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An Improvement to Multiple Criteria ABC Inventory Classification Using Shannon Entropy*

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Abstract This paper extends the Ng-model [Ng, 2007] for multiple criteria ABC inventory classification based upon Shannon entropy. The proposed approach determines the common weights associated with all criteria importance rankings, and provides a comprehensive scoring scheme by aggregating all rankings of the criteria importance. A numerical illustration is presented to compare the model with previous studies.

Keywords Inventory, multiple criteria analysis, Shannon entropy.

1 Introduction

In the domain of inventory management, the ABC classification is an effective technique to develop a mechanism for identifying inventory items that not only has a significant impact on the total inventory cost, but also requires different management schemes and controls. The extant literature has developed abundant of approaches to support multiple criteria ABC inventory classification. Flores, et al.[1, 2] pioneered the multiple criteria ABC inventory analysis. Guvenir and Erel[3] used a genetic algorithm to realize ABC analysis. Partovi and Burton[4] employed analytic hierarchy process for ABC classification. Partovi and Anandarajan[5] utilized artificial neutral network approach to classify inventory items. Ramanathan[6] proposed a DEA-like

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weighted linear optimization model (R-model) for multiple criteria ABC inventory classification. Zhou and Fan[7] extended the R-model using two sets of weights that are most and least favorable for each inventory item. Chen, et al.[8] proposed a case-based distance model for multiple criteria inventory ABC analysis. Chu, et al.[9] combined ABC classification and fuzzy analysis. Teunter, et al.[10] incorporated service level as a new criterion and investigate the relative effect on the classification.

Recently, Ng[11] presented a weighted linear optimization model (hereafter called the Ng-model) for multiple criteria ABC inventory classification, which converts all criteria measures of an inventory item into a scalar score. The Ng-model is well-known as simple-to-understand and easy-to-implement. The optimal scores of inventory items can be easily obtained without a linear optimizer. Despite its many advantages, the Ng-model requires the decision maker to subjectively rank the criteria importance in a sequence. Some questions inevitably arise: Which ranking is more suitable? Which viewpoint is more desirable? Are there common weights associated with each of the rankings?

Hadi-Vencheh[12] presented an improvement to the Ng-model by proposing a non-linear programming model (hereafter called the HV-model) to determine a common set of weights for all the items. However, the problem of subjective ranking of criteria importance is still ignored. It is clear that the optimal scores derived from different ranking may not be the same. Each of the aforementioned rankings and viewpoints has some valuable advantages which we could not ignore. While it is impossible to ignore any ranking completely, the best way to make decision is to accept all possible rankings first, and then aggregate the results of the different rankings and viewpoints.

The purpose of this short communication is to provide an extended version of the Ng-model based upon Shannon entropy, and to present a comprehensively scoring schemeby determining the common weights associated with all criteria importance rankings. Undoubtedly, combining the scores of different criteria sequence definitely provides a more realistic classification compared with employing any ranking individually.

Compared with the Ng-model, our proposed Shannon entropy method has at least three advantages. Firstly, our method proposes a unique ranking among the inventory items. Secondly, our method eliminates unrealistic subjective ranking of the criteria importance, without requiring the elicitation of ranking restrictions from industry experts. Thirdly, the comprehensive scoring scheme effectively distinguishes between good and poor performance.

2 Ng-Model

Assume that there are I items which are to be classified as A, B and C on the basis of their performance in terms of J criteria. Let y_{ij} denote the performance score of the item i in terms of criterion j , which are transformed to 0-1 scale for comparable purpose

$$\frac{y_{ij} - \min\limits_{i=1,2,\cdots,I}\{y_{ij}\}}{\max\limits_{i=1,2,\cdots,I}\{y_{ij}\} - \min\limits_{i=1,2,\cdots,I}\{y_{ij}\}}. \tag{1}$$

