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Radar Target Recognition Algorithm Based on RCS Observation Sequence — Set-Valued Identification Method*

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Abstract This paper studies the problem of radar target recognition based on radar cross section (RCS) observation sequence. First, the authors compute the discrete wavelet transform of RCS observation sequence and extract a valid statistical feature vector containing five components. These five components represent five different features of the radar target. Second, the authors establish a set-valued model to represent the relation between the feature vector and the authenticity of the radar target. By set-valued identification method, the authors can estimate the system parameter, based on which the recognition criteria is given. In order to illustrate the efficiency of the proposed recognition method, extensive simulations are given finally assuming that the true target is a cone frustum and the RCS of the false target is normally distributed. The results show that the set-valued identification method has a higher recognition rate than the traditional fuzzy classification method and evidential reasoning method.

Keywords Feature extraction, radar target recognition, RCS, set-valued identification, wavelet transform.

1 Introduction

Radar target recognition is an important part of modern radar technology development. Research on the radar target recognition at home and abroad has become a hot spot, but radar target identification problem has not got a satisfactory answer, or mature technology and methods because of the complexity of the problem itself, as well as multiple interfering signals, especially in complex electromagnetic environments with more noise sources. Therefore, the study of radar target recognition technology has important value.

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Radar signals that can be received in the modern technology are divided into four kinds: The echo signal, the signal pole distribution, the high resolution radar imaging signal and the polarization characteristics of the signal. Among them, the echo signal is easy to be gotten because there is no special requirements for the radar. So radar target recognition based on the echo signal is rather important. In the echo signal, the radar cross section (RCS) is an important physical quantities. RCS characterizes radar targets' scattering ability to incident electromagnetic wave. Long before the radar appeared, people have obtained several exact solutions of electromagnetic scattering from typical shapes targets and perfect conductor. After radar appearing in the 1930s, radar target becomes an important part in radar transponder closed loop. People need to learn more about radar targets and RCS is one of the most important and basic parameters. Identification and anti-stealth technology in the early 1960s put RCS study into the climax. Currently there are many books on RCS[1–3].

In general, the process of radar target recognition is as follows: First, extract the target features from RCS sequences with known target category, then give a recognition criteria based on the relation between the target and its feature. Finally, recognize the unknown object by the recognition criteria. For the feature extraction, there are two main ways. One is extracting physical features from the time domain, such as extracting the cyclical nature of the RCS sequence[4]. The other is extracting features from transform domain (such as Fourier transform, wavelet transform, Merlin transform)[5]. What is more, there are many methods to establish the recognition criteria, such as Bayesian methods[6–8], evidential reasoning method[9–11], fuzzy classification method[12, 13] and neural network method[13–15].

Bayesian method requires the knowledge of the prior distribution. Then the minimum error rate or the minimum risk criteria can be given and the target can be recognized by the criteria. If there is no knowledge of the prior distribution, we usually assume the prior distribution is uniform distribution. Evidential reasoning doesn't require prior knowledge of the probability distribution function. It is the method of fusing the different probability distribution functions given by different evidences. Then we can give a recognition criteria based on the new probability distribution after fusing. Fuzzy classification method can deal with some of the challenges encountered in conventional recognition pattern. In recent years, it has been widely studied. The main idea of this method is converting the target feature to fuzzy sets and membership functions, then determining the target category through the fuzzy relationship and fuzzy reasoning. Neural network has the capability of adaptive, self-organizing, e-learning. It can handle some problems with very complex environmental information or unclear background knowledge. By learning to build the sample memory, the unknown mode will be sentenced to the closest memories.

Set-valued model is a new class of systems emerging from network and information technology. Unlike traditional systems, the available information to set-valued system is not the accurate output of system. Specifically, the set-valued system output is in some set, even binary, which brought great difficulties for parameter identification. In recent years, through the tireless efforts of some researchers, many innovative identification methods have been successfully applied to set-valued system and some interesting results have been got[16, 17]. With the improvement of theoretical work, set-valued system has gradually been applied to various fields, such as complex diseases and genome-wide association analysis[18].

In this paper, we extract five statistical features after calculating the wavelet transform of the RCS firstly. Then we establish the set-valued model of the radar target recognition problem and give the recognition criteria for the first time. Finally, the simulation is given and the results is compared with the ones of fuzzy classification method and evidential reasoning method. The simulation results show the effectiveness and advantages of the proposed set-valued identification method.

This paper is structured as follows: Section 2 introduces the basic concept of RCS and its influencing factors; Section 3 describes the wavelet transform and the feature extraction on time - scale domain; Section 4 gives specific guidelines for object recognition – set-valued identification methods; Section 5 is to simulate the recognition of genuine and fake bullets. From the data generation, the data pre-processing (wavelet transform), and the feature extraction to recognition by set-valued identification method, each step has been simulated by Matlab. Finally, we compare the final results with the ones of fuzzy classification and evidential reasoning; Section 6 summarizes this paper.

2 Radar Cross Section

Radar cross section (RCS) is the measure of a target's ability to reflect radar signals in the direction of the radar receiver. It is defined in two ways: One is based on electromagnetic theory; the other definition is based on radar observations. But the basic concept of the two ways is unified. It is defined as 4π times the ratio of backscatter power per steradian (unit solid angle) in the direction of the radar (from the target) to the power density that is intercepted by the target. It is denoted as σ.

The definition based on electromagnetic theory is as follows[2]:

$$\sigma = 4\pi R^2 \left|\frac{E_s}{E_i}\right|^2, \tag{1}$$

where R is the distance from target to radar, E_i is the strength of the incident electric field, and E_s is the strength of scattering electric field. This definition is based on the assumption that the scatter is small enough to radiate energy uniformly in all directions. Indeed, except near the forward shadow zone, the fields scattered by a large, perfectly conducting sphere are also distributed uniformly in all directions. So the RCS of a target can be viewed as the projected area of a metal sphere that will scatter the same power in the same direction that the target does.

