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Sensor Fault Estimation and Fault-Tolerant Control for a Class of Takagi-Sugeno Markovian Jump Systems with Partially Unknown Transition Rates Based on the Reduced-Order Observer*

LI Xiaohang · LU Dunke · ZHANG Wei · ZHU Fanglai

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Abstract This paper addresses the problem on sensor fault estimation and fault-tolerant control for a class of Takagi-Sugeno Markovian jump systems, which are subjected to sensor faults and partially unknown transition rates. First, the original plant is extended to a descriptor system, where the original states and the sensor faults are assembled into the new state vector. Then, a novel reduced-order observer is designed for the extended system to simultaneously estimate the immeasurable states and sensor faults. Second, by using the estimated states obtained from the designed observer, a state-feedback fault-tolerant control strategy is developed to make the resulting closed-loop control system stochastically stable. Based on linear matrix inequality technique, algorithms are presented to compute the observer gains and control gains. The effectiveness of the proposed observer and controller are validated by a numerical example and a compared study, respectively, and the simulation results reveal that the proposed method can successfully estimate the sensor faults and guarantee the stochastic stability of the resulting closed-loop system.

Keywords Fault-tolerant control, Markovian jump system, partially unknown transition rates, reduced-order observer, sensor fault estimation, T-S fuzzy system.

LI Xiaohang
School of Electronic and Electric Engineering, Shanghai University of Engineering Science, Shanghai 201620, *China; School of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo* 454010, *China.*
Email: lixiaohang58@163.com.
LU Dunke
School of Electronic and Electric Engineering, Shanghai University of Engineering Science, Shanghai 201620, *China.*
ZHANG Wei
Engineering Training Center, Shanghai University of Engineering Science, Shanghai 201620, *China.*
ZHU Fanglai
College of Electronics and Information Engineering, Tongji University, Shanghai 200093, *China.*
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1 Introduction

Markovian jump systems (MJSs), known as very famous stochastic systems, play an important role in modeling, as it can describe plants with random abrupt changes. A Markovian jump system is governed by a Markov process and the switching mode ranges in a finite integer set. Due to the popularity of MJSs, tremendous results have been reported on MJSs[1−9].

With the increasing demand of reliability, safety and maintainability of automatic systems, fault estimation (FE) and fault-tolerant control (FTC) have become more and more important in both theoretical and practical areas. During the past few decades, FE and FTC have attracted considerable attention, and fruitful results can be found in literatures for MJSs[10−19]. Specially, [10] proposed a sensor-fault-estimation method and FTC for time-delayed MJSs with Lipschitz non-linearity. [11] developed a method on simultaneous estimations of the actuator and sensor faults by means of sliding mode observer, then designed an actuator fault-tolerant controller to ensure the stability of the overall system. [16] proposed an integrated FE and FTC strategy for time-delayed MJSs subjected to actuator fault and sensor fault, where the sufficient and necessary conditions of the existence of the designed observer were provided. [18] proposed a method on actuator fault estimation based on adaptive observer, and designed an FTC scheme by using a similar technique to [10, 11]. [19] considered actuator-fault-estimation problem for a kind of fuzzy MJSs without reference to sensor fault. In fact, autonomous systems depend very much on sensors acquisition of certain system information, and in addition, sensor signals often carry important information for feedback control systems[20]. However, a faulty sensor signal may lead to poor control performance. Therefore, sensor-fault-diagnoses are becoming increasingly important, and certain results have been reported[10,20−26]. For instance, for a class of Itô stochastic systems, [21] addressed the problems of FE and FTC against sensor faults.

On the other hand, since most practical systems are nonlinear in nature, FE/FTC applications to industrial and commercial processes should specifically take nonlinear models into account. Takagi-Sugeno fuzzy models have proven to be capable of approximating any smooth nonlinear functions with any specified accuracy. It is fortunate that the T-S model has a convenient and simple structure based on a set of IF-THEN rules. Hence, by means of T-S fuzzy models, the existing FE/FTC methods can be extensively performed in nonlinear systems directly[27−30].

Consider that, in real applications, information of each element of the transition rate matrix is hardly known[8], hence, the study of integrated sensor-fault-estimation and FTC for a class of T-S fuzzy MJSs with partial unknown transition rates (PUTRs) is of great significance. To our best knowledge, such a topic has not been studied yet. Hence, this paper will focus on the following topics: 1) Design a reduced-order observer for simultaneous estimations of state and sensor fault; 2) Design an observer-based fault-tolerant controller to guarantee the stochastic stability of the considered system. The main contributions are listed in three folds: 1) In this paper, we design a novel reduced-order observer to perform the state and sensor fault estimations. In comparison to full-order designs[10−19], our proposed method gives a simpler structure and easier implementation. 2) Unlike the preconditions of the faults considered in [10–12, 14], the sensor fault discussed in this paper has no constraints, which makes our proposed method less conservative. 3) The proposed method can be directly applied to MJSs with exact known transition rates, indicating an improved generality.

The rest of the paper is organized as follows. In Section 2, we formally state the problem and some preliminaries. The main results are discussed in Section 3. We present numerical simulations in Section 4 to demonstrate the performance of the proposed approach and draw conclusions in Section 5.

2 Problem Formulation and Preliminaries

In this section, some preliminaries and the system description are given. Let us consider the following Markovian jump linear systems described by T-S fuzzy models over the probability space:

Plant Rule i:

If $\theta_1(t)$ is ν_{i1} and $\theta_q(t)$ is ν_{iq}, then

$$\begin{cases} \dot{x}(t) = A_i(r(t))x(t) + B_i(r(t))u(t), \\ y(t) = C(r(t))x(t) + G(r(t))f_s(t), \end{cases} \quad i = 1, 2, \cdots, N, \tag{1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, $y(t) \in R^p$ is the measurement output. $A_i(r(t)) \in R^{n\times n}$, $B_i(r(t)) \in R^{n\times m}$, $C_i(r(t)) \in R^{p\times n}$ and $G_i(r(t)) \in R^{p\times w}$ are system matrices. $f_s(t) \in R^w$ is the sensor fault. Besides, we assume that $G(r)$ is of full column rank.

Next, we recall some basic of the Markov chain. Let $r(t)$, $t \geq 0$ be a right-continuous Markov chain on the probability space taking values in a finite state space $S = \{1, 2, \cdots, N_r\}$ with generator $\Pi = (\pi_{lj})$ $(l, j \in S)$, which is called transition rate matrix (TRM) given by

$$P\{r(t+\Delta) = j|r(t) = l\} = \begin{cases} \pi_{lj}\Delta + o(\Delta), & j \neq l, \\ 1 + \pi_{ll}\Delta + o(\Delta), & j = l, \end{cases}$$

where $\Delta > 0$, and $\lim_{t\to 0} \frac{o(\Delta)}{\Delta} = 0$, $\pi_{lj} \geq 0$ is the transition rate from state l to j if $j \neq l$ and $\pi_{ll} = -\sum_{j\neq l} \pi_{lj}$.

