems have been acquired. However, they are based on the quadratic Lyapunov function method and rather conservative. In addition, there are less results on discrete-time systems. This motivates us to do this work.

In this paper, the problems of admissibility and fuzzy control problems for uncertain discrete-time singular systems are investigated. The membership functions and fuzzy rules of the systems and the fuzzy controllers are not necessarily the same which is much helpful to practical application. New sufficient conditions for the admissibility of the considered systems is derived which can be expressed by LMIs, and the conditions are extended to design a fuzzy controller for the closed-loop systems. Comparing with the existing achievements, the obtained results have the following contributions.

- By introducing a new approximation technique, the initial membership functions are expressed in piecewise-linear functions. The approximation errors are reduced.

- By using the piecewise-linear membership functions, the fuzzy weighting-based Lyapunov function and the use of auxiliary matrices, the admissibility and stabilization conditions are less conservative.

2 Problem Formulation and Preliminaries

Consider the following T-S fuzzy discrete-time singular system:

$$Ex(k+1) = \sum_{i=1}^{s} h_i(\theta(k))(A_i + \Delta A_i(k))x(k) + B_i u(k)), \tag{1}$$

where $i \in \mathfrak{R} := \{1, 2, \cdots, s\}$, s is the number of IF-THEN rules, $\theta(k) = [\theta_1(k)\ \theta_2(k)\ \cdots\ \theta_s(k)]$ are the premise variables. $h_i(\theta(k))$ are the normalized membership functions. Hence, for all k, $h_i(\theta(k)) \geq 0$, $i = 1, 2, \cdots, s$, $\sum_{j=1}^{s} h_j(\theta(k)) = 1$. $x(k) \in \mathbb{R}^n$ is the state vector of the system, and $u(k) \in \mathbb{R}^p$ is the control input.

The matrix E is known. We assume that $\text{rank}(E) = r < n$. A_i, and B_i, $i = 1, 2, \cdots, s$, are known matrices. $\Delta A_i(k)$ is the unknown matrix representing time-varying norm-bounded parameter uncertainties, and is assumed to be

$$\Delta A_i(k) = M_i F_i(k) N_i, \quad i = 1, 2, \cdots, s, \tag{2}$$

where M_i and N_i are known real constant matrices and $F_i(\cdot) : \mathbb{N} \to \mathbb{R}^{l_1 \times l_2}$ are unknown real and possibly time-varying matrices satisfying

$$F_i(k)^{\mathrm{T}} F_i(k) \leq I, \quad \forall\ k. \tag{3}$$

The uncertainties $\Delta A_i(k)$ is admissible if Equations (2) and (3) hold.

Assumption 2.1 Suppose that the membership functions $h_i(\theta(t))$ satisfies

$$|h_i(\theta(k+1)) - h_i(\theta(k))| \leq \gamma_i, \quad i = 1, 2, \cdots, s, \tag{4}$$

where $\gamma_i \geq 0$, and the range of $\theta(t)$ is known.

Remark 2.1 For the membership functions satisfies $0 \leq h_i(\theta(t)) \leq 1$, the condition in Assumption 2.1 is not loss of generality in real-world processes.

For the singular system:

$$Ex(k+1) = Ax(k), \tag{5}$$

the following definition will be adopted.

Definition 2.2 (see [45]) The pair (E, A) is said to be regular if $\det(zE - A)$ is not identically zero. It is said to be causal if $\deg(\det(zE - A)) = \text{rank}(E)$. It is said to be stable if for any scalar $\varepsilon > 0$, there exists a scalar $\delta(\varepsilon) > 0$ such that the solution $x(k)$ to the system (5) satisfies $\| x(k) \| \leq \varepsilon$ for any $k \geq 0$, moreover $\lim_{k\to\infty} x(k) = 0$.

System (5) is said to be admissible if it is regular, causal and stable.

Moreover, for the pair $(E, \sum_{i=1}^{s} h_i(\theta(k))(A_i + \Delta A_i(k)))$, appropriate invertible matrices G and H can be chosen, and we have

$$\begin{aligned}
&\overline{E} := GEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \\
&\overline{A} = G\sum_{i=1}^{s}(A_i + \Delta A_i(k))H = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \\
&\overline{A_i} = GA_iH = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix}, \\
&\Delta\overline{A}_i(k) = G\Delta A_i(k)H, \quad \overline{M_i} = GM_i, \quad \overline{N_i} = N_iH,
\end{aligned} \tag{6}$$

where $A_{i11} \in R^{r\times r}$. It is stated in [37] that the system (1) with $u(k) = 0$ is admissible if and only if $\det(A_{22}) \neq 0$ and $A_{11} - A_{12}A_{22}^{-1}A_{21}$ is stable.

The aim of this paper is to design a c-rule state feedback controller

$$u(k) = \sum_{j=1}^{c} \eta_j(\overline{\theta}(k))K_j(\overline{\theta}(k))x(k), \tag{7}$$

where $\eta_j(\overline{\theta}(k))$ is the membership function which also satisfies Assumptions 2.1–2.2. $\overline{\theta}(k)$ is a function which can different from $\theta(k)$. $K_j(\overline{\theta}(k)) \in \mathbb{R}^{p\times n}$ are the controller gains, and $\eta_j(\overline{\theta}(k))$ satisfies $\eta_j(\overline{\theta}(k)) \geq 0$ and $\sum_{j=1}^{c} \eta_j(\overline{\theta}(k)) = 1$, such that the following closed-loop system

$$Ex(k+1) = \sum_{i=1}^{s}\sum_{j=1}^{c} h_i(\theta(k))\eta_j(\overline{\theta}(k))(A_i + \Delta A_i(k) + B_iK_j)x(k) \tag{8}$$

is admissible.

Next, we will give several useful lemmas to end this section.

Lemma 2.3 (see [46]) (*Schur's complement*) *For symmetric matrix* $S = \left[\begin{smallmatrix} A & B \\ B^{\mathrm{T}} & D \end{smallmatrix}\right]$, *the following inequalities are equivalent:*

$$\begin{aligned}
&\text{i)} \quad S > 0, \\
&\text{ii)} \quad A > 0,\ D - B^{\mathrm{T}}A^{-1}B > 0, \\
&\text{iii)} \quad D > 0,\ A - BD^{-1}B^{\mathrm{T}} > 0.
\end{aligned}$$

Lemma 2.4 *Let* M, N, *and* $F(k)$ *be real matrices of appropriate dimensions with* $F(k)^{\mathrm{T}} F(k) \leq I$. *Then, we have*

$$MF(k)N + N^{\mathrm{T}}F(k)^{\mathrm{T}}M^{\mathrm{T}} \leq MUM^{\mathrm{T}} + N^{\mathrm{T}}U^{-1}N, \tag{9}$$

where U *is an arbitrary invertible matrix.*

Lemma 2.5 (see [45]) *Let* $\Phi = \left[\begin{smallmatrix}\Phi_{11} & \Phi_{12}\\ \Phi_{21} & \Phi_{22}\end{smallmatrix}\right]$*, where* Φ_{11}*,* Φ_{12}*,* Φ_{21} *and* Φ_{22} *are any real matrices with appropriate dimensions such that* Φ_{22} *is invertible and* $\Phi + \Phi^{\mathrm{T}} < 0$*, we have*

$$\Phi_{11} + \Phi_{11}^{\mathrm{T}} - \Phi_{12}\Phi_{22}^{-1}\Phi_{21} - \Phi_{21}^{\mathrm{T}}\Phi_{22}^{-T}\Phi_{12}^{\mathrm{T}} < 0. \tag{10}$$

