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MULTI-AGENT BASED MODELING AND SIMULATING FOR EVACUATION PROCESS IN STADIUM*

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Abstract A multi-agent evacuation model is proposed in this paper to simulate the pedestrian evacuation process in stadium with or without obstacles. The authors give a multi-agent individual decision-making framework, in which the action direction of each pedestrian (called agent) is affected by the distance of the agent to the exits and the occupant number and density within the view field of the agent. Different from the existing results, the authors divide all the pedestrians in the stadium into four classes: Young male, young female, old male, and old female. In evacuation process, the weighting that affects individual decision-making between each class of agents is different. In the simulation, the authors present the effects of obstacles, crowd distribution and the exit position in evacuation process. Simulation results show that the proposed model can reproduce exactly the real evacuation process in stadium. Therefore, this method might be useful to assess public buildings design.

Keywords Cellular automaton, multi-agent, pedestrian evacuation.

1 Introduction

In recent years, with the emergence of more and more large public buildings, the pedestrian evacuation has become a very important problem. A large amount of research has been done to simulate the collective behavior of pedestrians inside a building or a room.

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The existing pedestrian evacuation models can be generally classified into macroscopic model and microscopic model. Macroscopic model usually takes nodes and connections as space unit. It treats individuals as typical molecules in fluid dynamics[1, 2]. Though the calculation of this model is easy, it can not describe the details of the pedestrian behaviors, so this model is currently replaced by other models.

Microscopic model focuses on each individual. The individual behaviors are affected by all kinds of factors such as individual ability, position, surrounding environment, the structure of the building, and so on. Microscopic model can be further divided into continuous model and discrete model. A typical continuous model is social force model proposed by Helbing, et al.[3, 4], in which the pedestrian movement is defined based on Newtonian mechanics and thus pedestrians can be treated as structureless particles. Some recent results on social force model can refer to [5, 6] and the references therein. Cellular automaton (CA) model and lattice gas model[7] are two well used discrete space models. CA model is based on individual characteristics. It allows pedestrians to be located at nodes of a fixed or adaptive grid, and pedestrian coordinates are updated at discrete time intervals. CA model is used successfully to describe pedestrian dynamics in complex situations because of its computational simplicity, flexibility, and efficiency[8]. In CA model, the floor field method is widely used, which includes the static floor field and the dynamic floor field[8–10]. Some extended floor field model can refer to [11, 12] and the literature therein. In addition, Fang, et al.[13] introduced a concept of space-time use efficiency for gauging the usage of space and time resources in an evacuation process, based on which a pedestrian waiting-time model was devised to improve evacuation performance. In [14], a logit-based discrete model is proposed to study the exit choice behaviour of evacuees in rooms with internal obstacles and multiple exits.

In this paper, the pedestrian evacuation problem in stadium is investigated from a new point of view. Different from the existing methods, we use the multi-agent based model. In this method, the action rule of each pedestrian at each time step is determined by the distance of the pedestrian to the exits and the occupant number and density within the view field of the pedestrian. In this model, there can be several exits and obstacles in the stadium. The pedestrians (called agents) are not identical, who are divided into four classes: Young male, young female, old male, and old female, according to individual gender and age. To evacuate from the exits, the weighting that affects individual decision-making between each class of agents is different. In the simulation, we consider two cases: (a) There is no obstacle and the agents are initially distributed randomly and (b) there are obstacles and the agents are initially arranged regularly. To study the effect of the exit position to evacuation efficiency, we put the two exits unilateral in the back wall and bilateral in the left and right wall, respectively. Simulation results show that the proposed model can reproduce exactly the real evacuation in stadium and reflect the effects of obstacles, crowd distribution, and the exit position in evacuation process. Therefore, this method might be useful to assess public buildings design and the ability to provide sufficient time for the pedestrians to evacuate safely in the event of an emergency.

The rest of the paper is organized as follows. Section 2 is our problem formulation and some necessary preliminaries. Section 3 describes the concrete model. First, to realize the evacuation process, we give an individual decision-making framework of the model, then the multi-agent based evacuation algorithm is proposed. In Section 4, according to the proposed evacuation algorithm, simulation results are presented using Visual C language. Section 5 is the conclusions.