Furthermore, all the criteria are assumed to be benefit-type criteria. More specifically, all these criteria are positively related to the importance level of an item. To realize the multiple criteria ABC inventory classification, Ng[11] defined a non-negative weight w_{ij} of contribution of performance of item i under criterion j to the score of the item. The criteria are assumed to be ranked in a descending order for any item i, i.e., $w_{i1} \geq w_{i2} \geq \cdots \geq w_{iJ}$. The score of item i is denoted as a weighted sum of performance measures under multiple criteria. Therefore, the Ng-model for aggregation purpose is presented as:

$$\begin{aligned} \max \ \ & S_i = \sum_{j=1}^{J} w_{ij} y_{ij} \\ \text{s.t.} \ \ & \sum_{j=1}^{J} w_{ij} = 1, \\ & w_{ij} \geq w_{i(j+1)} \geq 0, \quad j = 1, 2, \cdots, J-1, \\ & w_{ij} \geq 0, \quad j = 1, 2, \cdots, J. \end{aligned} \tag{2}$$

By employing the following transformations, namely, $u_{ij} = w_{ij} - w_{i(j+1)}$, $u_{iJ} = w_{iJ}$ and $x_{ij} = \sum_{k=1}^{j} y_{ik}$, the above model (2) is converted to the following formulations for each item i:

$$\begin{aligned} \max \ \ & S_i = \sum_{j=1}^{J} u_{ij} x_{ij} \\ \text{s.t.} \ \ & \sum_{j=1}^{J} j u_{ij} = 1, \\ & u_{ij} \geq 0, \quad j = 1, 2, \cdots, J. \end{aligned} \tag{3}$$

One can easily obtain the maximal score S_i by the dual of (3), which is

$$\begin{aligned} \min \ \ & z_i \\ \text{s.t.} \ \ & S_i \geq \frac{1}{j} x_{ij}, \quad j = 1, 2, \cdots, J. \end{aligned} \tag{4}$$

Finally, the maximal score S_i can be derived as

$$\max_{j=1,2,\cdots,J} \left\{ \frac{1}{j} \sum_{k=1}^{j} y_{ik} \right\}.$$

3 The Proposed Approach

In this section, we propose an approach based upon Shannon entropy to combine different subjective rankings of the importance of the criteria in the decision making process. Shannon entropy is a useful and effective concept in the field of information theory, and can be employed as a measure of uncertainty[13]. It is clear that each ranking has limited discrimination ability which we would like not to ignore. Therefore, Shannon entropy can be used to calculate the degrees of importance for different rankings, and then generate a comprehensive score for each item.

It is assumed that there are I inventory items to be classified as A, B and C based on J criteria. We also assume that these J criteria are sequenced by a set of different rankings, namely,

$$R = \{R_1, R_2, \cdots, R_k\}. \tag{5}$$

Therefore, the derived scores for each item are listed in the following matrix

$$S_{I\times K} = \begin{pmatrix} S_{11} & S_{12} & \cdots & S_{1K} \\ S_{21} & S_{22} & \cdots & S_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ S_{I1} & S_{I2} & \cdots & S_{IK} \end{pmatrix}. \tag{6}$$

Each row of $S_{I\times K}$ is corresponding to an item and each column of $S_{I\times K}$ is related to typical ranking of the importance of the criteria. Hence, S_{IK} exhibits the score of item R_K derived from ranking .

In line with the work of Soleimani-damaneh and Zarepisheh[14], we introduce the following five steps to determine the weights of each criterion on the basis of Shannon entropy.

Step 1 Normalize the matrix $S_{I\times K}$ by $s_{ik} = \frac{S_{ik}}{\sum_{i=1}^{I} S_{ik}}$;

Step 2 Determine the entropy for each ranking, $f_k = -\left[\ln(n)\right]^{-1} \sum_{i=1}^{I} S_{ik} \ln(S_{ik})$;

Step 3 Calculate the degree of discriminability for each ranking as $d_k = 1 - f_k$;

Step 4 Compute the weight λ_k for the ranking R_k in the comprehensive score by normalizing d_k, i.e., $\lambda_k = \frac{d_k}{\sum_{k=1}^{K} d_k}$;

Step 5 Calculate the comprehensive score for item i, $S_i = \sum_{k=1}^{K} \lambda_k S_{ik}, i = 1, 2, \cdots, I$.

