J Syst Sci Complex (2020) 33: 383–400

Exponential L_1 Filtering of Networked Linear Switched Systems: An Event-Triggered Approach*

DUAN Dandan · ZONG Guangdeng

DOI: 10.1007/s11424-019-8190-4
Received: 26 June 2018 / Revised: 10 October 2018

Abstract The issues of event-triggered exponential L_1 filtering are studied for a class of networked linear switched systems. An event-triggered mechanism is proposed to enhance resource utilization in transmission, and save the communication cost of systems as well. Then, the filtering error system is reconstructed as a switched delay system with bounded disturbance through the input delay system approach. By resorting to the Lyapunov-Krasovskii functional approach and the average dwell time (ADT) technique, some interesting results are derived to guarantee the exponential stability with a prescribed L_1 disturbance rejection level. Further, an event-triggered exponential L_1 filter is designed via solving a set of feasible linear matrix inequalities (LMIs). Finally, the efficiency of the proposed results is verified through a numerical example and a PWM-driven boost converter circuit system.

Keywords Exponential L_1 filtering, exponential stability, event-triggered mechanism, networked linear switched systems.

1 Introduction

As a special class of hybrid dynamical system, switched systems consist of certain subsystems and a rule orchestrating the switching among them. As it is pointed out that the study of switching signals is crucial in the stability analysis and stabilization of switched systems. For instance, appropriate switching signals can ensure the stability for the switched systems even though all subsystems are unstable[1, 2]. As the most widely used method, the ADT technique is first proposed in [2], which means that the switched system is stable if all subsystems are stable and each subsystem is satisfied with the ADT condition. Plenty of theoretical results in stability analysis and stabilization of switched systems have been reported under ADT switching[3–6]. However, few results on L_1 disturbance rejection performance analysis for switched systems

DUAN Dandan · ZONG Guangdeng
School of Engineering, Qufu Normal University, Rizhao 276826*, China.* Email: lovelyletian@gmail.com.
*This research was supported in part by the National Natural Science Foundation of China under Grant Nos. 61773225, 61773236, 61873331, 61803225, and in part by the Taishan Scholar Project of Shandong Province under Grant No. TSQN20161033.
◇ *This paper was recommended for publication by Editor FENG Gang.*

are reported. As pointed out in [7], when the switching signals meet the ADT conditions, the time-varying delay switched system is exponentially stable with L_1-gain performance.

Generally, the state information may not be completely available in practice due to the limitations of the devices. Therefore, filter design has received extensive attention, since filtering is considered one of the most prevailing state estimation methods for switched systems. There are quite a few types of filters being studied, such as H_∞ filter and $L_2 - L_\infty$ filter. In [8], the issues of $l_2 - l_\infty$ filtering and h_∞ filtering for the discrete-time switched systems are investigated, respectively. Based on the derived results in [8], the problems of robust $L_2 - L_\infty$ filtering for linear networked control systems are further investigated in [9]. In [10, 11], the event-triggered H_∞ filtering for continuous-time switched linear systems and discrete-time switched linear systems are considered, with an event-triggered scheme utilized to enhance the usage rate of communication resources.

As we all know, the external disturbances for many practical switched systems are sustained and amplitude bounded signals. For instance, sea wave clutter interference signals and unit step signal. In this scenario, the H_∞ filtering method and $L_2 - L_\infty$ filtering method are no longer viable. In this case, it may be meaningful to study the L_1 filtering problem; which can minimize the peak value of filtering error outputs in the worst case. In [12], a robust L_1 filter is designed to analyze the L_1 gain for a kind of nonlinear networked control system. The L_1 fixed-order filtering problem is addressed in [13] for switched linear parameter-varying systems.

Owing to the extremely rapid development of computer, microelectronics and communication networks, networked control systems receive wide attention. Many methods are introduced to study the problem of energy consumption in networked control systems, of which period sampling is the simplest one. Under discrete sampling conditions, there are three commonly used methods to analyze networked systems: Time-delay approach[14, 15], impulsive/hybrid system approach[16–18] and discrete-time approach[19, 20]. The time-delay approach is applicable to diverse types of systems, such as uncertain systems, networked control systems and sampled systems. In [21], the time-delay approach is extended to the event-triggered networked switched systems. Under an event-triggered communication scheme, the networked switched system is modeled as a kind of switched time-delay systems. By invoking a time-delay approach, the issues of non-parallel distribution compensation control for observer-based fuzzy systems are considered in [22], with an event-triggered scheme utilized to improve the use ratio of communication resources.

Whereas, the traditional time-triggered scheme may generate excessive redundant signals, and usually increases the network load and communication costs of the system[23–25]. In order to overcome the weakness of time-triggered scheme, the event-triggered mechanism is proposed, that is, only when the sampled date satisfies a prescribed event-triggered condition, they will be sent to the filter. Hence, in comparison with the time-triggered mechanism, event-triggered mechanism can significantly reduce the consumption of communication resources and improve the control performance of the system. Many reports on the event-triggered scheme are available[26–30]. By proposing a new scheme, an event-triggered control system is used for approximating the behavior of a continuous state-feedback system in [26].

For switched nonlinear multi-agent systems, the issue of leader-following consensus is studied in [27] through event-triggered protocols. Further, the problem of containment control is investigated in [29] for second-order nonlinear multi-agent systems. Under redesigned event-triggered protocols, all followers converged to the convex hull spanned asymptotically and the Zeno behavior was excluded. The event-triggered filtering problem for neural networked system is dealt with in [30], in which, some sufficient conditions of the generalized dissipativity for the filtering error system are presented. But until now, the use of event-triggered scheme in the switched systems is not common, especially the L_1 filtering problem.

In this paper, we pay close attention to the issues of event-triggered L_1 filtering for networked linear switched systems. Our target is to develop an event-triggered L_1 filter such that the filtering error systems are exponentially stable with a required L_1 disturbance rejection level. The main contributions are fourfold: (i) Different from the existing literatures[28, 29], a discrete time event-triggered detector is designed which can be used to monitor the event-triggered conditions periodically. Here, the discrete time event detector can reduce monitoring time and save communication costs. (ii) Under an event-triggered sampling scheme, a viable L_1 filter is designed which minimizes peak value of filtering error outputs in the worst case. (iii) Sufficient conditions are obtained to ensure the exponential stability and a given L_1 disturbance rejection performance. (iv) By means of the L_1 filter technique, the exponential stability problem of the PWM-driven boost converter circuit system is introduced, which verifies that the conclusions obtained in this paper are effective in practical application.

Notations Throughout this paper, $\mathbb{N}$ stands for the set of natural numbers. $\mathcal{R}^n$ represents the n-dimensional Euclidean space. For a matrix $\mathcal{P} \in R^{n\times n}$, the superscript T represents its transpose. $\mathcal{P} > 0$ means $\mathcal{P}$ is a symmetric matrix and positive definite matrix, the symmetric terms are represented by $*$, and diag$\{\cdot\}$ means a block-diagonal matrix.

2 Problem Formulation and Preliminaries

2.1 Networked Linear Switched Systems

Consider the networked linear switched system as follows

$$\begin{cases} \dot{x}(t) = \mathcal{A}_{\sigma(t)}x(t) + \mathcal{D}_{\sigma(t)}\omega(t), \\ y(t) = \mathcal{C}_{\sigma(t)}x(t), \\ z(t) = \mathcal{L}_{\sigma(t)}x(t), \end{cases} \tag{1}$$

where $x(t) \in \mathcal{R}^n$, $\omega(t) \in L_\infty[0,\infty)$, $y(t) \in \mathcal{R}^p$, $z(t) \in \mathcal{R}^p$ are the state vector, the disturbance input, the measured output vector, and the output signal to be estimated, respectively. $\mathcal{A}_{\sigma(t)}$, $\mathcal{C}_{\sigma(t)}$, $\mathcal{D}_{\sigma(t)}$, $\mathcal{L}_{\sigma(t)}$ are known real matrices. The function $\sigma(t) : [0,\infty) \to \mathcal{I} = \{1, 2, \cdots, N\}$, is a piecewise constant switching signal and continuous from the right everywhere. The switching sequence associated with switching signal $\sigma(t)$ is given below:

$$M = \{(t_0, \sigma(t_0)), (t_1, \sigma(t_1)), (t_2, \sigma(t_2)), \cdots (t_k, \sigma(t_k))|\sigma(t_k) \in \mathcal{I}, k \in \mathbb{N}\},$$

where $t_0, x(t_0)$ are the initial time and initial state, respectively. The switching points satisfy $t_0 < t_1 < \cdots < t_k < \cdots$. When $t \in [t_k, t_{k+1})$, the $\sigma(t_k)$-th subsystem will be activated.

In order to save communication costs, we will design an appropriate filter to estimate the output $z(t)$ of System (1) through an event-triggered scheme. Moreover, Figure 1 is the framework of the event-triggered filtering.

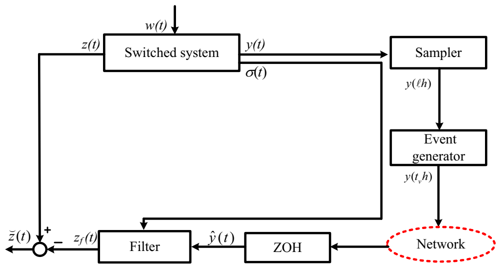

Figure 1 The framework of the event-triggered filtering

In Figure 1, the measured outputs $y(t)$ are sampled at a constant period $h > 0$, i.e., periodic sampling. And the set of sampled instants is given by $\mathcal{H} = \{\ell h | \ell \in \mathbb{N}\}$. Here, the event-triggered condition is critical in the event generator, since it can decide whether the sampled signal should be transferred to zero-order hoder (ZOH) or not. In other words, only those sampled signals that gratify the predetermined event-triggered condition are transferred through a network channel. This is vital from the perspective of improving the utilization ratio of communication resources and saving communication costs. The ZOH is event-driven, which is utilised to hold the signal generated from the event generator until a new signal generated.

