FINITE-TIME TRACKING CONTROL FOR MOTOR SERVO SYSTEMS WITH UNKNOWN DEAD-ZONES*

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Abstract A finite-time tracking control scheme is proposed in this paper based on the terminal sliding mode principle for motor servo systems with unknown nonlinear dead-zone inputs. By using the differential mean value theorem, the dead-zone is represented as a time-varying system and thus the inverse compensation approach is avoided. Then, an indirect terminal sliding mode control (ITSMC) is developed to guarantee the finite-time convergence of the tracking error and to overcome the singularity problem in the traditional terminal sliding mode control. In the proposed controller design, the unknown nonlinearity of the system is approximated by a simple sigmoid neural network, and the approximation error is diminished by employing a robust term. Comparative experiments on a turntable servo system are conducted to show the superior performance of the proposed method.

Key words Dead zone, finite-time control, neural network, servo system.

1 Introduction

As a nonlinear element, the dead-zone is widely encountered in motor servo systems and the existence of dead-zones may lead to the performance deterioration or even instability of the systems[1]. In order to improve the control performance of servo systems, many research works were proposed for the compensation and control of the dead-zones[2−7]. Tao and Kokotovic[2] built the dead-zone inverse model for linear systems and designed an adaptive controller to compensate for the negative effect of the dead-zone. However, it is difficult to obtain the precise inverse model of the dead-zone for nonlinear systems. Then, Wang, et al.[3] developed a robust adaptive control scheme for a class of nonlinear systems with a symmetric dead-zone by modeling

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the dead-zone as a linear time-varying system with a disturbance term. In References [4] and [5], the compensation methods of unknown non-symmetric dead-zones were further investigated based on the idea of [3]. However, among all of those schemes aforementioned, it is assumed that the maximum and the minimum values of dead-zone slopes should be known or estimated for the controller design. Recently, Zhang and Ge[6], and Na, et al.[7] transformed the unknown dead-zone into a time-varying system via the differential mean value theorem, and the inverse compensation approach was thus avoided. Moreover, the characteristic parameters of dead-zones were only used for the character analysis rather than the controller design.

Sliding mode control (SMC) scheme is one of the most useful approaches to deal with system uncertainties and bounded disturbances, and has been widely used in many fields, such as robots, motors, and so on[8,9]. The traditional linear sliding mode control scheme can guarantee the asymptotical convergence of tracking errors and thus the system states can converge to the desired trajectories when time goes to the infinity. Recently, many research works have focused on the finite-time convergence of tracking errors. Man and Yu[10] proposed a terminal sliding mode control (TSMC) scheme by introducing a nonlinear term in the SMC design and the tracking error can be guaranteed to converge in a finite time. Feng, et al.[11] and Yu, et al.[12] proposed nonsingular terminal sliding mode control methods to overcome the singularity problem, which was applied in the control of permanent magnet synchronous motor (PMSM) servo systems[13−16]. In [17] and [18], an indirect TSMC scheme was developed to avoid the singularity problem by switching from terminal to linear sliding manifold, but the fractional power $p = p_1/p_2$ may lead to the error term $e^p \notin R$ and thus $\dot{e} \notin R$ for $e < 0$.

However, most of the controllers mentioned above require that the model of mechanical system is known or partially known. Therefore, the approaches aforementioned cannot be used in the control of PMSM servo systems directly when both model uncertainties and nonlinear dead-zones are encountered. Due to the capability of approximating any smooth functions over a compact set to arbitrary accuracy, neural networks (NNs) have been widely employed to handle the system uncertainties and nonlinearities[19−24]. Inspired by previous work, this paper mainly focuses on the finite-time tracking control for a PMSM servo system with unknown dead-zone input as well as model uncertainties. By using the differential mean value theorem, the dead-zone is represented as a time-varying system and thus the inverse compensation approach is avoided. Then, an indirect terminal sliding mode control (ITSMC) with NN approximation is developed to guarantee the finite-time convergence of the tracking error and overcome the singularity problem in the traditional terminal sliding mode control.

The main contributions of this paper are listed as follows.

1) The singularity problem and the problem of $\dot{e} \notin R$ for $e < 0$ are both overcome by switching between the terminal and general sliding manifolds.

2) A finite-time tracking control is developed for PMSM servo systems with unknown dead-zone inputs. Experimental results validate the superior performance of the proposed control scheme by comparing it with a linear sliding mode (LSM) controller and a proportional-integral-derivative (PID) controller.

The rest of this paper is organized as follows. The PMSM servo system, nonlinear dead-zone and neural network are briefly described in Section 2. Section 3 proposes a finite-time tracking control scheme for the PMSM motor system with an unknown nonlinear dead zone. The stability analysis is given in Section 4. Comparative simulations and experimental results are provided in Section 5, followed by conclusions in Section 6.

