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Connecting levels of activity with classroom network technology

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Abstract Classroom activity traditionally takes one of three forms, variously oriented toward the levels of individual students, small groups, or the whole class. CSCL systems, however, may enable novel ways to facilitate instruction within or sequence activity across these different levels. Drawing on theoretical accounts of learning at and across different scales of social interaction, this paper examines episodes of classroom activity featuring two learning environment designs that leverage networked digital devices to support face-to-face collaboration. Analysis of these episodes focused on two questions: When did activity shift between small and whole-group levels, and what mechanisms enabled or supported those shifts? Findings suggest that classroom activity in these environments was sometimes characterized by frequent, rapid shifts between levels, as well as instances that suggested hybrid forms of small-group and whole-class interaction. These shifts between and overlaps across levels were enabled and sustained through mechanisms including teacher orchestration, mediating roles played by virtual mathematical objects, learners' appropriation of shared artifacts and resources, and emergent properties of these complex interactions among classroom participants.

Keywords Mathematics · Classroom networks · Sociocultural theory · Classroom orchestration

Introduction

CSCL systems enable new modes of interaction among learners and novel ways to facilitate instructional activity (Koschmann, 1996). CSCL environments span a wide range of contexts, including face-to-face and virtual modalities, and formal and informal learning settings. This paper focuses on conventional classrooms, which have over the past two decades seen the emergence of an array of CSCL tools with the potential to reshape learning interactions in those settings (Brady, White, Davis & Hegedus, 2013; Chen, Looi & Tan, 2010; Higgins,

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Mercier, Burd & Hatch, 2011; Roschelle & Pea, 2002; Roschelle et al., 2007; Schwarz, de Groot, Mavrikis & Dragon, 2015; Szewkis et al., 2011). Classroom instructional activity traditionally unfolds at one of three levels: individual students, small groups, or the whole class (Kaput, 2000; Stahl, 2012). CSCL systems have been harnessed to support each of these pedagogical modes, through a variety of tools for *classroom networking*—local or wide-area communication between students' and teachers' computing devices. Perhaps the most widely used form of classroom network technology—audience response systems, or "clickers"—centers on activity at the individual level, typically in the form of students' multiple-choice or short-answer input to an instructor prompt (Hunsu, Adesope, & Bayly, 2016; Roschelle, Penuel & Abrahamson, 2004).

However, the integration of classroom communication infrastructures with dynamic representation technologies that are increasingly central in mathematics teaching and learning also enable more interactive forms of activity at other levels (Hegedus & Moreno-Armella, 2009). Some learning activity designs have focused on the small-group level, exploring the potential for classroom networks to support cooperation and collaboration by distributing interdependent mathematical objects, representations, or roles (White, 2006; Zurita & Nussbaum, 2004). Another line of innovation has emphasized dynamic objects jointly produced and collectively engaged by the whole class as resources for supporting discussion and participation (Hegedus & Kaput, 2004; Stroup, Ares & Hurford, 2005).

In addition to supporting novel forms of small- and large-group learning activity, these integrated communication and representation infrastructures may also allow the boundaries between levels of instructional interaction to become more fluid. For example, Clark-Wilson (2010) has described the ways classroom network technology can enable novel forms of interplay between the individual student and whole-class levels of activity, such as when mathematics teachers used publicly displayed screen captures of all students' devices to showcase contrasting student solution strategies, or even to increase the sample size of classroom datasets in discussions of statistics topics by aggregating results from multiple student screens. Indeed, any audience response system can provide a ready means of linking the individual and whole class levels when the instructor uses real-time polling results as the basis for a classroom discussion.