Our aim is to design event-triggered L_1 filters to estimate the system output $z(t)$ based on the measured output vector $y(t)$ in order to enhance the network resource utility.

2.2 Event-Triggered Communication Mechanism

Here, the event-triggered condition is defined as follows:

$$t_{\nu+1}h = t_\nu h + \min_{\ell}\{\ell h | \mathrm{e}^{\mathrm{T}}((t_\nu+\ell)h)\varPhi_i e((t_\nu+\ell)h) \geq \delta_i y^{\mathrm{T}}((t_\nu+\ell)h)\varPhi_i y((t_\nu+\ell)h), \ell \in N\}, \quad (2)$$

where $e((t_\nu+\ell)h) = y((t_\nu+\ell)h) - y(t_\nu h)$, $\delta_i > 0$ is the coefficient of the threshold, $\varPhi_i > 0$ is a weighting matrix, $(t_\nu+\ell)h$ and t_ν denote the sampling point and triggering point of events, respectively.

Remark 2.1 In most existing literatures, event-triggered conditions are monitored continuously, or partially continuously, which may increase the computational burden. Meanwhile, the time interval between the two trigger events may tend to zero under the continuous event-trigger mechanism. It is highly undesirable since it may lead to the sensors sample infinitely fast. In order to reduce resource consumption in monitoring, and avoid the Zeno-sampling, here the event-trigger conditions are verified only periodically. The main difference compared with the conventional event monitoring mechanism, is that the event-triggered conditions are only verified at discrete sampling instants. In this scenario, the time interval between the two trigger events is greater than zero obviously, because it is equal to or greater than the verifying period. Therefore, the periodic event-triggered control used in this paper combines the advantages of periodic sampling and continuous event-triggered control. The event-trigger conditions are detected periodically, and the output signals are transmitted only when necessary.

2.3 Event-Triggered L_1 Filtering

In this subsection, for each value $\sigma(t) = i \in \mathcal{I}$, we take into account an event-triggered mode-dependent filter for System (1) described by the following form

$$\begin{cases} \dot{x}_f(t) = \mathcal{A}_{f_i} x_f(t) + \mathcal{B}_{f_i} \widehat{y}(t), \\ z_f(t) = \mathcal{L}_{f_i} x_f(t), \end{cases} \tag{3}$$

where $x_f(t) \in \mathcal{R}^n$, $z_f(t) \in \mathcal{R}^p$ are the filter state vector and filter output vector, respectively. $\mathcal{A}_{f_i}$, $\mathcal{B}_{f_i}$, $\mathcal{L}_{f_i}$ are the parameter matrices to be determined later. It is worth pointing out that $\widehat{y}(t)$ is the input vector of the filter instead of the measured output vector.

Remark 2.2 For H_∞ filters and $L_2 - L_\infty$ filters, the external disturbance must be energy-bounded. However, the disturbance of many practical control systems is not satisfied with that requirement. For instance, sea wave clutter interference signals and unit step signal. In this sense, the H_∞ filtering method and $L_2 - L_\infty$ filtering method are no longer workable. In this case, the L_1 filtering is better suited for engineering applications since the external disturbance of L_1 filtering is persistent and amplitude-bounded.

Defining $q(t) = t - t_{\nu,\ell}$, $t_{\nu,\ell} = (t_\nu + \ell)h$, we can see that $q(t)$ meet with $0 \le q(t) \le h$. Suppose delay $q(t)$ satisfies with the condition $\dot{q}(t) \le \widetilde{q} < 1$. $\phi(t)$ is an initial vector function in interval $[-h, 0]$. By utilizing the input delay system approach, for $t \in [t_\nu h, t_{\nu+\ell} h)$, we get $\widehat{y}(t) = y(t_\nu h) = y(t - q(t)) - e(t_{\nu,\ell})$. In conclusion, the filtering error system is obtained as following

$$\begin{cases} \dot{\check{x}}(t) = \check{\mathcal{A}}_{\sigma(t)} \check{x}(t) + \check{\mathcal{C}}_{\sigma(t)} \check{x}(t - q(t)) + \check{\mathcal{D}}_{\sigma(t)} w(t) + \check{\mathcal{B}}_{\sigma(t)} e(t_{\nu,\ell}), \\ \check{z}(t) = \check{\mathcal{L}}_{\sigma(t)} \check{x}(t), \\ \check{x}(t) = [\phi^{\mathrm{T}}(t) \ \ 0]^{\mathrm{T}}, \quad t \in [-h, 0], \end{cases} \tag{4}$$

where

$$\begin{aligned} &\check{\mathcal{A}}_{\sigma(t)} = \begin{bmatrix} \mathcal{A}_{\sigma(t)} & 0 \\ 0 & \mathcal{A}_{f\sigma(t)} \end{bmatrix}, \quad \check{\mathcal{C}}_{\sigma(t)} = \begin{bmatrix} 0 & 0 \\ \mathcal{B}_{f\sigma(t)} \mathcal{C}_{\sigma(t)} & 0 \end{bmatrix}, \\ &\check{\mathcal{D}}_{\sigma(t)} = \begin{bmatrix} \mathcal{D}_{\sigma(t)} \\ 0 \end{bmatrix}, \quad \check{\mathcal{B}}_{\sigma(t)} = \begin{bmatrix} 0 \\ -\mathcal{B}_{f\sigma(t)} \end{bmatrix}, \quad \check{\mathcal{L}}_{\sigma(t)} = \begin{bmatrix} \mathcal{L}_{\sigma(t)} & -\mathcal{L}_{f\sigma(t)} \end{bmatrix}, \\ &\check{x}(t) = [x^{\mathrm{T}}(t) \ x_f^{\mathrm{T}}(t)]^{\mathrm{T}}, \ \check{x}(t - q(t)) = [x^{\mathrm{T}}(t - q(t)) \ x_f^{\mathrm{T}}(t - q(t))]^{\mathrm{T}}, \ \check{z}(t) = z(t) - z_f(t). \end{aligned} \tag{5}$$

Remark 2.3 Combining (1) with (3), the filtering error system is reconstructed into a switched system with time-varying delay applying the input delay system approach. Further, some delay-dependent inequalities shall be established for the problems of exponential stability and L_1 disturbance rejection performance.

Definition 2.4 (see [31]) If the solution $x(t)$ of the filtering error system (4) satisfies

$$\|x(t)\| \le k \parallel x(t_0) \parallel_h \mathrm{e}^{-\lambda(t-t_0)}, \qquad \forall t \ge t_0, \tag{6}$$

for constants $k \ge 1$ and $\lambda > 0$, where

$$\|x(t_0)\|_h = \sup_{-h\le\theta\le 0}\{\parallel x(t+\theta) \parallel, \parallel \dot{x}(t+\theta) \parallel\}, \tag{7}$$

then the equilibrium $x^* = 0$ of the filtering error system (4) is said to be exponentially stable under switching signal $\sigma(t)$.

Definition 2.5 (see [13]) The event-triggered filtering error system is called exponentially stable with a given L_1 disturbance rejection performance if the following conditions are satisfied:

(i) The event-triggered filtering error system is exponentially stable with $\omega(t) = 0$.

(ii) Under the zero initial condition, $\forall\omega(t) \in L_\infty[0,\infty), \omega(t) \ne 0$, the following condition hold

$$\sup_{\omega(t)\in L_\infty[0,\infty)} \frac{\parallel z(t) \parallel_{L_\infty}}{\parallel \omega(t) \parallel_{L_\infty}} < \gamma.$$

Lemma 2.6 (see [28]) *For any matrix $\mathcal{P} > 0$, we have*

$$\int_{t-q}^{t}\int_{\tau}^{t} x^{\mathrm{T}}(\nu)\mathcal{P}x(\nu)d\nu d\tau \le q\int_{t-q}^{t} x^{\mathrm{T}}(\tau)\mathcal{P}x(\tau)d\tau. \tag{8}$$

3 Main Results

In this section, some sufficient conditions are obtained by Lyapunov-Krasovskii functional approach and the ADT technique, which can make the filtering error system exponentially stable with a given L_1 disturbance rejection level.

Theorem 3.1 *For given scalars $\alpha > 0$, $\beta > 0$, $\mu > 1$, $\gamma > 0$, $\delta_i > 0$, $\rho_1 > 0$, $\rho_2 > 0$, $\rho_3 > 0$, $\rho_4 > 0$, $\rho_5 > 0$, the event-triggered filtering error system* (4) *is exponentially stable with a prescribed L_1 disturbance rejection level, if there exist an event-triggered filter* (3) *and positive definite matrices $\mathcal{P}_i > 0$, $\mathcal{Q}_i > 0$, $\mathcal{Z}_i > 0$, $\mathcal{R}_i > 0$, $\forall(i,j) \in \mathcal{I}\times\mathcal{I}, i \ne j$ such that*

$$\begin{bmatrix} -\beta\mathcal{P}_i & 0 & \check{\mathcal{L}}_i^{\mathrm{T}} \\ * & -(\gamma-\beta)I & 0 \\ * & * & -\gamma I \end{bmatrix} < 0, \tag{9}$$

$$\begin{bmatrix} \Psi_{11} & \mathcal{P}_i\check{\mathcal{C}}_i & \mathcal{P}_i\check{\mathcal{D}}_i & \mathcal{P}_i\check{\mathcal{B}}_i & h\check{\mathcal{A}}_i^{\mathrm{T}} \\ * & \Psi_{22} & 0 & 0 & h\check{\mathcal{C}}_i^{\mathrm{T}} \\ * & * & -\beta I & 0 & h\check{\mathcal{D}}_i^{\mathrm{T}} \\ * & * & * & -\Phi_i & h\check{\mathcal{B}}_i^{\mathrm{T}} \\ * & * & * & * & -\mathcal{R}_i^{-1} \end{bmatrix} < 0, \tag{10}$$