2 Problem Formulation and Preliminaries

2.1 System Description

As shown in Figure 1, the motor servo system is composed of a permanent magnet synchronous motor (PMSM, HC-UFS13), an encoder and pulse width modulation (PWM) amplifiers in the motor drive card (MR-J2S-10A), a digital signal processing unit (DSP, TMS3202812) performing as the controller, and a Pentium 2.8-GHz PC operating for display. The schematic diagram of the proposed control system is depicted in Figure 2. The PMSM is driven by a PWM voltage source inverter, and the i_d and i_q control loops are controlled by two identical PI controllers which make the current transients negligible with respect to the mechanical dynamics (i.e., $i_d^* = i_d = 0$ and $i_q^* = i_q = u(t)$ where superscript '*' denotes reference signals and u indicates controller output). The output of current controllers are voltages which are applied to the motor by means of a PWM three phase inverter.

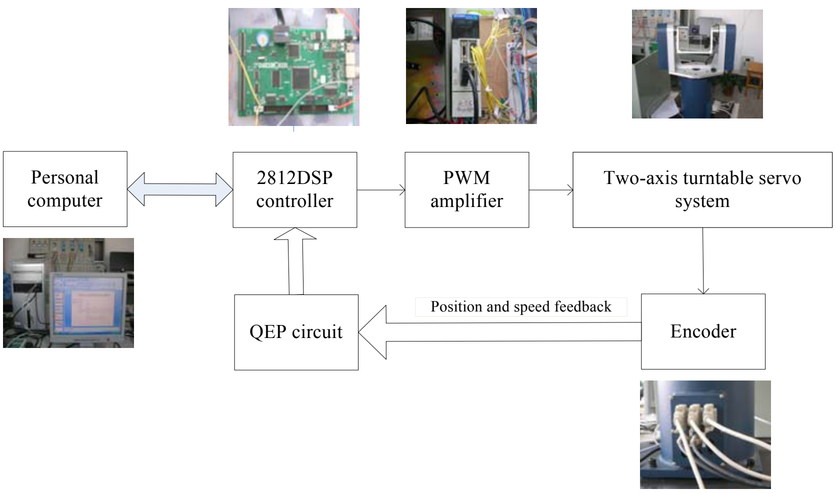

Figure 1 Two-axis turntable motor servo system

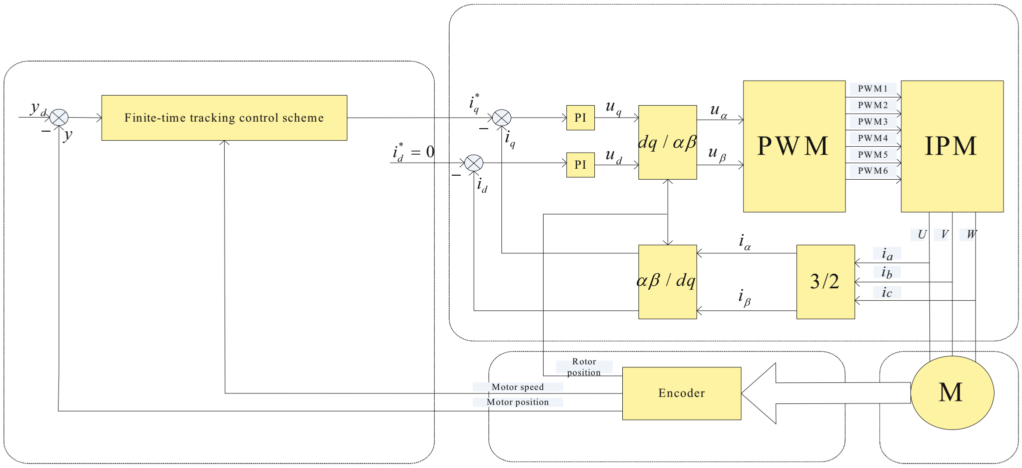

Figure 2 Schematic diagram of the proposed control system

The mechanical dynamics of the motor servo system can be described as follows:

$$m\ddot{x} + f^*(\overline{x}, t) + d^*(\overline{x}, t) = k_0^* u(t), \tag{1}$$

where $\overline{x} = [x, \dot{x}]^{\mathrm{T}} \in R^2$, $u(t) \in R$, $y = x(t) \in R$ are state variables, the control input voltage to the motor and the output from the motor, respectively; x is the position, m is the inertia, k_0^* is a positive control gain (the force constant), $f^*(\overline{x}, t)$ is the friction force; $d^*(\overline{x}, t)$ represents a bounded disturbance modeling nonlinear elastic forces generated by coupling and protective covers, measurement noise, power electronics disturbances, and other uncertainties.

