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Input-to-State Stability of Switched Nonlinear Delay Systems Based on a Novel Lyapunov-Krasovskii Functional Method*

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Abstract In this paper, the input-to-state stability (ISS) analysis is addressed for switched nonlinear delay systems. By introducing a novel Lyapunov-Krasovskii functional with indefinite derivative and the merging switching signal techniques, some new criteria are established for switched nonlinear delay systems under asynchronous switching, which extends the existing results to the nonlinear systems with switching rules and delays. The ISS problem is also considered under synchronous switching for switched nonlinear systems by employing the similar techniques. Finally, a nonlinear delay model is provided to show the effectiveness of the proposed results.

Keywords Asynchronous switching, average dwell time, input-to-state stability, Lyapunov-Krasovskii functional, switched delay system.

1 Introduction

The switched nonlinear system, typically, contains a family of nonlinear subsystems and a logical rule that handles the switching between the subsystems. It is used to model many physical or man-made systems displaying switching features and has been extensively studied in past years[1–5]. Because time delays often occur in many practical control systems such as communication systems, and often lead to poor system performance even instability of the systems[6, 7], the influence of time delays for system performance can not be ignored. Switched systems with time delays are referred to as switched delay systems, which are a brand new type of systems. A large number of actual systems belong to typical switched delay systems, such

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as network control systems and mechanical rotational cutting processes[8, 9]. The dynamic behaviour of switched delay systems is more complicated than that of non-switched delay systems or switched systems without time delays due to the interaction among continuous dynamics, discrete dynamics and time delays. For switched delay systems, there are a number of results with respect to stability analysis and controller design[10–15].

Last two decades witnessed the rapid progress of input-to-state stability (ISS) since it was first proposed in [16]. ISS characterizes that the dynamic behavior of the system remains bounded when its inputs are bounded, and tends to equilibrium when the inputs go to zero, which is proved to be a very popular and useful tool for robustness analysis of nonlinear control systems influenced by external inputs[17, 18]. Recently, ISS has been extended to switched delay systems by the use of the multiple Lyapunov-Krasovskii functionals[19–21]. However, it should be pointed out that all these results require that the Lyapunov-Krasovskii functional has a negative definite derivative, which is difficult in practice. In [22], a new criterion is proposed for the ISS of nonlinear time-varying systems through the construction a $\mathcal{KL}$ function and the use of a new type of Lyapunov function for which the derivative is allowed to be positive definite during some periods. Inspired by this work, we shall establish a new Lyapunov-Krasovskii functional with indefinite derivative checking the ISS property of switched delay systems.

Usually, by an ideal switched system, we mean that the controller has instant access to both the plant's state and the plant's switching signal. In this case, we see that the controller's switching and the plant's switching are synchronized[23]. However, when the system and the controller communicate via a communication channel and the current subsystem is switched to next one, it will take some time to identify the active subsystem and then switch the controller from the current one to the corresponding one. For this case, the closed-loop system will feature asynchronous switching signal[24–27]. In [28], ISS criteria of time-varying stochastic systems are obtained by applying the generalized Razumikhin and Krasovskii stability theorems. And, the time-derivatives of the Razumikhin functions and Krasovskii functionals are not required to be negative definite. In [29], the problem of ISS for switched nonlinear time-varying system is considered by Lyapunov functions that its derivative are allowed to be indefinite. However, in the above work, the switching signal available to the controller is synchronized with the switching signal of the subsystems. In [30], the problem of ISS for the switched nonlinear delay systems is investigated, where the Lyapunov-Krasovskii functional is required to be negative definite. Motivated by these observations, we focus our attentions upon the ISS issue of switched nonlinear systems with time delays using the Lyapunov-like functional with indefinite derivative under the asynchronous switching. Borrowing the merging switching signal technique reported in [31] and utilizing the average dwell time technique, we establish some ISS conditions ensuring the ISS of the given switched delay systems by constructing Lyapunov-like functional with indefinite derivative under the asynchronous switching. Our results are proved to be more relaxed than those in [30]. In addition, we also provide the ISS conditions for switched nonlinear delay systems for the case of synchronous switching. Finally, a nonlinear delay model is provided to show the effectiveness of the proposed result.

The rest of the paper is organized as follows: Section 2 first defines ISS and other basic concepts. Section 3 constructs some new criterions for the ISS of switched nonlinear delay systems with synchronous and asynchronous switching. Section 4 gives a numerical example that illustrate the effectiveness of results. Section 5 gives some concluding remarks.

Notations Throughout this paper, the symbol $|\cdot|$ denotes a real vector or induced matrix norm for vectors in the Euclidean space. $\mathbb{R}^+$ denotes the set of nonnegative real numbers. $\mathbb{R}^n$ is the n-dimensional vector space and $\mathbb{N}$ is the set of nonnegative integers. For a measurable and essentially bounded function $u:\mathbb{R}^+\to\mathbb{R}^m$, we define its infinity norm $\|u\|_\infty=\operatorname{ess\,sup}_{t\geq 0}|u(t)|$, where "ess sup" stands for essential supremum. If we have $\|u\|_\infty<\infty$, then we write $u\in L_\infty^m$. τ is the input delay satisfying $0\leq\tau\leq r$. $C([-r,0];\mathbb{R}^n)$ denotes the set of the continuous functions mapping from $[-r,0]$ to $\mathbb{R}^n$. Given $r>0$, a norm on $C([-r,0];\mathbb{R}^n)$ is defined as $\|\phi\|_{M_2}=(\phi^T(0)\phi(0)+\int_{-r}^0\phi^T(s)\phi(s)ds)^{\frac{1}{2}}$ for any $\phi\in C([-r,0];\mathbb{R}^n)$. For each $t\in\mathbb{R}^+$, $x_t\in C([-r,0];\mathbb{R}^n)$, is defined as $x_t(s):=x(t+s)$, $-r\leq s\leq 0$. For a continuous-time signal $w(t)$, set $\|w[t_1,t_2]\|_\infty=\sup_{t_1\leq s\leq t_2}\{|w(s)|\}$. And, $S_{ave}[\tau_a,N_0]$ denotes the class of switching signals with average dwell time τ_a and chatter bound N_0.