The fact that Definition (1) depends on the distance R is disturbing. This dependence on distance implies that the target characteristic we want to determine is a function of the measurement environment, clearly an undesirable qualification. In order to eliminate this undesirable influence, we "standardize" the definition of RCS by letting R to approach infinity:

$$\sigma = \lim_{R\to\infty} 4\pi R^2 \left|\frac{E_s}{E_i}\right|^2, \tag{2}$$

Equation (2) is the formal definition of RCS.

The definition based on radar observation is as follows:

$$\sigma = 4\pi \frac{P_r}{\frac{A_r}{r_r^2}} \frac{1}{\frac{P_t G_t}{4\pi r_t^2}}$$
$$= 4\pi[\text{scattering power of the receiver}]/[\text{the power density of the incident plane wave}],$$

where P_r is the power input of the receiver, P_t is the power output of the transmitter, $A_r = \frac{G_r \lambda_0^2}{4\pi}$ is the effective area of the receiving antenna, G_r, G_t are the gains of the receiving antenna and the transmitting antenna, r_t, r_r are the distances from transmit antenna to the target and the receiving antenna to the target.

RCS of the target is mainly related to the following factors: The frequency of the incident electromagnetic wave, the incident angle of the target, the polarization of the incident electromagnetic wave, the surface geometry of the target and its coating material. The RCS of the same object shows different scattering characteristics for different frequency of the incident electromagnetic wave. When the target is irradiated by electromagnetic waves of the same frequency from different directions, the backscattering strength (or RCS) are different. In other words, electromagnetic scattering has the properties of directionality. Figure 1 shows the RCS of a typical type of a naval craft in different incidence angle.

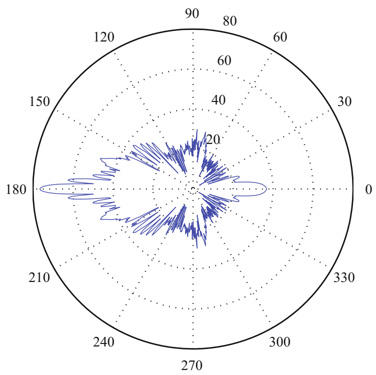


Figure 1 RCS in different incident angle

RCS is the essential characteristics of the object. In theory, the variety of the RCS along with the angle changing is determined when the physical properties of the target, materials and the frequency of the electromagnetic radiation are determined before. Therefore, the RCS of the target can be applied to radar target recognition.

3 Feature Extraction

This section introduces the concept of wavelet transform and how to feature extraction.

3.1 Wavelet Transform

Wavelet transform is similar to Fourier transform. They both study a function by a group of basic functions. However, the basic function of Fourier transform is a family of cosine function, while the basic function of wavelet transform is the wavelet function.

Given a basic function $\psi(t)$, let

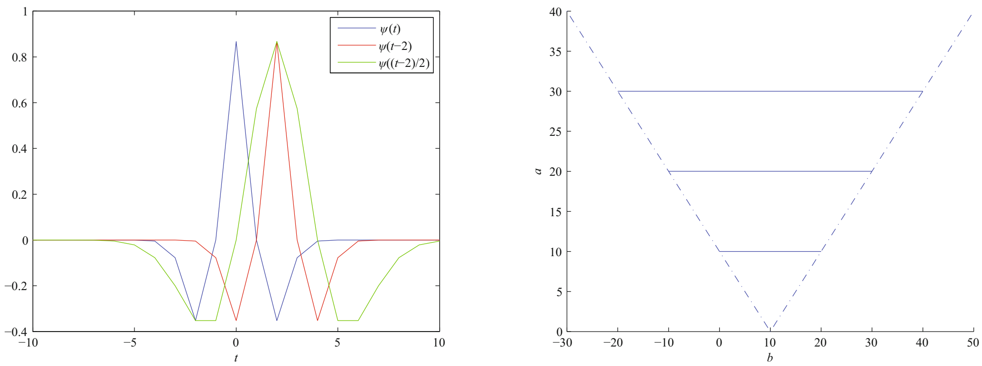
$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\Big(\frac{t-b}{a}\Big), \tag{3}$$

where a, b are constants, and $a > 0$. Clearly, $\psi(\frac{t-b}{a})$ is generated by doing first translation operations then expansion operations on the mother wavelet $\psi(t)$. If a and b are changing, we can get a family of $\psi(\frac{t-b}{a})$, which is called the wavelet basic function. For an L^2 integrable function $x(t)$, its wavelet transform with respect to the wavelet $\psi(t)$ is defined by

$$WT_x(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} x(t)\psi\Big(\frac{t-b}{a}\Big)dt, \tag{4}$$

where a, b and t are continuous variables. So Equation (4) is called continuous wavelet transform.

In the equation (3), b is to determine the center of the analysis time and a is to stretch the mother wavelet $\psi(t)$. The larger a, the support range of $\psi(\frac{t}{a})$ on the time-domain bigger. The center position and the time width in analysis can be determined by a and b. It is shown in Figure 2.