2 Problem Formulation and Preliminaries

In this paper, we consider the pedestrian evacuation problem of stadium with two exits and obstacles. Similar to cellular automaton model, the room is divided by a two-dimensional grid. Each cell can be empty, occupied by an obstacle (such as chair or wall) or by a pedestrian. At any time, one agent can only occupy one cell. If a cell is occupied by an agent, then this agent occupies the whole space of this cell. The size of each cell corresponds to $0.4 \times 0.4\text{m}^2$, the typical surface occupied by a person in a dense situation[9]. Assume the mean velocity of pedestrian is about 1m/s, then each time-step is 0.4s when moving 0.4m.

In this model, each pedestrian is regarded as an agent. We assume each agent can have nine possible action directions including stationary with the same probability (refer to Figure 1).

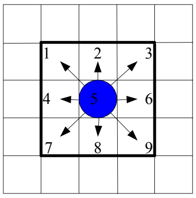


Figure 1 Nine possible action directions

In the existing simulation results on pedestrian evacuation, the action update rule for all the agents at each time step can be synchronous and asynchronous. In this paper, we adopt the asynchronous update rule. First, we order all the agents at each time step according to the distance of the agents to the target exit, then let the agent who lies in close to the exit take action first. The idea is inspired by the phenomenon that the vehicle waiting ahead moves first in the road when the traffic signal turns green.

In this paper, we assume all agents can obtain the information of the exits, that is, they can know the position of the exits. This is reasonable since everyone has known the entrance after they enter the stadium. But the view field of each agent is not global due to obstacles and the limited vision of the agent. The agent's view field is defined as follows. First draw a line between the center of the exit and the cell occupied by the agent. Then the view field is an angular domain with this agent as the vertex and 2α as the angle and the line as the bisector, which is shown in Figure 2. In this paper, we choose $\alpha = 45°$. Note that a cell belongs to the view field if and only if its center lies within the two boundary lines.

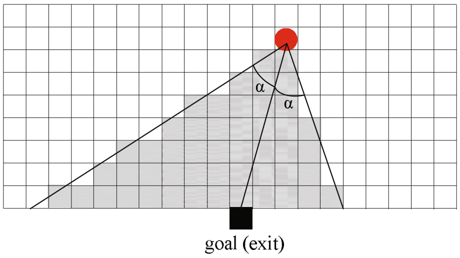


Figure 2 View field of agent to the exit

In this model, the pedestrians in stadium are not identical. The individual ability is different due to the gender, age, health status, and so on. According to individual gender and age, all the pedestrians are divided into four classes: Young male, young female, old male, and old female. During the evacuation process, the weighting factors that affect the agent's decision-making between each class are different.

3 Model Description

3.1 Individual Decision-Making Framework

It is natural that the pedestrians move and make decision spontaneously in evacuation process. At every time step, each pedestrian must decide where to move. Study has shown that the pedestrians do not always move towards the closest exit[10]. The individual movement direction is affected by many factors, such as the individual position, the individual ability, the position of the exit, the density of agents, and the obstacles in his view field, and so on. Due to these factors, the following phenomena may arise during evacuation process.

1) The pedestrians gather in front of the exit spontaneously.

2) The pedestrians present arched or semicircular shape close to the exit.

3) When the pedestrian finds that the evacuation velocity of the target exit is lower than other exits, he may change the target exit.

4) The pedestrians near close to the exit take action first so that they can give place to those far from the exit.

To realize the above evacuation process, we design the following individual decision-making framework as shown in Figure 3.

3.2 Multi-Agent Based Algorithm Description

According to the individual decision-making framework in Figure 3, the multi-agent based evacuation algorithms are as follows.

1) Decide the target exit.

If there are several feasible exits, each agent needs to decide the target exit first, then decide the action direction of next step. This process is affected by the distance to the exits, the number and the density of agents and obstacles within the view field of the agent.