2 Problem Formulation

Consider the following n-dimensional switched nonlinear delay system:

$$\dot{x}(t)=f_{\sigma(t)}(t,x_t,u(t)),\quad x_{t_0}(\theta)=\xi(\theta),\quad \theta\in[-r,0],\tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state; $u(t)\in\mathbb{R}^m$ is the input function; $\xi(\theta)\in C([-r,0];\mathbb{R}^n)$ is the initial data; $r\geq 0$ is the maximum involved delay. $\sigma(t):\mathbb{R}^+\to\mathcal{N}_c=\{1,2,\cdots,m\}$ specifies, at each time instant t, the index of the active subsystem. Corresponding to the switching signal $\sigma(t)$, we have the following switching sequence $\Sigma=\{\xi(\theta):(i_0,t_0),\cdots,(i_k,t_k),\cdots|i_k\in\mathcal{N}_c,k\in\mathbb{N}\}$, which means that the i_k-th subsystem is activated when $t\in[t_k,t_{k+1})$. It is assumed that no jump occurs in the state at a switching time and that only finitely many switchings can occur in any finite interval. For each $i_k\in\mathcal{N}_c$, $f_{i_k}:\mathbb{R}^+\times\mathbb{R}^n\times C([-r,0];\mathbb{R}^n)\times\mathbb{R}^m\to\mathbb{R}^n$ is completely continuous and locally Lipschitz so that the existence and uniqueness property holds for System (1). Throughout the whole paper, we assume that $f_{i_k}(0,0,0,0)=0$. In ideal cases, assume that $u(t)=g_{\sigma(t)}(t,x_t,w(t))$ with $w(t)\in L_\infty^m$, we have

$$\dot{x}(t)=f_{\sigma(t)}(t,x_t,g_{\sigma(t)}(t,x_t,w(t))),\quad t\geq t_0.$$

Thus, the corresponding closed-loop switched system can be written as

$$\dot{x}(t)=\overline{f}_{\sigma(t)}(t,x_t,w(t)).\tag{2}$$

We recall the following standard definitions. A continuous function $\gamma:[0,a)\to[0,\infty)$ is said to be of class $\mathcal{K}$ if it is strictly increasing and $\gamma(0)=0$. It is said to belong to class $\mathcal{K}_\infty$ if $a=\infty$ and $\gamma(r)\to\infty$ as $r\to\infty$. A continuous function $\beta:[0,a)\times[0,\infty)\to[0,\infty)$ is of class $\mathcal{KL}$ if, for each fixed s, the mapping $\beta(r,s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r,s)$ is decreasing with respect to s and $\beta(r,s)\to 0$ as $s\to\infty$.

Definition 2.1 (see [7]) For any $t \geq t_0$ and any switched signal $\sigma(\varsigma)$, $t_0 \leq \varsigma < t$, let $N_\sigma(t_0, t)$ mean the number of switchings of $\sigma(\varsigma)$ during the interval $[t_0, t)$. If there exist $N_0 > 0$ and $\tau_a > 0$ such that $N_\sigma(t_0, t) \leq N_0 + (t - t_0)/\tau_a$, then τ_a and N_0 are called average dwell time and the chatter bound, respectively.

Definition 2.2 (see [14]) System (2) is said to be ISS, if there exist a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function γ such that, for any initial state $\xi \in C([-r, 0]; \mathbb{R}^n)$, $w(t) \in L_\infty^m$ and all possible switching signals $\sigma(t)$, the solution of (2) exists globally and satisfies

$$|x(t)| \leq \beta(\|\xi\|_\infty, t - t_0) + \gamma(\|w[t_0, t]\|_\infty), \quad t \geq t_0 \geq 0. \tag{3}$$

To establish the ISS stability property of System (2), we consider a piecewise Lyapunov-Krasovskii functional $V(t, \phi) := V_{\sigma(t)}(t, \phi)$, where $V(t, \phi)$ is continuously differentiable and ϕ is the solution to System (2). For each $i \in \mathcal{N}_c$, define the upper right-hand derivative of $V_i(t, \phi)$ with respect to the i-th mode of System (2) as follows:

$$D^+ V_i(t, \phi) = \lim_{h \to 0^+} \sup \frac{V_i(t + h, \phi_h^*) - V_i(t, \phi)}{h}, \tag{4}$$

where $\phi_h^* \in C([-r, 0]; \mathbb{R}^n)$ is given by

$$\phi_h^*(s) = \begin{cases} \phi(s + h), & s \in [-r, -h]; \\ \phi(0) + f_i(t, \phi, w)(h + s), & s \in [-h, 0]. \end{cases}$$

In practical applications, the switching of the controller may not correspond precisely to the switching of the system because it is unavoidable to take some time to identify it and then switch the controller from the present one to the matched one. So, due to the existence of switching delay $\tau_s(t)$, we consider the following input:

$$u(t) = g_{\sigma(t - \tau_s(t))}(t, x_t, w(t)), \tag{5}$$

where $\tau_s(t)$ is the uncertain switching delay, satisfying $0 \leq \tau_s(t) \leq \overline{\tau}_s$. Here assume that the maximal switching delay $\overline{\tau}_s$ is known without loss of generality, which satisfies $2r \leq \tau_s + r \leq t_{k+1} - t_k$, $k \in \mathbb{N}$. As a result, we have the following switching sequence: $\{\xi(\theta) : (i_0, t_0 + \tau_s(t_0)), (i_1, t_1 + \tau_s(t_1)), \cdots, (i_k, t_k + \tau_s(t_k)), \cdots, | i_k \in \mathcal{N}_c, k \in \mathbb{N}\}$, which means that the i_k-th controller is active when $t \in [t_k + \tau_s(t_k), t_{k+1} + \tau_s(t_{k+1}))$, $k \in \mathbb{N}$.