(a) Stretch of the basic wavelet (b) The effect of a, b on the scope of the analysis

Figure 2

In the continuous wavelet transform, a, b and t are continuous variables. However, for the computer cumulation, we should discretize the parameters a and b. Let $a = a_j, j \in Z$, $b = b_j, j \in Z$, then we can get the discrete wavelet transform.

$$WT_x(a_j,b_k) = \frac{1}{\sqrt{a_j}}\int_{-\infty}^{+\infty} x(t)\psi\Big(\frac{t-b_k}{a_j}\Big)dt. \tag{5}$$

In this paper, the sequence of RCS is discrete variable $x(n), n = 1, 2, \cdots, N$. a and b are discretized as $a = 1, 2, \cdots, M$, $b = 1, 2, \cdots, N$. Therefore, the discrete wavelet transform of the RCS sequence $x(n), n = 1, 2, \cdots, N$ is as follows:

$$W_x(a,b) = \sum_{n=1}^{N} x(n)\psi\Big(\frac{n-b}{a}\Big). \tag{6}$$

3.2 Feature Extraction

After calculating the discrete wavelet transform of the RCS observation sequence, we can get a family of $W_x(a,b), a = 1, 2, \cdots, M, b = 1, 2, \cdots, N$. Let $A = [W_x(a,b)]$, namely the element in row a and column b of the matrix A is $[W_x(a,b)]$. Therefore, the rank of A is $M \times N$. It is obviously unrealistic to take A as the feature of the target since the amount of the data in matrix A is very large. Therefore, we need to extract features which have a smaller dimension from matrices A. Since the RCS observation sequence of low resolution radar is random, we consider the following five statistical features as the features of the target[12]:

1) Mean (ϕ_1): $\phi_1 = \frac{1}{M\times N}\sum_{i=1}^{M}\sum_{j=1}^{N} A(i,j)$;

2) Variance (ϕ_2): $\phi_2 = \frac{1}{M\times N}\sum_{i=1}^{M}\sum_{j=1}^{N}[A(i,j)-\phi_1]^2$;

3) Scale gravity (ϕ_3): $\phi_3 = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N}[iA(i,j)]}{\sum_{i=1}^{M}\sum_{j=1}^{N}A(i,j)}$;

4) The largest element (ϕ_4): $\phi_4 = \max_{i,j} A(i,j)$, $i = 1, 2, \cdots, M$, $j = 1, 2, \cdots, N$.;

5) The largest singular value (ϕ_5): Suppose the singular value decomposition of A is $A = U\Sigma V$, $\Sigma = \mathrm{diag}[\sigma_1, \sigma_2, \cdots, \sigma_h]$, $h = \min(M,N)$, then $\phi_5 = \max_i \sigma_i$, where $i = 1, 2, \cdots, h$.

As above, from the RCS observation sequence we can get a feature vector $\phi = [\phi_1, \phi_2, \cdots, \phi_5]^{\mathrm{T}}$, which will be used later in the target recognition.

4 Target Recognition Algorithm

This section describes how to build the model that shows the relation between the feature vector and the target category by set-valued identification. Then we can give the recognition criteria based on the model.

4.1 System Model

In the situation that the model is unknown, it is common to use a linear approximation model to approximate the model. So we assume that there exists an implicit index which is the linear combination of the different features and the target category is decided by the implicit index. Then, the relation between the feature vector and the target category can be described as set-valued system model as follows:

$$\begin{cases} y_k = \phi_k^{\mathrm{T}}\theta + d_k = \displaystyle\sum_{j=1}^{5}\phi_{kj}\theta_j + d_k, \\ s_k = I_{\{y_k \le C\}}, \quad k = 1, 2, \cdots, N, \end{cases} \tag{7}$$

where N is the number of the RCS sequence, $\phi_k = (\phi_{k1}, \phi_{k2}, \cdots, \phi_{k5})^{\mathrm{T}}$ is the feature vector of the kth RCS sequence, $\theta = (\theta_1, \theta_2, \cdots, \theta_5)^{\mathrm{T}}$ is the parameter to be identified, d_k is the system noise, $\{y_k, k = 1, 2, \cdots, N\}$ is the output which is not measurable, $\{s_k, k = 1, 2, \cdots, N\}$ is the set-valued output which can be observed. $s_k = 1$ means that the target corresponding to the kth RCS sequence is the true target. $s_k = 0$ means that the target is false.

Inevitably there will be some errors existing in the acquisition and processing of RCS data. And we will ignore the actual data of nonlinear dynamic systems by taking advantage of the linear system. Therefore the noise d_k is added as the supplements of other environment factors. According to the center limit theorem, we assume that d_k is distributed normally with mean 0 and variance σ^2.

4.2 Set-Valued Identification Algorithm

Set-valued identification is to use the feature vector $\{\phi_k, k = 1, 2, \cdots, N\}$ and its target category $\{s_k, k = 1, 2, \cdots, N\}$ to estimate the parameter θ.

Traditional set-valued identification method needs some requirements for the input variable $\{\phi_k, k \le N\}$, such as periodic input forms and so on. In recent years, this constraint has been overcome by using the maximum likelihood criterion, which leads set-valued identification to a wide range of applications in different fields. We use an iterative identification algorithm based on expectation maximum (EM) proposed in [19]. The specific form is as follows:

$$\begin{aligned}\widehat{\theta}^{i+1} = \widehat{\theta}^{i} + & \left(\sum_{k=1}^{N} \phi_k \phi_k^{\mathrm{T}}\right)^{-1} \\ & \cdot \left(\sum_{k=1}^{N} \phi_k \sigma^2 f(C - \phi_k^{\mathrm{T}} \widehat{\theta}^{i}) \times \left[-\frac{I_{\{s_k=1\}}}{F(C - \phi_k^{\mathrm{T}} \widehat{\theta}^{i})} + \frac{I_{\{s_k=0\}}}{1 - F(C - \phi_k^{\mathrm{T}} \widehat{\theta}^{i})}\right]\right), \end{aligned} \tag{8}$$

where $\widehat{\theta}^{i}$ is the estimated value of the parameter in the ith iteration, C is a known threshold, $F(\cdot)$, $f(\cdot)$ denote the probability distribution function of the standard normal distribution and probability density function respectively, $I(\cdot)$ is the indicator function.