This paper seeks to further explore this potential for integrating levels of instructional activity through a classroom network design that incorporates elements oriented toward individual students, small groups and the whole class. Any given daily session in a typical classroom might alternately feature activity at two or all three of these levels. These different segments of class activity might each stand more or less alone, or be intentionally grouped into successive steps of a coherent pedagogical scenario (Dillenbourg, 2015), such as individual practice followed by small group investigation and then whole-class reflection. In either case, instructional activity at these different levels is generally sequential rather than simultaneous—a few small groups engaged in collaborative inquiry while the teacher tries to lecture would likely be disruptive; a student completing a worksheet or solving a problem might have difficulty listening or contributing to a parallel class discussion. But the interweaving of face-to-face interactions with networked transactions such as the sharing of dynamic math representations may enable new scenarios for connecting levels of learning activity. Below, I begin by situating this inquiry among theoretical accounts of learning at and across different scales of social interaction. I then present two learning environment designs situated at the interstices between individual, small- and whole-group classroom activity, and report findings from a cycle of investigation in an ongoing design-based research project that explores ways networked digital devices might support mathematically rich classroom activities and interactions among students.

Theoretical perspectives on multilevel classroom activity

Different levels of classroom activity have often been informed by and investigated through correspondingly different theoretical perspectives on learning, ranging from individual cognition, to small group collaboration, to community participation (Stahl, 2012). To examine forms of classroom activity that may span these multiple levels, I consider theoretical frameworks and key constructs that offer more integrated accounts of action and interaction at individual, small- and large- group scales. In particular, sociocultural theory posits individual cognition and development as inherently social, grounded in interactions with others and shaped by the internalization of cultural tools (Vygotsky, 1978; Wertsch, 1985). On this account, individual and group processes are not distinct, but rather combine to form a single coherent unit of cultural-historical activity (Cole, 1996).

Nonetheless, there is analytic utility in alternating focus between different planes of activity; Rogoff (1995), for example, differentiates three critical planes of sociocultural activity: personal, interpersonal and community. Rogoff stresses that the planes are interdependent, but also argues that foregrounding each plane in turn lends salience to distinct aspects and mechanisms of learning and development. Similarly, Ludvigsen and Arnseth (2017) characterize sociocultural analyses of learning and development in terms of three nested and interdependent layers: individual, social interaction, and cultural practice. While acknowledging the primacy of the interactional layer in CSCL analyses, Ludvigsen and Arnseth's account stresses the importance of simultaneously attending to the other layers, as tool-mediated collaboration among peers creates conditions for individual learning and participation, even as those interactions also shape and are shaped by historical and institutional conditions.

Saxe (2002) has elaborated an analytic approach that draws on sociocultural theory in order to examine the development of mathematical practices within and across these three planes or layers in classroom settings. On Saxe's account, individual learners take up forms, such as digital artifacts and mathematical objects, through goal-directed interactions with peers. Over time, through successive cycles of social interaction, those forms take on particular meanings to collective practices established by the classroom community. Thus collaborative activity among small groups of learners provides a site and a task structure for sociogenetic mechanisms through which mathematical artifacts become meaningful to collective classroom practice. In the context of a classroom-based CSCL environment, Furberg, Kluge and Ludvigsen (2013) have likewise drawn on sociocultural theory to show how representational artifacts such as science diagrams serve as resources for individual learners to coordinate their conceptual sense-making activities through interactions with peers, and in relation to institutional practices and norms. Below, I elaborate the roles of key constructs from sociocultural theory as they further illuminate aspects of these interactions between levels of classroom activity.

Mediation and appropriation

Two mechanisms from sociocultural theory are particularly relevant to conceptualizing intersections among different levels of activity in settings such as classrooms. One, the concept of *mediation*, offers utility for examining the role of artifacts such as classroom network technology in classroom activity. Mediation generally refers to the ways physical or symbolic tools reshape relations between a human subject and the object of his or her activity. Mediation can describe activity on any plane; dynamic representation technologies, for example, might variously mediate individual students' problem-solving or proving activities (Guin & Trouche, 1998; Mariotti, 2000), peer interaction in collaborative learning tasks (Stahl, 2009), or teacher facilitation and student participation in whole-group classroom learning activities and mathematical discussions (Ares, Stroup & Schademan, 2009; Clark-Wilson, 2010). In each case, mediation is a two-way process; it has both an external orientation through which an artifact enables actions toward an object, and an internal orientation whereby the subject is transformed through participation in mediated activity.