For notational convenience, (1) can be normalized as:

$$\begin{aligned} \ddot{x} &= k_0 u(t) - f(\overline{x}, t) - d(\overline{x}, t), \\ y &= x(t), \end{aligned} \tag{2}$$

where $f(\overline{x}, t) = f^*(\overline{x}, t)/m$, $d(\overline{x}, t) = d^*(\overline{x}, t)/m$ and $k_0 = k_0^*/m$. It should be noted that k_0 is positive but unknown due to the change of the payload. Grouping the uncertain functions $f(\overline{x}, t)$ and $d(\overline{x}, t)$ in a single function $h(\overline{x}, t)$, we have

$$h(\overline{x}, t) = f(\overline{x}, t) + d(\overline{x}, t). \tag{3}$$

Substitute (3) into (2), and we can obtain:

$$\begin{aligned} \ddot{x} &= -h(\overline{x}, t) + k_0 u(t), \\ y &= x(t). \end{aligned} \tag{4}$$

2.2 Nonlinear Dead-Zone Model

As shown in Figure 3, the control input $u(t) \in R$ is the output of the following nonlinear dead-zone

$$u(t) = G(v(t)) = \begin{cases} g_r(v), & \text{if } \ v(t) \geq b_r, \\ 0, & \text{if } \ b_l < v(t) < b_r, \\ g_l(v), & \text{if } \ v(t) \leq b_l, \end{cases} \tag{5}$$

where $v(t) \in R$ is the input of the dead-zone (practical control signal), $g_l(v)$, $g_r(v)$ are unknown nonlinear smooth functions, and b_l, b_r are unknown width parameters of the dead-zone. Without loss of generality, it is assumed that $b_l < 0$, $b_r > 0$.

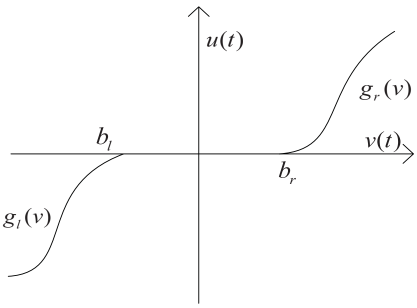

Figure 3 Nonlinear dead-zone model

To facilitate the control system design, the following assumption is needed.

Assumption 1 The functions $g_l(v)$ and $g_r(v)$ are smooth, and there exist unknown positive constants g_{l0}, g_{l1}, g_{r0}, and g_{r1} such that

$$0 < g_{l0} \leq g_l'(v) \leq g_{l1}, \quad \forall v \in (-\infty, b_l], \tag{6}$$

$$0 < g_{r0} \leq g_r'(v) \leq g_{r1}, \quad \forall v \in [b_r, +\infty), \tag{7}$$

where $g_l'(v) = dg_l(z)/dz|_{z=v}$ and $g_r'(v) = dg_r(z)/dz|_{z=v}$.

According to the differential mean value theorem, there exist $\xi_l \in (-\infty, b_l)$ and $\xi_r \in (b_r, +\infty)$ such that

$$g_l(v) = g_l(v) - g_l(b_l) = g_l'(\xi_l)(v - b_l), \quad \forall v \in (-\infty, b_l], \tag{8}$$

$$g_r(v) = g_r(v) - g_r(b_r) = g_r'(\xi_r)(v - b_r), \quad \forall v \in [b_r, +\infty). \tag{9}$$

As stated in [6], the definitions of $g_l(v)$ and $g_r(v)$ can be extended as:

$$g_l(v) = g_l'(b_l)(v - b_l), \qquad \forall v \in (b_l, b_r], \tag{10}$$

$$g_r(v) = g_r'(b_r)(v - b_r), \qquad \forall v \in [b_l, b_r). \tag{11}$$

From (8)–(11), it can be concluded that

$$g_l(v) = g_l'(\xi_l')(v - b_l), \qquad \forall v \in (-\infty, b_r] \tag{12}$$

with $\xi_l' \in (-\infty, b_l]$, and

$$g_r(v) = g_r'(\xi_r')(v - b_l), \qquad \forall v \in [b_l, +\infty) \tag{13}$$

with $\xi_r' \in [b_r, +\infty)$.

According to (12), (13) and Assumption 1, the dead zone (5) can be rewritten as:

$$u = \varphi(v)v + \rho(v), \qquad \forall t \geq 0, \tag{14}$$

where $|\rho(v)| \leq \rho_N$, ρ_N is an unknown positive constant with $\rho_N = (g_{r1} + g_{l1})\max\{b_r, -b_l\}$ and

$$\varphi(v) = \varphi_r(v) + \varphi_l(v), \tag{15}$$

where

$$\varphi_r(v) = \begin{cases} g_r'(\xi_r), & \text{if } v(t) \geq b_l, \\ 0, & \text{if } v(t) < b_l, \end{cases} \tag{16}$$

$$\varphi_l(v) = \begin{cases} g_l'(\xi_l), & \text{if } v(t) \leq b_r, \\ 0, & \text{if } v(t) > b_r, \end{cases} \tag{17}$$

$$\rho(v) = \begin{cases} -g_r'(\xi_r)b_r, & \text{if } v(t) \geq b_r, \\ -[g_l'(\xi_l) + g_r'(\xi_r)]v(t), & \text{if } b_l < v(t) < b_r, \\ -g_l'(\xi_l)b_l, & \text{if } v(t) \leq b_l. \end{cases} \tag{18}$$

Substituting (14) into (4), we have

$$\begin{aligned} \ddot{x} &= -h(\overline{x}, t) + k_0(\varphi(v)v + \rho(v)), \\ y &= x(t). \end{aligned} \tag{19}$$

From Assumption 1, it is easy to verify that $\varphi(v) \in [\varphi_0, \varphi_1] \subset (0, +\infty)$ with $\varphi_0 = \min(g_{l0}, g_{r0})$ and $\varphi_1 = g_{l1} + g_{r1}$, $|\rho(v)| \leq \rho_N$ and $\rho_N = (g_{l1} + g_{r1})\max\{b_r, -b_l\}$ being positive constants.