[19] has proved that if the corresponding maximum likelihood estimates exist, the iterative algorithm can converge to the point of maximum likelihood estimation and can achieve exponential convergence rate regardless of the initial value of the iteration.

In radar target recognition, the threshold C is unknown due to the lack of prior information. So we should estimate both the unknown parameter θ and the threshold C. This can be shown as follows:

$$\{(\phi_k, s_k), k \le N\} \xrightarrow[\text{identification}]{\text{system}} (\theta, C). \tag{9}$$

To estimate C, we change C into a component of the new parameters:

$$\underline{\phi}_k \triangleq (\phi_{k1}, \phi_{k2}, \cdots, \phi_{k5}, -1)^{\mathrm{T}}, \quad \underline{\theta} \triangleq (\theta_1, \theta_2, \cdots, \theta_5, C)^{\mathrm{T}}.$$

Then the model (7) is equal to the following model:

$$\begin{cases} \underline{y}_k = \underline{\phi}_k^{\mathrm{T}} \underline{\theta} + d_k = \displaystyle\sum_{j=1}^{5} \phi_{kj}\theta_j + d_k - C, \\ s_k = I_{\{\underline{y}_k \leq 0\}}, \quad k = 1, 2, \cdots, N. \end{cases} \tag{10}$$

By Equation (8), the estimate of the new parameter $\widehat{\underline{\theta}}$ can be give as follows:

$$\widehat{\underline{\theta}}^{i+1} = \widehat{\underline{\theta}}^{i} + \left(\sum_{k=1}^{N} \underline{\phi}_k \underline{\phi}_k^{\mathrm{T}}\right)^{-1} \left(\sum_{k=1}^{N} \underline{\phi}_k \sigma^2 f(-\underline{\phi}_k^{\mathrm{T}} \widehat{\underline{\theta}}^{i}) \times \left[-\frac{I_{\{s_k=1\}}}{F(-\underline{\phi}_k^{\mathrm{T}} \widehat{\underline{\theta}}^{i})} + \frac{I_{\{s_k=0\}}}{1 - F(-\underline{\phi}_k^{\mathrm{T}} \widehat{\underline{\theta}}^{i})} \right] \right), \tag{11}$$

where $\widehat{\underline{\theta}}^{i}$ is the estimate value of the new parameter in the ith iteration. The estimate of the new parameter $\widehat{\underline{\theta}}$ can determine the estimate of the original parameter $\widehat{\theta} = (\widehat{\theta}_1, \widehat{\theta}_2, \cdots, \widehat{\theta}_n)^{\mathrm{T}}$ and the estimate of the threshold $\widehat{C}$.

4.3 Target Recognition Criteria

Set-valued identification can give the estimated value of the parameter θ and the threshold C (i.e., $\widehat{\theta}$ and $\widehat{C}$). According to the model (7), we can get the following criteria by replacing the θ and C with $\widehat{\theta}$ and $\widehat{C}$, where ϕ is the feature vector of the unknown target and the noise is assumed to be zero.

$$\widehat{y} = \phi^{\mathrm{T}} \widehat{\theta} = \sum_{j=1}^{5} \phi_j \widehat{\theta}_j, \quad \widehat{s} = I_{\{\widehat{y} \leq \widehat{C}\}}. \tag{12}$$

If $\widehat{s} = 1$, the unknown target is the true target, otherwise false.

4.4 Other Target Recognition Methods

There are many ways to identify the target. Here we only introduce fuzzy classification method and evidential reasoning method.

4.4.1 Fuzzy Classification Method

In the fuzzy classification process, the feature extracted by numerical transformation is converted into a reasonable language tag characterized by fuzzy sets and its membership, which is equivalent to the reflection of human brain for object features on the concept of layers. The relationship between objects and language tag is characterized by simple and effective fuzzy relationship and fuzzy reasoning model in the fuzzy set theory. Varying fuzzy relationship between samples and patterns can be described by the similarity, membership, ambiguity, and closeness. Based on the statistical theory, when the target is characterized by a single parameter, the membership function v_{ij} can be considered as the normal distribution function, namely,

$$v_{ij} = \exp\left[-\frac{(\phi_j - m_{ij})^2}{2\sigma_{ij}^2} \right], \tag{13}$$

where ϕ_j is the jth feature of an unknown target, m_{ij} and σ_{ij} represent mean value of the jth feature of the ith target category and its corresponding standard deviation. $i = 1, 2, \cdots, I$

represent the different target category, I is the number of target categories. In this paper, $I = 2$, which means that there are two target category: True and false. $j = 1, 2, \cdots, J$ is the number of the feature. In this paper, $J = 5$, which means that each target has five features defined in Subsection 3.2. After the membership function v_{ij} is given, the fuzzy evaluation V_i which represents the degree of the unknown target belonging to the ith target category can be calculated as follows:

$$V_i = \sum_{j=1}^{5} w_j v_{ij}, \quad i = 1, 2, \tag{14}$$

where w_j reflecting the importance of the jth feature in the target recognition is defined by

$$w_j = \frac{p_j}{\sum_{k=1}^{5} p_k}, \quad j = 1, 2, \cdots, 5, \tag{15}$$

where p_j is the mean recognition rate by using only the jth feature. Finally the recognition criteria can be given as follows: If the fuzzy evaluation V_i is largest in $V_i, i = 1, 2, \cdots, N$, then the unknown target belongs to the ith target category.

Remark 1 In the fuzzy classification method, w_j which represents the importance of the jth feature in recognition is given by Equation (15). While θ_j in the set-valued identification method which has the same effect with w_j is given by Equation (8), using the system information. Therefore, the set-valued identification is more reasonable.