For $r(t) = r \in S$, the system matrices of rth mode are denoted by A_{ir}, B_{ir}, C_r and G_r, which are real and known. x_0 and r_0 stand for the initial values of $x(t)$ and $r(t)$, respectively.

By using a standard fuzzy singleton inference method[19], the overall fuzzy MJSs in (1) can be expressed as:

$$\begin{cases} \dot{x}(t) = \displaystyle\sum_{i=1}^{N} \rho_i(\theta(t))(A_{ir}x(t) + B_{ir}u(t)), \\ y(t) = C_r x(t) + G_r f_s(t), \end{cases} \tag{2}$$

where $\rho_i(\theta(t)) = \frac{\vartheta_i(\theta(t))}{\sum_{i=1}^{N} \vartheta_i(\theta(t))}$, $\vartheta_i(\theta(t)) = \prod_{g=1}^{q} \nu_{ig}(\theta_g(t))$, and $\nu_{ig}(\theta_g(t))$ is the grade of membership of $\theta_g(t)$ in ν_{ig}. According to the theory of fuzzy sets, it is known that $\rho_i(\theta(t)) \geq 0$ $(i = 1, 2, \cdots, N)$, and $\sum_{i=1}^{N} \rho_i(\theta(t)) = 1$.

Just as discussed in the introduction part, the transition rates are often partially unknown, and in this case, the transition rates of the jumping process $r(t)$ are considered to be partially accessed, i.e., some elements in matrix $\varPi$ are unknown. For instance, for System (2) with 2 operation modes, the TRM $\varPi$ may be as $\left[\begin{smallmatrix} \pi_{11} & ? \\ ? & ? \end{smallmatrix}\right]$, where ? represents the inaccessible element. For notational clarity, we denote $S = S_k^r + S_{uk}^r$ with

$$S_k^r = \{j : \pi_{rj} \text{ is known}\}, \quad S_{uk}^r = \{j : \pi_{rj} \text{ is unknown}\}.$$

Throughout this paper, the following definition will be adopted.

Definition 2.1 The Markovian jump system (2) is said to be stochastically stable if, for $u(t) \equiv 0$ and $f_s(t) \equiv 0$ and every initial condition $x_0 \in R^n$ and r_0, the following holds:

$$\mathrm{E}\left\{ \int_0^\infty \|x(t)\|^2 dt | x_0, r_0 \right\} < \infty.$$

Remark 2.2 The accessibility of the jumping process $r(t)$ in the existing literature is commonly assumed to be completely accessible ($S_k^r = S, S_{uk}^r = \emptyset$) or completely inaccessible ($S_{uk}^r = S, S_k^r = \emptyset$). Therefore, the considered TRM is a more general assumption to the Markovian jump systems thus covers the existing ones.

3 Main Results

If we extend the state $x(t)$ to $\overline{x}(t) = \left[\begin{smallmatrix} x(t) \\ f_s(t) \end{smallmatrix}\right] \in R^{n+w}$, and correspondingly, denote $E = \left[\begin{smallmatrix} I_n & 0_{n\times w} \end{smallmatrix}\right]$, $\overline{A}_{ir} = \left[\begin{smallmatrix} A_{ir} & 0_{n\times w} \end{smallmatrix}\right]$ and $\overline{C}_r = \left[\begin{smallmatrix} C_r & G_r \end{smallmatrix}\right]$, System (2) can be written in anther representation as

$$\begin{cases} E\dot{\overline{x}}(t) = \sum\limits_{i=1}^{N} \rho_i(\theta(t))(\overline{A}_{ir}\overline{x}(t) + B_{ir}u(t)), \\ y(t) = \overline{C}_r\overline{x}(t). \end{cases} \tag{3}$$

Now, System (2) is transformed into a descriptor system of which the state is composed of the original state and the sensor fault. In order to estimate the state and sensor fault simultaneously, a novel reduced-order observer will be designed for System (3) in the following discussion. After getting the estimated states, the observer-based fault-tolerant controller will be developed.

For design purpose, we denote a new variable as $\alpha(t)=M\overline{x}(t)$, where we assume $M = \left[\begin{smallmatrix} M_1 & 0 \\ 0 & I_w \end{smallmatrix}\right] \in R^{(n+w-p)\times(n+w)}$ and $M_1 \in R^{(n-p)\times n}$ is a known matrix which satisfies $S_r = \left[\begin{smallmatrix} C_r \\ M_1 \end{smallmatrix}\right] \in R^{n\times n}$ being nonsingular. By noticing $\alpha(t)$, the reduced-order observer is constructed as follows:

$$\begin{cases} \dot{z}(t) = \sum\limits_{i=1}^{N} \rho_i(\theta(t))(N_{ir}(r)z(t) + L_{ir}(r)y(t) + T_rB_{ir}u), \\ \widehat{\alpha}(t) = z(t) + Q_ry(t), \end{cases} \tag{4}$$

where $z(t) \in R^{n+w-p}$ and $\widehat{\alpha}(t) \in R^{n+w-p}$ are intermediate variables. $N_{ir} \in R^{(n-p+w)\times(n-p+w)}$, $L_{ir} \in R^{(n-p+w)\times p}$, $T_r \in R^{(n-p+w)\times m}$ and $Q_r \in R^{(n-p+w)\times p}$ are observer matrices to be determined later.

Define the observation error as $e(t) = \alpha(t) - \widehat{\alpha}(t)$, then we have

$$e(t) = M\overline{x}(t) - z(t) - Q_r y(t) = (M - Q_r\overline{C}_r)\overline{x}(t) - z(t).$$

If the observer matrices T_r and Q_r satisfy

$$T_r E + Q_r \overline{C}_r = M, \tag{5}$$

we can have

$$e(t) = T_r E\overline{x}(t) - z(t).$$

Then, the dynamic equation of the observer error $e(t)$ can be obtained by subtracting (4) from (3):

$$\begin{aligned}
\dot{e}(t) &= T_r E\dot{\overline{x}}(t) - \dot{z}(t)\\
&= \sum_{i=1}^{N} \rho_i(\theta(t))(T_r\overline{A}_{ir}\overline{x}(t) + T_r B_{ir}u(t) - N_{ir}z(t) - L_{ir}y(t) - T_r B_{ir}u(t))\\
&= \sum_{i=1}^{N} \rho_i(\theta(t))(T_r\overline{A}_{ir}\overline{x}(t) + T_r B_{ir}u(t) - N_{ir}z(t) - L_{ir}y(t) - T_r B_{ir}u(t)\\
&\quad + N_{ir}T_r E\overline{x}(t) - N_{ir}T_r E\overline{x}(t))\\
&= \sum_{i=1}^{N} \rho_i(\theta(t))(T_r\overline{A}_{ir}\overline{x}(t) - L_{ir}y(t) + N_{ir}(T_r E\overline{x}(t) - z(t)) - N_{ir}T_r E\overline{x}(t))\\
&= \sum_{i=1}^{N} \rho_i(\theta(t))(N_i(r)e(t) + (T_r\overline{A}_{ir} - N_{ir}T_r E - L_{ir}\overline{C}_r)\overline{x}(t)).
\end{aligned} \tag{6}$$

From observing the above equation, we can see that if the observer matrices N_{ir} and L_{ir} satisfy

$$T_r\overline{A}_{ir} - N_{ir}T_r E - L_{ir}\overline{C}_r = 0, \tag{7}$$

the observer error dynamic equation (6) becomes

$$\dot{e}(t) = \sum_{i=1}^{N} \rho_i(\theta(t))N_{ir}e(t). \tag{8}$$

Obviously, it is easy to find that N_{ir} and L_{ir} that satisfying

$$L_{ir} = K_{ir} + N_{ir}Q_r, \tag{9}$$

$$N_{ir}M + K_{ir}\overline{C}_r = T_r\overline{A}_{ir} \tag{10}$$

are a set of solutions of (7) with arbitrary compatible matrix K_{ir}.