Focusing on this latter, inward orientation, the concept of *appropriation* has been widely applied to instances where individuals take up and make use of symbolic or material resources they encountered in social settings. In mathematics education contexts, researchers have used appropriation to describe how learners come to adopt ways of using cultural tools—including mathematics concepts and ways of participating in mathematical forms of practice—by observing and interacting with teachers, tutors or peers in small-group and whole-class settings (Abrahamson, Trninic, Gutiérrez, Huth, & Lee, 2011; Carlsen, 2010; Lai & White, 2014; Moschkovich, 2004; Radford, 2000, 2013). Fundamentally, appropriation involves a transformation of activity at the individual level through participation in collective activity (including, but not limited to, the levels of small groups and classroom communities) (Rogoff, 1990, 1995).

Thus, the mechanisms of mediation and appropriation can be used *in tandem* to consider transitions between levels of mathematics classroom activity. When mediational means—mediating artifacts or forms of tool-mediated practice—are appropriated, they move between planes of activity. In a classroom environment replete with representation and communication technologies, and combining segments of instructional activity at different levels of interaction, individual students or small groups may regularly appropriate mediational means first demonstrated by others in the classroom community. As these transitions between planes occur, they also mark opportunities for transformation of those meditational means as they are taken up and potentially recast by actors at different levels.

Emergence

Another mechanism of interactions between levels or layers of sociocultural activity is emergence. The idea of emergence has been extensively developed in relation to the study of natural systems that are self-organizing, governed by rules or patterns of behavior at the level of individual elements or agents rather than imposed by any top-down structures or imperatives (Johnson, 2001). A key characteristic of these complex systems is *emergence*, a process by which properties at the global level arise from but do not duplicate and are not readily explained by events or interactions at the local level (Sawyer, 2005). Metaphorically extending this principle to classroom contexts, Cobb & Yackel (1996) draw on sociocultural theory to articulate an "emergent perspective" on mathematics classroom activity in which mathematical understandings or practices might emerge as properties of either individuals, or the classroom group as a whole, through ongoing interactions among members of the classroom community. In Saxe's (2002) framework, the goals of individual learners in classroom mathematics practices emerge from interactions with other participants, artifacts, and activity structures. In classrooms where conditions of complexity are nurtured (Davis & Simmt, 2003), and in collaborative knowledge building contexts like wikis (Cress and

Kimmerle 2008), learning can be characterized in terms of emergence—as a collective accomplishment of the group or community rather than a property of individuals.

Some design experiments have taken the analogy between classrooms and complex systems a step further, explicitly designing learning activities that invite students to engage in "participatory simulations" wherein handheld or wearable technologies, or simple rules governing individual actors' behaviors, allow a group of learners to collectively play out a scenario and then observe phenomena that emerge as a result (Colella, 2000; Klopfer, Yoon & Perry, 2005; Levy and Wilensky, 2008; Wilensky & Stroup, 1999a). These designs intentionally leverage the complexity inherent in the interactions of a classroom group as a resource for illustrating and inviting students to engage and reflect on emergent phenomena as they arise from other complex systems modeled by participants' interactions in the learning activity design. Prior work with participatory simulations in mathematics classrooms equipped with classroom network technology has emphasized the importance of reciprocities between mathematical and social structures in these settings. In this "generative design" approach (Stroup, Ares & Hurford, 2005), mathematical relationships are leveraged as resources for organizing social interactions in class-level activity, so that those emergent interactions might in turn enrich participants' experience of and insight into mathematical structure.