4.4.2 Evidential Reasoning Method

The basic strategy of evidential reasoning method is to divide evidence set into two or more unrelated parts, then use each part to judge the identification framework independently, finally fuse them by Dempster combination rule. Dempster combination rule determines the comprehensive degree of the trust that the assumption is correct under multiple evidence, which makes the judgment on the issue more rational and more reliable. Let Θ be the set of all the possible values of variable x. The elements in set Θ are all different. Then Θ is called the idenfication framework. In this paper, the identification framework $\Theta = \{T, D\}$, where T represents the true target, D represents the false target. If the number of elements in Θ is N, then power set of Θ has $2^N - 1$ elements. In our paper, the power set of Θ is $2^\Theta = \{\{T\}, \{D\}, \{T, D\}\}$. m is a set function defined on the power set 2^Θ, which satisfies: $m(\Phi) = 0$ and $\sum_{A \subseteq \Theta} m(A) = 1$. m is called the basic probability distribution function on the identification framework Θ. $m(A)$ represents the occurance of the event A supported by the evidence. Define the belief function Bel on R: $Bel(A) = \sum\{m(B) | B \subseteq A, B \neq \Phi\}$. Let $Bel_1, Bel_2, \cdots, Bel_n$ be the different belief function on the same identification framework Θ and $m_1, m_2, \cdots, m_n$ be the corresponding basic probability distribution function. If there exists basic probability distribution function m on $Bel_1 \oplus Bel_2 \oplus \cdots \oplus Bel_n$, then

$$m(A) = K^{-1} \sum_{A_1, A_2, \cdots, A_n \subset \Theta, A_1 \cap A_2 \cap \cdots \cap A_n = A} m_1(A_1) m_2(A_2) \cdots m_n(A_n), \tag{16}$$

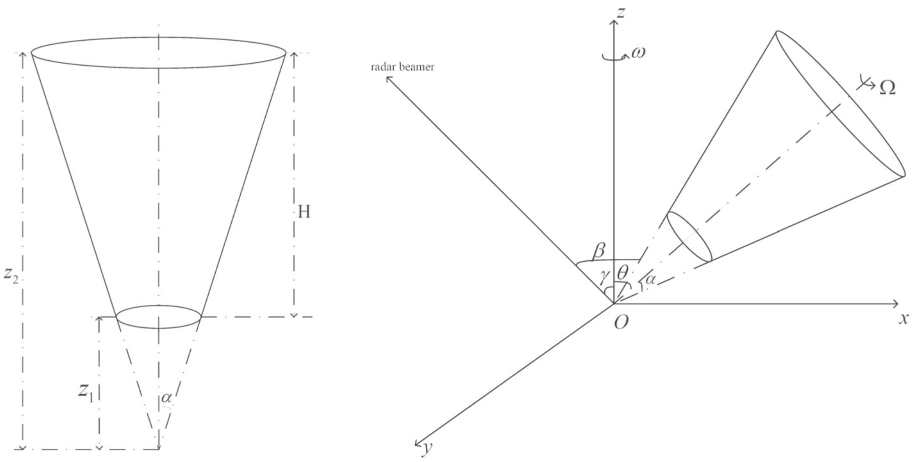
where $K = \sum_{A_1, A_2, \cdots, A_n \subset \Theta, A_1 \cap A_2 \cap \cdots A_n \neq \Phi} m_1(A_1) m_2(A_2) \cdots m_n(A_n)$. In our paper, the different evidences are the five different features. $m_1, m_2, \cdots, m_5$ are the corresponding basic probability distribution functions. Let $m_j(T) = \frac{v_{1j}}{v_{1j}+v_{2j}}, m_j(D) = \frac{v_{2j}}{v_{1j}+v_{2j}}, m_j(\Theta) = 0, j = 1, 2, \cdots, 5$, where v_{ij} is defined as Equation (13). Fusing the five different features by Equation (16), we can give the recognition criteria based on the value of $m(T), m(D), m(\Theta)$: If $m(T)$ is the largest, we judge that the unknown target is the true target; if $m(D)$ is the largest, the unknown target is false; otherwise we can judge the unknown target.

5 Simulation

This section gives a simulation example of the recognition process. In the example, the true target is a cone frustum and the RCS of the false target is distributed normally with mean 3 and variance 1. Firstly, we produce the RCS data of the true and false target by Matlab. Secondly, we calculate the wavelet transform of the RCS data and extract the feature vector. Finally, we use the set-valued identification method to recognize the target, and compare the results to the ones of the fuzzy classification method and the evidential reasoning method.

5.1 Produce of the RCS Data

Suppose the true target is a cone frustum. Its geometric character is as Figure 3(a), and Figure 3(b) gives the attitude change in the procession process of a pyramidal object.



(a) Geometric characteristic of a cone frustum

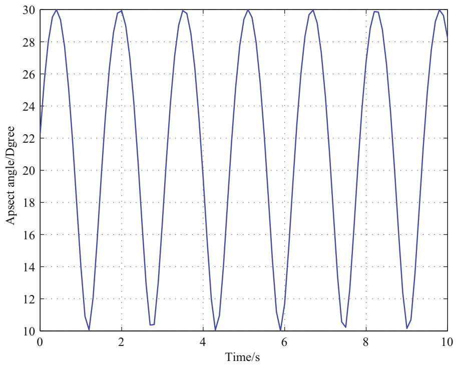
(b) the relationship of radar and target