Remark 3.1 Combining with (5), the left side of Equation (7) can be first written as

$$\begin{aligned}
&T_r\overline{A}_{ir} - N_{ir}T_r E - L_{ir}\overline{C}_r\\
&= T_r\overline{A}_{ir} - N_{ir}(M - Q_r\overline{C}_r) - L_{ir}\overline{C}_r\\
&= T_r\overline{A}_{ir} - N_{ir}M + N_{ir}Q_r\overline{C}_r - L_{ir}\overline{C}_r,
\end{aligned}$$

then substitute (9) and (10) into it, and we have

$$\begin{aligned}
&T_r\overline{A}_{ir} - N_{ir}M + N_{ir}Q_r\overline{C}_r - L_{ir}\overline{C}_r\\
&= T_r\overline{A}_{ir} - (T_r\overline{A}_{ir} - K_{ir}\overline{C}_r) + N_{ir}Q_r\overline{C}_r - (K_{ir} + N_{ir}Q_r)\overline{C}_r\\
&= T_r\overline{A}_{ir} - T_r\overline{A}_{ir} + K_{ir}\overline{C}_r + N_{ir}Q_r\overline{C}_r - K_{ir}\overline{C}_r - N_{ir}Q_r\overline{C}_r\\
&= 0.
\end{aligned}$$

So this just proves that (9) and (10) are a set of solutions of (7).

Because G_r is of full column rank, so is $\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix} = \begin{bmatrix} I_n & 0_{n\times w} \\ C_r & G_r \end{bmatrix}$. Since Equation (5) can be transformed into $\begin{bmatrix} T_r & Q_r \end{bmatrix}\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix} = M$, hence, we can calculate the solutions of T_r and Q_r from (5):

$$T_r = M\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}^+ \begin{bmatrix} I_n \\ 0_{p\times n} \end{bmatrix} - Z_r\left(I - \begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}^+\right)\begin{bmatrix} I_n \\ 0_{p\times n} \end{bmatrix}, \tag{11}$$

$$Q_r = M\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}^+ \begin{bmatrix} 0_{n\times p} \\ I_p \end{bmatrix} - Z_r\left(I - \begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}^+\right)\begin{bmatrix} 0_{n\times p} \\ I_p \end{bmatrix}, \tag{12}$$

where $\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}^+ = \left(\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}\right)^{-1}\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}^{\mathrm{T}}$ is the Penrose-Moore inverse of $\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}$. Further, we note (11) and (12) in another form as

$$T_r = T_{r1} - Z_rT_{r2}, \tag{13}$$

$$Q_r = Q_{r1} - Z_rQ_{r2}, \tag{14}$$

by denoting

$$T_{r1} = M\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}^+ \begin{bmatrix} I_n \\ 0_{p\times n} \end{bmatrix}, \quad T_{r2} = \left(I - \begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}^+\right)\begin{bmatrix} I_n \\ 0_{p\times n} \end{bmatrix},$$

$$Q_{r1} = M\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}^+ \begin{bmatrix} 0_{n\times p} \\ I_p \end{bmatrix}, \quad Q_{r2} = \left(I - \begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}\begin{bmatrix} E \\ \overline{C}_r \end{bmatrix}^+\right)\begin{bmatrix} 0_{n\times p} \\ I_p \end{bmatrix}.$$

It can be seen from (10) that

$$\begin{bmatrix} N_{ir} & K_{ir} \end{bmatrix}\begin{bmatrix} M \\ \overline{C}_r \end{bmatrix} = T_r\overline{A}_{ir} \tag{15}$$

and we know that since S_r is nonsingular, we have

$$\operatorname{rank}\left(\begin{bmatrix} M \\ \overline{C}_r \end{bmatrix}\right) = \operatorname{rank}\left(\begin{bmatrix} M_1 & 0 \\ 0 & I_w \\ C_r & G_r \end{bmatrix}\right) = \operatorname{rank}\left(\begin{bmatrix} 0 & I_w & 0 \\ I_{n-p} & 0 & 0 \\ 0 & -G_r & I_p \end{bmatrix}\begin{bmatrix} M_1 & 0 \\ 0 & I_w \\ C_r & G_r \end{bmatrix}\right)$$

$$= \operatorname{rank}\left(\begin{bmatrix} 0 & I_w \\ M_1 & 0 \\ C_r & 0 \end{bmatrix}\right) = n + w,$$

which implies that $\left[\begin{smallmatrix} M \\ \overline{C}_r \end{smallmatrix}\right]$ is also nonsingular. Then, we can get the solutions of N_{ir} and K_{ir} by solving (15):

$$N_{ir} = T_r\overline{A}_{ir}\begin{bmatrix} M \\ \overline{C}_r \end{bmatrix}^{-1}\begin{bmatrix} I_{n-p+w} \\ 0_{p\times(n-p+w)} \end{bmatrix}, \quad K_{ir} = T_r\overline{A}_{ir}\begin{bmatrix} M \\ \overline{C}_r \end{bmatrix}^{-1}\begin{bmatrix} 0_{(n-p+w)\times p} \\ I_p \end{bmatrix}.$$

Substitute (13) into the above equation, one can obtain

$$N_{ir} = N_{ir1} - Z_r N_{ir2}, \tag{16}$$

$$K_{ir} = K_{ir1} - Z_r K_{ir2}, \tag{17}$$

where

$$N_{ir1} = T_{r1}\overline{A}_{ir}\begin{bmatrix} M \\ \overline{C}_r \end{bmatrix}^{-1}\begin{bmatrix} I_{n-p+w} \\ 0_{p\times(n-p+w)} \end{bmatrix}, \quad N_{ir2} = T_{r2}\overline{A}_{ir}\begin{bmatrix} M \\ \overline{C}_r \end{bmatrix}^{-1}\begin{bmatrix} I_{n-p+w} \\ 0_{p\times(n-p+w)} \end{bmatrix},$$

$$K_{ir1} = T_{r1}\overline{A}_{ir}\begin{bmatrix} M \\ \overline{C}_r \end{bmatrix}^{-1}\begin{bmatrix} 0_{(n-p+w)\times p} \\ I_p \end{bmatrix}, \quad K_{ir2} = T_{r2}\overline{A}_{ir}\begin{bmatrix} M \\ \overline{C}_r \end{bmatrix}^{-1}\begin{bmatrix} 0_{(n-p+w)\times p} \\ I_p \end{bmatrix}.$$

From the previously discussions we can see that the observer matrices N_{ir}, L_{ir}, T_{ir} and Q_{ir} can be computed out once Z_r is determined. The selection of suitable Z_r that makes the error dynamic equation (8) is stable will be discussed later.