In order to present the main result, the merging switching technique proposed in [31] will be introduced. Given two mismatched switching signals such as $\sigma_1(t), \sigma_2(t)$, we can create a virtual switching signal $\sigma'(t) : [0, \infty) \to \overline{\mathcal{N}}_c = \mathcal{N}_c \times \mathcal{N}_c$ as follows: $\sigma'(t) = (\sigma_1(t), \sigma_2(t))$. The merging action is denoted by $\oplus$ such that $\sigma'(t) = \sigma_1(t) \oplus \sigma_2(t)$. From the definition, it follows that the set of switching times of $\sigma'(t)$ is the union of the sets of switching times of $\sigma_1(t)$ and $\sigma_2(t)$.

Lemma 2.3 (see [31]) *Let* $\sigma_1(t) \in S_{ave}[\tau_a, N_0]$, *and* $\sigma_2(t) = \sigma_1(t - \tau_s(t))$, *it has* $\sigma_2(t) \in S_{ave}[\tau_a, N_0 + \frac{\overline{\tau}_s}{\tau_a}]$, $\sigma'(t) \in S_{ave}[\overline{\tau}_a, \overline{N}_0]$, *where* $\overline{\tau}_a = \frac{\tau_a}{2}$, $\overline{N}_0 = 2N_0 + \frac{\overline{\tau}_s}{\tau_a}$.

Lemma 2.4 (see [31]) *Let* $\sigma_1(t) \in S_{ave}[\tau_a, N_0]$, *and* $\sigma_2(t) = \sigma_1(t-\tau_s(t))$, *for some positive function* $\tau_s(t)$. *For an interval* (t_0, t) , *let* $m_{(t_0,t)}$ *be the total time for which* $\sigma_1(t) = \sigma_2(t)$, *and let* $\overline{m}_{(t_0,t)} = t - t_0 - m_{(t_0,t)}$. *Suppose that* $0 \le \tau_s(t) \le \overline{\tau}_s$ *for all* t. *If*

$$\overline{\tau}_s(\lambda_s + \lambda_u) \le (\lambda_s - \lambda)\tau_a \tag{6}$$

for some positive constants λ_s, λ_u *and* $\lambda \in [0, \lambda_s]$, *then*

$$-\lambda_s m_{(t_0,t)} + \lambda_u \overline{m}_{(t_0,t)} \le c_T - \lambda(t - t_0), \quad \forall t \ge t_0, \tag{7}$$

where $c_T = (\lambda_s + \lambda_u)N_0\overline{\tau}_s$.

Now, rewriting the system and integrating the switching sequence of the system with the switching sequence of the controller, we can derive

$$\dot{x}(t) = \overline{f}_{\sigma'(t)}(t, x_t, w(t)). \tag{8}$$

3 Main Results

Theorem 3.1 *Consider the switched nonlinear time-delay system* (8) *with the disturbance input* $w(t) \in L^m_\infty$. *Suppose there exists a piecewise Lyapunov-Krasovskii functional* $V(t, \phi) = V_{\sigma'(t)}(t, \phi)$, *and functions* $\alpha_1, \alpha_2, \rho \in \mathcal{K}_\infty$, *continuous function* $\lambda_{ii}(t), \varphi_{ij}(t)$, *constants* $\delta_{ii} > 0$, $\delta_{ij} > 0$ *and* $T > t_0$, c_{ii}, c_{ij} *and* $\mu \ge 1$, *such that, for all* $i, j \in \mathcal{N}_c, i \ne j$ *we have*

(a) $\alpha_1(|\phi(0)|) \le V(t, \phi) \le \alpha_2(\|\phi\|_{M_2}), \qquad \forall (t, \phi) \in \mathbb{R}^+ \times \mathbb{R}^n;$

(b) $\|\phi\|_{M_2} \ge \rho(|w(t)|) \Rightarrow \begin{cases} D^+V_{i,i}(t,\phi) \le \lambda_{ii}(t)V_{i,i}(t,\phi), \\ D^+V_{i,j}(t,\phi) \le \varphi_{ij}(t)V_{i,j}(t,\phi); \end{cases}$

(c) $V_{i,i}(t,\phi) \le \mu V_{i,j}(t,\phi), \quad V_{i,j}(t,\phi) \le \mu V_{j,j}(t,\phi);$

(d) $\displaystyle\int_{t_0}^{\infty} \lambda^+_{ii}(\tau)d\tau \le c_{ii} < \infty, \qquad \int_{t_0}^{\infty} \varphi^-_{ij}(\tau)d\tau \le c_{ij} < \infty, \qquad \forall\ t > T, \quad T > t_0;$

(e) $\displaystyle\int_{t_0}^{t} \lambda^-_{ii}(\tau)d\tau \ge \delta_{ii}(t-t_0), \quad \int_{t_0}^{t} \varphi^+_{ij}(\tau)d\tau \ge \delta_{ij}(t-t_0), \qquad \forall\ t > T, \quad T > t_0,$ (9)

where

$$\lambda^+_{ii}(t) = \max\{\lambda_{ii}(t), 0\}, \qquad \lambda^-_{ii}(t) = \max\{-\lambda_{ii}(t), 0\};$$
$$\varphi^+_{ij}(t) = \max\{\varphi_{ij}(t), 0\}, \qquad \varphi^-_{ij}(t) = \max\{-\varphi_{ij}(t), 0\}.$$

If the switching signal satisfies average dwell time $\tau_a > \tau^*_a = \frac{2\ln\mu + M\overline{\tau}_s}{\delta_s}$, *where*

$$M = \sup_{t \ge t_0}\{\lambda^-_s(t) + \varphi^+_u(t)\}, \quad \lambda_s(t) = \min_{i \in \mathcal{N}_c}\{\lambda_{ii}(t)\},$$
$$\varphi_u(t) = \max_{i,j \in \mathcal{N}_c, i \ne j}\{\varphi_{ij}(t)\}, \quad \delta_s = \min_{i \in \mathcal{N}_c}\{\delta_{ii}\},$$

then the system (8) *is ISS.*

Proof Let $t_0 \geq 0$ be arbitrary. For $t \geq t_0$, let

$$\nu(t) := \rho(\|w[t_0,t]\|_\infty), \qquad c(t) := \alpha_1^{-1}\Big(g_0 \mathrm{e}^{\delta_s(T-t_0)}\alpha_2(\nu(t))\Big),$$

where $g_0 = C\mu^{\overline{N}_0}\mathrm{e}^{MN_0\overline{\tau}_s}$ and $C = \max_{i\in\mathcal{N}_c}\{\mathrm{e}^{\int_{t_0}^{\infty}\lambda_{ii}^+(\tau)d\tau}\}$.