Figure 3

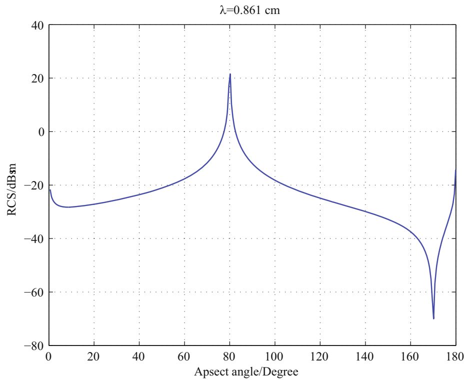
In Figure 3(b), the height of the cone frustum is H and its half cone angle is $\alpha(0 \leq \alpha \leq \pi/2)$. It is processing with angular velocity ω around the z axis. $\theta(0 \leq \theta \leq \pi/2)$ is the precession angle. While it spin around its own with angular velocity Ω. Let the angle between the radar beam and the precession axis z be γ, and the angle between the radar beam and the target axis be the aspect angle β. If we start timing when the cone frustum axis located in the xoz plane, then the aspect angle β can be calculated in [20]:

$$\beta(t) = \arccos(\cos\theta\cos\gamma - \sin\theta\sin\gamma\sin\omega t). \tag{17}$$

If $\theta = 10°, \gamma = 20°, \omega = 4$ rad/s, the simulation of the aspect angle of a cone frustum is as Figure 4(a).



(a) The aspect angle of a cone frustum over time



(b) The RCS over aspect angle

Figure 4

Let λ be the incident wavelength and α be the half cone angle of the cone frustum. z_1 and z_2 are defined as Figure 3(a). When the polarization is linear polarization, the RCS of the cone frustum can be calculated by [21]:

$$\mathrm{RCS}(\beta) = \begin{cases} \dfrac{\lambda z_2 \tan\alpha \tan^2(\alpha+\beta)}{8\pi \sin\beta}, & \beta \in (0,\pi) \ \text{ and } \ \beta \neq \pi/2-\alpha, \\ \dfrac{8\pi(z_2^{\frac{3}{2}} - z_1^{\frac{3}{2}})^2 \sin\alpha}{9\lambda \cos^4\alpha}, & \beta = \pi/2-\alpha. \end{cases} \tag{18}$$

Consider a cone frustum with $z_1 = 11.67$cm, $z_2 = 32.61$cm, $\alpha = 10°$. When the incident electromagnetic with wavelength $\lambda = 0.861$cm radiate, the RCS over aspect angle is as Figure 4(b).

Substituting (17) into Equation (18), we can get the RCS of the cone frustum over time as follows:

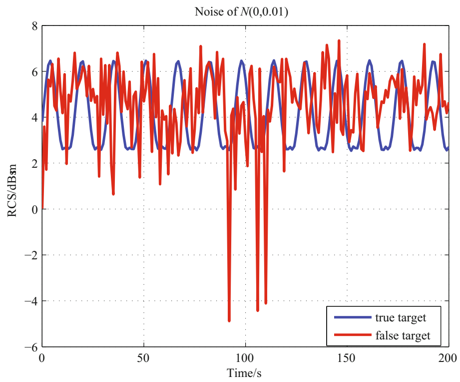
$$\mathrm{RCS}_{th}(t) = \begin{cases} \dfrac{\lambda z_2 \tan\alpha \tan^2(\alpha + \arccos(\cos\theta\cos\gamma - \sin\theta\sin\gamma\sin\omega t))}{8\pi \sin(\arccos(\cos\theta\cos\gamma - \sin\theta\sin\gamma\sin\omega t))}, \\ \qquad \text{If } \arccos(\cos\theta\cos\gamma - \sin\theta\sin\gamma\sin\omega t) \in (0,\pi) \text{ and } \beta \neq \pi/2-\alpha, \\ \dfrac{8\pi(z_2^{\frac{3}{2}} - z_1^{\frac{3}{2}})\sin\alpha}{9\lambda\cos^4\alpha}, \\ \qquad \text{If } \arccos(\cos\theta\cos\gamma - \sin\theta\sin\gamma\sin\omega t) = \pi/2-\alpha. \end{cases}$$

The model for the actual observed RCS is as follows:

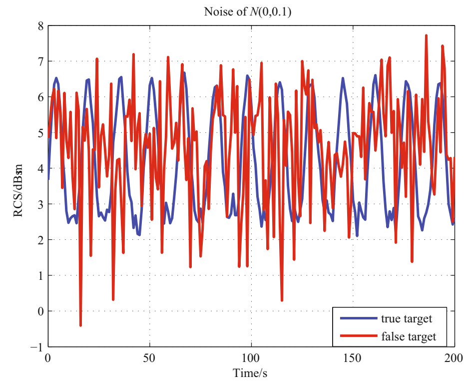
$$\mathrm{RCS}_{ob}(t) = \mathrm{RCS}_{th}(t) + v(t), \tag{19}$$

where $\mathrm{RCS}_{ob}(t)$ is the observed value of RCS, $\mathrm{RCS}_{th}(t)$ is the theoretical value, $v(t)$ is Gaussian white noise with mean 0 and variance σ^2.

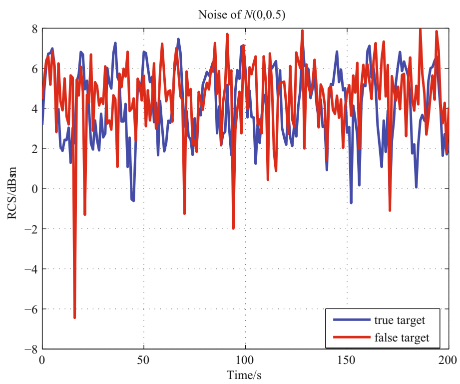
We can get 100 RCS sequences of the true target (false target, respectively) by Matlab. The length of each RCS sequence is $N = 201$. Figure 5 is the 100th RCS sequences of true target and false target under different noises.