After getting the estimation of α based on the reduced-order observer (4), we obtain the estimated sensor fault from $\widehat{f}_s(t) = \left[\, 0_{w\times(n-p)} \;\, I_w \,\right]\widehat{\alpha}(t)$. Extend $M_1\widehat{x}(t)$ as $\left[\begin{smallmatrix} M_1\widehat{x}(t) \\ C_r\widehat{x}(t) \end{smallmatrix}\right]$, and we can have $\left[\begin{smallmatrix} M_1\widehat{x}(t) \\ C_r\widehat{x}(t) \end{smallmatrix}\right] = S_r\widehat{x}(t) = \left[\begin{smallmatrix} [\, I_{n-p} \;\, 0_{(n-p)\times w} \,]\widehat{\alpha}(t) \\ y(t) - G_r\widehat{f}_s(t) \end{smallmatrix}\right]$. Then, the asymptotical estimations of the state can be obtained from

$$\widehat{x}(t) = S_r^{-1}\begin{bmatrix} \begin{bmatrix} I_{n-p} & 0_{(n-p)\times w} \end{bmatrix}\widehat{\alpha}(t) \\ y(t) - G_r\widehat{f}_s(t) \end{bmatrix}$$

by noticing that S_r is nonsingular.

Next, we shall focus on the observer-based fault-tolerant controller design for plant (2). We construct the following observer-based controller as:

$$u(t)=\sum_{i=1}^{N}\rho_i(\theta(t))\widehat{K}_{ir}\widehat{x}(t), \tag{18}$$

where $\widehat{x}(t)$ is the estimated state provided by (4), and $\widehat{K}_{ir}$ will be designed later in the following analysis.

Substitute (18) into (2), and we can obtain the whole resulting closed-loop system

$$\dot{x}(t)=\sum_{i=1}^{N}\rho_i(\theta(t))\sum_{j=1}^{N}\rho_j(\theta(t))((A_{ir}+B_{ir}\widehat{K}_{jr})x(t)-B_{ir}\widehat{K}_{jr}e_x(t)), \tag{19}$$

where $e_x(t)=x(t)-\widehat{x}(t)$. If we can find suitable matrix Z_r such that system (8) is stable, then we have $e_x(t)\rightarrow 0$. Hence, if $\widehat{K}_{jr}$ can be designed to ensure the stability of the following system, $x(t)$ will be stable

$$\dot{x}(t)=\sum_{i=1}^{N}\rho_i(\theta(t))\sum_{j=1}^{N}\rho_j(\theta(t))((A_{ir}+B_{ir}\widehat{K}_{jr})x(t)). \tag{20}$$

The following theorem is the major result of the present paper which provides the existence conditions of the proposed observer and controller.

Theorem 3.2 *If there exist symmetric positive definite matrices $P_r\in R^{(n+w-p)\times(n+w-p)}$, $R_r\in R^{n\times n}$, symmetric matrices $X_{xr}\in R^{n\times n}$ and $X_{er}\in R^{(n+w-p)\times(n+w-p)}$, and matrices $Y_r\in R^{(n+w-p)\times(n+p)}$ such that the following matrix inequalities hold*

$$\begin{bmatrix} \begin{array}{l} R_r(A_{ir}+B_{ir}\widehat{K}_{jr})+(A_{ir}+B_{ir}\widehat{K}_{jr})^{\mathrm{T}}R_r \\ +\sum\limits_{k'\in S_k^r}\pi_{rk'}(R_{k'}-X_{xr}) \end{array} & 0 \\ 0 & \begin{array}{r} P_rN_{ir1}-Y_rN_{ir2}+N^{\mathrm{T}}{}_{ir1}P_r \\ -(Y_rN_{ir2})^{\mathrm{T}}+\sum\limits_{k'\in S_k^r}\pi_{rk'}(P_{k'}-X_{er}) \end{array} \end{bmatrix}<0, \tag{21}$$

$$R_{k'}-X_{xr}\geq 0,\quad k'\in S_{uk}^r,\quad k'=r, \tag{22}$$

$$R_{k'}-X_{xr}\leq 0,\quad k'\in S_{uk}^r,\quad k'\neq r, \tag{23}$$

$$P_{k'}-X_{er}\geq 0,\quad k'\in S_{uk}^r,\quad k'=r, \tag{24}$$

$$P_{k'}-X_{er}\leq 0,\quad k'\in S_{uk}^r,\quad k'\neq r, \tag{25}$$

where $Z_r=P_r^{-1}Y_r$, then the observer error system (8) *and closed-loop system* (19) *is stochastically stable.*

Proof Consider the Lyapunov candidate function as $V(x(t),e(t),r(t))=x^{\mathrm{T}}(t)R(r(t))x(t)+e^{\mathrm{T}}(t)P(r(t))e(t)$, and for each $r\in S$, take the weak infinitesimal operator along the trajectories of (8) and (20), then it follows that

$$\begin{aligned}
&\ell V(x(t),e(t),r)\\
&=\sum_{i=1}^{N}\rho_i(\theta(t))\sum_{j=1}^{N}\rho_j(\theta(t))\Bigg(x^{\mathrm{T}}(t)(R_r(A_{ir}+B_{ir}\widehat{K}_{jr})+(A_{ir}+B_{ir}\widehat{K}_{jr})^{\mathrm{T}}R_r)x(t)\\
&\quad+x^{\mathrm{T}}(t)\sum_{k'=1}^{N_r}\pi_{rk'}R_{k'}x(t)+e^{\mathrm{T}}(t)(P_rN_{ir}+N_{ir}^{\mathrm{T}}P_r)e(t)+e^{\mathrm{T}}(t)\sum_{k'=1}^{N_r}\pi_{rk'}P_{k'}e(t)\Bigg)\\
&=\sum_{i=1}^{N}\rho_i(\theta(t))\sum_{j=1}^{N}\rho_j(\theta(t))(x^{\mathrm{T}}(t)\Bigg(R_r(A_{ir}+B_{ir}\widehat{K}_{jr})+(A_{ir}+B_{ir}\widehat{K}_{jr})^{\mathrm{T}}R_rx(t)\\
&\quad+x^{\mathrm{T}}(t)\sum_{k'\in S_{uk}^r}\pi_{rk'}R_{k'}x(t)+x^{\mathrm{T}}(t)\sum_{k'\in S_k^r}\pi_{rk'}R_{k'}x(t)+e^{\mathrm{T}}(t)(P_rN_{ir}+N_{ir}^{\mathrm{T}}P_r)e(t)\\
&\quad+e^{\mathrm{T}}(t)\sum_{k'\in S_{uk}^r}\pi_{rk'}P_{k'}e(t)+e^{\mathrm{T}}(t)\sum_{k'\in S_k^r}\pi_{rk'}P_{k'}e(t)\Bigg),
\end{aligned}\tag{26}$$