Let y_d be a given twice differentiable desired trajectory and define the tracking error as

$$e = y_d - y. \tag{20}$$

The control objective is to design an adaptive finite-time neural network controller v for System (19) such that the tracking error e converges to zero within finite time, while all signals in the closed-loop systems are bounded.

2.3 Neural Network Approximation

Due to the good capabilities in function approximation, neural networks (NNs) are usually used for the approximation of nonlinear functions. The following neural network with a simple structure and a fast convergence property will be used to approximate the continuous function $H(X) : R^{n_1} \to R^{n_2}$:

$$H(X) = W^{*\mathrm{T}}\phi(X) + \varepsilon, \tag{21}$$

where $W^* \in R^{n_1 \times n_2}$ is the ideal weight matrix, $\phi(X) \in R^{n_1 \times 1}$ is the basis function of the neural network, ε is the neural network approximation error satisfying $|\varepsilon| \le \varepsilon_N$, $\phi(X)$ can be chosen as the commonly used sigmoid function, which is in the following form:

$$\phi(X) = \frac{a}{b + \mathrm{e}^{(-X/c)}} + d \tag{22}$$

with a, b, c, and d being appropriate parameters.

Remark 1 The employed neural network with sigmoid function represents a class of linearly parameterized approximation methods, and can be replaced by any other approximation approaches such as spline functions, RBF functions or fuzzy systems. However, the structure of the employed neural network in the this paper is simpler than the other neural networks that are commonly used in other works. There is no hidden layer in the employed NN, in which five inputs and one output are included and the corresponding weight matrix is 5×1. Although consuming a little more time than PID method, the proposed method is still easy to run in the DSP (TMS3202812) unit.

3 Finite Time Tracking Control Design

In this section, a finite-time tracking control scheme is designed based on the terminal sliding mode principle and neural network approximation.

3.1 Terminal Sliding Manifold

As shown in Figure 4, a terminal sliding manifold is defined as:

$$s = \dot{e} + \lambda|e|^{\gamma}\mathrm{sgn}(e), \tag{23}$$

where $e \in R$, $\lambda > 0$, $\gamma = q/p$, $p, q > 0$ are positive odd numbers satisfying $q < p$.

According to finite-time stability theory[25], the equilibrium point $e = 0$ of differential equation (23) is globally finite-time stable, i.e., for any given initial condition $e(0) = e_0$, the tracking error can converge to zero in finite time:

$$T = \frac{1}{\lambda(1-\gamma)}|e_0|^{1-\gamma}. \tag{24}$$

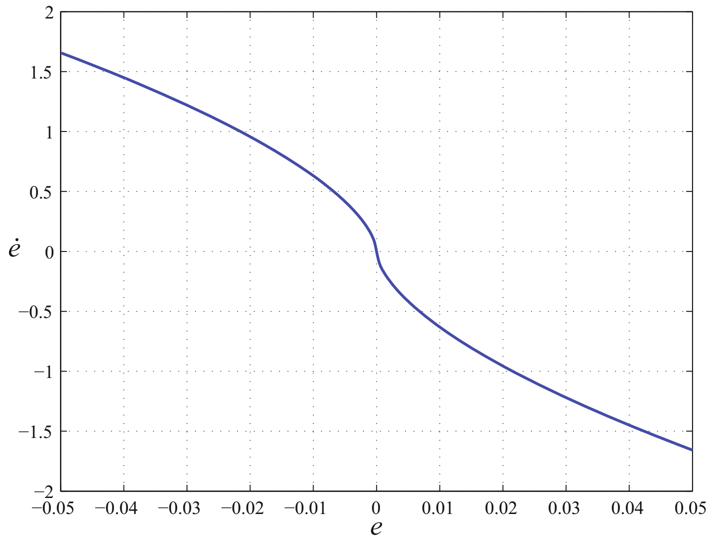

Figure 4 The terminal sliding manifold

To facilitate the controller design, a new command vector x_r and its time derivative $\dot{x}_r$ are defined as:

$$x_r = \dot{y}_d + \lambda|e|^{\gamma}\mathrm{sgn}(e) \tag{25}$$

and

$$\dot{x}_r = \ddot{y}_d + \lambda\gamma|e|^{\gamma-1}\dot{e}, \tag{26}$$

where $e \in R$, $\lambda > 0$, $\gamma = q/p$, $p, q > 0$ are positive odd numbers satisfying $q < p$.