Furthermore, introduce the sets $\mathcal{B}_\nu(t) := \{\phi \in C([-r,0];\mathbb{R}^n) : \|\phi\|_{M_2} \leq \nu(t)\}$ as well as $\mathcal{B}_c(t) := \{\phi \in C([-r,0];\mathbb{R}^n) : \|\phi\|_{M_2} \leq c(t)\}$. Note that $\nu(t)$, and thus also $c(t)$, are non-decreasing functions of time, and therefore the ball $\mathcal{B}_\nu$ and $\mathcal{B}_c$ has non-decreasing volume.

If $\|\phi\|_{M_2} \geq \nu(t) \geq \rho(|w(t)|)$ during some time interval $t \in [t', t'']$, and $[t_k, t_{k+1}) \cap [t', t''] \neq 0$, where t_k is the k-th switching instant of the corresponding subsystem. Define the piecewise Lyapunov-Krasovskii functional $V_{\sigma'(t)}(t,x_t)$. Let the function $W(t) = \mathrm{e}^{-\int_{t_0}^{t}\lambda_{ii}(\tau)d\tau}V_{\sigma'(t)}(t,x_t)$. When $\sigma_1(t) = \sigma_2(t)$, from Condition (b), we have

$$\dot{W}(t) = -\lambda_{ii}(t)\mathrm{e}^{-\int_{t_0}^{t}\lambda_{ii}(\tau)d\tau}V_{\sigma'(t)}(t,x_t) + \mathrm{e}^{-\int_{t_0}^{t}\lambda_{ii}(\tau)d\tau}D^+V_{\sigma'(t)}(t,x_t) \leq 0, \tag{10}$$

that is $W(t)$ is monotone decreasing. And, from (10) and Condition (c), we have

$$W(t_{i+1}) \leq \mu W(t_{i+1}^-) \leq \mu W(t_i). \tag{11}$$

In a similar way, when $\sigma_1(t) \neq \sigma_2(t)$, we get

$$\begin{aligned}\dot{W}(t) &\leq -\lambda_{ii}(t)\mathrm{e}^{-\int_{t_0}^{t}\lambda_{ii}(\tau)d\tau}V_{\sigma'(t)}(t,x_t) + \varphi_{ij}(t)\mathrm{e}^{-\int_{t_0}^{t}\lambda_{ii}(\tau)d\tau}V_{\sigma'(t)}(t,x_t)\\ &= (-\lambda_{ii}(t) + \varphi_{ij}(t))W(t).\end{aligned} \tag{12}$$

And, from (12) and Condition (c), we have

$$W(t_{i+1}) \leq \mu W(t_{i+1}^-) \leq \mu \mathrm{e}^{\int_{t_i}^{t_{i+1}}(-\lambda_{ii}(\tau)+\varphi_{ij}(\tau))d\tau}W(t_i). \tag{13}$$

In any interval $[t', t'']$, let $t_1, t_2, \cdots, t_k$ denote the switching times of the corresponding subsystem in (t', t''), $t_0 = t'$, $t_{k+1} = t''$. For any $t \in [t_k, t_k + \tau_s(t_k)]$, by using the iteration, (11) and (13) yield

$$\begin{aligned}W(t) &\leq \mu \mathrm{e}^{\int_{t_k}^{t}(-\lambda_{ii}(\tau)+\varphi_{ij}(\tau))d\tau}W(t_k)\\ &\leq \mu^2\mathrm{e}^{\int_{t_k}^{t}(-\lambda_{ii}(\tau)+\varphi_{ij}(\tau))d\tau}W(t_{k-1}+\tau_s(t_{k-1}))\\ &\leq \mu^3\mathrm{e}^{\int_{t_k}^{t_{k-1}+\tau_s(t_{k-1})}(-\lambda_{ii}(\tau)+\varphi_{ij}(\tau))d\tau}\mathrm{e}^{\int_{t_k}^{t}(-\lambda_{ii}(\tau)+\varphi_{ij}(\tau))d\tau}W(t_{k-1})\\ &\leq \cdots \leq\\ &\leq \mu^{N_{\sigma'}(t_0,t)}\mathrm{e}^{\sum\limits_{q=0}^{k-1}\int_{t_q}^{t_q+\tau_s(t_q)}(-\lambda_{ii}(\tau)+\varphi_{ij}(\tau))d\tau}\mathrm{e}^{\int_{t_k}^{t}(-\lambda_{ii}(\tau)+\varphi_{ij}(\tau))d\tau}W(t_0)\\ &\leq \mu^{N_{\sigma'}(t_0,t)}\mathrm{e}^{\sum\limits_{q=0}^{k-1}\int_{t_q}^{t_q+\tau_s(t_q)}(\lambda_s^-(\tau)+\varphi_u^+(\tau))d\tau}\mathrm{e}^{\int_{t_k}^{t}(\lambda_s^-(\tau)+\varphi_u^+(\tau))d\tau}W(t_0),\end{aligned} \tag{14}$$

where

$$\begin{aligned}&\lambda_s^-(t) = \max\{-\lambda_s(t), 0\}, \quad \varphi_u^+(t) = \max\{\varphi_u(t), 0\},\\ &\lambda_s(t) = \min_{i\in\mathcal{N}_c}\{\lambda_{ii}(t)\}, \quad \varphi_u(t) = \max_{i,j\in\mathcal{N}_c, i\neq j}\{\varphi_{ij}(t)\}.\end{aligned}$$