Lemma 2.6 (see [47]) *Consider the following inequality in the variable* U*:*

$$BUC + (BUC)^{\mathrm{T}} + \Phi < 0, \tag{11}$$

which has a solution U *if and only if*

$$N_B\Phi N_B^{\mathrm{T}} < 0 \quad \text{and} \quad N_C^{\mathrm{T}}\Phi N_C < 0, \tag{12}$$

where N_B *and* N_C *are bases of the null spaces of* B *and* C*, respectively.*

3 Main Results

3.1 Piecewise-Linear Membership Functions

Similar to the method in [24], we use the PLMF to facilitate the stability analysis. Let us consider the membership functions depending on the function of $\theta(k)$, as defined in Assumption 2.1, and the range of the function $\theta(k)$ is known as $[a, b]$. By choosing suitable points in the range of the function $\theta(k)$, the membership functions $h_i(\theta(k))$, $i = 1, 2, \cdots, s$ are divided into several pieces. Then, by the following optimization, the PLMF $\overline{h}_i(\theta(k))$, $i = 1, 2, \cdots, s$, can be found.

$$\begin{aligned}
\min_{\{\alpha_i^{\{j\}}, \beta_i^{\{j\}}\}} \quad & \sup_{\theta(k)}(|\ h_i(\theta(k)) - \overline{h}_i(\theta(k))\ |)\\
\text{s.t.} \quad & \overline{h}_i(\theta(k)) = \alpha_i^{\{j\}}\theta(k) + \beta_i^{\{j\}},\ \theta(k) \in [\theta^{\{j-1\}}\ \theta^{\{j\}}], \quad j = 2, 3, \cdots, t,\\
& 0 \le \overline{h}_i(\theta(k)) \le 1, \quad \text{for all} \ \ \theta(k),
\end{aligned}$$

where $\theta^{\{j\}}$, $j = 1, 2, \cdots, t$ are the piecewise points of $h_i(\theta(k))$, and $\alpha_i^{\{j\}}\theta(k) + \beta_i^{\{j\}}$ is the PLMF.

Nevertheless, the optimization is nonlinear and can not be solved directly. Instead, we use a practical optimal approach to obtain a solution. It contains four steps (see Figure 1 for example):

- Choosing the intersection abscissas of the endpoints of PLFM $\theta^{\{1\}}$, $\theta^{\{2\}}$, $\cdots$, $\theta^{\{t\}}$. The grades of membership functions are denoted as $h_i^{\{1\}}$, $h_i^{\{2\}}$, $\cdots$, $h_i^{\{t\}}$.
- Choosing the points on the membership functions. The inflection points, extremal points, and boundary points are normally required. Drawing the tangent lines of membership curves through the chosen points.
- Finding the endpoints of the piecewise liner membership functions in each region.
  - Calculating the intersection ordinates of tangent lines $h_i^{\{j\}'}$, $h_i^{\{j\}''}$ associated with each intersection abscissas of the endpoints.

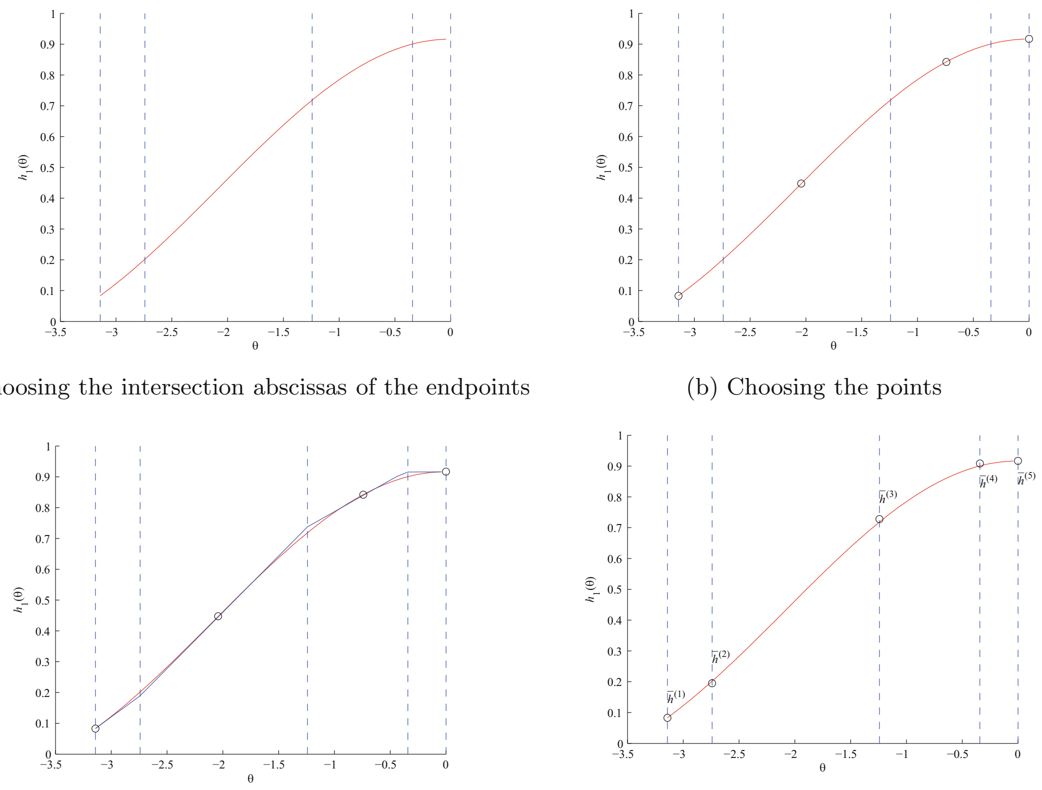
– By $\overline{h}_i^{\{j\}'} = (h_i^{\{j\}} + (h_i^{\{j\}'} + h_i^{\{j\}''})/2)/2$, and $\overline{h}_i^{\{j\}} = \frac{\overline{h}_i^{\{j\}'}}{\sum_{i=1}^{s} \overline{h}_i^{\{j\}'}}$, the endpoints are obtained as $(\theta^{\{j\}}, \overline{h}_i^{\{j\}})$, $j = 1, 2, \cdots, t$.

- Connecting the endpoints in a piecewise line.

The PLMFs can be extended to approximate higher dimensional membership functions $h\eta_{ij}(\theta(k), \overline{\theta}(k)) = h_i(\theta(k))\eta_j(\overline{\theta}(k))$. The PLFM is defined as

$$\overline{h\eta}_{ij}(\theta(k), \overline{\theta}(k)) = \sum_{l=1}^{2}\sum_{\iota=1}^{2} \nu_l(\theta(k))\upsilon_\iota(\overline{\theta}(k))\overline{h}_i^{\{l\}}\overline{\eta}_j^{\{\iota\}}, \tag{13}$$

where $\sum_{\iota=1}^{2} \upsilon_\iota(\overline{\theta}(k)) = 1$, $\upsilon_1(\overline{\theta}(k)) = \frac{\overline{\theta}_{\max} - \overline{\theta}(k)}{\overline{\theta}_{\max} - \overline{\theta}_{\min}}$, and $\upsilon_2(\overline{\theta}(k)) = 1 - \upsilon_1(\overline{\theta}(k))$. Outside this range, both υ_1 and υ_2 have to be set to 0. $\overline{\theta}_{\max}$ and $\overline{\theta}_{\min}$ denote the minimum and maximum values of $\overline{\theta}(k)$ in the corresponding region, respectively, and $\overline{\eta}_j^{\{1\}}$ and $\overline{\eta}_j^{\{2\}}$ are the corresponding endpoints of PLMFs of $\eta_j(\overline{\theta}(k))$.



(a) Choosing the intersection abscissas of the endpoints (b) Choosing the points

(c) Drawing the tangent lines of membership curve (d) Finding the endpoints of PLFM

Figure 1 The steps to find a PLFM $\overline{h}(f)$

Remark 3.1 The PLMF method is effective to deal with the membership functions which are deterministic, and satisfy Assumption 2.1. Otherwise, the above method is inapplicable.

Remark 3.2 The PLMF obtained by the the practical optimal approach is more close to the real membership function than the method in [24]. Take the case of the function in Figures 1–2, the maximum value of $|h(\theta(k)) - \overline{h}(\theta(k))|$ is 0.0132 by using the practical optimal approach. If we adopt the method in [24], the maximum value is 0.0227.