Then, the terminal sliding mode s and its time derivative $\dot{s}$ are given as

$$s = x_r - \dot{x} \tag{27}$$

and

$$\dot{s} = \dot{x}_r - \ddot{x}. \tag{28}$$

3.2 Controller Design

From (19), the equation (28) can be rewritten as

$$\begin{aligned}\dot{s} &= \dot{x}_r + h(\overline{x}, t) - k_0\varphi(v)v - k_0\rho(v)\\ &= -k_0\varphi(v)v + \kappa,\end{aligned} \tag{29}$$

where the nonlinear function is

$$\kappa = \dot{x}_r + h(\overline{x}, t) - k_0\rho(v). \tag{30}$$

Without loss of generality, two technical assumptions are made to pose the problem in a tractable manner:

1) The desired position trajectory y_d, the time derivative $\dot{y}_d$ and $\ddot{y}_d$ are both bounded and smooth signals.

2) The angular position and velocity, x and $\dot{x}$, are measurable.

Since k_0, $\varphi(t)$, and κ are not easily known, the model-based controllers cannot be applied directly. Hence, we adopt a neural network to approximate the nonlinear function $H = \kappa/(k_0\varphi(v))$.

Assume that there exists a constant ideal weight matrix W^* so that the nonlinear function H can be expressed as

$$H = W^{*\mathrm{T}}\phi(X) + \varepsilon, \tag{31}$$

where the input vector $X = [x^{\mathrm{T}}, \dot{x}^{\mathrm{T}}, y_d^{\mathrm{T}}, \dot{y}_d^{\mathrm{T}}, \ddot{y}_d^{\mathrm{T}}]^{\mathrm{T}} \in R^5$.

In the following, an adaptive finite-time neural control approach is developed for tracking control of the PMSM servo system described by Equation (19). The expression of the designed controller is given by:

$$v = v_0 + v_1 + v_2 \tag{32}$$

with

$$\begin{cases} v_0 = \widehat{W}^{\mathrm{T}}\phi(x), \\ v_1 = k_1|s|^{\gamma}\mathrm{sgn}(s), \\ v_2 = (\delta_1 + \delta_2)\mathrm{sgn}(s), \end{cases} \tag{33}$$

where $\widehat{W}$ is the estimate of the ideal weight W^*, v_0 is the NN uncertainty estimator, v_1 is a feedback control for guaranteeing the finite-time convergence of sliding mode s, v_2 is a robust term which is designed to provide robustness in the presence of the approximation and weight estimation errors of NN, $k_1 > 0$ are control parameters, $\delta_1 > \varepsilon_N$, and δ_2 is a positive constant satisfying $\delta_2 > \|\widetilde{W}^{\mathrm{T}}\phi(x)\|_F$ where $\widetilde{W} = W^* - \widehat{W}$ is the weight estimation error of the neural network.

The weight update law is provided by

$$\dot{\widehat{W}} = \Gamma\phi(x)s, \tag{34}$$

where Γ is a positive definite and diagonal matrix.

Substituting (32), (33) into (29) yields the following equation:

$$\dot{s} = k_0\varphi(v)\big[\widetilde{W}^{\mathrm{T}}\phi(x) - k_1|s|^{\gamma}\mathrm{sgn}(s) + \varepsilon - (\delta_1 + \delta_2)\mathrm{sgn}(s)\big]. \tag{35}$$

From the expression (26), it can be seen that $|e|^{\gamma-1}\dot{e}$ is included in the design of $\dot{x}_r$. Due to $\gamma - 1 < 0$, singularity will occur as $e = 0$ and $\dot{e} \neq 0$, that is, $\lim_{e\to 0}|e|^{\gamma-1}\dot{e} \to \infty$, $i = 1, 2, \cdots, n$.

To overcome the singularity problem, the following definition $e_r \in R$ is defined as

$$e_r = \begin{cases} |e|^{\gamma-1}\dot{e}, & e \neq 0 \quad \text{and} \quad \dot{e} \neq 0, \\ |\Delta|^{\gamma-1}\dot{e}, & e = 0 \quad \text{and} \quad \dot{e} \neq 0, \\ 0, & \dot{e} = 0, \end{cases} \tag{36}$$

where $\Delta > 0$ is a small positive constant.

Remark 2 According to the definition of e_r, the singularity problem can be avoided in the design of command vector $\dot{x}_r$. By using a small positive number Δ, the proposed method is different from the conventional TSMC[10], in which the command vector is set to be 0 as arbitrary position error $e = 0$. The switch scheme is an approximation method to avoid the singularity when $e = 0$ and $\dot{e} \neq 0$, so the main disadvantage is that the selection of the positive number Δ is a little sensitive. If the value of Δ is set too high, the finite-time convergence speed will become lower, while if is selected too small, the singularity problem will not be avoided well. Thus, the selection of Δ should be careful and an appropriately small positive value of Δ is needed to avoid the singularity.

Under the definition of e_r, the time derivative of command vector $\dot{x}_r$ can be rewritten as follows:

$$\dot{x}_r = \ddot{y}_d + \lambda\gamma e_r. \tag{37}$$

Remark 3 In order to avoid the chattering problem caused by the use of signum functions in the controller design, we employ the following continuous saturation function instead to design the controller in the experiment section:

$$sat(s) = \begin{cases} \mathrm{sgn}(s), & \text{if} \quad |s| > \zeta, \\ \dfrac{s}{\zeta}, & \text{if} \quad |s| \leq \zeta, \end{cases} \tag{38}$$

where ζ is a small positive constant.