Similarly, for any $t \in [t_k + \tau_s(t_k), t_{k+1})$, we have

$$\begin{aligned} W(t) &\le \mu^{N_{\sigma'}(t_0,t)} e^{\sum_{q=0}^{k} \int_{t_q}^{t_q+\tau_s(t_q)} (-\lambda_{ii}(\tau)+\varphi_{ij}(\tau))d\tau} W(t_0) \\ &\le \mu^{N_{\sigma'}(t_0,t)} e^{\sum_{q=0}^{k} \int_{t_q}^{t_q+\tau_s(t_q)} (\lambda_s^-(\tau)+\varphi_u^+(\tau))d\tau} W(t_0). \end{aligned} \tag{15}$$

Substituting $W(t) = e^{-\int_{t_0}^{t} \lambda_{ii}(\tau)d\tau} V_{\sigma'(t)}(t, x_t)$ into (14) and (15), respectively, yields

$$\begin{aligned} V_{\sigma'(t)}(t, x_t) &\le \mu^{N_{\sigma'(t)}(t_0,t)} e^{\int_{t_0}^{t} \lambda_{ii}^+(\tau)d\tau} e^{-\int_{t_0}^{t} \lambda_{ii}^-(\tau)d\tau} e^{M\overline{m}_{(t_0,t)}} V_{\sigma'(t_0)}(t_0, x_{t_0}) \\ &\le C\mu^{N_{\sigma'(t)}(t_0,t)} e^{-\int_{t_0}^{t} \lambda_{ii}^-(\tau)d\tau} e^{(M-\delta_s)\overline{m}_{(t_0,t)} - \delta_s m_{(t_0,t)} + \delta_s(t-t_0)} V_{\sigma'(t_0)}(t_0, x_{t_0}), \end{aligned} \tag{16}$$

where $\delta_s = \min_{i\in\mathcal{N}_c}\{\delta_{ii}\}$ and $C = \max_{i\in\mathcal{N}_c}\left\{e^{\int_{t_0}^{\infty} \lambda_{ii}^+(\tau)d\tau}\right\}$, for all $t > t_0$.

Bearing Condition (e) into mind, one yields that

$$\begin{aligned} V_{\sigma'(t)}(t, x_t) &\le C\mu^{N_{\sigma'(t)}(t_0,t)} e^{(M-\delta_s)\overline{m}_{(t_0,t)} - \delta_s m_{(t_0,t)}} V_{\sigma'(t_0)}(t_0, x_{t_0}) \\ &\le C\mu^{N_{\sigma'(t)}(t_0,t)} e^{(M-\delta_s)\overline{m}_{(t_0,t)} - \delta_s m_{(t_0,t)}} e^{\delta_s(T-t_0)} V_{\sigma'(t_0)}(t_0, x_{t_0}) \end{aligned} \tag{17}$$

holds for all $t > T$, where $T \ge t_0$ is a finite constant.

Due to

$$\tau_a > \tau_a^* = \frac{2\ln\mu + M\overline{\tau}_s}{\delta_s}, \tag{18}$$

which implies the existence of δ such that

$$\frac{2\ln\mu}{\tau_a} < \delta < \delta_s - \frac{M\overline{\tau}_s}{\tau_a}, \tag{19}$$

which can be rewritten as

$$M\overline{\tau}_s < (\delta_s - \delta)\tau_a, \tag{20}$$

$$\delta > \frac{\ln\mu}{\overline{\tau}_a}. \tag{21}$$

Then, by Lemma 2.4, we get

$$-\delta_s m_{(t_0,t)} + (M - \delta_s)\overline{m}_{(t_0,t)} \le c_T - \delta(t - t_0), \tag{22}$$

where $c_T = MN_0\overline{\tau}_s$.

From the above discussions, we can derive

$$\begin{aligned} V_{\sigma'(t)}(t, x_t) &\le C\mu^{N_{\sigma'(t)}(t_0,t)} e^{c_T - \delta(t-t_0)} e^{\delta_s(T-t_0)} V_{\sigma'(t_0)}(t_0, x_{t_0}) \\ &\le g_0 e^{\delta_s(T-t_0)} e^{-(\delta - \frac{\ln\mu}{\overline{\tau}_a})(t-t_0)} V_{\sigma'(t_0)}(t_0, x_{t_0}), \end{aligned} \tag{23}$$

where $g_0 = C\mu^{\overline{N}_0} e^{MN_0\overline{\tau}_s}$.

From Condition (a), there holds

$$\begin{aligned} |x(t)| &\le \alpha_1^{-1}\Big(g_0 e^{\delta_s(T-t_0)} e^{-(\delta - \frac{\ln\mu}{\overline{\tau}_a})(t-t_0)} \alpha_2(\|x_{t_0}\|_{M_2})\Big) \\ &\le \overline{\beta}(\|\xi\|_{M_2}, t - t_0), \end{aligned} \tag{24}$$

where $\overline{\beta} \in \mathcal{KL}$ is constructed as follows

$$\overline{\beta}(r,t) = \begin{cases} \alpha_1^{-1}\big(g_0 \mathrm{e}^{\delta_s(T-t_0)}\alpha_2(r)\big), & t_0 \le t \le T, \\ \alpha_1^{-1}\big(g_0 \mathrm{e}^{\delta_s(T-t_0)}\alpha_2(r)\mathrm{e}^{(-\delta+\frac{\ln\mu}{\overline{\tau}_a})t}\big), & t \ge T+1, \\ \alpha_1^{-1}\big(g_0 \mathrm{e}^{\delta_s(T-t_0)}\alpha_2(r)(T+1-t+\mathrm{e}^{(-\delta+\frac{\ln\mu}{\overline{\tau}_a})(T+1)}(t-T))\big), & T < t < T+1, \end{cases} \tag{25}$$

where δ and T are ensured by Condition (e). For fixed r, it is obvious that $\overline{\beta}(r,t)$ is continuous in t. Moreover, to derive the $\mathcal{KL}$ property of function $\overline{\beta}(r,t)$, letting $q := \delta - \frac{\ln\mu}{\overline{\tau}_a}$ and bearing the convexity of e^{-qt}, we have

$$\begin{aligned} \mathrm{e}^{-qt} &\le \mathrm{e}^{-qT}(T+1-t) + \mathrm{e}^{-q(T+1)}(t-T) \\ &\le (T+1-t) + \mathrm{e}^{-q(T+1)}(t-T), \qquad T < t < T+1. \end{aligned} \tag{26}$$