4 Numerical Illustration

For the purpose of illustrating the effectiveness of our proposed method, we investigate the same multiple criteria ABC inventory classification problem as discussed by Ng[11]. Three criteria, namely, Annual Dollar Usage (ADU), Average Unit Cost (AUC) and Lead Time (LT) are considered for inventory classification. All the criteria are benefit-type criteria, which are positively related to the importance level of an item. An inventory with 47 items and the measures of each criterion are listed in the following Table 1.

In order to compare our results with the previous studies[11, 12], we maintain the same distribution of class A, B and C, i.e., 10 in class A, 14 in class B and 23 in class C. Since the multiple criteria ABC inventory classification is processed with three criteria, we comprehensively investigate $A_3^3 = 6$ preferences of the criteria, i.e., AUC$\succ$ADU$\succ$LT, AUC$\succ$LT$\succ$ADU, ADU$\succ$AUC$\succ$LT, ADU$\succ$LT$\succ$AUC, LT$\succ$AUC$\succ$ADU and LT$\succ$ADU $\succ$AUC. The final results and related comparison are summarized in Table 2 and Figure 1.

Table 1 Measures of inventory items and transformed measures

Item	AR(1)			MA(1)		
	AUC	ADU	LT	AUC (Transformed)	ADU (Transformed)	LT (Transformed)
1	49.92	5840.64	2	0.22	1.00	0.17
2	210	5670	5	1.00	0.97	0.67
3	23.76	5037.12	4	0.09	0.86	0.50
4	27.73	4769.56	1	0.11	0.82	0.00
5	57.98	3478.8	3	0.26	0.59	0.33
6	31.24	2936.67	3	0.13	0.50	0.33
7	28.2	2820	3	0.11	0.48	0.33
8	55	2640	4	0.24	0.45	0.50
9	160.5	2407.5	4	0.33	0.41	0.83
10	73.44	2423.52	6	0.76	0.41	0.50
11	86.5	1038	7	0.00	0.18	0.17
12	5.121	1075.2	2	0.08	0.18	0.67
13	20.87	1043.5	5	0.40	0.17	1.00
14	110.4	883.2	5	0.51	0.15	0.67
15	71.2	854.4	3	0.32	0.14	0.33
16	45	810	3	0.19	0.13	0.33
17	14.66	703.68	4	0.05	0.12	0.50
18	49.5	594	6	0.22	0.1.	0.83
19	47.5	570	5	0.21	0.09	0.67
20	58.45	467.6	4	0.26	0.08	0.50
21	65	455	4	0.09	0.08	0.50
22	86.5	432.5	4	0.29	0.07	0.50
23	24.4	463.6	4	0.40	0.07	0.50
24	33.2	398.4	3	0.14	0.06	0.33
25	84.03	336.12	1	0.16	0.05	0.00
26	134.34	268.68	7	0.14	0.05	0.33
27	37.05	370.5	1	0.39	0.05	0.00
28	78.4	313.6	6	0.36	0.04	0.83
29	33.84	338.4	3	0.63	0.03	1.00
30	72	216	5	0.25	0.03	0.00
31	56	224	1	0.33	0.03	0.67
32	53.02	212.08	2	0.23	0.03	0.17
33	49.48	197.92	5	0.22	0.03	0.67
34	60.6	181.8	3	0.01	0.03	1.00

Table 1 (continued) Measures of inventory items and transformed measures

Item	AR(1)			MA(1)		
	AUC	ADU	LT	AUC (Transformed)	ADU (Transformed)	LT (Transformed)
35	48.82	163.28	3	0.27	0.03	0.33
36	7.07	190.89	7	0.17	0.02	0.33
37	67.4	134.8	3	0.12	0.02	0.67
38	30	150	5	0.30	0.02	0.33
39	59.6	119.2	5	0.27	0.02	0.67
40	51.68	103.36	6	0.23	0.01	0.83
41	37.7	75.4	2	0.07	0.01	0.17
42	19.8	79.2	2	0.16	0.01	0.17
43	48.3	48.3	3	0.12	0.01	0.67
44	29.89	59.78	5	0.21	0.00	0.33
45	34.4	34.4	7	0.14	0.00	1.00
46	28.8	28.8	3	0.12	0.00	0.33
47	8.46	25.38	5	0.02	0.00	0.67