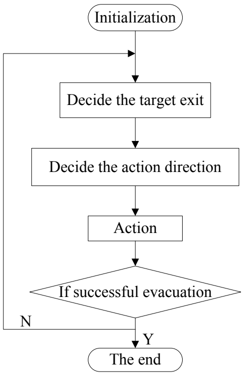


Figure 3 Individual decision-making framework

Step 1 Calculate the distance of each agent to all the exits.

The distance in this paper refers to the Euclidean distance. Assume there are N exits in the stadium and the width of each exit is equal to n-grid. In this paper, we use E_i to denote the i-th "big" exit and E_{i_j} the i_j-th "small" exit, which means the j-th grid of the i-th "big" exit, where $i = 1, 2, \cdots, N,\ j = 1, 2, \cdots, n$.

Then the distance of the agent to the exit E_{i_j} is

$$D_{i_j} = \sqrt{(x - x_e^{i_j})^2 + (y - y_e^{i_j})^2}, \tag{1}$$

where $(x, y), (x_e^{i_j}, y_e^{i_j})$ are the position coordinates of the agent and the exit $E_{i_j}, i = 1, 2, \cdots, N$, $j = 1, 2, \cdots, n$, respectively. Note that in this paper the position coordinate refers to the coordinate of the occupied cell center.

Step 2 Calculate the number and the density of the cells occupied by pedestrians or obstacles within the view field of each agent.

Denote the grid coordinate set within the view field of the agent at present position to the i_j-th exit E_{i_j} by U_{i_j}. Then the number of the cells occupied by agent or obstacle in this view field is

$$P_{i_j} = \sum_{(x',y') \in U_{i_j}} f(x', y'), \tag{2}$$

where

$$f(x', y') = \begin{cases} 1, & \text{the cell } (x', y') \text{ is occupied by certain agent or obstacle,} \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Assume the area of each cell is S, $|U_{i_j}|$ is the cardinality of the set U_{i_j}, then the density of the cells occupied by agent or obstacle within the view field of each agent can be

$$Q_{i_j} = \frac{P_{i_j}}{|U_{i_j}| \cdot S}, \tag{4}$$

where P_{i_j} is defined in (2), $i = 1, 2, \cdots, N,\ j = 1, 2, \cdots, n$.

Step 3 Choose the ultimate target exit.

Each agent will choose the ultimate exit according to the distance to the exits and the complexity to the exits. The complexity to the exits is affected by the number and the density of pedestrians and obstacles within the view field of it. So the target exit $E_{i^*_{j^*}}$ is decided as follows. Denote

$$i^*_{j^*} = \arg\left\{\min_{1\leq i\leq N,\ 1\leq j\leq n}\left\{\frac{\alpha_1 D_{i_j} + \alpha_2 P_{i_j} + \alpha_3 Q_{i_j}}{\alpha_1+\alpha_2+\alpha_3}\right\}\right\}, \tag{5}$$

then $(x_e^{i^*_{j^*}}, y_e^{i^*_{j^*}})$ is the position coordinate of the target exit $E_{i^*_{j^*}}$.

In (5), $\alpha_1, \alpha_2, \alpha_3$ are the weighting coefficients of the decision-making factors to decide the target exit. The values of $\alpha_1, \alpha_2, \alpha_3$ reflect the weigh of the corresponding factors in the choice of the target exit.

Since the distance and the complexity to the exit will reflect the evacuation speed, (5) can show that when the agent finds the evacuation speed of the target exit is lower than other exits, he will change the target exit.

2) Decide the action direction of next step.

After the target exit is decided, the agent will choose the action direction of next step from the nine possible feasible directions as shown in Figure 1. Similar to the choice of the target exit, the choice of the action direction is also affected by the distance and the complexity to the target exit. Since the process is similar to last part, we just list the steps and omit the details.

Step 1 Calculate the distance of each feasible direction to the target exit.

Denote the distance of the j-th feasible direction of the agent to the target exit by D^j_{TE}. The distance formula is similar to Equation (1).