It is easily verified that $\overline{\beta}(r,t)$ is a decreasing function for a fixed r and $\overline{\beta}(r,t)$ is a $\mathcal{KL}$ function. From the above discussions, we can obtain

$$|x(t)| \le \overline{\beta}(\|x_{t'}\|_{M_2}, t-t'), \tag{27}$$

for all $t \ge t'$.

Denote the first time when $\phi \in \mathcal{B}_\nu(t)$ by $\check{t}_1$, i.e., $\check{t}_1 := \inf\{t \ge t_0 : \|\phi\|_{M_2} \le v(t)\}$. For $t_0 \le t \le \check{t}_1$ we get

$$|x(t)| \le \overline{\beta}(\|\xi\|_{M_2}, t-t_0), \tag{28}$$

according to (24). For $t > \check{t}_1$, $|x(t)|$ can be bounded above in terms of $\nu(t)$. Namely, let $\widehat{t}_1 := \inf\{t > \check{t}_1 : \|\phi\|_{M_2} > \nu(t)\}$. If this is an empty set, let $\widehat{t}_1 := \infty$. Clearly, for all $t \in [\check{t}_1, \widehat{t}_1)$, it holds that $|x(t)| \le \|\phi\|_{M_2} \le v(t) \le c(t)$. For the case of $\widehat{t}_1 < \infty$, due to the continuity of $x(\cdot)$ and monotonicity of $\nu(\cdot)$ it holds that $\|x_{\widehat{t}_1}\|_{M_2} = \nu(\widehat{t}_1)$. Furthermore, for all $\tau > \widehat{t}_1$, if $\|x_\tau\|_{M_2} > \nu(\tau)$ define

$$\widehat{t} := \sup\{t < \tau : \|\phi\|_{M_2} \le \nu(t)\}, \tag{29}$$

which can be interpreted as the previous exit time of the trajectory $x(\cdot)$ from the ball $\mathcal{B}_\nu$. Again, due to the same argument as above, one obtains that $\|x_{\widehat{t}}\|_{M_2} = \nu(\widehat{t})$. But then, according to (25), it holds that

$$\begin{aligned} |x(\tau)| &\le \overline{\beta}(\|x(\widehat{t})\|_{M_2}, \tau - \widehat{t}) \\ &\le \alpha_1^{-1}\Big(g_0 \mathrm{e}^{\delta_s(T-t_0)}\alpha_2(\|x(\widehat{t})\|_{M_2})\Big) \\ &= c(\widehat{t}) \le c(\tau). \end{aligned} \tag{30}$$

Thus, for all $t \ge \check{t}_1$, it holds that

$$\begin{aligned} |x(t)| &\le c(t) \\ &= \alpha_1^{-1}\Big(g_0 \mathrm{e}^{\delta_s(T-t_0)}\alpha_2(\rho(\|w[t_0,t]\|_\infty))\Big) \\ &= \gamma(\|w[t_0,t]\|_\infty). \end{aligned} \tag{31}$$

Combining (24) with (31) arrives

$$\begin{aligned}|x(t)| &\leq \overline{\beta}((1+r)^{1/2}\|\xi\|_\infty, t-t_0) + \gamma(\|w[t_0,t]\|_\infty) \\ &= \beta(\|\xi\|_\infty, t-t_0) + \gamma(\|w[t_0,t]\|_\infty), \end{aligned} \tag{32}$$

for all $t \geq t_0$, where $\beta(\|\xi\|_\infty, t-t_0) = \overline{\beta}((1+r)^{1/2}\|\xi\|_\infty, t-t_0)$. The proof is completed. ∎

Remark 3.2 Because $\overline{m}_{t_0,t}$ is the total time for which $\sigma_1(t) \neq \sigma_2(t)$ and

$$M = \sup_{t\geq t_0}\{\lambda_s^-(t) + \varphi_u^+(t)\},$$

in (14) and (15), we can get

$$\int_{t_k}^{t}(\lambda_s^-(\tau) + \varphi_u^+(\tau))d\tau + \sum_{q=0}^{k-1}\int_{t_q}^{t_q+\tau_s(t_q)}(\lambda_s^-(\tau) + \varphi_u^+(\tau))d\tau \leq M\overline{m}_{t_0,t},$$

$$\sum_{q=0}^{k}\int_{t_q}^{t_q+\tau_s(t_q)}(\lambda_s^-(\tau) + \varphi_u^+(\tau))d\tau \leq M\overline{m}_{t_0,t}.$$

Thus, (16) is obtained for all $t > t_0$.