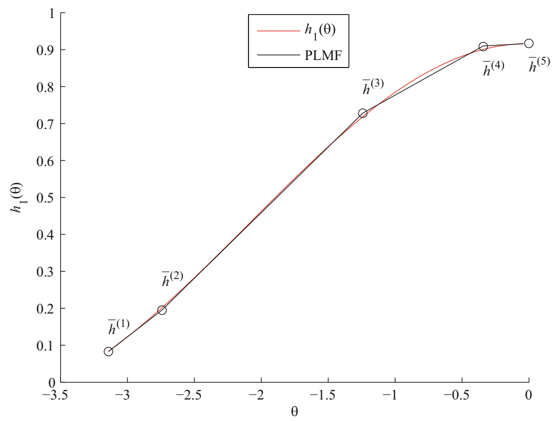


Figure 2 Membership function of $h(f)$ and PLFM $\overline{h}(f)$

By the PLMF in (13), and noted that

$$\sum_{i=1}^{s}\sum_{j=1}^{c}(h_i(\theta(k))\eta_j(\overline{\theta}(k)) - \overline{h\eta}_{ij}(\theta(k),\overline{\theta}(k)))T = 0 \tag{14}$$

holds for any symmetric matrix T. The condition

$$x^{\mathrm{T}}\varOmega x = x^{\mathrm{T}}\sum_{i=1}^{s}\sum_{j=1}^{c} h_i(\theta(k))\eta_j(\overline{\theta}(k))\varOmega_{ij}x < 0 \tag{15}$$

can be transformed as follows.

$$\begin{aligned}
& x^{\mathrm{T}}\sum_{i=1}^{s}\sum_{j=1}^{c} h_i(\theta(k))\eta_j(\overline{\theta}(k))\varOmega_{ij}x \\
={}& x^{\mathrm{T}}\sum_{i=1}^{s}\sum_{j=1}^{c}\big(\overline{h\eta}_{ij}(\theta(k),\overline{\theta}(k))\varOmega_{ij} + (h_i(\theta(k))\eta_j(\overline{\theta}(k)) - \overline{h\eta}_{ij}(\theta(k),\overline{\theta}(k)))(\varOmega_{ij}+T) \\
& + (\delta_{ij}-\delta_{ij})(\varOmega_{ij}+T)\big)\,x \\
={}& x^{\mathrm{T}}\varTheta x + x^{\mathrm{T}}\sum_{i=1}^{s}\sum_{j=1}^{c}(h_i(\theta(k))\eta_j(\overline{\theta}(k)) - \overline{h\eta}_{ij}(\theta(k),\overline{\theta}(k)) - \delta_{ij})(\varOmega_{ij}+T)x,
\end{aligned}$$

where $\varTheta = \sum_{i=1}^{s}\sum_{j=1}^{c}(\overline{h\eta}_{ij}(\theta(k),\overline{\theta}(k)) + \delta_{ij})\varOmega_{ij} + \delta_{ij}T = \sum_{l=1}^{2}\sum_{\iota=1}^{2}\nu_l(\theta(k))\upsilon_\iota(\overline{\theta}(k))\varPhi_{l\iota}$, and $\varPhi_{l\iota} = \sum_{i=1}^{s}\sum_{j=1}^{c}(\overline{h}_i^{\{l\}}\overline{\eta}_j^{\{\iota\}} + \delta_{ij})\varOmega_{ij} + \delta_{ij}T$. Thus, we have the following lemma.

Lemma 3.3 *The inequality* (15) *holds, if there exists symmetric matrix* T, *such that*

$$\begin{aligned}&\Omega_{ij}+T<0,\\&\sum_{i=1}^{s}\sum_{j=1}^{c}(\overline{h}_i^{\{l\}}\overline{\eta}_j^{\{\iota\}}+\delta_{ij})\Omega_{ij}+\delta_{ij}T<0,\\&h_i(\theta(k))\eta_j(\overline{\theta}(k))-\overline{h\eta}_{i,j}(\theta(k),\overline{\theta}(k))-\delta_{ij}>0,\ \textit{for all } l,\iota.\end{aligned}\tag{16}$$

3.2 Admissibility and Stabilization

In this subsection, a new admissibility condition for the system (1) with $u(k)=0$ is derived and the controller synthesis is addressed. For simplicity, in the rest of this section, we will consider the equivalent transformation of the system (1) and the closed-loop system (8) as

$$\overline{E}\overline{x}(k+1)=\sum_{i=1}^{s}h_i(\theta(k))((\overline{A}_i+\Delta\overline{A}_i(k))\overline{x}(k)+\overline{B}_iu(k))\tag{17}$$

$$=\sum_{i=1}^{s}\sum_{j=1}^{c}h_i(\theta(k))\eta_j(\overline{\theta}(k))(\overline{A}_i+\Delta\overline{A}_i(k)+\overline{B}_i\overline{K}_j)\overline{x}(k),\tag{18}$$

respectively, where $\overline{x}(k)=H^{-1}x(k)$, $\overline{B}_i=GB_i$, and $\overline{K}_j=K_jH$. For the system (17) with $u(k)=0$, the following admissible condition is proposed.

Theorem 3.4 *System* (17) *with* $u(k)=0$ *under Assumption* 2.1 *is admissible, if there exist symmetric matrices* $P_i>0$, $U_i>0$, $i=1,2,\cdots,s$, T, X, *matrices* Q, R, S *such that*

$$\begin{aligned}&P_i-X\geq 0,\\&\begin{bmatrix}\Omega_i+T & *\\ \Pi_{1i} & -U_i\end{bmatrix}<0,\quad i=1,2,\cdots,s,\\&\begin{bmatrix}\sum_{i=1}^{s}(\overline{h}_i^{\{l\}}+\delta_i)\Omega_i+\delta_iT & *\\ \Pi_2 & \Pi_3\end{bmatrix}<0,\quad \textit{for all } l\end{aligned}\tag{19}$$

hold, where $h_i(\theta(k))-\overline{h}_i(\theta(k))-\delta_i\geq 0$, *and*

$$\Omega_i=\begin{bmatrix}-\frac{1}{2}Q-\frac{1}{2}Q^{\mathrm{T}} & * & *\\ A_i^{\mathrm{T}}[Q\ R]^{\mathrm{T}} & \Pi_{22i} & *\\ P_i-\frac{1}{2}Q-Q^{\mathrm{T}}+\sum_{j=1}^{s}\gamma_j(P_j-X) & [Q\ R]A_i & -Q-Q^{\mathrm{T}}\end{bmatrix},$$

$$\Pi_{22i}=A_i^{\mathrm{T}}\begin{bmatrix}0&0\\0&S\end{bmatrix}^{\mathrm{T}}+\begin{bmatrix}0&0\\0&S\end{bmatrix}A_i-\begin{bmatrix}P_i&0\\0&0\end{bmatrix}+N_i^{\mathrm{T}}U_iN_i,$$

$$\Pi_{1i}=\begin{bmatrix}([Q\ R]M_i)^{\mathrm{T}} & \left(\begin{bmatrix}0&0\\0&S\end{bmatrix}M_i\right)^{\mathrm{T}} & ([Q\ R]M_i)^{\mathrm{T}}\end{bmatrix},$$

$$\Pi_2=[(\overline{h}_1^{\{l\}}+\delta_1)\Pi_{11}^{\mathrm{T}},(\overline{h}_2^{\{l\}}+\delta_2)\Pi_{12}^{\mathrm{T}},\cdots,(\overline{h}_s^{\{l\}}+\delta_s)\Pi_{1s}^{\mathrm{T}}]^{\mathrm{T}},\quad \Pi_3=-\mathrm{diag}\{U_1,U_2,\cdots,U_s\}.$$