Table 2 A comparison of our proposed model, Ng-model and HV-model

Item	AUC	ADU	LT	Score	Proposed model	Ng-model	HV-model
1	49.92	5840.64	2	0.7260	A	A	A
2	210	5670	5	0.9520	A	A	A
3	23.76	5037.12	4	0.6738	A	A	A
4	27.73	4769.56	1	0.5539	B	A	A
5	57.98	3478.8	3	0.4900	B	A	A
6	31.24	2936.67	3	0.4115	B	A	B
7	28.2	2820	3	0.3981	C	A	B
8	55	2640	4	0.4572	B	B	A
9	160.5	2407.5	4	0.6490	A	A	A
10	73.44	2423.52	6	0.6279	A	A	C
11	86.5	1038	7	0.1609	C	C	C
12	5.121	1075.2	2	0.4576	B	B	B
13	20.87	1043.5	5	0.7114	A	A	A
14	110.4	883.2	5	0.5438	B	B	C
15	71.2	854.4	3	0.3004	C	C	C
16	45	810	3	0.2613	C	C	C
17	14.66	703.68	4	0.3366	C	C	C

Table 2 (continued) A comparison of our proposed model, Ng-model and HV-model

Item	AUC	ADU	LT	Score	Proposed model	Ng-model	HV-model
18	49.5	594	6	0.5615	B	B	B
19	47.5	570	5	0.4610	B	B	B
20	58.45	467.6	4	0.3653	C	C	C
21	65	455	4	0.3350	C	C	C
22	86.5	432.5	4	0.3692	C	C	C
23	24.4	463.6	4	0.4066	C	B	C
24	33.2	398.4	3	0.2370	C	C	C
25	84.03	336.12	1	0.1026	C	C	C
26	134.34	268.68	7	0.2348	C	C	C
27	37.05	370.5	1	0.2339	C	C	C
28	78.4	313.6	6	0.5733	A	B	B
29	33.84	338.4	3	0.7403	A	A	A
30	72	216	5	0.1494	C	C	C
31	56	224	1	0.4693	B	B	B
32	53.02	212.08	2	0.1796	C	C	C
33	49.48	197.92	5	0.4497	B	B	B
34	60.6	181.8	3	0.6113	A	B	B
35	48.82	163.28	3	0.2682	C	C	C
36	7.07	190.89	7	0.2336	C	C	C
37	67.4	134.8	3	0.4297	B	C	C
38	30	150	5	0.2770	C	C	C
39	59.6	119.2	5	0.4564	B	B	B
40	51.68	103.36	6	0.5435	B	B	B
41	37.7	75.4	2	0.1171	C	C	C
42	19.8	79.2	2	0.1445	C	C	C
43	48.3	48.3	3	0.4275	B	C	C
44	29.89	59.78	5	0.2436	C	C	C
45	34.4	34.4	7	0.6728	A	B	B
46	28.8	28.8	3	0.2203	C	C	C
47	8.46	25.38	5	0.4075	C	C	C

Compared with the Ng-model, 10 out of the 47 items are classified differently. More specifically, 6 of 10 items are classified as class A in both models, 10 of 14 items are classified as class B in both models, and 21 of 23 items are classified as class C in both models.

Compared with the HV-model, 11 out of the 47 items are classified differently. Similarly, 7 of 10 items are classified as class A in both models, 10 of 14 items are classified as class B in both models, and 19 of 23 items are classified as class C in both models.

The difference in classification derived from the above two comparisons is resulted from the newly employed scoring method based upon Shannon entropy.

The weighting for each ranking of importance of the criteria is depicted in the following Figure 2, in which 1, 2, 3, 4, 5 and 6 denote the preferences AUC≻ADU≻LT, AUC≻LT≻ADU, ADU≻AUC≻LT, ADU≻LT≻AUC, LT≻AUC≻ADU and LT≻ADU≻AUC, respectively.