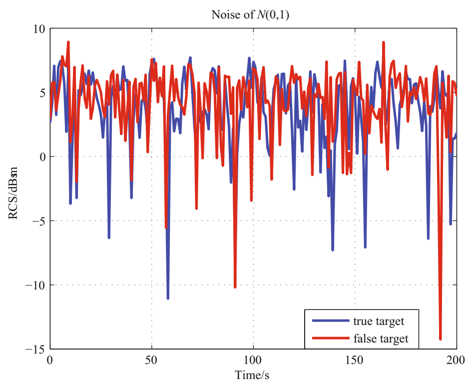
(a) Noise of $N(0, 0.01)$



(b) Noise of $N(0, 0.1)$



(c) Noise of $N(0, 0.5)$



(d) Noise of $N(0, 1)$

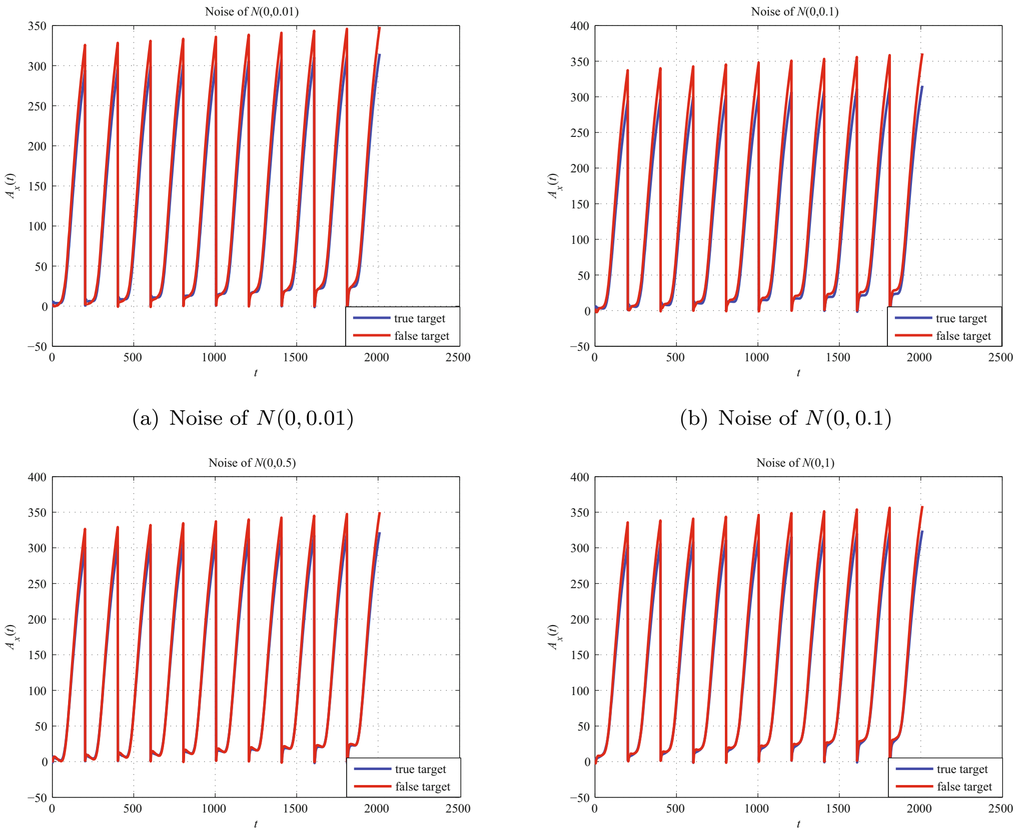
Figure 5 RCS of true and false target under different noises

5.2 Wavelet Transform and Feature Extraction

Calculate the wavelet transform of each RCS sequence of true and false target by Equation (6), where $M = 10$, $N = 201$ and the wavelet function is Mexican Hat function. Let $A = [W_x(a, b)]$. Rearranging the matrix A line by line, we can get the vector A_x, namely $A(i, j) = A_x((i-1)N + j)$. Figure 6 is the vector A_x of the 100th RCS sequence of the true target (false target, respectively) under different noises.

Figure 6 shows that the true target and the false target can be clearly separated by A_x. For another, it is the reason why we extract the five features after calculating the wavelet transform of the RCS sequence.

Now we should extract the feature vector ϕ from the wavelet matrix A of true target (false target, respectively). Since the number of the RCS sequence of the true target (false target, respectively) is 100, we can get 100 feature vectors $\phi_k, k = 1, 2, \cdots, 100$ of the true target (false target, respectively).