Proof By Lemma 2.4 and Equations (2)–(3), we obtain

$$\begin{bmatrix} 0 \\ \Delta A_i(k)^{\mathrm{T}} \\ 0 \end{bmatrix} \begin{bmatrix} [Q\ R]^{\mathrm{T}} \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix}^{\mathrm{T}} [Q\ R]^{\mathrm{T}} \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} Q\ R \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix} \\ \begin{bmatrix} Q\ R \end{bmatrix} \end{bmatrix} \begin{bmatrix} 0\ \Delta A_i(k)\ 0 \end{bmatrix}$$

$$\le \begin{bmatrix} 0 \\ N_i^{\mathrm{T}} \\ 0 \end{bmatrix} U_i[0\ N_i\ 0] + \begin{bmatrix} \begin{bmatrix} Q\ R \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix} \\ \begin{bmatrix} Q\ R \end{bmatrix} \end{bmatrix} M_i U_i^{-1} M_i^{\mathrm{T}} \begin{bmatrix} [Q\ R]^{\mathrm{T}} \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix}^{\mathrm{T}} [Q\ R]^{\mathrm{T}} \end{bmatrix}, \tag{20}$$

where $U_i > 0$. By (20) and Schur's complement, (19) gives

$$\begin{aligned} & P_i - X \ge 0, \\ & \overline{\Omega}_i + \overline{T} < 0, \quad i = 1, 2, \cdots, s, \\ & \sum_{i=1}^{s} (\overline{h}_i^{\{l\}} + \delta_i)\overline{\Omega}_i + \delta_i \overline{T} < 0, \quad \text{for all } l, \end{aligned} \tag{21}$$

where

$$\overline{\Omega}_i = \begin{bmatrix} -\frac{1}{2}Q - \frac{1}{2}Q^{\mathrm{T}} & * & * \\ (A_i + \Delta A_i(k))^{\mathrm{T}}[Q\ R]^{\mathrm{T}} & \overline{\Pi}_{22i} & * \\ P_i - \frac{1}{2}Q - Q^{\mathrm{T}} + \sum_{j=1}^{s} \gamma_j (P_j - X) & \begin{bmatrix} Q\ R \end{bmatrix} (A_i + \Delta A_i(k)) & -Q - Q^{\mathrm{T}} \end{bmatrix},$$

$$\overline{\Pi}_{22i} = (A_i + \Delta A_i(k))^{\mathrm{T}} \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix} (A_i + \Delta A_i(k)) - \begin{bmatrix} P_i & 0 \\ 0 & 0 \end{bmatrix}.$$

By Lemma 3.3 and the condition (21), we have

$$\sum_{i=1}^{s} h_i(\theta(k))\overline{\Omega}_i := \overline{\Omega} < 0, \tag{22}$$

where

$$\overline{\Omega} = \begin{bmatrix} -\frac{1}{2}Q - \frac{1}{2}Q^{\mathrm{T}} & * & * \\ \overline{A}^{\mathrm{T}}[Q\ R]^{\mathrm{T}} & \overline{\Pi}_{22} & * \\ \overline{\boldsymbol{P}}_k - \frac{1}{2}Q - Q^{\mathrm{T}} & [Q\ R]\overline{A} & -Q - Q^{\mathrm{T}} \end{bmatrix},$$

$$\overline{\boldsymbol{P}}_k = \boldsymbol{P}_k + \sum_{j=1}^{s} \gamma_j (P_j - X), \quad \overline{\Pi}_{22} = \overline{A}^{\mathrm{T}} \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix} \overline{A} - \begin{bmatrix} \boldsymbol{P}_k & 0 \\ 0 & 0 \end{bmatrix},$$

$\overline{A}$ is defined in (6), and $\boldsymbol{P}_k = \sum_{i=1}^{s} h_i(\theta(k))P_i$.

Next, we will decompose (22) by (6) and obtain

$$\widetilde{\Omega} < 0, \tag{23}$$

where

$$\widetilde{\Omega} = \begin{bmatrix} -\frac{1}{2}Q - \frac{1}{2}Q^{\mathrm{T}} & * & * & * \\ A_{11}^{\mathrm{T}}Q^{\mathrm{T}} + A_{21}^{\mathrm{T}}R^{\mathrm{T}} & -\boldsymbol{P}_k & A_{21}^{\mathrm{T}}S^{\mathrm{T}} & * \\ A_{12}^{\mathrm{T}}Q^{\mathrm{T}} + A_{22}^{\mathrm{T}}R^{\mathrm{T}} & SA_{21} & A_{22}^{\mathrm{T}}S^{\mathrm{T}} + SA_{22} & * \\ \overline{\boldsymbol{P}}_k - \frac{1}{2}Q - Q^{\mathrm{T}} & QA_{11} + RA_{21} & QA_{12} + RA_{22} & -Q - Q^{\mathrm{T}} \end{bmatrix}.$$

Left and right-multiplying (23) by $\begin{bmatrix} I_r & & & \\ & I_r & & \\ & & & I_r \\ & & I_{n-r} & \end{bmatrix}$ and its transpose, we obtain that

$$W + W^{\mathrm{T}} < 0, \tag{24}$$

where

$$W = \begin{bmatrix} -\frac{1}{2}Q & 0 & 0 & 0 \\ A_{11}^{\mathrm{T}}Q^{\mathrm{T}} + A_{21}^{\mathrm{T}}R^{\mathrm{T}} & -\frac{1}{2}\boldsymbol{P}_k & A_{11}^{\mathrm{T}}Q^{\mathrm{T}} + A_{21}^{\mathrm{T}}R^{\mathrm{T}} & A_{21}^{\mathrm{T}}S^{\mathrm{T}} \\ \overline{\boldsymbol{P}}_k - \frac{1}{2}Q - Q^{\mathrm{T}} & 0 & -Q & 0 \\ A_{12}^{\mathrm{T}}Q^{\mathrm{T}} + A_{22}^{\mathrm{T}}R^{\mathrm{T}} & 0 & A_{12}^{\mathrm{T}}Q^{\mathrm{T}} + A_{22}^{\mathrm{T}}R^{\mathrm{T}} & A_{22}^{\mathrm{T}}S^{\mathrm{T}} \end{bmatrix},$$

which implies that $A_{22}^{\mathrm{T}}S^{\mathrm{T}} + SA_{22} < 0$. Therefore, the system (17) with $u(k) = 0$ is regular, and causal.

Applying Lemma 2.5 to (24), we obtain that

$$\begin{bmatrix} -\frac{1}{2}Q - \frac{1}{2}Q^{\mathrm{T}} & * & * \\ \widehat{A}^{\mathrm{T}}Q^{\mathrm{T}} & -\boldsymbol{P}_k & * \\ \overline{\boldsymbol{P}}_k - \frac{1}{2}Q - Q^{\mathrm{T}} & Q\widehat{A} & -Q - Q^{\mathrm{T}} \end{bmatrix} < 0, \tag{25}$$

where $\widehat{A} = A_{11} - A_{12}A_{22}^{-1}A_{21}$. Noted that Equation (25) can be rewritten as

$$\Xi + \xi^{\mathrm{T}}Q^{\mathrm{T}}\psi + \psi^{\mathrm{T}}Q\xi < 0, \tag{26}$$

where $\Xi = \begin{bmatrix} & & \overline{\boldsymbol{P}}_k \\ & -\boldsymbol{P}_k & \\ \overline{\boldsymbol{P}}_k & & \end{bmatrix}$, $\psi = [I_r\ 0\ I_r]^{\mathrm{T}}$, and $\xi = [-\frac{1}{2}I_r\ \widehat{A}\ -I_r]$. By Lemma 2.6, (26) is equivalent to

$$\Upsilon\psi^{\mathrm{T}}\Xi\Upsilon\psi \quad \text{and} \quad \Upsilon_{\xi}^{\mathrm{T}}\Xi\Upsilon_{\xi} < 0, \tag{27}$$

where

$$\Upsilon\psi = \begin{bmatrix} 0 & -I_r \\ I_r & 0 \\ 0 & I_r \end{bmatrix}, \quad \Upsilon_{\xi} = \begin{bmatrix} I_r & 0 \\ 0 & I_r \\ -\frac{1}{2}I_r & \widehat{A} \end{bmatrix},$$

which can be rewritten as

$$\begin{bmatrix} -\boldsymbol{P}_k & \\ & -2\overline{\boldsymbol{P}}_k \end{bmatrix} < 0, \quad \begin{bmatrix} -\overline{\boldsymbol{P}}_k & \overline{\boldsymbol{P}}_k \widehat{A}^{\mathrm{T}} \\ \widehat{A}\overline{\boldsymbol{P}}_k & -\boldsymbol{P}_k \end{bmatrix} < 0. \tag{28}$$