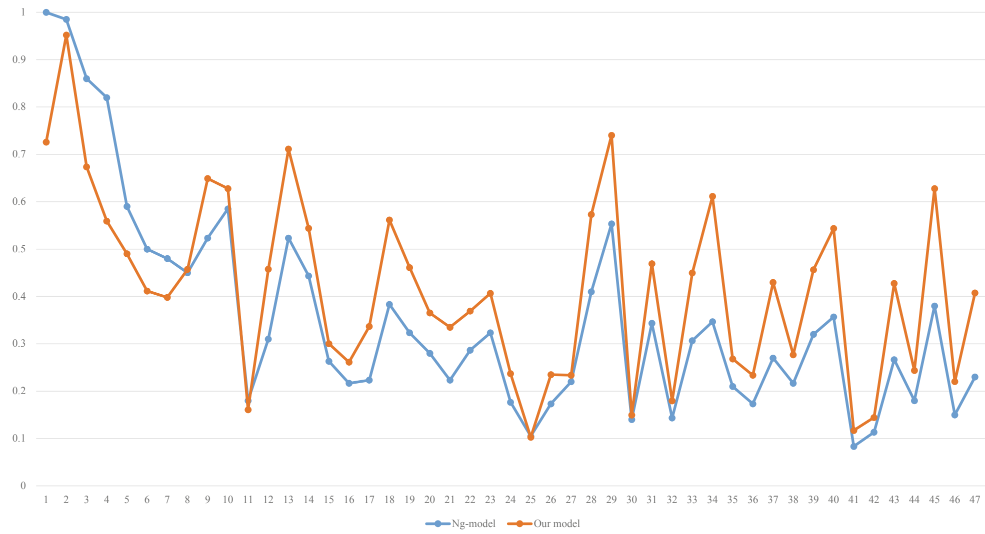

Figure 1

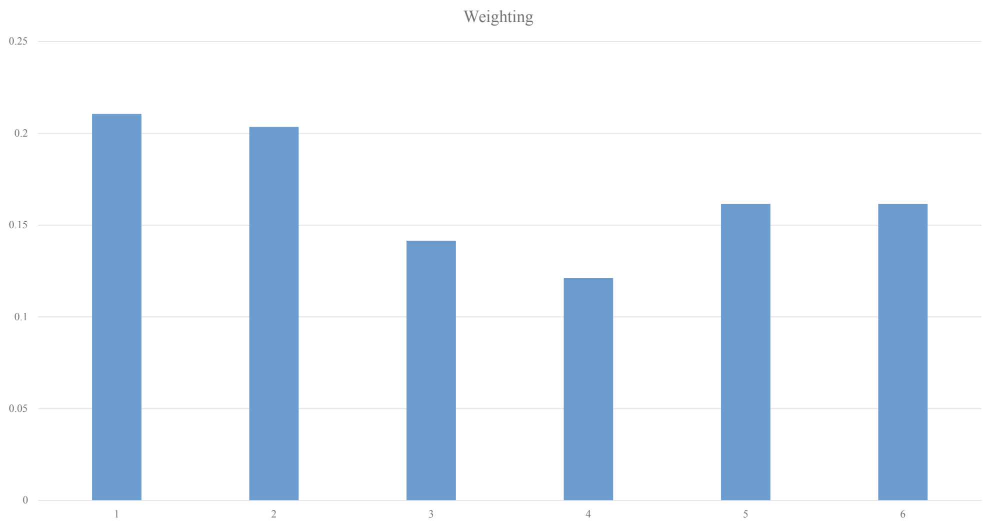

Figure 2

5 Concluding Remarks

The present short paper provides an extended version of the Ng-model for multiple criteria ABC inventory classification. The contribution of our paper is to present a model for comprehensively classify all inventory items, which improves Ng-model by aggregating all criteria sequences based upon Shannon entropy. A numerical illustration is conducted to compare our model with the Ng-model and HV model.

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