4 Stability Analysis

Theorem 1 *Consider the motor servo system* (19)*, the terminal sliding manifold* (23)*, the controllers* (32) *and* (33)*, and the weight update law* (34)*, then all signals of the closed loop system are bounded.*

Proof Select the following Lyapunov function candidate:

$$V(t)=\frac{1}{2k_0\varphi_0}s^2+\frac{1}{2}\widetilde{W}^{\mathrm{T}}\Gamma^{-1}\widetilde{W}. \tag{39}$$

Differentiating (39) with respect to time and using (35), we have

$$\begin{aligned}\dot V(t)&=\frac{1}{k_0\varphi_0}s\dot s+\widetilde{W}^{\mathrm{T}}\Gamma^{-1}\dot{\widetilde{W}}\\&=\frac{1}{k_0\varphi_0}s\big\{k_0\varphi(v)\big[\widetilde{W}^{\mathrm{T}}\phi(x)+\varepsilon-k_1|s|^{\gamma}\mathrm{sgn}(s)\big]-(\delta_1+\delta_2)\mathrm{sgn}(s)\big\}-\widetilde{W}^{\mathrm{T}}\Gamma^{-1}\dot{\widehat{W}}\\&\le\widetilde{W}^{\mathrm{T}}\phi(x)s-(\delta_1+\delta_2)|s|-k_1|s|^{\gamma+1}+s\varepsilon-\widetilde{W}^{\mathrm{T}}\Gamma^{-1}\dot{\widehat{W}}\\&=-\widetilde{W}^{\mathrm{T}}\Gamma^{-1}\big[\dot{\widehat{W}}-\Gamma\phi(x)s\big]-(\delta_1+\delta_2)|s|+s\varepsilon-k_1|s|^{\gamma+1}.\end{aligned} \tag{40}$$

Substituting (34) into (40) yields

$$\dot V(t)\le-k_1|s|^{\gamma+1}-\delta_2|s|\le 0. \tag{41}$$

Inequality (41) implies that both s and $\widetilde{W}$ are bounded. Meanwhile, considering (23) and the boundedness of W^*, we can conclude e, $\dot e$, and $\widehat{W}$ are bounded, and thus v is bounded from (32) and (33). Furthermore, the boundedness of y_d, $\dot y_d$, and $\ddot y_d$ can lead to the boundedness of x_r and $\dot x_r$ according to (25) and (26). As a result, $\dot s$ is bounded due to the boundedness of $\varphi(v)$. Therefore, all signals of the closed loop system are bounded. ▮

In Theorem 1, the stability of the system (19) with control laws (32), (33) and weight update law (34) has been proved. However, it is not necessary for the terminal sliding manifold s to converge to zero in finite time. Therefore, a second theorem is given to guarantee that the terminal sliding manifold s converge to zero in finite time.

From (22), we can see that the sigmoid function $\phi(x)$ is bounded by $0<\phi_i(x)<n_0$, $i=1,2,\cdots,n_1$, with $n_0=\max\{|\frac{a}{b}+d|,|\frac{a}{b+1}+d|\}$. Therefore, $\phi(x)$ is bounded by

$$\|\phi(x)\|\le n_0\sqrt{n_1}, \tag{42}$$

where $\|\cdot\|$ denotes the Euclidean norm of a vector, $\phi(x)=[\phi_1(x),\phi_2(x),\cdots,\phi_{n_1}(x)]^{\mathrm{T}}$.

From the property of Forensics norm, it can be obtained that

$$\|\widetilde{W}^{\mathrm{T}}\phi(x)\|_F\le\|\widetilde{W}\|_F\|\phi(x)\|. \tag{43}$$

According to (42), (43), and Theorem 1, we can concluded $\|\widetilde{W}^{\mathrm{T}}\phi(x)\|_F$ is bounded.

Lemma 1[12] *Suppose that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:*

$$\dot V(t)\le-\alpha V^{\eta}(t),\quad\forall t\ge t_0,\ V(t_0)\ge 0, \tag{44}$$

where $\alpha>0$, $0<\eta<1$ are constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - \alpha(1-\eta)(t-t_0), \quad t_0 \leq t \leq t_1 \tag{45}$$

and

$$V(t) \equiv 0, \quad \forall t \geq t_1 \tag{46}$$

with t_1 *given by*

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{\alpha(1-\eta)}. \tag{47}$$

Theorem 2 is provided to guarantee the terminal sliding manifold s converge to zero in finite time by using controllers (32) and (33).