$$\mathcal{P}_i \le \mu \mathcal{P}_j, \quad \mathcal{Q}_i \le \mu \mathcal{Q}_j, \quad \mathcal{Z}_i \le \mu \mathcal{Z}_j, \quad \mathcal{R}_i \le \mu \mathcal{R}_j, \tag{11}$$

$$\mathcal{P}_i \ge \rho_5 I, \quad \mathcal{P}_i \le \rho_1 I, \quad \mathcal{Q}_i \le \rho_2 I, \quad \mathcal{Z}_i \le \rho_3 I, \quad \mathcal{R}_i \le \rho_4 I, \tag{12}$$

$$\tau_a > \tau_a^* = \frac{\ln \mu}{\alpha}, \quad \beta \mathcal{Q}_i + (\beta h - 1)\mathcal{Z}_i < 0, \quad \beta \mathcal{Q}_i + (\beta h - 1)\mathcal{R}_i < 0, \tag{13}$$

where

$$\Psi_{11} = \mathcal{P}_i \check{\mathcal{A}}_i + \check{\mathcal{A}}_i^{\mathrm{T}} \mathcal{P}_i + (\alpha + \beta)\mathcal{P}_i + \mathcal{Q}_i + h\mathcal{Z}_i,$$
$$\Psi_{22} = \delta_i \widehat{\mathcal{C}}_i - (1 - \widetilde{q})\mathrm{e}^{-\alpha h}\mathcal{Q}_i.$$

Proof Choose the following mode-dependent Lyapunov-Krasovskii functional

$$\mathcal{V}_i(\check{x}(t)) = \mathcal{V}_{1,i}(\check{x}(t)) + \check{x}_{2,i}(\check{x}(t)) + \mathcal{V}_{3,i}(\check{x}(t)) + \mathcal{V}_{4,i}(\check{x}(t)), \tag{14}$$

where

$$\mathcal{V}_{1,i}(\check{x}(t)) = \check{x}^{\mathrm{T}}(t)\mathcal{P}_i \check{x}(t), \qquad \mathcal{V}_{3,i}(\check{x}(t)) = \int_{t-h}^{t}\int_{s}^{t} \mathrm{e}^{-\alpha(t-\vartheta)}\check{x}^{\mathrm{T}}(\vartheta)\mathcal{Z}_i \check{x}(\vartheta)d\vartheta ds,$$
$$\mathcal{V}_{2,i}(\check{x}(t)) = \int_{t-q(t)}^{t} \mathrm{e}^{-\alpha(t-s)}\check{x}^{\mathrm{T}}(s)\mathcal{Q}_i \check{x}(s)ds, \mathcal{V}_{4,i}(\check{x}(t)) = h\int_{t-h}^{t}\int_{s}^{t} \mathrm{e}^{-\alpha(t-\vartheta)}\dot{\check{x}}^{\mathrm{T}}(\vartheta)\mathcal{R}_i \dot{\check{x}}(\vartheta)d\vartheta ds.$$

Taking the time derivative of Lyapunov-Krasovskii functional $\mathcal{V}_i(\check{x}(t))$ along solutions of System (4), we get

$$\begin{aligned}
\dot{\mathcal{V}}_i(\check{x}(t)) \le\ & \dot{\check{x}}^{\mathrm{T}}(t)\mathcal{P}_i \check{x}(t) + \check{x}^{\mathrm{T}}(t)\mathcal{P}_i \dot{\check{x}}(t) - \alpha \int_{t-q(t)}^{t} \mathrm{e}^{-\alpha(t-s)}\check{x}^{\mathrm{T}}(s)\mathcal{Q}_i \check{x}(s)ds + \check{x}^{\mathrm{T}}(t)\mathcal{Q}_i \check{x}(t) \\
& -\mathrm{e}^{-\alpha q(t)}\check{x}^{\mathrm{T}}(t-q(t))\mathcal{Q}_i \check{x}(t-q(t))(1-\widetilde{q}) - \alpha \int_{t-h}^{t}\int_{s}^{t} \mathrm{e}^{-\alpha(t-\vartheta)}\check{x}^{\mathrm{T}}(\vartheta)\mathcal{Z}_i \check{x}(\vartheta)d\vartheta ds \\
& + \int_{t-h}^{t} \check{x}^{\mathrm{T}}(t)\mathcal{Z}_i \check{x}(t)ds - \int_{t-h}^{t} \mathrm{e}^{-\alpha(t-\vartheta)}\check{x}^{\mathrm{T}}(\vartheta)\mathcal{Z}_i \check{x}(\vartheta)d\vartheta + h\int_{t-h}^{t} \dot{\check{x}}^{\mathrm{T}}(t)\mathcal{R}_i \dot{\check{x}}(t)ds \\
& -\alpha h \int_{t-h}^{t}\int_{s}^{t} \mathrm{e}^{-\alpha(t-\vartheta)}\dot{\check{x}}^{\mathrm{T}}(\vartheta)\mathcal{R}_i \dot{\check{x}}(\vartheta)d\vartheta ds - h\int_{t-h}^{t} \mathrm{e}^{-\alpha(t-\vartheta)}\dot{\check{x}}^{\mathrm{T}}(\vartheta)\mathcal{R}_i \dot{\check{x}}(\vartheta)d\vartheta.
\end{aligned} \tag{15}$$

Considering Lemma 2.6 and $\beta \mathcal{Q}_i + (\beta h - 1)\mathcal{Z}_i < 0$, $\beta \mathcal{Q}_i + (\beta h - 1)\mathcal{R}_i < 0$, we have

$$\begin{aligned}
& -\int_{t-h}^{t} \mathrm{e}^{-\alpha(t-\vartheta)}\check{x}^{\mathrm{T}}(\vartheta)\mathcal{Z}_i \check{x}(\vartheta)d\vartheta \\
\le\ & -\beta \int_{t-h}^{t} \mathrm{e}^{-\alpha(t-\vartheta)}\check{x}^{\mathrm{T}}(\vartheta)\mathcal{Q}_i \check{x}(\vartheta)d\vartheta - \beta h \int_{t-h}^{t} \mathrm{e}^{-\alpha(t-\vartheta)}\check{x}^{\mathrm{T}}(\vartheta)\mathcal{Z}_i \check{x}(\vartheta)d\vartheta \\
\le\ & -\beta \int_{t-q(t)}^{t} \mathrm{e}^{-\alpha(t-\vartheta)}\check{x}^{\mathrm{T}}(\vartheta)\mathcal{Q}_i \check{x}(\vartheta)d\vartheta - \beta \int_{t-h}^{t}\int_{\vartheta}^{t} \mathrm{e}^{-\alpha(t-\vartheta)}\check{x}^{\mathrm{T}}(\vartheta)\mathcal{Z}_i \check{x}(\vartheta)d\vartheta ds \\
=\ & -\beta \mathcal{V}_{2i}(\check{x}(t)) - \beta \mathcal{V}_{3i}(\check{x}(t)),
\end{aligned} \tag{16}$$

$$
\begin{aligned}
&-h\int_{t-h}^{t}\mathrm{e}^{-\alpha(t-\vartheta)}\dot{\check{x}}^{\mathrm{T}}(\vartheta)R_i\dot{\check{x}}(\vartheta)d\vartheta\\
\le& -\beta h\int_{t-h}^{t}\mathrm{e}^{-\alpha(t-\vartheta)}\dot{\check{x}}^{\mathrm{T}}(\vartheta)\mathcal{Q}_i\dot{\check{x}}(\vartheta)d\vartheta-\beta h^2\int_{t-h}^{t}\mathrm{e}^{-\alpha(t-\vartheta)}\dot{\check{x}}^{\mathrm{T}}(\vartheta)\mathcal{R}_i\dot{\check{x}}(\vartheta)d\vartheta\\
\le& -\beta h\int_{t-h}^{t}\int_{s}^{t}\mathrm{e}^{-\alpha(t-\vartheta)}\dot{\check{x}}^{\mathrm{T}}(\vartheta)\mathcal{R}_i\dot{\check{x}}(\vartheta)d\vartheta ds\\
=& -\beta\mathcal{V}_{4i}(\check{x}(t)).
\end{aligned}
\tag{17}
$$

From the event-triggered condition (2), it is clear that

$$
\delta_i\check{x}^{\mathrm{T}}(t-q(t))\widehat{\mathcal{C}}\check{x}(t-q(t))-\mathrm{e}^{\mathrm{T}}(t_{\nu,\ell})\Phi_i e(t_{\nu,\ell})>0,\quad \forall t\in[t_\nu h,t_{\nu+1}h),
\tag{18}
$$

where $\widehat{\mathcal{C}}=\left[\begin{smallmatrix}\mathcal{C}_i^{\mathrm{T}}\Phi_i\mathcal{C}_i & 0\\ 0 & 0\end{smallmatrix}\right]$.