by noticing $S=S_k^r+S_{uk}^r$. It is the fact that

$$\sum_{k'\in S}\pi_{rk'}x^{\mathrm{T}}(t)X_{xr}x(t)=0\quad\text{and}\quad\sum_{k'\in S}\pi_{rk'}e^{\mathrm{T}}(t)X_{er}e(t)=0,$$

where X_{xr} and X_{er} are symmetric matrices, since $\sum_{k'\in S}\pi_{rk'}=0$. Thus, we add the term $-\sum_{k'\in S}\pi_{rk'}x^{\mathrm{T}}(t)X_{xr}x(t)$ and $-\sum_{k'\in S}\pi_{rk'}e^{\mathrm{T}}(t)X_{er}e(t)$ to the right side of (26), and it becomes

$$\begin{aligned}
&\ell V(x(t),e(t),r)\\
&=\sum_{i=1}^{N}\rho_i(\theta(t))\sum_{j=1}^{N}\rho_j(\theta(t))(x^{\mathrm{T}}(t)(R_r(A_{ir}+B_{ir}\widehat{K}_{jr})+(A_{ir}+B_{ir}\widehat{K}_{jr})^{\mathrm{T}}R_rx(t)\\
&\quad+x^{\mathrm{T}}(t)\sum_{k'\in S_{uk}^r}\pi_{rk'}R_{k'}x(t)+x^{\mathrm{T}}(t)\sum_{k'\in S_k^r}\pi_{rk'}R_{k'}x(t)+e^{\mathrm{T}}(t)(P_rN_{ir}+N_{ir}^{\mathrm{T}}P_r)e(t)\\
&\quad+e^{\mathrm{T}}(t)\sum_{k'\in S_{uk}^r}\pi_{rk'}P_{k'}e(t)+e^{\mathrm{T}}(t)\sum_{k'\in S_k^r}\pi_{rk'}P_{k'}e(t))-x^{\mathrm{T}}(t)\sum_{k'=1}^{N_r}\pi_{rk'}X_{xr}x(t)\\
&\quad-e^{\mathrm{T}}(t)\sum_{k'=1}^{N_r}\pi_{rk'}X_{er}e(t)\\
&=\sum_{i=1}^{N}\rho_i(\theta(t))\sum_{j=1}^{N}\rho_j(\theta(t))(x^{\mathrm{T}}(t)(R_r(A_{ir}+B_{ir}\widehat{K}_{jr})+(A_{ir}+B_{ir}\widehat{K}_{jr})^{\mathrm{T}}R_rx(t)\\
&\quad+x^{\mathrm{T}}(t)\sum_{k'\in S_{uk}^r}\pi_{rk'}R_{k'}x(t)+x^{\mathrm{T}}(t)\sum_{k'\in S_k^r}\pi_{rk'}R_{k'}x(t)+e^{\mathrm{T}}(t)(P_rN_{ir}+N_{ir}^{\mathrm{T}}P_r)e(t)\\
&\quad+e^{\mathrm{T}}(t)\sum_{k'\in S_{uk}^r}\pi_{rk'}P_{k'}e(t)+e^{\mathrm{T}}(t)\sum_{k'\in S_k^r}\pi_{rk'}P_{k'}e(t)\\
&\quad-x^{\mathrm{T}}(t)\Bigg(\sum_{k'\in S_k^r}\pi_{rk'}X_{xr}+\sum_{k'\in S_{uk}^r}\pi_{rk'}X_{xr}\Bigg)x(t)\\
&\quad-e^{\mathrm{T}}(t)\Bigg(\sum_{k'\in S_{uk}^r}\pi_{rk'}X_{er}+\sum_{k'\in S_k^r}\pi_{rk'}X_{er}\Bigg)e(t)
\end{aligned}$$

$$
\begin{aligned}
&= \sum_{i=1}^{N} \rho_i(\theta(t)) \sum_{j=1}^{N} \rho_j(\theta(t)) (x^{\mathrm{T}}(t)(R_r(A_{ir} + B_{ir}\widehat{K}_{jr}) + (A_{ir} + B_{ir}\widehat{K}_{jr})^{\mathrm{T}} R_r x(t) \\
&\quad + x^{\mathrm{T}}(t) \sum_{k' \in S_{uk}^r} \pi_{rk'}(R_{k'} - X_{xr})x(t) + x^{\mathrm{T}}(t) \sum_{k' \in S_k^r} \pi_{rk'}(R_{k'} - X_{xr})x(t) \\
&\quad + e^{\mathrm{T}}(t)(P_r N_{ir} + N_{ir}^{\mathrm{T}} P_r)e(t) + e^{\mathrm{T}}(t) \sum_{k' \in S_{uk}^r} \pi_{rk'}(P_{k'} - X_{er})e(t) \\
&\quad + e^{\mathrm{T}}(t) \sum_{k' \in S_k^r} \pi_{rk'}(P_{k'} - X_{er})e(t).
\end{aligned} \tag{27}
$$

For unknown the term $x^{\mathrm{T}}(t)\sum_{k'\in S_{uk}^r} \pi_{rk'}(R_{k'} - X_{xr})x(t)$ and $e^{\mathrm{T}}(t)\sum_{k'\in S_k^r} \pi_{rk'}(P_{k'} - X_{er})e(t)$, we will argue in four cases:

(I) When $r = k'$, we have $\pi_{rk'} \leq 0$, then (22) implies $x^{\mathrm{T}}(t)\sum_{k'\in S_{uk}^r} \pi_{rk'}(R_{k'} - X_{xr})x(t) \leq 0$.

(II) When $r \neq k'$, we have $\pi_{rk'} \geq 0$, then (23) implies $x^{\mathrm{T}}(t)\sum_{k'\in S_{uk}^r} \pi_{rk'}(R_{k'} - X_{xr})x(t) \leq 0$.

(III) When $r = k'$, we have $\pi_{rk'} \leq 0$, then (24) implies $e^{\mathrm{T}}(t)\sum_{k'\in S_{uk}^r} \pi_{rk'}(P_{k'} - X_{er})e(t) \leq 0$.

(IV) When $r \neq k'$, we have $\pi_{rk'} \geq 0$, then (25) implies $e^{\mathrm{T}}(t)\sum_{k'\in S_{uk}^r} \pi_{rk'}(P_{k'} - X_{er})e(t) \leq 0$.