Step 2 Calculate the number and the density of the cells occupied by agent or obstacle within the view field of each feasible direction to the target exit, denote by P^j_{TE} and $Q^j_{TE}, j = 1, 2, \cdots, 9$, respectively, which is similar to Equations (2) and (4).

Note that the definition of the view field of each feasible direction to the target exit is the same as that in Section 2.

Step 3 Choose the action direction.

Each agent will choose the ultimate action direction according to the distance to the target exit and the number and the density of the cells occupied by agent or obstacle within the view field. The optimal action direction is decided as

$$J_o = \arg\left\{\min_{1\leq j\leq 9}\left\{\frac{\alpha_1 D^j_{TE} + \alpha_2 P^j_{TE} + \alpha_3 Q^j_{TE}}{\alpha_1+\alpha_2+\alpha_3}\right\}\right\}, \tag{6}$$

and the suboptimal action direction is

$$J_{so} = \arg\left\{\min_{j\neq J_o}\left\{\frac{\alpha_1 D^j_{TE} + \alpha_2 P^j_{TE} + \alpha_3 Q^j_{TE}}{\alpha_1+\alpha_2+\alpha_3}\right\}\right\}, \tag{7}$$

where $\alpha_1, \alpha_2, \alpha_3$ in (6) and (7) are the same weighting coefficients as in (5).

The agent will choose the next action direction from the optimal and suboptimal action direction by certain probability. For example, he will choose the optimal action direction by the 95% probability and choose the suboptimal one by the 5% probability.

3) Action.

For each single agent, the action process refers to each agent moving to the action direction chosen above. For all the agents in this model, in the action process, we first calculate the distance of each agent to the target exit, and sort them in ascending order, then decide the action order for all the agents according to this order. If the distance is the same for some agents, then their action order is chosen randomly.

4 Simulation Results

In this section, the pedestrian evacuation process is simulated by Visual C language. To study the effects of the exit position and obstacles to evacuation process, we assume there are two exits in the simulated stadium, and they are located at different positions. We consider two cases: (I) Two exits lie in the back wall of the stadium, and (II) two exits lie in the left and right wall, respectively. In each case, we will consider two typical scenes: (a) The room with no obstacle and agents distributed randomly, and (b) the room with obstacles and agents distributed regularly. The four kinds of dots, red, green, blue, and black, represent four kinds of pedestrians, young male, young female, old male, and old female, respectively. The black grids are obstacles, such as chairs, wall, panels, and so on. The two nicks in the wall represent two exits.

In the two typical scenes, we have the following basic assumptions.

1) There are 31 × 27 cells in the room.

2) The size of each cell is $0.4 \times 0.4\text{m}^2$.

3) There are two exits in the room. The width of each exit is 3-grid.

4) 255 agents are initially put in the room.

5) The agents representing young male, young female, old male, and old female are chosen randomly with the proportion of 4 : 3 : 2 : 1.

In the algorithm, the choice of the parameters $\alpha_i, i = 1, 2, 3$ will affect the evacuation process, which are regulated to realize the desired state. The parameters α_i of four kinds of pedestrians are listed in Tables 1 and 2. Table 1 is the case without obstacle (Cases I (a) and II (a)) and Table 2 is the case with obstacles (Cases I (b) and II (b)). The values of α_1 are chosen the same for all four kinds of agents in each case. Then the larger value of α_2, the higher probability to move towards the exit with fewer agents and obstacles. The larger value of α_3, the higher probability to move towards the exit with lower density of occupied cells.

Table 1 The parameters in the case without obstacle

parameter	young male	young female	old male	old female
α_1	20	20	20	20
α_2	20	10	5	1
α_3	40	20	10	2