By Schur's complement, (28) is equivalent to

$$\widehat{A}^{\mathrm{T}} \left(\boldsymbol{P}_k + \sum_{j=1}^{s} \gamma_j (P_j - X) \right) \widehat{A} - \boldsymbol{P}_k < 0, \tag{29}$$

where $\boldsymbol{P}_k > 0$, and $\boldsymbol{P}_k + \sum_{j=1}^{s} \gamma_j (P_j - X) > 0$. Now by Assumption 2.1, and noted that $\sum_{j=1}^{s} (h_j(\theta(k+1)) - h_j(\theta(k)))X = 0$, we obtain that

$$\begin{aligned} \boldsymbol{P}_{k+1} &:= \sum_{i=1}^{s} h_i(\theta(k+1)) P_i \\ &= \sum_{i=1}^{s} h_i(\theta(k)) P_i + \sum_{j=1}^{s} (h_j(\theta(k+1) - h_j(\theta(k))) P_j \\ &\leq \boldsymbol{P}_k + \sum_{j=1}^{s} \gamma_j (P_j - X) \\ &= \overline{\boldsymbol{P}}_k, \end{aligned} \tag{30}$$

where $P_j - X > 0$, for $j = 1, 2, \cdots, s$. Thus, (29) implies that

$$\widehat{A}^{\mathrm{T}} \boldsymbol{P}_{k+1} \widehat{A} - \boldsymbol{P}_k < 0, \tag{31}$$

which ensures the stability of the system. Taken together, the conditions in Theorem 3.4 can guarantee the admissibility of the system (17) with $u(k) = 0$. ▮

Remark 3.5 The PFMB system, with the membership functions satisfying $h_i(\theta(k)) - \overline{h}_i(\theta(k)) - \delta_i \geq 0$ for all i, is guaranteed to be admissible. It is easy to see that the value of δ_i is greatly related to the value of $\max_{\theta(k)} |h_i(\theta(k)) - \overline{h}_i(\theta(k))|$. By the practical optimal approach, the value of $\max_{\theta(k)} |h_i(\theta(k)) - \overline{h}_i(\theta(k))|$ can be effectively reduced. Thus, larger values of δ_i can be obtained by the practical optimal approach which can reduce the conservatism of the conditions.

Remark 3.6 For the conditions (19) in Theorem 3.4, the admissible condition $\sum_{i=1}^{s} h_i(\theta(k)) \overline{\Omega}_i < 0$ can be achieved if it satisfied for all values of PLMFs in sample points. Therefore, an infinite number of LMIs can be solved by the finite ones with the PLMFs. Furthermore, If $T = 0$ in Theorem 3.4, the conditions reduce to the common PDC cases as in the following corollarys.

Corollary 3.7 *The discrete-time T-S fuzzy singular system* (17) *with* $u(k) = 0$ *under Assumption* 2.1 *is admissible, if there exist symmetric matrices* $P_i > 0$, $U_i > 0$, $i = 1, 2, \cdots, s$, *X, matrices Q, R, S, such that*

$$P_i - X \geq 0,$$
$$\begin{bmatrix} \Omega_i & * \\ \Pi_{1i} & -U_i \end{bmatrix} < 0, \quad i = 1, 2, \cdots, s \tag{32}$$

hold, where Ω_i, and Π_{1i} are defined in Theorem 3.4.

Corollary 3.8 *System* (17) *with $u(k) = 0$ under Assumption* 2.1 *is admissible, if there exist symmetric matrices $P > 0$, $U_i > 0$, $i = 1, 2, \cdots, s$, matrices Q, X, matrices R, S, such that*

$$\begin{bmatrix} \widetilde{\Omega}_i & * \\ \Pi_{1i} & -U_i \end{bmatrix} < 0, \quad i = 1, 2, \cdots, s \tag{33}$$

hold, where

$$\widetilde{\Omega}_i = \begin{bmatrix} -\frac{1}{2}Q - \frac{1}{2}Q^{\mathrm{T}} & * & * \\ A_i^{\mathrm{T}}[Q\ R]^{\mathrm{T}} & \widetilde{\Pi}_{22i} & * \\ P - \frac{1}{2}Q - Q^{\mathrm{T}} & [Q\ R]A_i & -Q - Q^{\mathrm{T}} \end{bmatrix},$$
$$\widetilde{\Pi}_{22i} = A_i^{\mathrm{T}} \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix} A_i - \begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix} + N_i^{\mathrm{T}} U_i N_i,$$

Π_{1i} is defined in Theorem 3.4.

Remark 3.9 If $T = 0$ in Theorem 3.4, the conditions in (19) reduce to the conditions in (32) of Corollary 3.7. Thus, Corollary 3.7 is a special case of Theorem 3.4. A fuzzy weighting-based Lyapunov function is used in Theorem 3.4 and Corollary 3.7. Its applicability relies on $|h_i(\theta(k+1)) - h_i(\theta(k))|$. Thus, the conditions in Theorem 3.4 and Corollary 3.7 are less conservative than that in Corollary 3.8. This is because the LMIs in (32) reduce to (33) by setting $P_i = P$, $i = 1, 2, \cdots, s$, and $X = P$. If the number of subsystem is set by $s = 1$, it reduces to the method in [37] for linear discrete-time singular systems.

Next, we will further deal with the state feedback stabilization problem for Systems (18) and the following theorem is obtained.

Theorem 3.10 *System* (18) *under Assumption* 2.1 *is admissible, if there exist matrices $P_i > 0$, $U_i > 0$, Y_i, Q, R, S, X, T, $i = 1, 2, \cdots, s$, such that*

$$P_i - X \geq 0,$$
$$\begin{bmatrix} \Phi_{ij} + T & * \\ \widehat{\Pi}_{1i} & -U_i \end{bmatrix} < 0, \quad i = 1, 2, \cdots, s, \quad j = 1, 2, \cdots, c,$$
$$\begin{bmatrix} \sum_{i=1}^{s} \sum_{j=1}^{c} (\overline{h}_i^{\{l\}} \overline{\eta}_j^{\{\iota\}} + \delta_{ij}) \Phi_{ij} + \delta_{ij} T & * \\ \widehat{\Pi}_2 & \widehat{\Pi}_3 \end{bmatrix} < 0,$$
$$for\ all\ l, \iota, \tag{34}$$

hold, where $h_i(\theta(k))\eta_j(\overline{\theta}(k)) - \overline{h\eta}_{ij}(\theta(k),\overline{\theta}(k)) - \delta_{ij} \geq 0$, *and*

$$\Phi_{ij} = \begin{bmatrix} -\frac{1}{2}Q - \frac{1}{2}Q^{\mathrm{T}} & * & * \\ A_i[Q\ R]^{\mathrm{T}} + B_iY_j[I_r\ 0] & \widehat{\Pi}_{22ij} & * \\ P_i - \frac{1}{2}Q - Q^{\mathrm{T}} + \sum_{m=1}^{s}\gamma_m(P_m - X) & \begin{bmatrix} Q & R\end{bmatrix} A_i^{\mathrm{T}} + [I_r\ 0]^{\mathrm{T}}Y_j^{\mathrm{T}}B_i^{\mathrm{T}} & -Q - Q^{\mathrm{T}} \end{bmatrix},$$