Substituting (16) and (17) into (15), and combining (18) implies

$$
\begin{aligned}
&\dot{\mathcal{V}}_i(\check{x}(t))\\
\le&\check{x}^{\mathrm{T}}(t)[\mathcal{P}_i\check{\mathcal{A}}_i+\check{\mathcal{A}}_i^{\mathrm{T}}\mathcal{P}_i+(\alpha+\beta)\mathcal{P}_i+\mathcal{Q}_i+h\mathcal{Z}_i]\check{x}(t)+h^2\dot{\check{x}}^{\mathrm{T}}(t)\mathcal{R}_i\dot{\check{x}}(t)-\beta\check{x}^{\mathrm{T}}(t)\mathcal{P}_i\check{x}(t)\\
&-h\int_{t-h}^{t}\mathrm{e}^{-\alpha(t-\vartheta)}\dot{\check{x}}^{\mathrm{T}}(\vartheta)\mathcal{R}_i\dot{\check{x}}(\vartheta)d\vartheta+\delta_i\check{x}^{\mathrm{T}}(t-q(t))\widehat{\mathcal{C}}_i\check{x}(t-q(t))+2\check{x}^{\mathrm{T}}(t)\mathcal{P}_i\check{\mathcal{B}}_i e(t_{\nu,\ell})\\
&+2\check{x}^{\mathrm{T}}(t)\mathcal{P}_i\check{\mathcal{D}}_i\omega(t)-\mathrm{e}^{-\alpha h}\check{x}^{\mathrm{T}}(t-q(t))\mathcal{Q}_i\check{x}(t-q(t))(1-\widetilde{q})+2\check{x}^{\mathrm{T}}(t)\mathcal{P}_i\check{\mathcal{C}}_i\check{x}(t-q(t))\\
&+\beta\omega^{\mathrm{T}}(t)\omega(t)-\beta\omega^{\mathrm{T}}(t)\omega(t)-\int_{t-h}^{t}\mathrm{e}^{-\alpha(t-\vartheta)}\check{x}^{\mathrm{T}}(\vartheta)\mathcal{Z}_i\check{x}(\vartheta)d\vartheta-\mathrm{e}^{\mathrm{T}}(t_{\nu,\ell})\Phi_i e(t_{\nu,\ell})-\alpha\mathcal{V}_i(\check{x}(t)).
\end{aligned}
$$

Defining $\xi^{\mathrm{T}}(t)=[\check{x}^{\mathrm{T}}(t)\ \check{x}^{\mathrm{T}}(t-q(t))\ \omega^{\mathrm{T}}(t)\ \mathrm{e}^{\mathrm{T}}(t_{\nu,\ell})]$, we get

$$
\begin{aligned}
\dot{\mathcal{V}}_i(\check{x}(t))\le&\ \check{x}^{\mathrm{T}}(t)[\mathcal{P}_i\check{\mathcal{A}}_i+\check{\mathcal{A}}_i^{\mathrm{T}}\mathcal{P}_i+(\alpha+\beta)\mathcal{P}_i+\mathcal{Q}_i+h\mathcal{Z}_i]\check{x}(t)+2\check{x}^{\mathrm{T}}(t)\mathcal{P}_i\check{\mathcal{C}}_i\check{x}(t-q(t))\\
&+2\check{x}^{\mathrm{T}}(t)\mathcal{P}_i\check{\mathcal{B}}_i e(t_{\nu,\ell})+2\check{x}^{\mathrm{T}}(t)\mathcal{P}_i\check{\mathcal{D}}_i\omega(t)-\mathrm{e}^{-\alpha h}\check{x}^{\mathrm{T}}(t-q(t))\mathcal{Q}_i\check{x}(t-q(t))(1-\check{x})\\
&-(\alpha+\beta)\mathcal{V}_i(\check{x}(t))+\delta_i\check{x}^{\mathrm{T}}(t-q(t))\widehat{\mathcal{C}}_i\check{x}(t-q(t))-\mathrm{e}^{\mathrm{T}}(t_{\nu,\ell})\Phi_i e(t_{\nu,\ell})+\beta\omega^{\mathrm{T}}(t)\omega(t)\\
&-\beta\omega^{\mathrm{T}}(t)\omega(t)+h^2\xi^{\mathrm{T}}(t)\left[\ \check{\mathcal{A}}_i^{\mathrm{T}}\ \check{\mathcal{C}}_i^{\mathrm{T}}\ \check{\mathcal{D}}_i^{\mathrm{T}}\ \check{\mathcal{B}}_i^{\mathrm{T}}\right]^{\mathrm{T}}\mathcal{R}_i\left[\ \check{\mathcal{A}}_i\ \check{\mathcal{C}}_i\ \check{\mathcal{D}}_i\ \check{\mathcal{B}}_i\right]\xi(t)\\
=&\ \xi^{\mathrm{T}}(t)\check{\mathcal{E}}_i\xi(t)+\beta\omega^{\mathrm{T}}(t)\omega(t)-(\alpha+\beta)\mathcal{V}_i(\check{x}(t)).
\end{aligned}
$$

If (10) is satisfied, then there holds

$$
\dot{\mathcal{V}}_i(\check{x}(t))\le\beta\omega^{\mathrm{T}}(t)\omega(t)-(\alpha+\beta)\mathcal{V}_i(\check{x}(t)).
\tag{19}
$$

In the sequel, we will prove from two aspects: (I) When $\omega(t)\neq 0$, System (4) is exponentially stable. (II) Under zero initial condition, System (4) has L_1 disturbance rejection performance when $\omega(t)\neq 0$.

Case I For all $\xi(t)\neq 0$ and $\omega(t)=0$, (19) implies

$$
\dot{\mathcal{V}}_i(\check{x}(t))+\alpha\mathcal{V}_i(\check{x}(t))\le-\beta\mathcal{V}_i(\check{x}(t))\le 0,
$$

which together with the inequality conditions (11), for $\forall t \in [t_k, t_{k+1})$ yields

$$\begin{aligned}\mathcal{V}_{\sigma(t)}(\check{x}(t)) &\le \mathrm{e}^{-\alpha(t-t_k)}\mathcal{V}_{\sigma(t_k)}(\check{x}(t_k)) \le \mu \mathrm{e}^{-\alpha(t-t_k)}\mathrm{e}^{-\alpha(t-t_{k-1})}\mathcal{V}_{\sigma(t_{k-1})}(\check{x}(t_{k-1}))\\ &\le \mu^2\mathrm{e}^{-\alpha(t-t_{k-2})}\mathcal{V}_{\sigma(t_{k-2})}(\check{x}(t_{k-2})) \le \cdots\\ &\le \mu^k\mathrm{e}^{-\alpha(t-t_0)}\mathcal{V}_{\sigma(t_0)}(\check{x}(t_0)) \le \mathrm{e}^{-(t-t_0)(\alpha-\frac{\ln\mu}{\tau_a})}\mathcal{V}_{\sigma(t_0)}(\check{x}(t_0)).\end{aligned}$$

From (12), we get

$$\begin{aligned}\mathcal{V}_{\sigma(t_0)}(\check{x}(t_0)) &\le \lambda_{\max}(\mathcal{P}_{\sigma(t_0)})\|\check{x}(t_0)\|^2 + h\lambda_{\max}(\mathcal{Q}_{\sigma(t_0)})\|\check{x}(t_0)\|^2 + \frac{1}{2}h^2\lambda_{\max}(\mathcal{Z}_{\sigma(t_0)})\|\check{x}(t_0)\|^2\\ &\quad+\frac{1}{2}h^3\lambda_{\max}(\mathcal{R}_{\sigma(t_0)})\|\dot{\check{x}}(t_0)\|^2\\ &\triangleq \left(\rho_1 + h\rho_2 + \frac{1}{2}h^2\rho_3 + \frac{1}{2}h^3\rho_4\right)\|\check{x}_{t_0}\|_h^2, \qquad (20)\\ \mathcal{V}_{\sigma(t)}(\check{x}(t)) &\ge \lambda_{\min}(\mathcal{P}_{\sigma(t)})\|\check{x}(t)\|^2 \triangleq \rho_5\|\check{x}(t)\|^2, \qquad (21)\end{aligned}$$

where $\|\check{x}(t_0)\|_h = \sup_{-h\le\vartheta\le 0}\{\| \check{x}(t+\vartheta) \|, \| \dot{\check{x}}(t+\vartheta) \|\}$. From (20) and (21), for $t \ge t_0$ the following inequalities are obtained

$$\|\check{x}(t)\|^2 \le M_0\|\check{x}_{t_0}\|_h^2\mathrm{e}^{-\varepsilon(t-t_0)},$$

where $M_0 = \frac{1}{\rho_5}(\rho_1 + h\rho_2 + \frac{1}{2}h^2\rho_3 + \frac{1}{2}h^3\rho_4)$, $\varepsilon = \alpha - \frac{\ln\mu}{\tau_a}$. By Definition 2.4, the filtering error system (4) is exponentially stable under the event-triggered filter (3).

Case II Further, under the zero initial condition, we shall demonstrate that the filtering error system (4) has a given L_1 disturbance rejection level γ for all nonzero $\|\omega(t)\|_{L_\infty} \le 1$. The inequality (19) will be discussed under Cases (i) and (ii).

(i) For $\dot{\mathcal{V}}_{\sigma(t)}(\check{x}(t)) \ge 0$, there holds

$$\mathcal{V}_{\sigma(t)}(\check{x}(t)) \le \frac{\beta}{\alpha+\beta}\omega^{\mathrm{T}}(t)\omega(t) \le \frac{\beta}{\alpha+\beta} \le 1.$$

(ii) For $\dot{\mathcal{V}}_{\sigma(t)}(\check{x}(t)) < 0$ and $\mathcal{V}_{\sigma(t_0)}(\check{x}(t_0)) \mid_{t_0=0} = 0$, $t > 0$, we can get

$$\mathcal{V}_{\sigma(t)}(\check{x}(t^+)) < \mathcal{V}_{\sigma(t)}(\check{x}(t)),$$

obviously, $\mathcal{V}_{\sigma(t)}(\check{x}(t)) \le 1$. Hence, according to the conditions above in cases (i) and (ii), we conclude that $\mathcal{V}_{\sigma(t)}(\check{x}(t)) \le 1$. Define

$$\begin{aligned}J &= \gamma^{-1}\check{z}^{\mathrm{T}}(t)\check{z}(t) - (\gamma-\beta)\omega^{\mathrm{T}}(t)\omega(t) - \beta\check{x}^{\mathrm{T}}(t)\mathcal{P}_i\check{x}(t)\\ &= \begin{bmatrix} \check{x}^{\mathrm{T}}(t) & \omega^{\mathrm{T}}(t)\end{bmatrix}\begin{bmatrix} \gamma^{-1}\check{\mathcal{L}}_i^{\mathrm{T}}\check{\mathcal{L}}_i - \beta\mathcal{P}_i & 0\\ 0 & -(\gamma-\beta)I\end{bmatrix}\begin{bmatrix}\check{x}(t)\\ \omega(t)\end{bmatrix}.\end{aligned}$$

From (9), we get $J < 0$ and

$$\check{z}^{\mathrm{T}}(t)\check{z}(t) < \gamma[\beta\check{x}^{\mathrm{T}}(t)\mathcal{P}_i\check{x}(t) + (\gamma-\beta)\omega^{\mathrm{T}}(t)\omega(t)].$$

Since $\check{x}^{\mathrm{T}}(t)\mathcal{P}_i\check{x}(t) \le 1$ is implied by $\mathcal{V}_{\sigma(t)}(\check{x}(t)) \le 1$, we further have $\sup_{\|\omega(t)\|_{L_\infty}\le 1}\|\check{z}(t)\|_{L_\infty} < \gamma$. The proof is completed. ∎

Remark 3.2 From the above derivation process, the event-triggered L_1 filter makes the following property hold:

1) The event-triggered filtering error system (4) is exponentially stable with $\omega(t) = 0$.