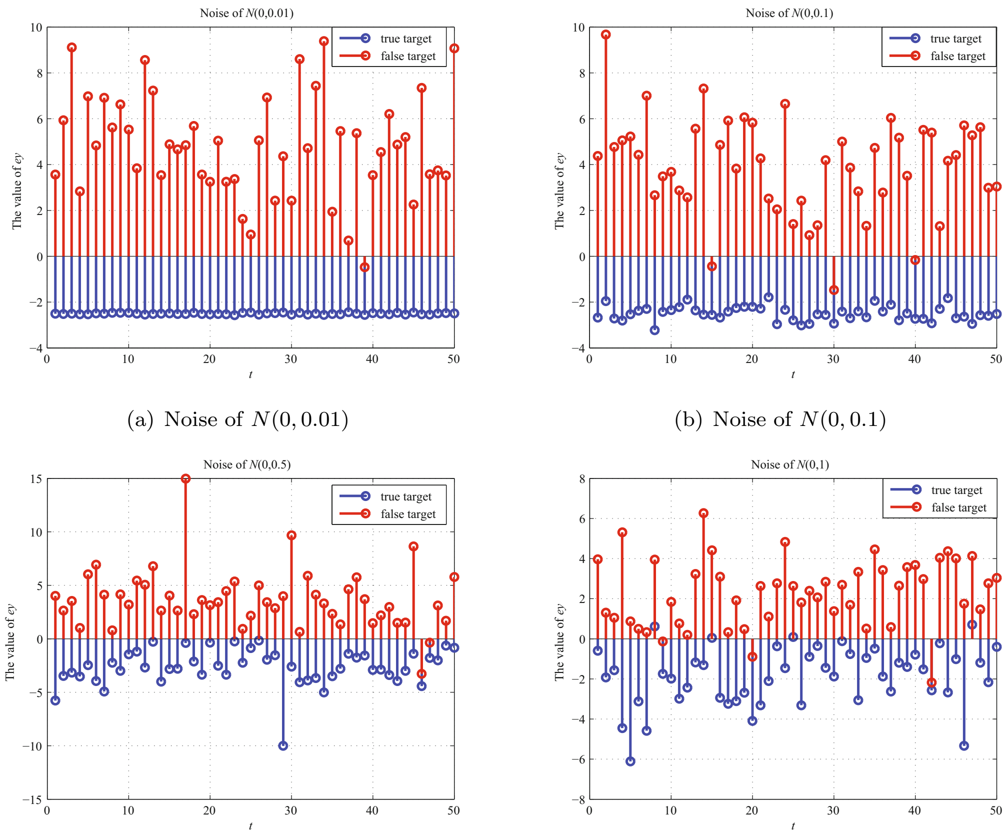
(a) Noise of $N(0, 0.01)$ (b) Noise of $N(0, 0.1)$

(c) Noise of $N(0, 0.5)$ (d) Noise of $N(0, 1)$

Figure 6 A_x of the RCS under different noises

5.3 Comparison of Set-Valued Identification Method and Other Methods

Having got 100 groups of feature vector of the true target (false target, respectively), we should classify these groups. The odd groups are used as training data to identify the parameter θ and C. We can calculate the estimation of the parameter by 100 iteration: $\widehat{\theta} = [6.3129, -5.9773, -116.1323, 0.3120, -0.0184]^{\mathrm{T}}$, $\widehat{C} = -627.7962$ with the noise of $N(0, 0.01)$; $\widehat{\theta} = [8.7971, 3.4688, -39.9522, 0.6771, -0.2209]^{\mathrm{T}}$, $\widehat{C} = -167.9744$ with the noise of $N(0, 0.1)$; $\widehat{\theta} = [16.6912, 5.9260, -93.2404, 0.0578, -0.3636]^{\mathrm{T}}$, $\widehat{C} = -499.7196$ with the noise of $N(0, 0.5)$; $\widehat{\theta} = [-1.6892, -3.6159, -21.8552, 0.2652, 0.07955]^{\mathrm{T}}$, $\widehat{C} = -73.4912$ with the noise of $N(0, 1)$. The even groups are used as test data to calculate the corresponding value of $\phi^{\mathrm{T}}\widehat{\theta} - \widehat{C}$, which is denoted as ey. Figure 7 is the value of ey with different noises from the test data of the true target (false target, respectively). At last, we can recognize the target based on the the value of ey (i.e., $\phi^{\mathrm{T}}\widehat{\theta} - \widehat{C}$).



(a) Noise of $N(0, 0.01)$ (b) Noise of $N(0, 0.1)$

(c) Noise of $N(0, 0.5)$ (d) Noise of $N(0, 1)$

Figure 7 The value of ey with different noises

Figure 7 shows the true target and the false target can be almost separated by the recognition criteria in Subsection 4.3. In Figure 7(b), we will make 3 wrong judgment by the recognition criteria. The false target at points 7, 10 and 34 will be judged as true target for the values of ey are less than 0. So the recognition rate is 0.97.

Table 1 shows the recognition rate of fuzzy classification method, evidential reasoning method and set-valued identification method under different noises. From the table, we can conclude that

1) Under different noises, set-value identification method has higher recognition rate than the ones of fuzzy classification method and evidential reasoning method.

2) When the noise is relatively large, the recognition rate of fuzzy classification and evidential reasoning method are very low, but set-valued identification method still has high recognition rate.

6 Summary

This paper studies the radar target recognition problem. A radar target recognition algorithm based on RCS observation sequence — Set-value identification method is proposed. The algorithm is to calculate the discrete wavelet transform of the observed RCS sequence firstly, and then extract five valid statistical features on the time-scale plane. Finally, recognize the unknown target by set-valued identification method. The method is applied to the situation that the true target is a cone frustum and the RCS of the false target is distributed normally. The simulation results in Matlab shows that the set-valued identification method performs much better than other methods. On the basis of the work in this paper, we can study more meaningful problems. For example, how to use set-valued identification method to solve multi-target recognition problem.

Table 1 The results of 3 different methods under different noises

Noise Variance	Test Number	Fuzzy Classification	Evidential Reasoning	Set-valued Idenfication
0.01	1	0.96	0.94	0.99
	2	0.93	0.92	0.97
	3	0.92	0.92	0.99
	4	0.95	0.94	0.98
0.1	1	0.78	0.85	0.97
	2	0.94	0.86	0.99
	3	0.90	0.87	0.99
	4	0.80	0.88	0.96
0.5	1	0.75	0.75	0.98
	2	0.71	0.75	0.98
	3	0.85	0.86	0.97
	4	0.78	0.88	0.98
1	1	0.78	0.80	0.93
	2	0.76	0.76	0.91
	3	0.75	0.85	0.92
	4	0.66	0.77	0.96

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