Thus, from analyzing Cases (I)–(IV), we can conclude that

$$
x^{\mathrm{T}}(t) \sum_{k' \in S_{uk}^r} \pi_{rk'}(R_{k'} - X_{xr})x(t) \leq 0, \tag{28}
$$

$$
e^{\mathrm{T}}(t) \sum_{k' \in S_{uk}^r} \pi_{rk'}(P_{k'} - X_{er})e(t) \leq 0. \tag{29}
$$

Substitute (28) and (29) into (27) we have

$$
\begin{aligned}
&\ell V(x(t), e(t), r) \\
&\leq \sum_{i=1}^{N} \rho_i(\theta(t)) \sum_{j=1}^{N} \rho_j(\theta(t)) \Big(x^{\mathrm{T}}(t)(R_r(A_{ir} + B_{ir}\widehat{K}_{jr}) + (A_{ir} + B_{ir}\widehat{K}_{jr})^{\mathrm{T}} R_r)x(t) \\
&\quad + e^{\mathrm{T}}(t)(P_r N_{ir} + N_{ir}^{\mathrm{T}} P_r)e(t) + x^{\mathrm{T}}(t) \sum_{k' \in S_k^r} \pi_{rk'}(R_{k'} - X_{xr})x(t) \\
&\quad + e^{\mathrm{T}}(t) \sum_{k' \in S_k^r} \pi_{rk'}(P_{k'} - X_{er})e(t) \Big) \\
&= \sum_{i=1}^{N} \rho_i(\theta(t)) \sum_{j=1}^{N} \rho_j(\theta(t)) \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}^{\mathrm{T}} \varOmega_{ijr} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix},
\end{aligned} \tag{30}
$$

where $\varOmega_{ijr} = \left[\begin{smallmatrix} \varTheta_{ijr} & 0 \\ 0 & \varPhi_{ijr} \end{smallmatrix}\right]$, $\varTheta_{ijr} = R_r(A_{ir} + B_{ir}\widehat{K}_{jr}) + (A_{ir} + B_{ir}\widehat{K}_{jr})^{\mathrm{T}} R_r + \sum_{k'\in S_k^r} \pi_{rk'}(R_{k'} - X_{xr})$, and $\varPhi_{ijr} = P_r N_{ir} + N_{ir}^{\mathrm{T}} P_r + \sum_{k'\in S_k^r} \pi_{rk'}(P_{k'} - X_{er})$.

Substitute (16) into (30) and let $Y_r = P_r Z_r$, and one have

$$
\ell V(x(t), e(t), r) \leq \sum_{i=1}^{N} \rho_i(\theta(t)) \sum_{j=1}^{N} \rho_j(\theta(t)) \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}^{\mathrm{T}} \varOmega_{ijr} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} < 0.
$$

According to (21). Further we can deduce that

$$\ell V(x(t),e(t),r) \leq -\sum_{i=1}^{N}\rho_i(\theta(t))\sum_{j=1}^{N}\rho_j(\theta(t))\delta\begin{bmatrix} x(t)\\ e(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} x(t)\\ e(t)\end{bmatrix}, \tag{31}$$

where $\delta = \min_{i,j,r}\lambda_{\min}(-\Omega_{ijr})$.

Integrate on both sides of (31) under zero initial condition, and apply Dynkin's formula $\int_0^\infty \ell V(x(t),e(t),r) = \mathrm{E}\{V(x(\infty),e(\infty),r)\} - \mathrm{E}\{V(x(0),e(0),r_0)\}$, we can yield

$$\begin{aligned}
&\int_0^\infty \ell V(x(t),e(t),r)\\
&= E\{V(x(\infty),e(\infty),r)\} - \mathrm{E}\{V(x(0),e(0),r_0)\}\\
&\leq \int_0^\infty -\sum_{i=1}^{N}\rho_i(\theta(t))\sum_{j=1}^{N}\rho_j(\theta(t))\delta\begin{bmatrix} x(t)\\ e(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} x(t)\\ e(t)\end{bmatrix}dt\\
&= -\delta\int_0^\infty \begin{bmatrix} x(t)\\ e(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} x(t)\\ e(t)\end{bmatrix}dt.
\end{aligned}$$

By noticing $\sum_{i=1}^{N}\rho_i(\theta(t)) = 1$. As $V(x(\infty),e(\infty),r) > 0$, we can have from the above inequality

$$\int_0^\infty \left(\begin{bmatrix} x(t)\\ e(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} x(t)\\ e(t)\end{bmatrix}\right)dt \leq \frac{\mathrm{E}\{V(x(0),e(0),r_0)\}}{\delta} < \infty,$$

which implies that $\int_0^\infty x^{\mathrm{T}}(t)x(t)dt < \infty$ and $\int_0^\infty e^{\mathrm{T}}(t)e(t)dt < \infty$. According to Definition 2.1, we can see that both (8) and (20) are stochastically stable, which in turn ensures the stability of (19). This completes the proof. ∎

We can obviously find that the matrix inequality conditions (21) are not linear, which puts obstacle in computing matrices Z_r and K_{ir}. Hence, next by using some subtle skills, we transform (21)–(25) into linear matrix inequalities those can be easily solved to provide the appropriate Z_r and K_{ir}.

Theorem 3.3 *If there exist symmetric positive definite matrices $P_r \in R^{(n+w-p)\times(n+w-p)}$, $U_r \in R^{n\times n}$, symmetric matrices $\widetilde{X}_{xr} \in R^{n\times n}$ and $X_{er} \in R^{(n+w-p)\times(n+w-p)}$, and matrices $Y_r \in R^{(n+w-p)\times(n+p)}$ such that the following linear matrix inequalities hold:*

(i) *For $r \notin S_k^r$,*

$$\begin{bmatrix}
\Im_{ijr} & 0 & U_r & U_r & U_r\\
* & \wp_{ir} & 0 & \cdots & 0\\
* & * & -\pi_{rl_1^r}^{-1}U_{l_1^r} & \ddots & 0\\
* & * & 0 & \ddots & 0\\
* & * & 0 & 0 & -\pi_{rl_m^r}^{-1}U_{l_m^r}
\end{bmatrix} < 0; \tag{32}$$

(ii) *For* $r \in S_k^r$, $l_m^r \neq r$,

$$\begin{bmatrix} \overline{\Im}_{ijr} & 0 & U_r & U_r & U_r \\ * & \wp_{ir} & 0 & \cdots & 0 \\ * & * & -\pi_{rl_1^r}^{-1} U_{l_1^r} & \ddots & 0 \\ * & * & 0 & \ddots & 0 \\ * & * & 0 & 0 & -\pi_{rl_m^r}^{-1} U_{l_m^r} \end{bmatrix} < 0, \tag{33}$$

$$U_{k'} - \widetilde{X}_{xr} \geq 0, \quad k' \in S_{uk}^r, \quad k' = r, \tag{34}$$

$$\begin{bmatrix} -\widetilde{X}_{xr} & U_r \\ U_r & -U_{k'} \end{bmatrix} \leq 0, \quad k' \in S_{uk}^r, \quad k' \neq r, \tag{35}$$