Remark 3.3 In this Theorem, the structure of the $\mathcal{KL}$ function $\overline{\beta}(r,t)$ is complicated but reasonable, which can be divided into three sections by the time. When $t \in [t_0, T]$, $\overline{\beta}(r,t) = \alpha_1^{-1}\Big(g_0 \mathrm{e}^{\delta_s(T-t_0)}\alpha_2(r)\Big)$. And, when $t > T$, we choose

$$\overline{\beta}(r,t) = \alpha_1^{-1}\Big(g_0 \mathrm{e}^{\delta_s(T-t_0)}\alpha_2(r)\mathrm{e}^{(-\delta+\frac{\ln\mu}{\overline{\tau}_a})t}\Big).$$

However, the continuity of the function $\overline{\beta}(r,t)$ at the time instant T can not be guaranteed. Further, we apply the convexity of e^{-qt} (with $q := \delta - \frac{\ln\mu}{\overline{\tau}_a}$) during the time interval $t \in [T, T+1]$ to get

$$\overline{\beta}(r,t) = \alpha_1^{-1}\Big(g_0 \mathrm{e}^{\delta_s(T-t_0)}\alpha_2(r)(T+1-t+\mathrm{e}^{(-\delta+\frac{\ln\mu}{\overline{\tau}_a})(T+1)}(t-T))\Big).$$

Remark 3.4 Consider the asynchronous switching between the controllers and the systems, the merging switching signal technique is used to guarantee the ISS of the switched nonlinear delay system. In the above proof, it can be verified that $M - \delta_s > 0$ from Condition (e) after some manipulations. Therefore, taking $\lambda_s := \delta_s, \lambda_u := M - \delta_s$ and by Lemma 2.4, we obtain (6).

Remark 3.5 Theorem 3.1 extends the results in [22] to nonlinear switched systems with delays. It is not trivial to do so since it is difficult to deal with the switching signals for nonlinear systems especially when asynchronous switching occurs.

If $\tau_s(t) = 0$, the asynchronous switching signal changes to the synchronous switching signal. Then, we can derive the following Corollary 3.6.

Corollary 3.6 *Consider the switched nonlinear time-delay system* (2) *with the disturbance input* $w(t) \in L_\infty^m$. *Suppose that there exists a piecewise Lyapunov-Krasovskii functional*

$V_i(t,\phi) = V_{\sigma(t)}(t,\phi)$, and functions $\alpha_1, \alpha_2, \rho \in \mathcal{K}_\infty$, continuous functions $\lambda_i(t)$, constants c_i, $\delta_i > 0$ and $\mu \geq 1$, such that, for all $i, j \in \mathcal{N}_c$, we have

(f) $\alpha_1(|\phi(0)|) \leq V_i(t,\phi) \leq \alpha_2(\|\phi\|_{M_2}), \qquad \forall (t,\phi) \in \mathbb{R}^+ \times \mathbb{R}^n$;

(g) $\|\phi\|_{M_2} \geq \rho(|w(t)|) \Rightarrow D^+V_i(t,\phi) \leq \lambda_i(t)V_i(t,\phi)$;

(h) $V_i(t,\phi) \leq \mu V_j(t,\phi)$;

(i) $\displaystyle\int_{t_0}^{\infty} \lambda_i^+(\tau)d\tau \leq c_i < \infty, \qquad \forall\ t > T, \quad T > t_0$;

(j) $\displaystyle\int_{t_0}^{\infty} \lambda_i^-(\tau)d\tau \geq \delta_i(t - t_0), \qquad \forall t > T, \quad T > t_0$,

where $\lambda_i^+(t) = \max\{\lambda_i(t), 0\}$, $\lambda_i^-(t) = \max\{-\lambda_i(t), 0\}$. If the switching signal satisfies with the average dwell time $\tau_a > \tau_a^* = \frac{\ln \mu}{\delta}$, where $\delta = \min_{i\in\mathcal{N}_c}\{\delta_i\}$, then System (2) is ISS.

Remark 3.7 If the condition (h) holds for $\mu = 1$, then the condition $\tau_a > \tau_a^* = \frac{\ln\mu}{\delta}$ reduces to $\tau_a > 0$. This means that the system is ISS for arbitrarily small dwell time. Actually, $\mu = 1$ in condition (h) implies the existence of a common ISS-Lyapunov functional for switched system (2), and thus it is in fact ISS for any arbitrary switching.

4 Simulation Example

Consider the following switched delay systems:

$$\begin{aligned} \dot{x} &= f_\sigma(t, x(t), x_t, u(t)) \\ &= \begin{pmatrix} x_2 \\ -\rho_3 x_1 - \rho_{1,\sigma} x_2 - \rho_{2,\sigma} x_1(t-1) - \rho_3 x_1^3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t), \end{aligned} \tag{33}$$

where the parameters are given by $\rho_{1,1} = 1$, $\rho_{1,2} = 0.5$, $\rho_{2,1} = 0.6$, $\rho_{2,2} = 0.3$, $\rho_3 = 1$, $r = 1$ and $\sigma(t) : \mathbb{R}^+ \to S = \{1,2\}$. The control input $u = g_{\sigma(t-\tau_s)}(t, x(t), w)$ with

$$\begin{aligned} g_1 &= x_1^2 x_2 - 4x_1 - x_2 + \frac{x_2}{1+t^2} + w, \\ g_2 &= x_1^2 x_2 - x_1 - \frac{x_2}{1+t^2} + w. \end{aligned}$$

Choose the Lyapunov-Krasovskii functional

$$V(t, x_t) = V_{\sigma'}(t, x_t) \tag{34}$$

with

$$V_{1,1}(t, x_t) = x_1^2 + x_2^2 + c_{1,1}\int_{t-1}^{t} \mathrm{e}^{-\lambda_{1,1}(t-s)}\left(x_1^2 + \frac{1}{9}x_2^2\right)ds, \tag{35}$$

$$V_{2,2}(t, x_t) = x_1^2 + x_2^2 + c_{2,2}\int_{t-1}^{t} \mathrm{e}^{-\lambda_{2,2}(t-s)}(x_1^2 + x_2^2)ds, \tag{36}$$

$$V_{i,j}(t, x_t) = x_1^2 + x_2^2 + c_{i,j}\int_{t-1}^{t} \mathrm{e}^{\mu_{i,j}(t-s)}(x_1^2 + x_2^2)ds, \quad i \neq j, \quad i, j \in S, \tag{37}$$

where $c_{1,1} = 0.9, c_{2,2} = 0.39, c_{1,2} = 0.6, c_{2,1} = 0.6, \lambda_{1,1} = 0.5, \lambda_{2.2} = 0.25, \mu_{1,2} = 0.1, \mu_{2,1} = 0.1$.