Theorem 2 *Considering the dynamic model* (19)*, the controllers are chosen as* (32) *and* (33)*, and the NN weight update law is chosen as* (34)*. If the design parameters* δ_1 *and* δ_2 *satisfy* $\delta_1 > \varepsilon_N$*, and* $\delta_2 \geq \|\widetilde{W}^{\mathrm{T}}\phi(x)\|_F$*, respectively, then the terminal sliding manifold* s *can converge to zero in finite time.*

Proof Select another Lyapunov function candidate

$$V_1 = \frac{1}{2k_0\varphi_0}s^2. \tag{48}$$

Differentiating (48) with respect to time and using (35), we have

$$\begin{aligned}\dot{V}_1 &= \frac{1}{k_0\varphi_0}s\big\{k_0\varphi(v)\big[\widetilde{W}^{\mathrm{T}}\phi(x) + \varepsilon - (\delta_1+\delta_2)\mathrm{sgn}(s) - k_1|s|^{\gamma}\mathrm{sgn}(s)\big]\big\}\\ &\leq s\widetilde{W}^{\mathrm{T}}\phi(x) + s\varepsilon - (\delta_1+\delta_2)|s| - k_1|s|^{\gamma+1}\\ &= -k_1|s|^{\gamma+1} + \big[s\widetilde{W}^{\mathrm{T}}\phi(x) + s\varepsilon - (\delta_1+\delta_2)|s|\big]\\ &\leq -k_1|s|^{\gamma+1} < 0.\end{aligned} \tag{49}$$

Furthermore, (49) can be rewritten as

$$\begin{aligned}\dot{V}_1 &\leq -k_1|s|^{\gamma+1}\\ &\leq -k_1(2k_0\varphi_0)^{\frac{\gamma+1}{2}}\left(\frac{1}{2}\frac{1}{k_0\varphi_0}s^2\right)^{\frac{\gamma+1}{2}}\\ &\leq -kV_1^{\frac{\gamma+1}{2}}\end{aligned} \tag{50}$$

with $k = k_1(2k_0\varphi_0)^{\frac{\gamma+1}{2}}$.

From Lemma 1, it can be obtained that the terminal sliding manifold s can converge to zero in finite time t_1 given by

$$t_1 = \frac{V_1^{[1-(1+\gamma)/2]}(t_0)}{k[1-(1+\gamma)/2]}. \tag{51}$$

▮

Remark 4 From (51), we can see that the reaching time t_1 is independent of the error dynamics, but only related to the constant k_0 and φ_0.

Theorem 3 *When the terminal sliding manifold s reaches zero, the tracking error e will converge to the neighborhood of the equilibrium point in finite time.*

Proof Once the states arrive at the sliding surface $s = 0$, they will remain on it and the system has invariant properties. On the sliding surface $s = 0$, we can obtain

$$\dot{e} = -\lambda|e|^{\gamma}\mathrm{sgn}(e). \tag{52}$$

Consider the following Lyapunov function:

$$V_2 = \frac{1}{2}e^2. \tag{53}$$

Differentiating V_2 along (52) yields:

$$\dot{V}_2 = -\lambda|e|^{\gamma+1} = -\lambda 2^{\frac{\gamma+1}{2}} V_2^{\frac{\gamma+1}{2}}. \tag{54}$$

Set $\beta_1 = \lambda 2^{\frac{\gamma+1}{2}}$ and $\beta_2 = \frac{\gamma+1}{2}$, (54) can be rewritten as

$$\dot{V}_2 + \beta_1 V_2^{\beta_2} \leq 0. \tag{55}$$

From Lemma 1, it can be concluded that the tracking error e can converge to the neighborhood of the equilibrium point in finite time t_2 given by

$$t_2 = \frac{1}{\beta_1(1-\beta_2)} V_2^{1-\beta_2}(t_0). \tag{56}$$

■

5 Experimental Results

In this section, experiments are performed on the two-axis turntable motor servo system to evaluate the superior performance of the proposed finite-time control scheme. Besides, an LSM control scheme and a PID control scheme are also given for the comparison.

The detailed implementation for the PMSM servo system is presented as follows:

1) Initialize the system conditions, neural network, and some relevant control parameters;
2) Derive the tracking error $e = y_d - y$ and the corresponding terminal sliding mode s based on (20) and (23);
3) Calculate the neural network estimator output according to (21)–(22) and the control input signal v via (32)–(33);
4) Update the neural network weight based on (34) and go back to Step 2) for the next sampling interval.

The proposed tracking control algorithm is implemented by a C-program in CCS3.0 programming environment. The total run time is 15s with the step size 0.01. Some initial conditions and states of the system are set as $(x(0), \dot{x}(0)) = (0, 0)$, $y_d = \sin 0.4\pi t$, $\dot{y}_d = 0.4\pi \cos 0.4\pi t$, and $\ddot{y}_d = -0.16\pi^2 \sin 0.4\pi t$, respectively. In the proposed control scheme, the relevant control parameters are chosen as $k_1 = 0.6$, $\gamma = 9/11$, $\delta_1 = \delta_2 = 0.001$, $\lambda = 10$, $\Delta = 0.01$, and $\zeta = 0.0001$. The parameters of NN are given by $\Gamma = 0.05$, $a = 2$, $b = 10$, $c = 1$, and $d = -10$.