$$\widehat{\Pi}_{22ij} = A_i\begin{bmatrix} 0 & 0 \\ 0 & S\end{bmatrix}^{\mathrm{T}} + B_iY_j\begin{bmatrix} 0 & 0 \\ 0 & I_{n-r}\end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & S\end{bmatrix}A_i^{\mathrm{T}} + \begin{bmatrix} 0 & 0 \\ 0 & I_{n-r}\end{bmatrix}Y_j^{\mathrm{T}}B_i^{\mathrm{T}} - \begin{bmatrix} P_i & 0 \\ 0 & 0\end{bmatrix} + M_iU_iM_i^{\mathrm{T}},$$

$$\widehat{\Pi}_{1i} = \left[[Q\ R]^{\mathrm{T}}N_i, \begin{bmatrix} 0 & 0 \\ 0 & S\end{bmatrix}^{\mathrm{T}} N_i, [Q\ R]^{\mathrm{T}}N_i \right],$$

$$\widehat{\Pi}_2 = \left[\sum_{j=1}^{c}(\overline{h}_1^{\{l\}}\overline{\eta}_j^{\{\iota\}} + \delta_{1j})\widehat{\Pi}_{11}^{\mathrm{T}}, \sum_{j=1}^{c}(\overline{h}_2^{\{l\}}\overline{\eta}_j^{\{\iota\}} + \delta_{2j})\widehat{\Pi}_{12}^{\mathrm{T}}, \cdots, \sum_{j=1}^{c}(\overline{h}_s^{\{l\}}\overline{\eta}_j^{\{\iota\}} + \delta_{sj})\widehat{\Pi}_{1s}^{\mathrm{T}}\right]^{\mathrm{T}},$$

$$\widehat{\Pi}_3 = -\mathrm{diag}\{U_1, U_2, \cdots, U_s\}.$$

If it is the case, the controller gain can be solved by $K_i = Y_i \begin{bmatrix} Q^{-1} & -Q^{-1}RS^{-1} \\ 0 & S^{-1}\end{bmatrix}^{\mathrm{T}} H^{-1}$.

Proof Replacing A_i^{T} into $A_i + B_iK_j$, and letting $Y_j = K_j \begin{bmatrix} Q & R \\ 0 & S\end{bmatrix}^{\mathrm{T}}$. Using the similar procedure as the proof in Theorem 3.4, this theorem can be proved easily. ∎

4 Numerical Examples

Example 4.1 Consider the system (1) with the parameters[29]

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1\end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.3 & 0.1 \\ a & 0.9\end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.9 & 0.5 \\ a & 0.5\end{bmatrix},$$

where $a \in (-\infty, 0]$.

The system is stable in the region $a \in [\alpha, 0]$ where $\alpha(< 0)$ is the undetermined parameter. Now we will determine the minimum value $\alpha_{\min}$ of α. By Corollary 3.8, the value of $\alpha_{\min}$ is -0.3773 which is equal to the case of $g = 0$, $d = 0$ or $d = 1$, and $m = 1$ in [29]. Set $\gamma_1 = \gamma_2 = 0.2$ in Corollary 3.7, the minimum value of α is obtained as -0.7325, which is better than the case of $m = 1$ in [29]. The Lyapunov matrices are

$$P_1 = \begin{bmatrix} 30.5925 & 8.5995 \\ 8.5995 & 87.8845\end{bmatrix}, \quad P_2 = \begin{bmatrix} 43.9164 & -15.1394 \\ -15.1394 & 62.6322\end{bmatrix}.$$

If $\gamma_1 = \gamma_2 = 0.1$ in Corollary 3.7, we can obtain that $\alpha_{\min} = -0.8752$, which is better than all the cases in Table 1 of [29]. In this case, the Lyapunov matrices are

$$P_1 = \begin{bmatrix} 51.9476 & 21.3634 \\ 21.3634 & 175.1358\end{bmatrix}, \quad P_2 = \begin{bmatrix} 72.7435 & -25.4349 \\ -25.4349 & 127.6076\end{bmatrix}.$$

Example 4.2 Consider the truck-trailer model[48] in this simulation.

$$\begin{aligned}
&x_1(k+1) = x_1(k) + v \cdot t/l \cdot u(k),\\
&x_2(k) = x_1(k) - x_3(k),\\
&x_3(k+1) = x_3(k) + v \cdot t/L \cdot \sin(x_2(k)),\\
&x_4(k+1) = x_4(k) + v \cdot t \cdot \cos(x_2(k)) \sin[\{x_3(k+1) + x_3(k)\}/2],\\
&x_5(k+1) = x_5(k) + v \cdot t \cdot \cos(x_2(k)) \cos[\{x_3(k+1) + x_3(k)\}/2].
\end{aligned}$$

l is the length of truck, L is the length of trailer, t is sampling time, and v is the constant speed of backing up. In this note, $l = 2.8$ m, $L = 5.5$ m, $v = -1.0$ m/s, and $t = 2.0$ s.

Let $\theta(k) = x_3(k)+v\cdot t/2L\cdot x_2(k)$, and assume $-179.997^\circ < \theta(k) < 179.997^\circ$, the membership functions are defined as follows:

$$\begin{aligned}
&h_1(\theta(k)) = \begin{cases} \dfrac{\sin(\theta(k)) + 0.1\theta(k)}{1.2\theta(k)}, & \text{if } \theta(k) \neq 0,\\ 0.9167, & \text{if } \theta(k) = 0,\end{cases}\\
&h_2(\theta(k)) = 1 - h_1(\theta(k)).
\end{aligned}$$

The T-S fuzzy model that represents the nonlinear system is as follows:

$$\begin{aligned}
&\text{Plant Rule 1: If } \theta(k) \ is\ h_1\\
&\text{Then } Ex(k+1) = A_1x(k) + Bu(k),\\
&\text{Plant Rule 2: If } \theta(k) \ is\ h_2\\
&\text{Then } Ex(k+1) = A_2x(k) + Bu(k),
\end{aligned}$$

where

$$E = \begin{bmatrix} 1\,0\,0\,0\\ 0\,0\,0\,0\\ 0\,0\,1\,0\\ 0\,0\,0\,1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 1 & 0 & 0 & 0\\ 1 & -1 & -1 & 0\\ 0 & v\cdot t/L & 1 & 0\\ 0 & 1.1v^2\cdot t^2/(2L) & 1.1v\cdot t & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0\\ 1 & -1 & -1 & 0\\ 0 & v\cdot t/L & 1 & 0\\ 0 & -0.1\cdot v^2\cdot t^2/(2L) & -0.1\cdot v\cdot t & 1 \end{bmatrix},$$

$B = [v \cdot t/l\ 0\ 0\ 0]^{\rm T}$, $M = [0\ 0\ 0.3v \cdot t/L\ 0]^{\rm T}$, and $N = [0\ 1\ 0\ 0]$.

The methods in [29, 39, 49] is infeasible for the above fuzzy model. If the piecewise-linear membership functions are chosen as the method in [24], the controller can not be solved by Theorem 3.10. Now, we use the practical optimal approach to obtain the PLMF of the membership functions. We define the sample points as $\theta(k) = [-180^\circ, -157.08^\circ, -71.14^\circ, -19.57^\circ, 0^\circ]$. The memberships at the sample points are $h_1(\theta(k)) = [0.0833\ 0.2017\ 0.7185\ 0.9006\ 0.9164]$, and $\overline{h}_1(\theta(k)) = [0.0833\ 0.1955\ 0.7281\ 0.9160\ 0.9164]$, respectively. The PLMFs are obtained in Figure 3.