Table 2 The parameters in the case with obstacles

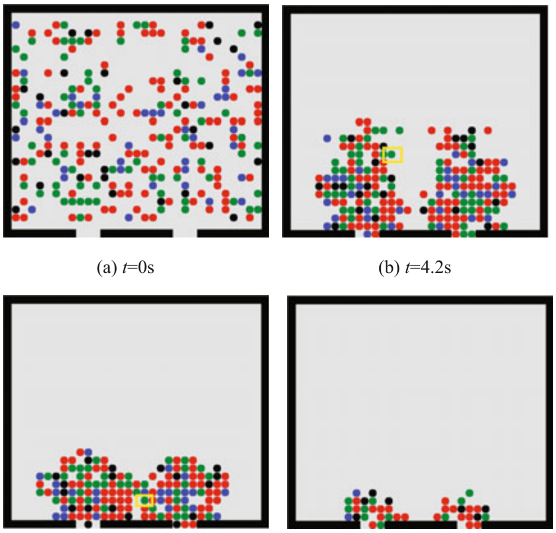
parameter	young male	young female	old male	old female
α_1	30	30	30	30
α_2	40	20	5	1
α_3	80	40	10	2

4.1 Case I: Two Exits Lie in the Back Wall

First we consider the case that two exits lie in the back wall of the stadium. Two evacuation scenes are designed.

(a) There is no obstacle and the agents are initially distributed randomly.

In this scene, we choose the parameters $\alpha_i,\ i = 1, 2, 3$ in Table 1, then the evacuation process is shown in Figure 4.


(a) *t*=0s (b) *t*=4.2s

(c) *t*=7.2s (d) *t*=13.8s

Figure 4 Evacuation state in Case I (a): Unilateral exits and without obstacle

Figure 4(a) is the initial state, Figures 4(b) and 4(c) are two middle states at $t = 4.2$s and $t = 7.2$s, respectively, and Figure 4(d) is the state close to the end of the simulation. From the simulation, we can see that in evacuation process:

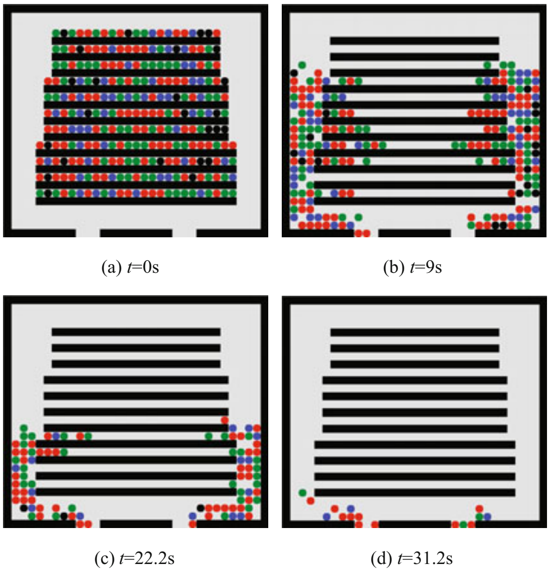
1) The agents gather in front of the exits spontaneously and evacuate regularly;

2) The arched shape arises close to the exit;

3) Some agents may change the exit when they find that the number of persons near the target exit is larger than other exits (refer to the agent within the rectangle in Figures 4(b) and 4(c)).

(b) There are obstacles and the agents are initially arranged regularly.

In this scene, the parameters $\alpha_i, i = 1, 2, 3$ are chosen in Table 2. The simulation results refer to Figure 5. In this scene, the agents first move to the bilateral aisle from their chairs and they are separated two parts. Then they move on along the aisle to the corresponding exit. We can see that the agents near close to the exit evacuate first so that they can give place to those far from the exit. The evacuation velocity of two exits is relatively uniform such that the evacuation is completed almost simultaneously.


(a) t=0s (b) t=9s

(c) t=22.2s (d) t=31.2s

Figure 5 Evacuation state in Case I (b): Unilateral exits and without obstacle

4.2 Case II: Two Exits Lie in the Left and Right Wall

In this part, we simulate the case that two exits lie in the left and right wall of the stadium, respectively. We also consider two scenes similar to Case I.

(a) There is no obstacle and the agents are initially distributed randomly.

The simulation results refer to Figure 6. We can see that the evacuation process is similar to Case I (a). All the agents move towards the target exit spontaneously and present arched shape near the exit. The parameters $\alpha_i,\ i = 1, 2, 3$ in this scene is chosen in Table 1.