$$P_{k'} - X_{er} \geq 0, \quad k' \in S_{uk}^r, \quad k' = r, \tag{36}$$

$$P_{k'} - X_{er} \leq 0, \quad k' \in S_{uk}^r, \quad k' \neq r, \tag{37}$$

where $\Im_{ijr} = A_{ir}U_r + B_{ir}\widehat{K}_{jr}U_r + (A_{ir}U_r + B_{ir}\widehat{K}_{jr}U_r)^{\mathrm{T}} - \sum_{k' \in S_k^r} \pi_{rk'}\widetilde{X}_{xr}$, $\wp_{ir} = P_r N_{ir1} - Y_r N_{ir2} + N^{\mathrm{T}}{}_{ir1} P_r - (Y_r N_{ir2})^{\mathrm{T}} + \sum_{k' \in S_k^r}^{S} \pi_{rk'}(P_{k'} - X_{er})$, $\overline{\Im}_{ijr} = A_{ir}U_r + B_{ir}\widehat{K}_{jr}U_r + (A_{ir}U_r + B_{ir}\widehat{K}_{jr}U_r)^{\mathrm{T}} - \sum_{k' \in S_k^r} \pi_{rk'}\widetilde{X}_{xr} + \pi_{rr}U_r$, $\widetilde{X}_{xr} = U_r X_{xr} U_r$ *and* l_m^r *represents the* m*th known element in* S_k^r*, then the observer error system* (8) *and closed-loop system* (19) *is stochastically stable.*

Proof Let $U_r = R_r^{-1}$, and pre- and post-multiply $\left[\begin{smallmatrix} U_r & 0 \\ 0 & I \end{smallmatrix}\right]$ and U_r on both sides of (21) and (22)–(23), we have

$$\begin{bmatrix} \begin{matrix} A_{ir}U_r + B_{ir}\widehat{K}_{jr}U_r + (A_{ir}U_r + B_{ir}\widehat{K}_{jr}U_r)^{\mathrm{T}} \\ -\sum_{k' \in S_k^r} \pi_{rk'} U_r X_{xr} U_r + \sum_{k' \in S_k^r} \pi_{rk'} U_r R_{k'} U_r \end{matrix} & 0 \\ 0 & \begin{matrix} P_r N_{ir1} - Y_r N_{ir2} + N^{\mathrm{T}}{}_{ir1} P_r \\ -(Y_r N_{ir2})^{\mathrm{T}} + \sum_{k' \in S_k^r} \pi_{rk'}(P_{k'} - X_{er}) \end{matrix} \end{bmatrix} < 0,$$

$$U_{k'} R_{k'} U_{k'} - U_r X_{xr} U_r \geq 0, \quad k' \in S_{uk}^r, \quad k' = r,$$

$$U_r R_{k'} U_r - U_r X_{xr} U_r \leq 0, \quad k' \in S_{uk}^r, \quad k' \neq r.$$

Using Schur complement lemma, (32)–(37) can be easily accessible to by noticing $\widetilde{X}_{xr} = U_r X_{xr} U_r$ and $\pi_{rr} < 0$. This completes the proof. ∎

The whole design procedures are summarized as follows:

Step 1 Solve LMIs (32)–(37) to get $\widehat{K}_{ir}$ and Z_r.

Step 2 Substitute Z_r into (13)–(14) and (16) to obtain T_r, Q_r, N_{ir} and K_{ir}.

Step 3 Substitute K_{ir} into (17) to get L_{ir}.

Now the observer coefficient matrices T_r, Q_r, N_{ir}, L_{ir} and observer-based fault-tolerant control gain $\widehat{K}_{ir}$ are all obtained. The proposed reduced-order observer and fault-tolerant control can be constructed.

Remark 3.4 Although Reference [10–19] concerned the problems of simultaneous fault estimation methods, they were on the basis of designing full-order (augmented) observers. However, in this paper, we design a novel reduced-order observer to provide state and sensor fault estimations, which involves less integrators, compared with adaptive ones[15,16,19], and has a simpler structure, thus can be easier to implement.

Remark 3.5 Reference [10–12, 14] required the exact boundaries of the faults and their derivatives, which brought some conservatism, as it is usually difficult to access to the exact information of the faults in practice. However, in our method, we do not need any priori information of the faults, thus has a looser pre-conditions.

Remark 3.6 To our best knowledge, most of the fault estimation and fault-tolerant control methods for MJSs were designed under the assumptions of exact knowing of the transition rates, however, our proposed method is designed with partial unknown transition rates, which ensures generality as it can be directly applied to MJSs with exact transition rates, as shown in the simulation section.

Remark 3.7 In the existing observer-based fault-tolerant control works, the control gain matrices were determined before the designs, which led to that some control performances can not be guaranteed well, such as. In the proposed method, the control gain matrix can be computed online, which brings some freedom to the design.

Remark 3.8 In practical control systems, although sensor faults are common, actuator faults also occur frequently, however, this manuscript only discusses sensor fault estimation, which seems limited. So it will make big significance to study simultaneous actuator and sensor fault estimation based on reduced-order observer. In our next work, we intend to combine the proposed method with descriptor system theory to accomplish the simultaneous estimation problems.

4 Simulation

4.1 Numerical Study

We consider the plant (2) associates with Modes (1) and (2) and two fuzzy rules, and the system data are chosen as follows:

$$A_{11} = \begin{bmatrix} -1.5 & -1.3 \\ -1.1 & -2 \end{bmatrix}, \ A_{21} = \begin{bmatrix} -1.5 & -0.3 \\ -3 & -1 \end{bmatrix}, \ A_{12} = \begin{bmatrix} -5 & -0.4 \\ -1 & -1.06 \end{bmatrix}, \ A_{22} = \begin{bmatrix} -2.5 & -1.4 \\ -2.1 & -2.06 \end{bmatrix},$$

$$B_{11} = B_{12} = B_{21} = B_{22} = \begin{bmatrix} -1 \\ 0.5 \end{bmatrix}, \quad C_1 = C_2 = \begin{bmatrix} 0.6 & 0.4 \end{bmatrix}, \quad G_1 = G_2 = 0.1.$$

The TRM has the following form $\mathit{\Pi} = (\pi_{rj})_{2\times 2} = \left[\begin{smallmatrix} -0.5 & 0.5 \\ ? & ? \end{smallmatrix}\right]$. We take the premise variable $\theta(t) = \left[\begin{smallmatrix} \theta_1(t) & \theta_2(t) & \cdots & \theta_q(t) \end{smallmatrix}\right]^{\mathrm{T}}$ as output, which can be measured online and assume that the membership functions are $\rho_1(\theta(t)) = \frac{1-\sin(y(t))}{2}$ and $\rho_2(\theta(t)) = 1 - \rho_1(\theta(t))$, respectively. We choose $M = \left[\begin{smallmatrix} -2 & 0 & 0 \\ 0 & 0 & 1 \end{smallmatrix}\right]$ to make sure S_r is nonsingular. According to Step 1, the control gains are obtained as $K_{11} = K_{12} = \begin{bmatrix} 2.1367 & -2.1289 \end{bmatrix}$, $K_{21} = \begin{bmatrix} 2.7909 & 0.4619 \end{bmatrix}$, $K_{22} = \begin{bmatrix} 3.2145 & 1.7863 \end{bmatrix}$, and we can also get

$$Z_1 = 10^4 \times \begin{bmatrix} -1.1235 & -1.0867 & -2.0012 \\ -1.0765 & -4.8970 & -3.2156 \end{bmatrix}$$

and

$$Z_2 = 10^4 \times \begin{bmatrix} -2.1256 & 1.6987 & -1.6578 \\ -8.7485 & 3.2356 & -1.2354 \end{bmatrix}.$$

Finally, we can compute the other observer matrices T_r, Q_r, N_{ir} and L_{ir} by following Steps 2–3.