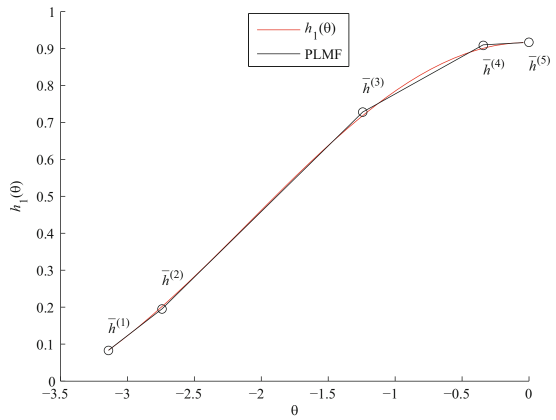


Figure 3 The PLMF of $h_1(\theta(k))$

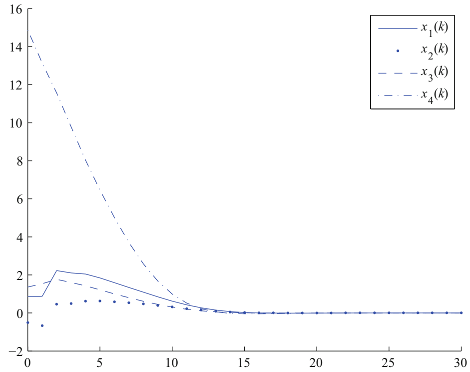
Next, we will design the robust state feedback controller of this system. The membership function of $\eta_i(\overline{\theta}(k))$ is chosen as $h_i(\theta(k))$, $i = 1, 2$. For $B_1 = B_2 = B$, the function $\sum_{i=1}^{s}\sum_{j=1}^{c}\overline{h}_i^{\{l\}}\overline{\eta}_j^{\{\iota\}}$ in Theorem 3.10 reduces to $\sum_{i=1}^{2}\overline{h}_i^{\{l\}}$, and δ_{ij} reduces to δ_i. It can be found numerically that $\delta_1 = -0.0132$, and $\delta_2 = -0.0096$ such that $h_i - \overline{h}_i - \delta_i > 0$, $i = 1, 2$, are satisfied. Let $G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, and $H = G^{\mathrm{T}}$. If we build the stabilization conditions by Corollaries 3.7–3.8, the conditions are fail to achieve the feasible solutions. Now, we solve the conditions in Theorem 3.10, let $\gamma_1 = \gamma_2 = 0.1$, and obtain the state feedback controller gains as

$$K_1 = [1.3650\ \ 1.9134\ \ -5.4150\ \ 0.5162],\quad K_2 = [1.2765\ \ 1.5975\ \ -3.9488\ \ 0.3424].$$

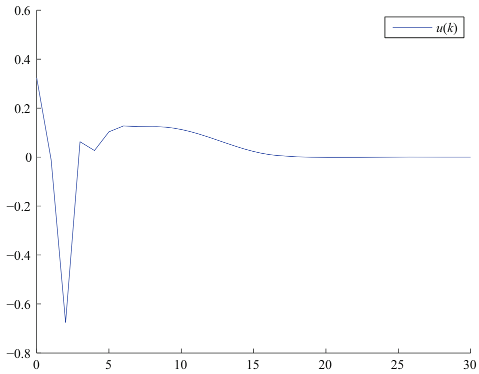
In this case, the Lyapunov matrices are

$$P_1 = \begin{bmatrix} 144.0101 & 41.1091 & 22.9274 \\ 41.1091 & 15.7136 & 23.0178 \\ 22.9274 & 23.0178 & 179.8645 \end{bmatrix},\quad P_2 = \begin{bmatrix} 124.8407 & 37.5273 & 23.0371 \\ 37.5273 & 16.1596 & 23.0315 \\ 23.0371 & 23.0315 & 179.8653 \end{bmatrix},$$

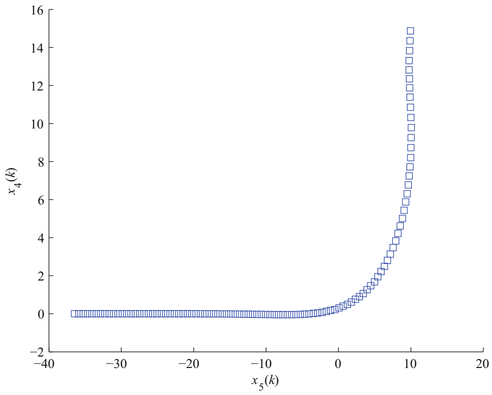
and $Y_1 = [106.5224\ 19.8468\ -0.2968\ 46.4164]$, $Y_2 = [113.2999\ 24.6251\ 0.0249\ 38.7546]$, $Q = \begin{bmatrix} 88.0224 & 22.8588 & 8.6930 \\ 25.6959 & 10.0781 & 13.9035 \\ 15.3471 & 15.3520 & 119.9053 \end{bmatrix}$, $R = [55.2243\ 16.8123\ -0.0036]^{\mathrm{T}}$, $S = 24.2589$. By the above controller gains, and let $x(0) = [0.3684\ -0.9943\ 1.3627\ 14.8691\ 9.9463]^{\mathrm{T}}$, the control responses are illustrated in Figure 4. Figure 4 (a) shows the state $x_i(k)$, $i = 1, 2, 3, 4$, of the closed-loop system, where the admissibility is achieved. Figure 4 (b) shows the input signal of $u(k)$, and the position of rear end of trailer is shown in Figure 4 (c).



(a) State response of the closed-loop system



(b) Input signal $u(k)$



(c) position of rear end of trailer

Figure 4 Control results of Example 4.2

Therefore, from Figure 4, it is obviously that the discrete T-S fuzzy singular system is admissible.

5 Conclusion

The admissibility and robust control problems of discrete-time T-S fuzzy singular systems are studied in this paper. In order to facilitate the analysis, we use a practical optimal approach to obtain a PLMF to approximate the membership function of the control systems. By applying the PLMFs, the fuzzy weighting-based Lyapunov function, and the use of auxiliary matrices, new sufficient conditions with less conservatism for the admissibility of the singular systems are derived. Then, the conditions are extended to check the admissibilization for the closed-loop systems. Numerical examples demonstrate the effectiveness and advantage of the proposed method. Possible research topics in the future study are to further reduce the conservatism produced in the admissible conditions, and to extend the results to nonlinear singular systems with time delay, or singular systems with input and state constraints.

References

[1] Takagi T and Sugeno M, Fuzzy identification of systems and its applications to modeling and control, *IEEE Transactions on Systems Man and Cybernetics*, 1985, **15**(1): 116–132.

[2] Ma H, Li H, Liang H, et al., Adaptive fuzzy event-triggered control for stochastic nonlinear systems with full state constraints and actuator faults, *IEEE Transactions on Fuzzy Systems*, 2019, **27**(11): 2242–2254.

[3] Liang H, Zhang L, Sun Y, et al., Containment control of semi-markovian multiagent systems with switching topologies, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2019, 10.1109/TSMC.2019.2946248.

[4] Zhu W, Wang D, and Zhou Q, Leader-following consensus of multi-agent systems via adaptive event-based control, *Journal of Systems Science and Complexity*, 2019, **32**(3): 846–856.

[5] Xu X, Liu L, and Feng G, Consensus of single integrator multi-agent systems with unbounded transmission delays, *Journal of Systems Science and Complexity*, 2019, **32**(3): 778–788.

[6] Zhang Z, Lin C, and Chen B, New decentralized H_∞ filter design for nonlinear interconnected systems based on Takagi-Sugeno fuzzy models, *IEEE Transactions on Cybernetics*, 2015, **45**(12): 2914–2924.

[7] Dong J, Wu Y, and Yang G H, A new sensor fault isolation method for T-S fuzzy systems, *IEEE Transactions on Cybernetics*, 2017, **47**(9): 2437–2447.

[8] Zhang L, Lam H K, Sun Y, et al., Fault detection for fuzzy semi-Markov jump systems based on interval type-2 fuzzy approach, *IEEE Transactions on Fuzzy Systems*, 2019, 10.1109/TFUZZ.2019.2936333.

[9] Yu J, Shi P, Dong W, et al., Command filtering-based fuzzy control for nonlinear systems with saturation input, *IEEE Transactions on Cybernetics*, 2017, **47**(9): 2472–2479.