(b) There are obstacles and the agents are initially arranged regularly.

The simulation results are shown in Figure 7. Though there exist obstacles, all the agents can also be evacuated successfully and the approximate arched shape is presented close to the exits.

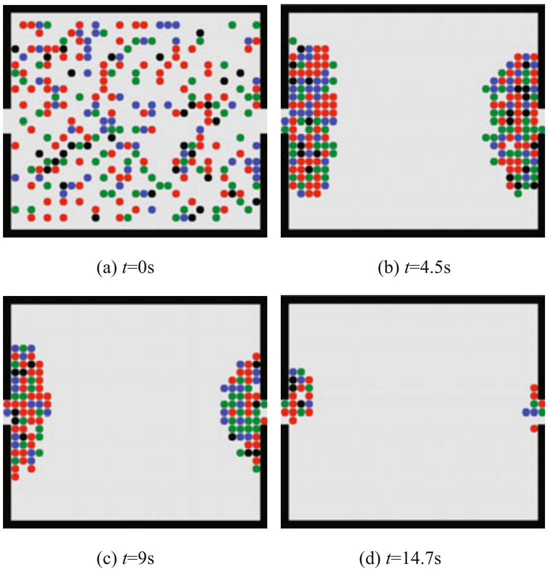

(a) t=0s (b) t=4.5s (c) t=9s (d) t=14.7s

Figure 6 Evacuation state in Case II (a): Bilateral exits and without obstacle

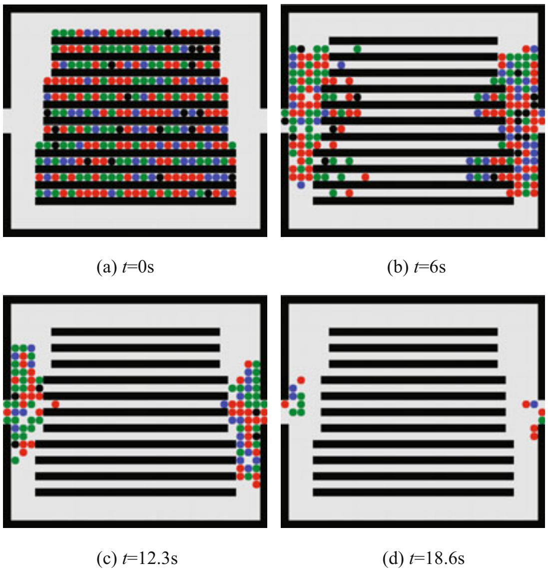

(a) t=0s (b) t=6s (c) t=12.3s (d) t=18.6s

Figure 7 Evacuation state in Case II (b): Bilateral exits and with obstacle

4.3 Data Analysis

Now, we summarize some data on evacuation number, evacuation time, and step number in Table 3 to Table 6, which are average results of 5 measurements.

In each table, the second row is the initial number of pedestrians of young male, young female, old male, and old female, respectively, which is chosen randomly with the proportion of 4 : 3 : 2 : 1. The third and the fourth row are the number of each class to left and right exit, respectively, from which we can see that for all four cases the number of each class to choose each exit is nearly the same. The difference of the total number to choose each exit is no more than 10, which can also reflect the evacuation of two exits is relatively uniform such that the they are completed almost simultaneously.

The last two rows in Table 3 to Table 6 represent the average evacuation time and step number. The first four columns are the average values of per person in each class and the last column are the total evacuation time and step number of all pedestrians. These data show that

1) The difference of evacuation time and steps among each class of pedestrians is not very obvious. Maybe this is because that the evacuation is under normal mode and there is no congestion.

2) When there are obstacles, the position of the exit has great effect to the evacuation efficiency. The evacuation for bilateral exits case is faster than that for unilateral exits case (see Tables 4 and 6). But if there is no obstacle, the evacuation efficiency is appropriate for the two cases (see Tables 3 and 5).

3) The obstacles can lower the evacuation efficiency on matter where the exits are.