For simulation, the sensor fault is set to be $f_s(t) = 3\cos(t) + \sin(t)$. The initial state of System (2) and observer (4) are $x(0) = \begin{bmatrix} 2 & 1 \end{bmatrix}$ and $z(0) = \begin{bmatrix} 0 & 4 \end{bmatrix}$, respectively. The estimations of state, which are activated by the fault-tolerant controller, are shown in Figures 1–2, illustrating that the closed-loop system is stochastically stable under the designed controller (18). The sensor fault estimation performance is depicted in Figure 3. Figure 4 shows the switching signal $r(t)$. It can be seen that both the states and sensor fault can be accurately estimated and controlled using the proposed method, thus proving the feasibility of the discussed method.

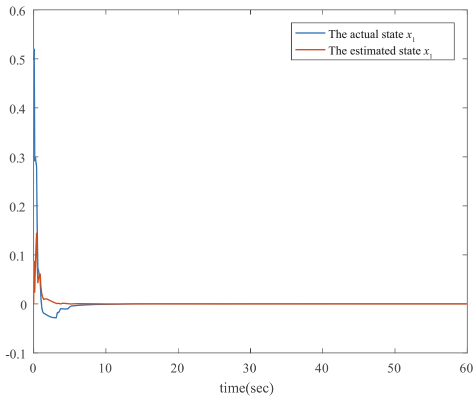

Figure 1 Stabilization and estimation of $x_1(t)$

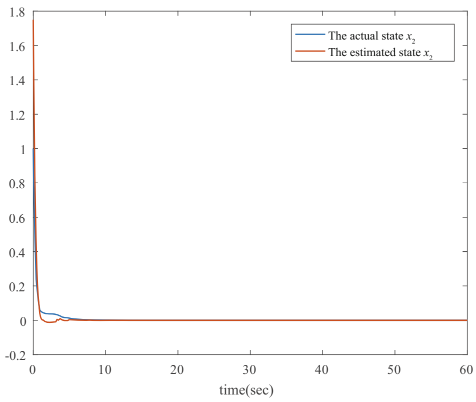

Figure 2 Stabilization and estimation of $x_2(t)$

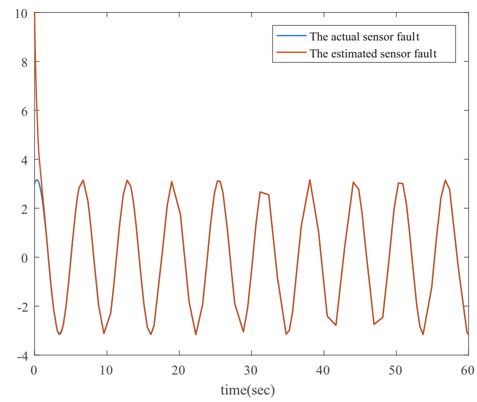

Figure 3 Stabilization and estimation of $f_s(t)$

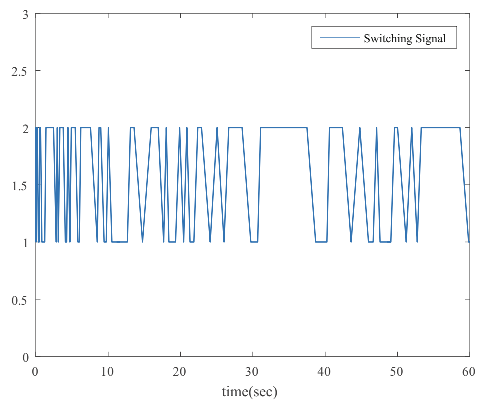

Figure 4 Switching signal $r(t)$

4.2 Compared Study

In order to testify the generality of the proposed method, we assume the transition rate matrix to be exact known as $\varPi = \left[\begin{smallmatrix} -1 & 1 \\ 2 & -2 \end{smallmatrix}\right]$. The existence conditions of the proposed observer and controller in Theorem 3.2 will become (33) with $k' \in S$. We obtain that

$$Z_1 = 10^3 \times \begin{bmatrix} 0.0014 & -0.3529 & -0.1504 \\ -0.0896 & -2.2666 & -1.0226 \end{bmatrix},$$

$$Z_2 = 10^4 \times \begin{bmatrix} -0.5293 & 0.7723 & 0.0283 \\ -1.7858 & 2.4672 & 0.0368 \end{bmatrix},$$

from solving the corresponding linear matrix inequalities. The simulation performance is depicted in Figures 5–6. From the simulation performance we can clearly see that the proposed method can be put into use for the MJSs with exact transition rates, as discussed in Remark 3.6.

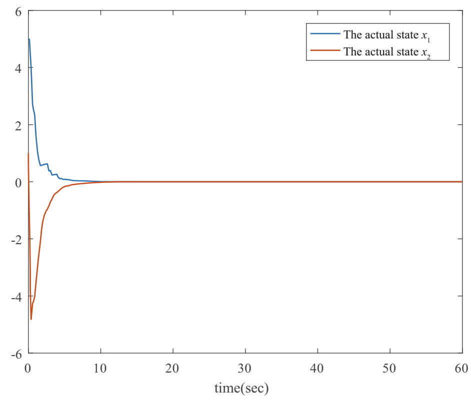

Figure 5 Stabilization of the states

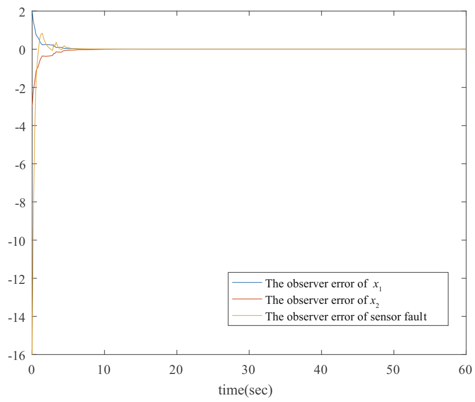

Figure 6 Observer errors of state and sensor fault stabilization of the states

5 Conclusions

In this paper, simultaneous estimations of state and sensor fault, together with the design of fault-tolerant control, are studied for a class of T-S fuzzy MJSs with PUTRs. A novel reduced-order observer is proposed to estimate the states and sensor faults simultaneously. Based on the state estimation, a fault-tolerant state-feedback controller is designed to ensure the stochastic stability for the resulting closed-loop system. Sufficient conditions of the existences of the observer and controller are given in terms of linear matrix inequalities. The simulation results show the validation and effectiveness of the proposed observer and controller.

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