2) Under the zero initial condition, the event-triggered filtering error system (4) has a prescribed L_1 disturbance rejection performance for all nonzero $\omega(t) \in L_\infty[0, \infty)$.

Theorem 3.3 *Consider switched systems* (1)*, for given constants* $\rho_1 > 0$, $\rho_2 > 0$, $\rho_3 > 0$, $\rho_4 > 0$, $\rho_5 > 0$, $\beta > 0$, $\mu > 1$, $\gamma > 0$, $\delta_i > 0$, $\alpha > 0$. *The event-triggered filtering error system* (4) *is exponentially stable with a given* L_1 *disturbance rejection performance, if there exist an event-triggered filter* (3) *and positive definite matrices* $\mathcal{P}_{1i} > 0, \mathcal{P}_{2i} > 0, \mathcal{Q}_{1i} > 0, \mathcal{Q}_{2i} > 0, \mathcal{Z}_{1i} > 0, \mathcal{Z}_{2i} > 0, \mathcal{R}_{1i} > 0, \mathcal{R}_{2i} > 0$, $\forall (i,j) \in \mathcal{I} \times \mathcal{I}, i \neq j$ *such that*

$$\begin{bmatrix} -\beta\mathcal{P}_{1i} & 0 & 0 & \mathcal{L}_i^{\mathrm{T}} \\ * & -\beta\mathcal{P}_{2i} & 0 & -\mathcal{L}_{fi}^{\mathrm{T}} \\ * & * & -(\gamma-\beta)I & 0 \\ * & * & * & -\gamma I \end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} \Lambda_{11} & 0 & 0 & 0 & \mathcal{P}_{1i}\mathcal{D}_i & 0 & h\mathcal{A}_i^{\mathrm{T}} & 0 & 0 \\ * & \Lambda_{22} & \widehat{\mathcal{B}}_{fi}\mathcal{C}_i & 0 & 0 & -\widehat{\mathcal{B}}_{fi} & 0 & h\widehat{\mathcal{A}}_{fi}^{\mathrm{T}} & 0 \\ * & * & \Lambda_{33} & 0 & 0 & 0 & 0 & \Lambda_{38} & \mathcal{C}_i^{\mathrm{T}}\Phi_i \\ * & * & * & \Lambda_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\beta I & 0 & h\mathcal{D}_i^{\mathrm{T}} & 0 & 0 \\ * & * & * & * & * & -\Phi_i & 0 & -h\widehat{\mathcal{B}}_{fi}^{\mathrm{T}} & 0 \\ * & * & * & * & * & * & \Lambda_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Lambda_{88} & 0 \\ * & * & * & * & * & * & * & * & \Lambda_{99} \end{bmatrix} < 0, \tag{23}$$

$$\mathcal{P}_{1i} < \mu\mathcal{P}_{1j}, \quad \mathcal{P}_{2i} < \mu\mathcal{P}_{2j}, \quad \mathcal{Q}_{1i} < \mu\mathcal{Q}_{1j}, \quad \mathcal{Q}_{2i} < \mu\mathcal{Q}_{2j}, \tag{24}$$

$$\mathcal{Z}_{1i} < \mu\mathcal{Z}_{1j}, \quad \mathcal{Z}_{2i} < \mu\mathcal{Z}_{2j}, \quad \mathcal{R}_{1i} < \mu\mathcal{P}_{1j}, \quad \mathcal{R}_{2i} < \mu\mathcal{P}_{2j}, \tag{25}$$

$$\mathcal{P}_i \leq \rho_1 I, \quad \mathcal{P}_i \geq \rho_5 I, \quad \mathcal{Q}_i \leq \rho_2 I, \quad \mathcal{Z}_i \leq \rho_3 I, \quad \mathcal{R}_i \leq \rho_4 I, \tag{26}$$

$$\beta\mathcal{Q}_i + (\beta h - 1)\mathcal{Z}_i < 0, \quad \beta\mathcal{Q}_i + (\beta h - 1)\mathcal{R}_i < 0, \quad \tau_a > \tau_a^* = \frac{\ln\mu}{\alpha}, \tag{27}$$

where

$$\Lambda_{11} = \mathcal{A}_i^{\mathrm{T}}\mathcal{P}_{1i} + \mathcal{P}_{1i}\mathcal{A}_i + (\alpha+\beta)\mathcal{P}_{1i} + \mathcal{Q}_{1i} + h\mathcal{Z}_{1i}, \quad \Lambda_{38} = h\mathcal{C}_i^{\mathrm{T}}\widehat{\mathcal{B}}_{fi}^{\mathrm{T}},$$
$$\Lambda_{22} = \widehat{\mathcal{A}}_{fi}^{\mathrm{T}} + \widehat{\mathcal{A}}_{fi} + (\alpha+\beta)\mathcal{P}_{2i} + \mathcal{Q}_{2i} + h\mathcal{Z}_{2i}, \quad \Lambda_{33} = \delta_i\mathcal{C}_i^{\mathrm{T}}\Phi_i\mathcal{C}_i - (1-\widetilde{q})\mathrm{e}^{-\alpha h}\mathcal{Q}_{1i},$$
$$\Lambda_{77} = \mathcal{R}_{1i} - 2I, \quad \Lambda_{44} = -(1-\widetilde{q})\mathrm{e}^{-\alpha h}\mathcal{Q}_{2i}, \quad \Lambda_{88} = \mathcal{R}_{2i} - 2\mathcal{P}_{2i}, \quad \Lambda_{99} = -\delta_i^{-1}\Phi_i.$$

Moreover, a feasible event-triggered filter can be structured by

$$\begin{bmatrix} \mathcal{A}_{fi} & \mathcal{B}_{fi} \\ \mathcal{L}_{fi} & 0 \end{bmatrix} = \begin{bmatrix} \mathcal{P}_{2j}^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \widehat{\mathcal{A}}_{fi} & \widehat{\mathcal{B}}_{fi} \\ \mathcal{L}_{fi} & 0 \end{bmatrix}. \tag{28}$$

Proof Assume

$$\mathcal{P}_i = \begin{bmatrix} \mathcal{P}_{1i} & 0 \\ 0 & \mathcal{P}_{2i} \end{bmatrix}, \quad \mathcal{Q}_i = \begin{bmatrix} \mathcal{Q}_{1i} & 0 \\ 0 & \mathcal{Q}_{2i} \end{bmatrix}, \quad \mathcal{Z}_i = \begin{bmatrix} \mathcal{Z}_{1i} & 0 \\ 0 & \mathcal{Z}_{2i} \end{bmatrix}, \quad \mathcal{R}_i = \begin{bmatrix} \mathcal{R}_{1i} & 0 \\ 0 & \mathcal{R}_{2i} \end{bmatrix}. \tag{29}$$

Substituting (5) and (29) into (10), and bearing $\widehat{\mathcal{C}} = \left[\begin{smallmatrix} \mathcal{C}_i^{\mathrm{T}} \varPhi_i \mathcal{C}_i & 0 \\ 0 & 0 \end{smallmatrix}\right]$ in mind, we see that (10) can be rewritten as

$$\begin{bmatrix} \varPi_{11} & 0 & 0 & 0 & \mathcal{P}_{1i}\mathcal{D}_i & 0 & h\mathcal{A}_i^{\mathrm{T}} & 0 \\ * & \varPi_{22} & \mathcal{P}_{2i}\mathcal{B}_{fi}\mathcal{C}_i & 0 & 0 & -\mathcal{P}_{2i}\mathcal{B}_{fi} & 0 & h\mathcal{A}_{fi}^{\mathrm{T}} \\ * & * & \varPi_{33} & 0 & 0 & 0 & 0 & h\mathcal{C}_i^{\mathrm{T}}\mathcal{B}_i^{\mathrm{T}} \\ * & * & * & \varPi_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & -\beta I & 0 & h\mathcal{D}_i^{\mathrm{T}} & 0 \\ * & * & * & * & * & -\varPhi_i & 0 & -h\mathcal{B}_{fi}^{\mathrm{T}} \\ * & * & * & * & * & * & -\mathcal{R}_{1i}^{-1} & 0 \\ * & * & * & * & * & * & * & -\mathcal{R}_{2i}^{-1} \end{bmatrix} < 0, \tag{30}$$

where

$$\begin{aligned} \varPi_{11} &= \mathcal{A}_i^{\mathrm{T}}\mathcal{P}_{1i} + \mathcal{P}_{1i}\mathcal{A}_i + (\alpha+\beta)\mathcal{P}_{1i} + \mathcal{Q}_{1i} + h\mathcal{Z}_{1i}, \\ \varPi_{22} &= \mathcal{P}_{2i}\mathcal{A}_{fi} + \mathcal{A}_{fi}^{\mathrm{T}}\mathcal{P}_{2i} + (\alpha+\beta)\mathcal{P}_{2i} + \mathcal{Q}_{2i} + h\mathcal{Z}_{2i}, \\ \varPi_{33} &= \delta_i\mathcal{C}_i^{\mathrm{T}}\varPhi_i\mathcal{C}_i - (1-\widetilde{q})\mathrm{e}^{-\alpha h}\mathcal{Q}_{1i}, \ \ \varPi_{44} = -(1-\widetilde{q})\mathrm{e}^{-\alpha h}\mathcal{Q}_{2i}. \end{aligned}$$