The compared LSM controller is expressed as

$$v = \widehat{W}^{\mathrm{T}}\phi(x) + k_1 s + (\delta_1 + \delta_2)\mathrm{sgn}(s), \tag{57}$$

where the linear sliding mode is selected as

$$s = \dot{e} + \lambda e \tag{58}$$

with the parameters $\widehat{W}$, k_1, δ_1, δ_2, and λ being chosen the same as those of the proposed TSM scheme for fair comparison.

The expression of PID controller is given by

$$v = k_P e + k_D \dot{e} + k_I \int_0^t e(t)dt, \tag{59}$$

where $k_P = 12$, $k_D = 0.6$, and $k_I = 0.4$.

The experimental results are shown in Figures 5–7. Figure 5 describes the tracking performance of the proposed control scheme. The tracking errors and control signals of three different schemes are depicted by Figures 6 and 7, respectively. Through the comparison, we can see that the proposed method can provide better tracking performance than the other two controllers for the motor servo systems with nonlinear dead-zones. The proposed ITSMC scheme has a faster convergence speed and smaller tracking error in comparison with LSM and PID control schemes.

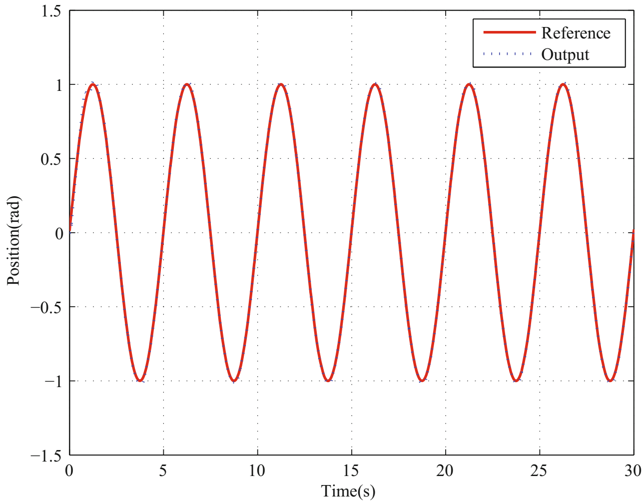

Figure 5 Tracking performance of the proposed scheme

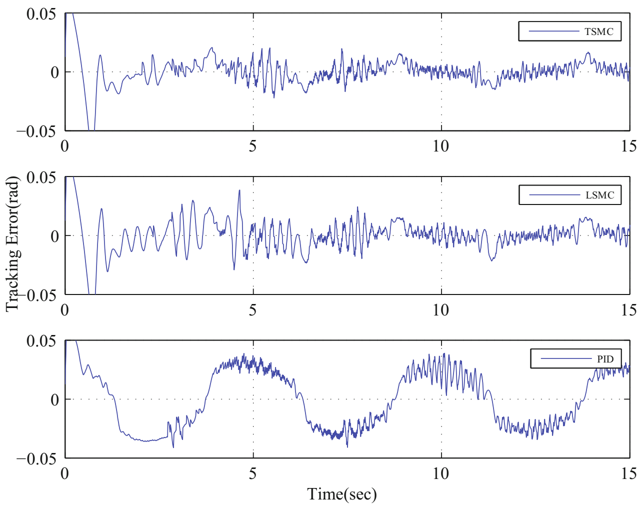

Figure 6 Tracking errors of three different schemes

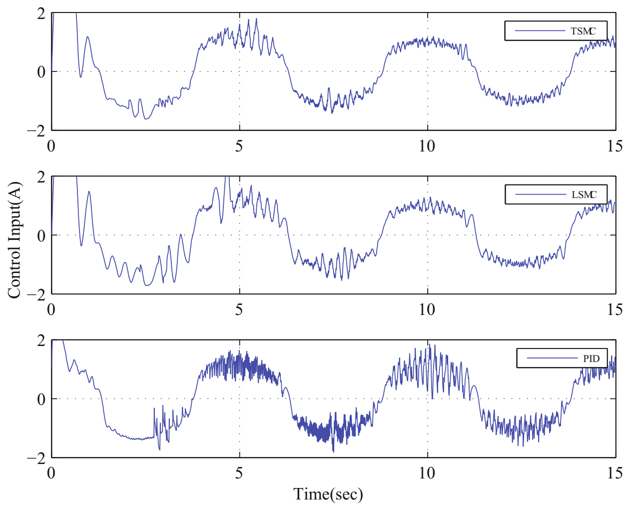

Figure 7 Control signals of three different schemes

6 Conclusions

In this paper, we present a finite-time tracking control for motor servo systems with unknown input dead-zones. None of dead-zone inverse compensation approach is needed by regarding the dead-zone as a simple linear time-varying system. Based on the terminal sliding mode principle, the finite-time controller is designed by using a simple neural network to approximate the unknown nonlinearities. With the proposed control approach, the singularity problem in the initial TSMC is eliminated and the approximation error is compensated by employing a robust term. Finite time convergence and stability of the closed loop system can be guaranteed based on the Lyapunov theory. Experiments results show that the proposed method has better tracking performance in comparison with LSM and PID controls.

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