Table 3 Evacuation number, time, and steps for Case I (a): Unilateral exits and without obstacle

	young male	young female	old male	old female	total
number of pedestrians	95	76	56	28	255
pedestrians to left exit	52	38	27	10	127
pedestrians to right exit	43	38	29	18	128
evacuation time (s)	7.78	7.73	8.64	7.75	18.6
evacuation steps	26	26	29	26	7071

Table 4 Evacuation number, time, and steps for Case I (b): Unilateral exits and with obstacle

	young male	young female	old male	old female	total
number of pedestrians	101	77	56	21	255
pedestrians to left exit	54	30	26	13	123
pedestrians to right exit	56	37	23	16	132
evacuation time (s)	17.70	18.54	18.17	17.06	35.7
evacuation steps	59	62	60	57	15573

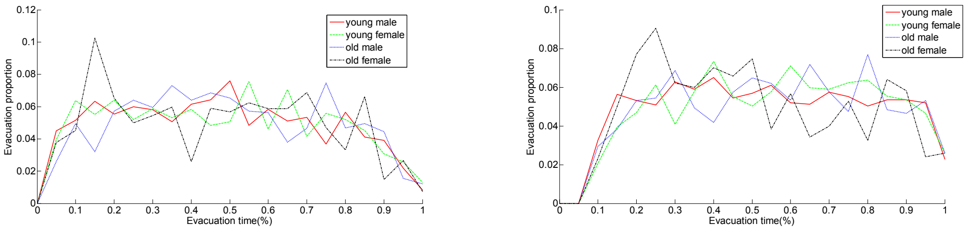
Table 5 Evacuation number, time, and steps for Case II (a): Bilateral exits and without obstacle

	young male	young female	old male	old female	total
number of pedestrians	110	78	41	26	255
pedestrians to left exit	55	40	20	15	130
pedestrians to right exit	46	37	36	6	125
evacuation time (s)	8.00	7.61	8.11	7.66	18.7
evacuation steps	27	25	27	26	6730

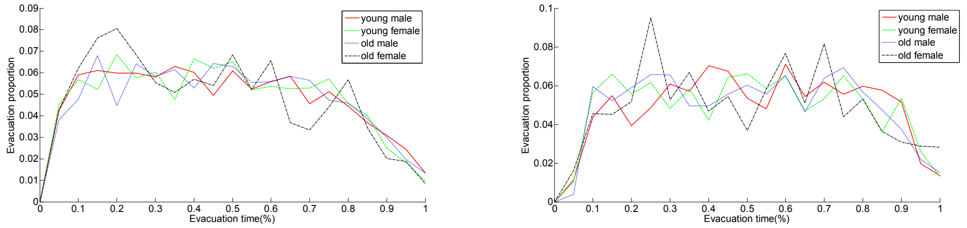
Table 6 Evacuation number, time, and steps for Case II (b): bilateral exits and with obstacle

	young male	young female	old male	old female	total
number of pedestrians	96	80	57	22	255
pedestrians to left exit	43	43	29	8	123
pedestrians to right exit	53	37	28	14	132
evacuation time (s)	11.07	8.82	9.51	11.13	21.5
evacuation steps	37	30	32	37	8540

The evacuation proportion of each class of agents during evacuation process are shown in Figure 8, which are the average of 50 experiments under the same parameters and settings. Since the evacuation time of every experiment is different, in each experiment we divide the total evacuation time into 20 parts and every part is 5% (0.05). Then we calculate the evacuation proportion in each time period and get the average value of 50 experiments. In Figure 8, the x-axis denotes the evacuation time partition with the step 0.05 (5%) and the y-axis is the average evacuation proportion of 50 experiments within the corresponding period of time. From Figure 8, we can see that, in all four cases, the evacuation proportion fluctuation of young male during every period of time is relative weak, showing that young male suffer small influence from others and can evacuate according to their own wills. The evacuation proportion of old female during every period of time has relative big changes. Basically, in all four cases, the evacuation proportion of old female is large during the initial and late stage, and small during the middle stage. This may be because the ability of old female is the weakest. During the initial and late stage, the occupant density near the exits is small, so they can act as their own wills. While during the middle stage the density is large, they suffer great influence from others and lower the evacuation. The evacuation efficiency of young female and old male is between young male and old female.