Noting $(\mathcal{P}_{2i} - \mathcal{R}_{2i})\mathcal{R}_{2i}^{-1}(\mathcal{P}_{2i} - \mathcal{R}_{2i}) \geq 0$ and $\mathcal{R}_{2i}^{-1} > 0$, we can derive

$$-\mathcal{P}_{2i}\mathcal{R}_{2i}^{-1}\mathcal{P}_{2i} \leq \mathcal{R}_{2i} - 2\mathcal{P}_{2i}, \qquad -R_{1i}^{-1} \leq R_{1i} - 2I.$$

Define $\widehat{\mathcal{B}}_{fi} = \mathcal{P}_{2i}\mathcal{B}_{fi}, \widehat{\mathcal{A}}_{fi} = \mathcal{P}_{2i}\mathcal{A}_{fi}$. Pre- and post-multiplying (30) by $\mathrm{diag}\{I, I, I, I, I, I, I, \mathcal{P}_{2i}\}$ and its transpose, respectively; one can get that Inequality (10) is implied by inequality (23).

In addition, due to $\mathcal{P}_i = \left[\begin{smallmatrix} \mathcal{P}_{1i} & 0 \\ 0 & \mathcal{P}_{2i} \end{smallmatrix}\right]$ and $\check{\mathcal{L}}_{\sigma(t)} = \left[\begin{smallmatrix} \mathcal{L}_{\sigma(t)} & \mathcal{L}_{f\sigma(t)} \end{smallmatrix}\right]$, we can obtain that (9) is equivalent to (22). We can also verify from (29) that the constraints (11) can be equivalently rewritten as (24) and (25). Therefore, from (22)–(27), we conclude that the event-triggered filtering error systems (4) is exponentially stable with a prescribed L_1 disturbance rejection performance. Moreover, the corresponding event-triggered filter gain parameters can be given in (28). The proof is completed. ∎

Remark 3.4 The L_1 filter, compared with H_∞ and $L_2 - L_\infty$ filters, is better able to address the persistent and amplitude bounded signals that extensive exist in engineering systems, which by means of minimizing the worst case peak-peak gain of dynamic systems. Under an event-triggered scheme, output signals of the L_1 filter are transmitted only when necessary, therefore, the event-triggered L_1 filter can significantly reduce the consumption of communication resources while the L_1 disturbance rejection level of switched systems is promised.

4 Simulation Examples

Example 4.1 Consider the switched system (4) with two subsystems, where

$$\mathcal{A}_1 = \begin{bmatrix} -0.7 & -0.2 \\ 1 & -0.6 \end{bmatrix}, \quad \mathcal{D}_1 = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, \quad \mathcal{L}_1 = \begin{bmatrix} -0.2 & 0.1 \end{bmatrix}, \quad \mathcal{C}_1 = \begin{bmatrix} -0.1 & 0.2 \end{bmatrix},$$
$$\mathcal{A}_2 = \begin{bmatrix} -0.5 & 0.2 \\ -0.5 & -0.4 \end{bmatrix}, \quad \mathcal{D}_2 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \quad \mathcal{L}_2 = \begin{bmatrix} 0.1 & 0.4 \end{bmatrix}, \quad \mathcal{C}_2 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}.$$

Given $\mu = 1.1$, $\alpha = 0.9$, $\beta = 0.92$, $h = 0.03$, $\Phi_1 = \Phi_2 = 1$, $\delta_1 = 0.1$ and $\delta_2 = 0.06$, solving (22)–(27) through Matlab LMI Toolbox, a set of feasible filter parameters are obtained

$$\mathcal{A}_{f1} = \begin{bmatrix} -9.4260 & -0.0986 \\ -0.3009 & -9.8031 \end{bmatrix}, \quad \mathcal{B}_{f1} = \begin{bmatrix} -0.3638 \\ -0.3303 \end{bmatrix}, \quad \mathcal{L}_{f1} = \begin{bmatrix} -0.1689 & -0.6751 \end{bmatrix},$$
$$\mathcal{A}_{f2} = \begin{bmatrix} -3.5948 & 0.3481 \\ 0.0518 & -9.5601 \end{bmatrix}, \quad \mathcal{B}_{f2} = \begin{bmatrix} -0.5446 \\ -0.2270 \end{bmatrix}, \quad \mathcal{L}_{f2} = \begin{bmatrix} -0.3735 & -0.8967 \end{bmatrix}.$$

Since $\tau_a > \tau_a^* = \frac{\ln \mu}{\alpha}$, we get the ADT as $\tau_a^* = 0.1059$. Choose $\tau_a = 0.11$, the initial state of system as $\breve{x}(t) = [0.5\ 0.2\ 0.1 - 0.1]^{\mathrm{T}}$ and $\gamma = 1.025$, then we obtain the following Figures 2–5. From Figure 2, the output signal $z(t)$ of the switched system (1) is successfully estimated by the event-triggered L_1 filter. From Figure 3, we can see that filtering state $x_f(t)$ can well estimate the system state $x(t)$; i.e., the filtering error of System (4) goes to zero exponentially. Figure 4 shows the switching signal under $\tau_a = 0.11$. Figure 5 depicts the corresponding event-triggered release intervals of the filtering error system (4); which shows that less information are needed when the filtering error system enters steady state.

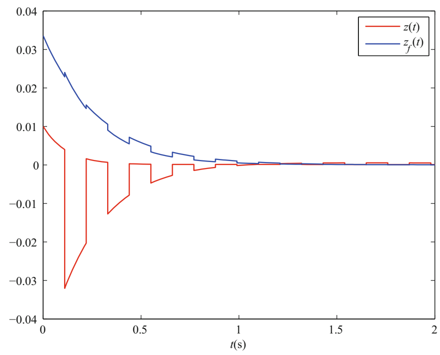

Figure 2 Signal $z(t)$ and its estimation $z_f(t)$

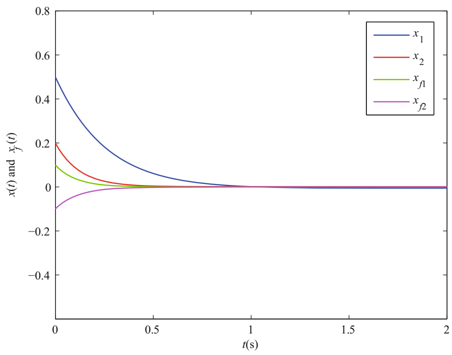

Figure 3 State trajectories of $x(t)$ and $x_f(t)$

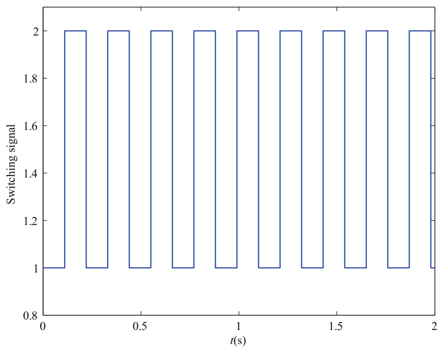

Figure 4 The switching signal $\sigma(t)$

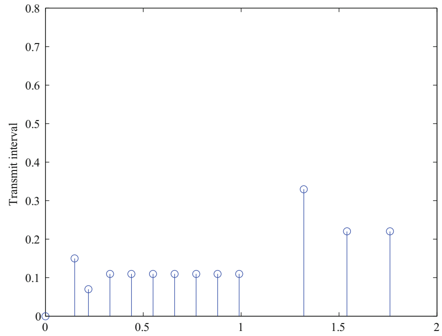

Figure 5 Release time intervals

Remark 4.2 By calculating $\frac{2}{h} = \frac{2}{0.03}$, one can get about 66 sampling points in the time interval $[0, 2)$. As can be seen from Figure 5, only 13 sampled data are transmitted to the filter. As a result, the communication resources of the system can be reduced by 80% under the event-triggered communication scheme. We conclude that the event-triggered filter is efficient in reducing the frequency of data transmission and saving communication costs.

Example 4.3 In this section, the boost converter example as provided in Figure 6 is borrowed from [32]. It is PWM (Pulse-Width-Modulation)-driven and can be modelled as a switched system with two subsystems. $\mathcal{L}$, $\mathcal{C}$, $\mathcal{R}$ and $e_s(t)$ are the inductance, the capacitance, the load resistance, and the source voltage, respectively.

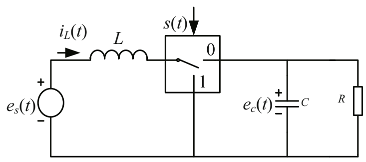

Figure 6 A boost converter circuit system

The differential equation of the boost converter circuit system is

$$\dot{e}_{\mathcal{C}}(v) = -\frac{1}{\mathcal{R}\mathcal{C}_1} e_{\mathcal{C}}(v) + (1 - s(v))\frac{1}{\mathcal{C}_1} i_{\mathcal{L}}(v), \tag{31}$$

$$\dot{i}_{\mathcal{L}}(v) = -(1 - s(v))\frac{1}{\mathcal{L}_1} e_{\mathcal{C}}(v) + s(v)\frac{1}{\mathcal{L}_1} e_{\mathcal{S}}(v), \tag{32}$$

where $v = t/\mathcal{T}$, $\mathcal{L}_1 = \mathcal{L}/\mathcal{T}$, and $\mathcal{C}_1 = \mathcal{C}/\mathcal{T}$.