[10] Vrkalovic S, Teban T A, and Borlea L D, Stable Takagi-Sugeno fuzzy control designed by optimization, *International Journal of Artificial Intelligence*, 2017, **15**(2): 17–29.

[11] Zhao T and Dian S, Fuzzy static output feedback H_∞ control for nonlinear systems subject to parameter uncertainties, *Journal of Systems Science and Complexity*, 2018, **31**(2): 343–371.

[12] Taniguchi T, Tanaka K, and Wang H, Fuzzy descriptor systems and nonlinear model following control, *IEEE Transactions on Fuzzy Systems*, 2000, **8**: 442–452.

[13] Wang H O, Tanaka K, and Griffin M F, An approach to fuzzy control of nonlinear systems: Stability and design issues, *IEEE Transactions on Fuzzy Systems*, 1996, **4**(1): 14–23.

[14] Precup R E, Doboli S, and Preitl S, Stability analysis and development of a class of fuzzy control systems, *Engineering Applications of Artificial Intelligence*, 2000, **13**(3): 237–247.

[15] Hao Y, Structure and stability analysis of general Mamdani fuzzy dynamic models, *International Journal of Intelligent Systems*, 2005, **20**(1): 103–125.

[16] Zeng H B, Teo K L, He Y, et al., Sampled-data-based dissipative control of T-S fuzzy systems, *Applied Mathematical Modelling*, 2019, **65**: 415–427.

[17] Li X, Luo X, Li S, et al., Output consensus for heterogeneous nonlinear multi-agent systems based on T-S fuzzy model, *Journal of Systems Science and Complexity*, 2017, **30**(5): 1042–1060.

[18] Feng G, Stability analysis of discrete-time fuzzy dynamic systems based on piecewise Lyapunov functions, *IEEE Transactions on Fuzzy Systems*, 2014, **12**(1): 22–28.

[19] Lin C, Wang Q G, Lee T H, et al., Fuzzy weighting-dependent approach to H_∞ filter design for time-delay fuzzy systems, *IEEE Transactions on Signal Processing*, 2007, **55**: 2746–2751.

[20] Rhee B J and Won S, A new fuzzy Lyapunov function approach for a Takagi-Sugeno fuzzy control system design, *Fuzzy Sets and Systems*, 2016, **157**(9): 1211–1228.

[21] Zhang Z, Lin C, and Chen B, New stability and stabilization conditions for T-S fuzzy systems with time delay, *Fuzzy Sets and Systems*, 2015, **263**: 82–91.

[22] Bernal M, Guerra T M, and Kruszewski A, A membership-function-dependent approach for stability analysis and controller synthesis of Takagi-Sugeno medels, *Fuzzy Sets and Systems*, 2009, **160**(19): 2776–2795.

[23] Lam H K and Narimani M, Stability analysis and performance design for fuzzy-model-based control system under imperfect premise matching, *IEEE Transactions on Fuzzy Systems*, 2009, **17**: 949–961.

[24] Lam H K, Polynomial fuzzy-model-based control systems: Stability analysis via piecewise-linear membership functions, *IEEE Transactions on Fuzzy Systems*, 2011, **19**: 588–593.

[25] Xiao B, Lam H K, and Li H, Stabilization of interval type-2 polynomial-fuzzy-model-based control systems, *IEEE Transactions on Fuzzy Systems*, 2017, **25**: 205–217.

[26] Lam H K and Tsai S H, Stability analysis of polynomial-fuzzy model-based control systems with mismatched premise membership functions, *IEEE Transactions on Fuzzy Systems*, 2014, **22**: 223–229.

[27] Lam H K and Li H, Output-feedback tracking control for polynomial fuzzy-model-based control systems, *IEEE Transactions on Industrial Electronics*, 2013, **60**(12): 5830–5840.

[28] Lee D H, Joo J H, and Tak M H, Local stability analysis of continuous-time Takagi-Sugeno fuzzy systems: A fuzzy Lyapunov function approach, *Information Sciences*, 2014, **257**: 163–175.

[29] Chen J, Xu S, Zhang B, et al., Novel stability conditions for discrete-time T-S fuzzy systems: A Kronecker-product approach, *Information Sciences*, 2016, **337–338**: 72–81.

[30] Chen J, Lin C, Chen B, et al., Regularization and stabilization for rectangular T-S fuzzy discrete-time systems with time delay, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2019, **49**(4): 833–842.

[31] Chen J, Zhang T, Zhang Z, et al., Stability and output feedback control for singular markovian jump delayed systems, *Mathematical Control & Related Fields*, 2018, **8**: 475–490.

[32] Chen J, Lin C, Chen B, et al., Mixed H_∞ and passive control for singular systems with time delay via static output feedback, *Applied Mathematics & Computation*, 2017, **293**: 244–253.

[33] Gu P, Tian S, and Liu Q, Closed-loop iterative learning control for discrete singular systems with fixed initial shift, *Journal of Systems Science and Complexity*, 2019, **32**(2): 577–587.

[34] Long S and Zhong S, H_∞ control for a class of discrete-time singular systems via dynamic feedback controller, *Applied Mathematics Letters*, 2016, **58**: 110–118.

[35] Ma Y, Jia X, and Liu D, Finite-time dissipative control for singular discrete-time Markovian jump systems with actuator saturation and partly unknown transition rates, *Applied Mathematical Modelling*, 2018, **53**: 49–70.

[36] Hsiung K L and Lee L, Lyapunov inequality and bounded real lemma for discrete-time descriptor systems, *IEE Proceedings Control Theory and Applications*, 1999, **146**: 327–331.

[37] Feng Y and Yagoubi M, On state feedback H_∞ control for discrete-time singular systems, *IEEE Transactions on Automatic Control*, 2013, **58**: 2674–2679.

[38] Feng Z and Lam J, Dissipative control and filtering of discrete-time singular systems, *International*

Journal of Systems Science, 2016, **47**: 2532–2542.

[39] Xu S, Song B, Lu J, et al., Robust stability of uncertain discrete-time singular fuzzy systems, *Fuzzy Sets and Systems*, 2007, **158**: 2306–2316.

[40] Huang C P, Stability analysis of discrete singular fuzzy systems, *Fuzzy Sets and Systems*, 2005, **151**: 155–165.

[41] Peng T, Han C, Xiong Y, et al., Filter design for discrete-time singularly perturbed T-S fuzzy systems, *Journal of the Franklin Institute*, 2013, **350**: 3011–3028.

[42] Chen J, Lin C, Chen B, et al., Fuzzy-model-based admissibility analysis and output feedback control for nonlinear discrete-time systems with time-varying delay, *Information Sciences*, 2017, **412–413**: 116–131.

[43] Zhang D, Jing Y, Zhang Q, et al., Stabilization of singular T-S fuzzy Markovian jump system with mode-dependent derivative-term coefficient via sliding mode control, *Applied Mathematics and Computation*, 2020, **364**, Article 124643.

[44] Wang J, Ma S, and Zhang C, Finite-time H_∞ control for T-S fuzzy descriptor semi-Markov jump systems via static output feedback, *Fuzzy Sets and Systems*, 2019, **365**: 60–80.

[45] Xu S and Lam J, *Robust Control and Filtering of Singular Systems*, Springer, Berlin, 2006.

[46] Lin C, Wang Q G, Lee T H, et al., *LMI Approach to Analysis and Control of Takagi-Sugeno Fuzzy Systems with Time Delay*, Springer-Verlag, Berlin Heidelberg, 2007.

[47] Boyd S, Ghaoui L El, Feron E, et al., *Linear Matrix Inequalities in System and Control Theory*, Philadelphia, PA: SIAM, 1994.

[48] Park G K and Sugeno M, Learning based on linguistic instructions using fuzzy theory, 8*th Fuzzy System Symp.*, 1992, 561–564 (in Japanese).

[49] Chadli M, Karimi H R, and Shi S, On stability and stabilization of singular uncertain Takagi-Sugeno fuzzy systems, *Journal of the Franklin Institute*, 2014, **351**: 1453–1463.