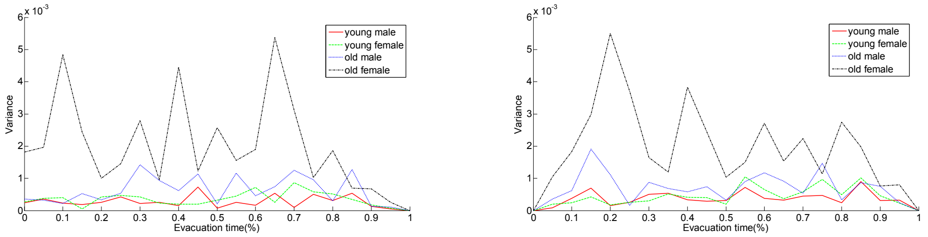
(a) Case I (a): Unilateral exits and without obstacle (b) Case I (b): Unilateral exits and with obstacle



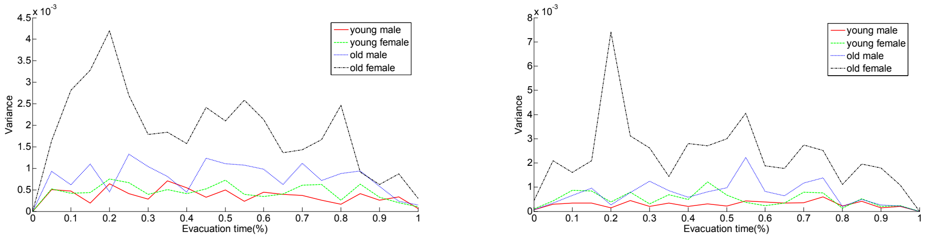
(c) Case II (a): Bilateral exits and without obstacle (d) Case II (b): Bilateral exits and with obstacle

Figure 8 Average evacuation proportion within each period of time

Figure 9 shows the variance of the evacuation proportion for each class of agents, which is also the result of 50 experiments. The x-axis is the same as in Figure 8 and the y-axis is the variance of the evacuation proportion of 50 experiments during the corresponding period of time. From Figure 9, we can see that in all four cases the variance value is very small. Though all the 50 experiments are repeated under the same parameters and settings, the initial positions of the agents are random at every experiment, which lead to the difference of individual decision making. So the variance has some mild fluctuations. We can also find that the variance for young male is the smallest, showing that the evacuation of young male is the most stable. This is because that the autonomy to act by their own will for young male is the strongest.



(a) Case I (a): Unilateral exits and without obstacle (b) Case I (b): Unilateral exits and with obstacle



(c) Case II (a): Bilateral exits and without obstacle (d) Case II (b): Bilateral exits and with obstacle

Figure 9 Variance of evacuation proportion of 50 experiments

5 Conclusions

In this paper, we investigated the pedestrian evacuation problem in stadium using a multi-agent model from a new point of view. The model is designed in such a way that the agent makes decision in each step considering the distance to the exits, the number and the density of agents and obstacles within its view field. Moreover, the pedestrians are not identical and are divided into four classes according to individual gender and age. During the evacuation process, the weighting that affects individual decision-making between each class of agents is different. Our simulations show that in evacuation process: 1) The agents gather in front of the exits spontaneously; 2) The agents present arched or semicircular shape close to the exits if there is no obstacle; 3) Some agents may change the target exit when they find that the number of persons near the target exit is larger than other exits; 4) The exit position has some effect to evacuation efficiency. For the case with obstacles, the evacuation is faster when the two exits lie in bilateral walls than that the two exits lie in unilateral wall. These phenomena are exact reproduction of the actual pedestrian evacuation process. Therefore, the proposed method might be useful to support the real pedestrian evacuation and assess public buildings design.

Further work will focus on the case of crowd and death since it may be inevitable due to crowd and the difference of individual ability. Furthermore, the sub-group behavior in evacuation process will also be considered.

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