From (31) and (32), we can get

$$\dot{x} = \mathcal{A}_\sigma^{\mathcal{C}} x(t), \quad \sigma \in \{1, 2\}, \tag{33}$$

where $x = [e_{\mathcal{C}}, i_{\mathcal{L}}, 1]^{\mathrm{T}}$ and

$$A_1^{\mathcal{C}} = \begin{bmatrix} -\frac{1}{\mathcal{R}\mathcal{C}_1} & -\frac{1}{\mathcal{C}_1} & 0 \\ -\frac{1}{\mathcal{L}_1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{A}_2^{\mathcal{C}} = \begin{bmatrix} -\frac{1}{\mathcal{R}\mathcal{C}_1} & 0 & 0 \\ 0 & 0 & \frac{1}{\mathcal{L}_1} \\ 0 & 0 & 0 \end{bmatrix}.$$

As in [32], it is assumed that the control matrices for (33) to be $\mathcal{B}_1^{\mathcal{C}} = \mathcal{B}_2^{\mathcal{C}} = [-0.1\ 0.4\ 0.5]^{\mathrm{T}}$ and controller gains can be obtained as $\mathcal{K}_1 = [-6.61\ -1.07\ -9.32]$, $\mathcal{K}_2 = [-5.37\ -12.42\ -10.07]$. Accordingly, we can obtain the closed-loop system (33) with matrices

$$\mathcal{A}_1 = \begin{bmatrix} -0.34 & 1.11 & 0.93 \\ -3.65 & -0.43 & -3.73 \\ -3.30 & -0.54 & -4.66 \end{bmatrix}, \quad \mathcal{A}_2 = \begin{bmatrix} -0.46 & 1.24 & 1.00 \\ -2.15 & -4.97 & -3.03 \\ -2.68 & -6.21 & -5.03 \end{bmatrix}.$$

Suppose that the parameters for the measured output, disturbance input and the output signal to be estimated are

$$\mathcal{D}_1 = \begin{bmatrix} -0.300.200.10 \end{bmatrix}^{\mathrm{T}}, \quad \mathcal{C}_1 = \begin{bmatrix} 0.10\ -0.10\ 0.10 \end{bmatrix}, \quad \mathcal{L}_1 = \begin{bmatrix} 0.70\ 0\ 0.30 \end{bmatrix},$$
$$\mathcal{D}_2 = \begin{bmatrix} -1.40 - 0.300.20 \end{bmatrix}^{\mathrm{T}}, \quad \mathcal{C}_2 = \begin{bmatrix} 0.30\ -0.40\ 0.10 \end{bmatrix}, \quad \mathcal{L}_2 = \begin{bmatrix} 0.20\ 0\ 0.40 \end{bmatrix}.$$

Then, by giving $\rho_1 = 2.5$, $\rho_2 = 6$, $\rho_3 = 4$, $\rho_4 = 1.8$, $\rho_5 = 0.18$, $\beta = 0.8$, $\gamma = 2.95$, $\mu = 1.18$, $\alpha = 0.75$, $h = 0.15$ and solving a set of LMIs in Theorem 2, we can get $\tau_a^* = 0.2207$ and filter gains as

$$\mathcal{A}_{f1} = \begin{bmatrix} -2.5492 & -0.1377 & -0.0079 \\ 0.5173 & -4.4215 & 0.3213 \\ -0.4588 & 0.3216 & -6.4559 \end{bmatrix}, \quad \mathcal{B}_{f1} = \begin{bmatrix} -0.4695 \\ -0.3960 \\ -0.0206 \end{bmatrix},$$
$$\mathcal{A}_{f2} = \begin{bmatrix} -6.4493 & -0.1369 & -0.0078 \\ 0.4360 & -5.3787 & -0.0064 \\ -0.1559 & 0.0669 & -3.1171 \end{bmatrix}, \quad \mathcal{B}_{f2} = \begin{bmatrix} 0.1460 \\ 0.1319 \\ -0.0595 \end{bmatrix},$$
$$\mathcal{L}_{f1} = \begin{bmatrix} 0.2183\ 5.4661\ 22.8747 \end{bmatrix}, \quad \mathcal{L}_{f2} = \begin{bmatrix} -0.2183\ -5.4661\ -22.8747 \end{bmatrix}.$$

Choose $\overline{x}(0) = [0.1\ 0.06\ 0.01\ -0.025\ -0.01\ -0.01]^{\mathrm{T}}$ and $\tau_a = 0.25$. Given the event-triggered parameters as $\Phi_1 = \Phi_2 = 0.8$, $\delta_1 = 0.02$, $\delta_2 = 0.015$, the simulation results show that filtering error system (4) is exponentially stable with a given L_1 disturbance rejection performance. It is clear from Figure 7 that the L_1 filter produces a good estimation $z_f(t)$ of the system output $z(t)$. Figure 8 shows state trajectories of System (1) and filter (3) eventually converge to zero. Figure 9 presents the switching signal with $\tau_a = 0.25$. The corresponding release time intervals are plotted in Figure 10. From Figure 10, we can see that the event-triggered mechanism is extremely efficient in increasing the utilization rate of networked resources.

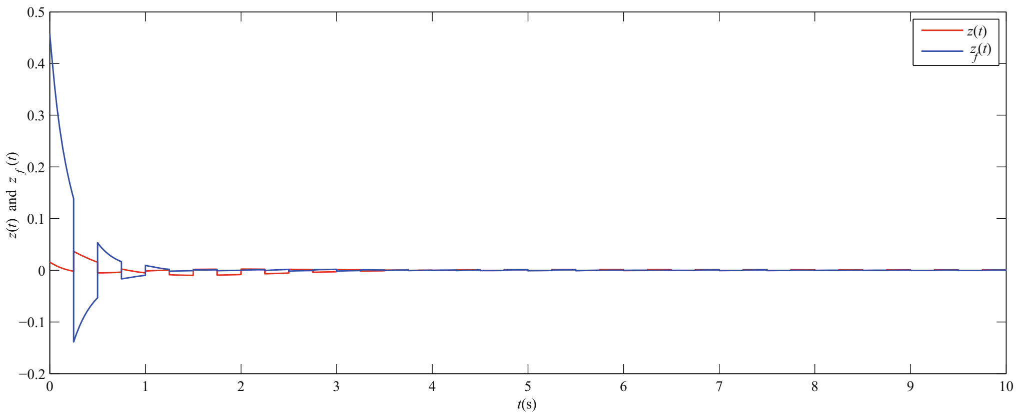

Figure 7 Signal $z(t)$ and its estimation $z_f(t)$

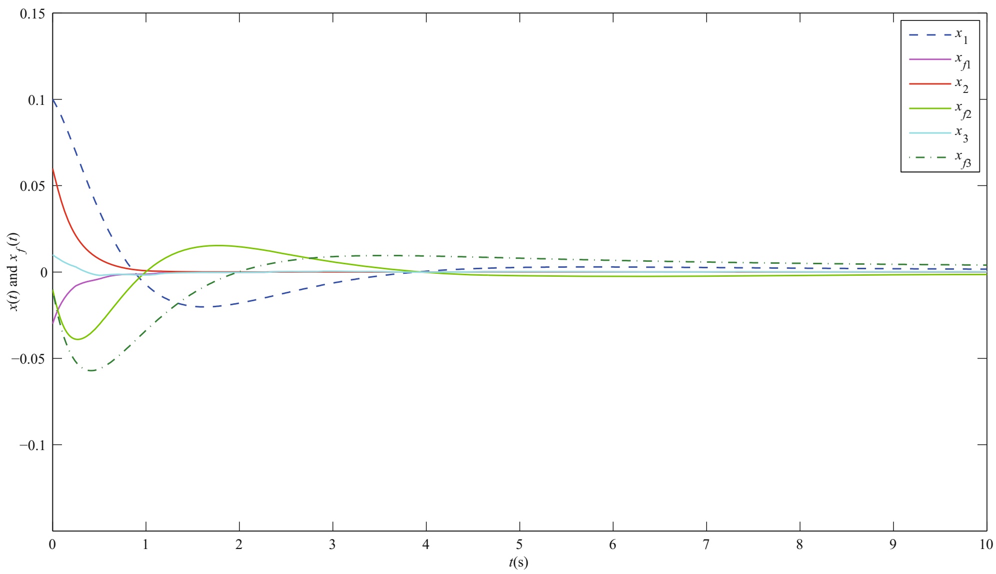

Figure 8 Responses of $x(t)$ and $x_f(t)$

Remark 4.4 There are 66 sampling points in the time interval $[0, 10)$ by calculating $\frac{10}{0.15}$ and the introduction of event-triggered scheme makes only 27 sampled data to be transmitted to the filter, which takes only 41% of total sampled signals. Obviously, the presented event-triggered scheme is highly effective in improving the utilization of communication resources.