Conditions (a) of Theorem 3.1 are clearly satisfied with $\alpha_1(\|\phi(0)\|) = \|\phi(0)\|^2, \alpha_2(\|\phi\|_{M_2}) = \|\phi\|^2_{M_2}$. Next, verify the remainder of Condition (c) in Theorem 3.1, we have $\mu = 2.19$. Now, we check Condition (b). Observe that

$$D^+V_{1,1} \le \left(\frac{1}{1+t^2} - 0.31\right)V_{1,1}, \tag{38}$$

where the relation $2\sqrt{5}|w| \le \|\phi\|_{M_2}$ is used and

$$\lambda_{11}(t) = \frac{1}{1+t^2} - 0.31. \tag{39}$$

Then one derives

$$\int_{t_0}^{\infty} \lambda_{11}^+(\tau)d\tau \le \frac{\pi}{2} < \infty, \qquad \int_{t_0}^{t} \lambda_{11}^-(\tau)d\tau \ge \delta_{11}(t - t_0), \tag{40}$$

where $\delta_{11} = 0.0235$.

Similarly, for $2\sqrt{5}|w| \le \|\phi\|_{M_2}$, we can derive $D^+V_{2,2} \le \lambda_{22}(t)V_{2,2}$, $D^+V_{2,1} \le \varphi_{21}(t)V_{2,1}$, $D^+V_{1,2} \le \varphi_{12}(t)V_{1,2}$, where $\lambda_{22}(t) = \frac{1}{1+t^2} - 0.09, \varphi_{12}(t) = \varphi_{21}(t) = \frac{1}{1+t^2} + 0.16$. From the above analysis and based on Theorem 3.1, we get $\int_{t_0}^{t} \lambda_{22}^-(\tau)d\tau \ge \delta_{22}(t - t_0), \delta_{22} = 0.0786$ and $M = \sup_{t\ge t_0}\{\lambda_{ii}^-(t) + \varphi_{ij}^+(t)\} = 0.47$. From the proof of the Theorem 3.1, we get $\delta_s = 0.0786$. So, if switching delay $\overline{\tau}_s = 1$, we obtain $\tau_a^* = \frac{2\ln\mu + M\overline{\tau}_s}{\delta_s} = 25.9262$. Figure 1 gives the state response of the closed-loop system with $w(t) = \sin 0.1t$ and Figure 2 shows the asynchronous switching signal, respectively.

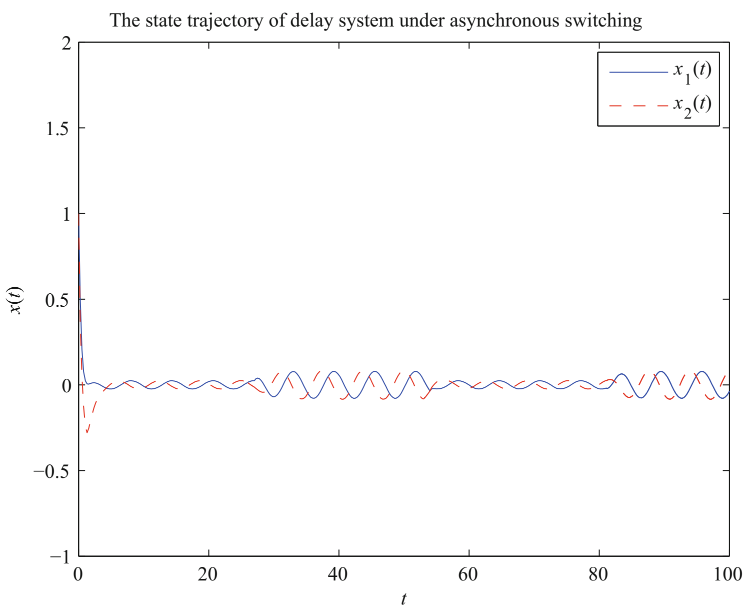

Figure 1 The trajectory of $x(t)$

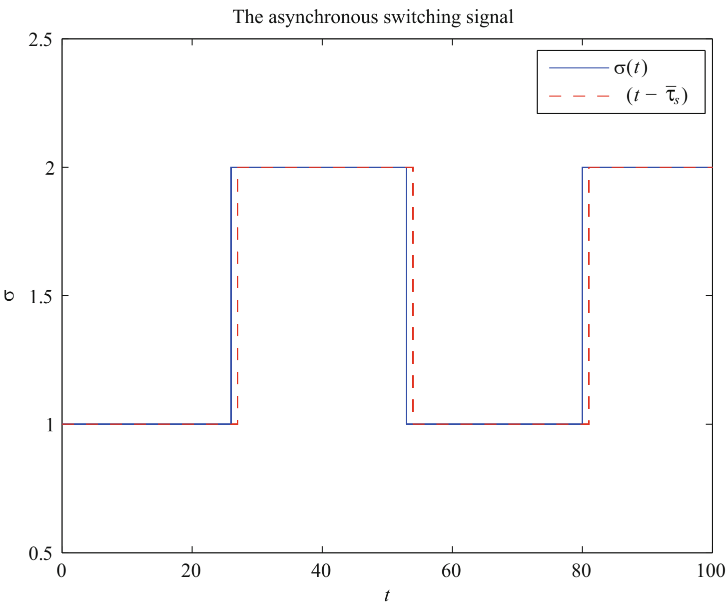

Figure 2 The switching signal of the system

5 Conclusions

In this paper, we have investigated the ISS problem for a class of switched nonlinear delay systems with synchronous or asynchronous switching. By introducing a novel Lyapunov-Krasovskii functional with indefinite derivative and the merging switching signal techniques, we have established some new criteria for switched nonlinear delay systems and asynchronous switching controllers; which generalized the existing results to the nonlinear systems with switching rules and delays. We have also considered the ISS property under synchronous switching control for switched nonlinear systems by employing the similar techniques. Borrowing the delay model with only one cubic structural nonlinearity of the form[21], we have demonstrated the effectiveness of the proposed results.

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