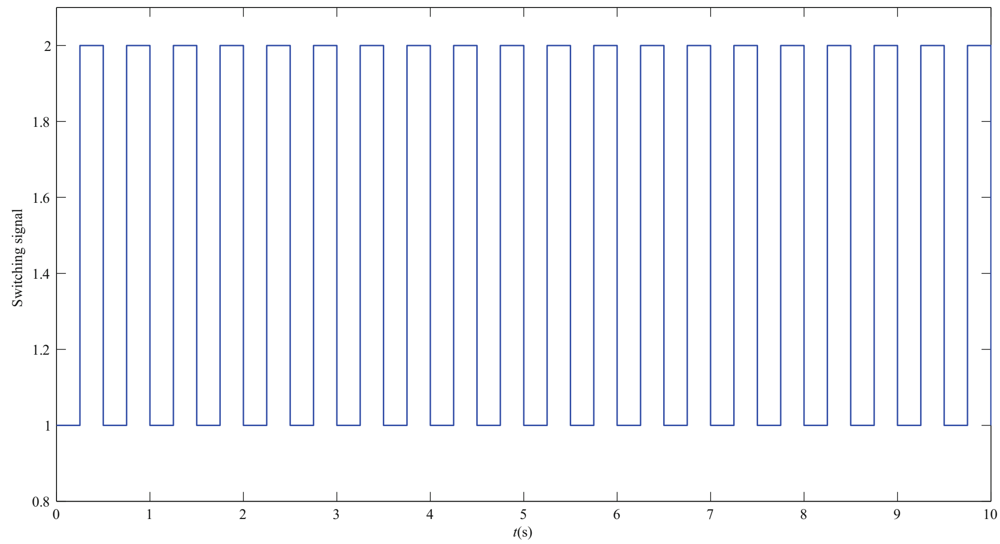

Figure 9 The switching signal $\sigma(t)$

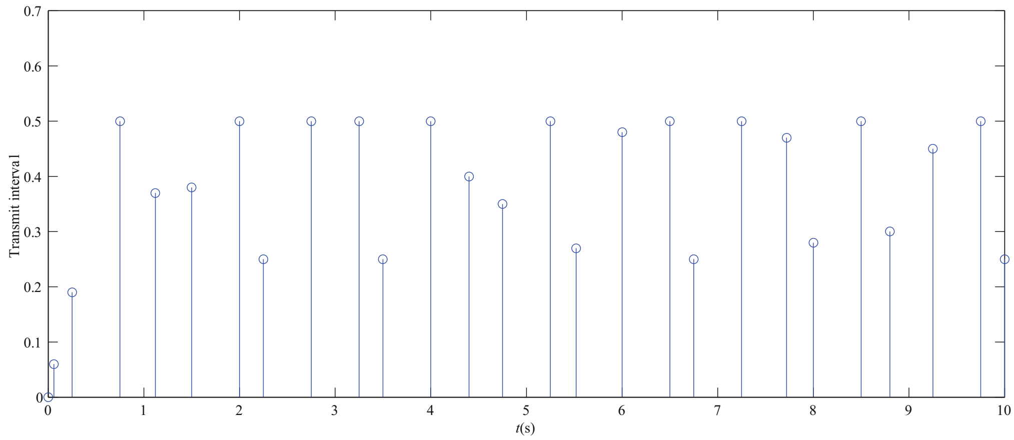

Figure 10 Release time intervals

5 Conclusions

In this paper, we have investigated the event-triggered exponential L_1 filtering issue for networked linear switched systems. An event-triggered mechanism has been put forward to save communication costs and improve the resources utilization ratio. Then, the filtering error system has been reconstructed as a switched delay system with bounded disturbance. By virtue of the ADT technique, a stability criterion has been derived such that the filtering error system is exponentially stable with a given L_1 disturbance rejection performance. The event-triggered L_1 filtering parameters are available by solving a set of strict LMIs; for which the filtering information is updated only when the given event-triggered conditions happen. Eventually, two examples have been presented to verify the efficiency of the acquired results.

References

[1] DeCarlo R A, Branicky M S, Pettersson S, et al., Perspectives and results on the stability and stabilizability of hybrid systems, *Proceedings of the IEEE*, 2000, **88**(7): 1069–1082.

[2] Hespanha J P and Morse A S, Stability of switched systems with average dwell-time, *Proceedings of the 38th IEEE Conference on Decision and Control*, 1999, **3**: 2655–2660.

[3] Zong G D, Wang R H, Zheng W X, et al., Finite-time H_∞ control for discrete-time switched nonlinear systems with time delay, *International Journal of Robust and Nonlinear Control*, 2015, **25**(6): 914–936.

[4] Zong G D and Wang Q Z, Robust resilient control for impulsive switched systems under asynchronous switching, *International Journal of Computer Mathematics*, 2015, **92**(6): 1143–1159.

[5] Benzaouia A and Eddoukali Y, Robust fault detection and control for continuous-time switched systems with average dwell time, *Circuits Systems and Signal Processing*, 2018, **37**(6): 2357–2373.

[6] Jiang Z Y and Yan P, asynchronous switching control of discrete impulsive switched delay systems with average dwell time, *IET Control Theory and Applications*, 2018, **12**(7): 992–999.

[7] Li S, Xiang Z, and Karimi H R, Stability and L_1-gain controller design for positive switched systems with mixed time-varying delays, *Applied Mathematics and Computation*, 2013, **222**: 507–518.

[8] Zhang L, Shi P, Boukas E K, et al., Robust l_2-l_∞ filtering for switched linear discrete time-delay systems with polytopic uncertainties, *IET Control Theory and Applications*, 2007, **1**(3): 722–730.

[9] Zhang H, Shi Y, and Saadat Mehr A, Robust energy-to-peak filtering for networked systems with time-varying delays and randomly missing data, *IET Control Theory and Applications*, 2010, **4**(12): 2921–2936.

[10] Xiao X Q, Zhou L, and Lu G P, Event-triggered H_∞ filtering of continuous-time switched linear systems, *Signal Processing*, 2017, **141**: 343–349.

[11] Xiao X Q, Park J H, and Zhou L, Event-triggered H_∞ filtering of discrete-time switched linear systems, *ISA Transactions*, 2018, **77**: 112–121.

[12] Li Y H, Zhang Q, Zhou X J, et al., Basis-dependent and delay-dependent results on robust L_1 filtering for networked control systems, *Journal of the Franklin Institute*, 2016, **353**(14): 3368–3385.

[13] Li Y H, Shi Q, and Karimi H R, Robust L_1 fixed-order filtering for switched LPV systems with parameter-dependent delays, *Journal of the Franklin Institute*, 2015, **352**(3): 761–775.

[14] Ren H L, Zong G D, and Karimi H R, Asynchronous finite-time filtering of networked switched systems and its application: An event-driven method, *IEEE Transactions on Circuits and Systems — I: Regular Papers*, 2018, DOI: 10.1109/TCSI.2018.2857771.

[15] Fridman E, Seuret A, and Richard J P, Robust sampled-data stabilization of linear systems: An input delay approach, *Automatica*, 2004, **40**(8): 1441–1446.

[16] Zhu B L, Suo M L, Zhang Z P, et al., L_2-L_∞ filtering for a class of stochastic nonlinear systems with aperiodic sampling, *Optimal Control Applications and Methods*, 2018, **39**: 1158–1167.

[17] Zhu B L, Ma J, and Li S L, A novel $L_2 - L_\infty$ filtering strategy for two kinds of network-based linear time-invariant systems, *Circuits Systems and Signal Processing*, 2017, **36**(8): 3098–3113.

[18] Zhu B L, Suo M L, Chen Y, et al., Mixed H_∞ and passivity control for a class of stochastic nonlinear sampled-data systems, *Journal of the Franklin Institute*, 2018, **355**(7): 3310–3329.

[19] Ramezanifar A, Mohammadpour J, and Grigoriadis K M, Sampled-data filtering for linear parameter varying systems, *International Journal of Systems Science*, 2015, **46**(3): 474–487.

[20] Souza M, Fioravanti A R, and Geromel J C, H_2 sampled-data filtering of linear systems, *IEEE Trans Signal Process*, 2014, **62**(18): 4839–4846.

[21] Wang S, Zeng M, Park J H, et al., Finite-time control for networked switched linear systems with an event-driven communication approach, *International Journal of Systems Science*, 2017, **48**(2): 236–246.

[22] Peng C, Ma S D, and Xie X P, Observer-based non-PDC control for networked T-S fuzzy systems with an event-triggered communication, *IEEE transactions on Cybernetics*, 2017, **47**(8): 2279–2287.

[23] Heemels W P M H, Donkers M C F, and Teel A R, Periodic event-triggered control for linear systems, *IEEE Transactions on Automatic Control*, 2013, **58**(4): 847–861.

[24] Wang Z D, Yang F W, Ho D W, et al., Robust H_∞ control for networked systems with random packet losses, *IEEE Transactions on Systems, Man, and Cybernetics, Part B* (*Cybernetics*), 2007, **37**(4): 916–924.

[25] Han Q L, A discrete delay decomposition approach to stability of linear retarded and neutral systems, *Automatica*, 2009, **45**(2): 517–524.

[26] Lunze J and Lehmann D, A state-feedback approach to event-based control, *Automatica*, 2010, **46**(1): 211–215.

[27] Zou W C and Xiang Z R, Event-triggered leader-following consensus of non-linear multi-agent systems with switched dynamics, *IET Control Theory and Applications*, 2018, DOI: 10.1049/iet-cta.2018.5126.

[28] Ren H L, Zong G D, and Li T S, Event-triggered finite-time control for networked switched linear systems with asynchronous switching, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2018, **48**(11): 1874–1884.

[29] Zou W C and Xiang Z R, Event-triggered containment control of second-order nonlinear multi-agent systems, *Journal of the Franklin Institute*, 2018, DOI: 10.1016/j.jfranklin.2018.05.060.

[30] Wang J, Zhang X M, and Han Q L, Event-triggered generalized dissipativity filtering for neural networks with time-varying delays, *IEEE Transactions on Neural Networks and Learning Systems*, 2016, **27**(1): 77–88.

[31] Sun X M, Zhao J, and Hill D J, Stability and L_2 gain analysis for switched delay systems: A delay-depent method, *Automatica*, 2006, **42**(10): 1769–1774.

[32] Zhang L X, Cui N G, Liu M, et al., Asynchronous filtering of discrete-time switched linear systems with average dwell time, *IEEE Transactions on Circuits and Systems I: Regular Papers*, 2011, **58**(5): 1109–1118.