Participatory simulations showcase a pedagogical strategy for intentionally intermixing two levels of instructional activity, simultaneously foregrounding the individual and the whole class. Notably, however, in studying student learning about emergent phenomena in the context of participatory simulation activities, Levy and Wilensky (2008) report on a widespread strategy adopted by learners for reasoning about complex systems, whereby they make use of "mid level" constructions—imagined subgroups of a small number of agents that help them to envision and describe phenomena emerging in a larger interacting group. This finding echoes an argument from the field of computer-supported collaborative learning that small groups of learners function as crucial intermediaries between individuals and larger communities (Stahl, 2006). Moreover, it also suggests an interesting possibility for design: that the analogies between classroom interactions and complex systems, and between social and mathematical structures, each might be extended to learning environments that intentionally utilize small as well as whole-class groups of actors in modeling and enacting phenomena of interest.

Teachers and classroom orchestration

A final note in the application of sociocultural theory to the examination of multilevel mathematics classroom activity involves the role of the teacher. Saxe (2002) stresses that traditional and reform-oriented mathematics classrooms have different activity structures—different ways of framing possibilities for individual participation and goal-directed interaction in collective classroom practice. These structures entail correspondingly different roles for a teacher, as presenter of information or as facilitator of joint inquiry. In a compatible account, Enyedy (2005) details the critical role of the teacher in transforming individual students' personal acts of invention into shared conventions for the classroom community. So teachers play a pivotal part in providing both the structures and the mechanisms that enable connections across individual, social and collective layers of classroom activity.

In CSCL contexts, several researchers have adopted the metaphor of *orchestration* as a means of conceptualizing the critical role of a teacher in both organizing instructional activity at individual, small and whole-group scales, and integrating technological resources and other material artifacts across those levels (Dillenbourg, 2012; Drijvers, Doorman, Boon, Reed, &

Gravemeijer, 2010; Roschelle, Dimitriadis & Hoppe, 2013). Though it does not originate from the sociocultural tradition, I likewise take up orchestration as a means of naming teachers' facilitation moves that bridge levels of classroom activity. Adopting the term provides a means of incorporating specific tools for the examination of classroom levels into a broader framework for the analysis of layered sociocultural activity.

In particular, Dillenbourg (2015) elaborates an approach to modeling instructional sessions through *orchestration graphs* reflecting the ways a learning activity sequence is intentionally structured to vary between work at individual, team and whole class levels. Figure 1 illustrates a sample orchestration graph, in which the instructional sequence opens with an activity at the whole class level, followed by a pair of activities at the small group level, then a return to the class level, then a series of tasks at the individual, small group and whole class levels. Shifts between levels might be part of a planned sequence, or they might reflect in-the-moment instructional decisions ("I'm running out of time, so we'll skip the next team activity"). Dillenbourg proposes that any pedagogical scenario, whether in a conventional classroom or a massive open online context, can be modeled in this way. Moreover, attending to orchestration in this way can inform the design of tools to support learning activity within and transitions between levels (Dillenbourg, 2012).

Summary

Collectively, these theoretical and analytic resources point toward a framework for conceptualizing interplay between levels of activity in technology-mediated mathematics classrooms. Appropriation generally describes movement downward (from whole to small group or individual, from whole or small group to individual) between levels, and emergence describes movement upward between levels. And mediation emphasizes the roles played by artifacts, such as digital devices and dynamic representation tools, in those forms of activity that might emerge or be appropriated through interaction within and across levels, whereas orchestration emphasizes the role played by a teacher in facilitating instructional activity and interaction within and across levels. The next section introduces an approach to collaborative mathematics learning activity design that may lend salience to these transactions between levels.

Collective mathematical objects and multi-level learning designs

Prior investigations of classroom network technology have introduced learning activity designs in which a whole class group collectively constructs a set of mathematical objects—each student might use his or her device to contribute a distinct member of a family of functions, a

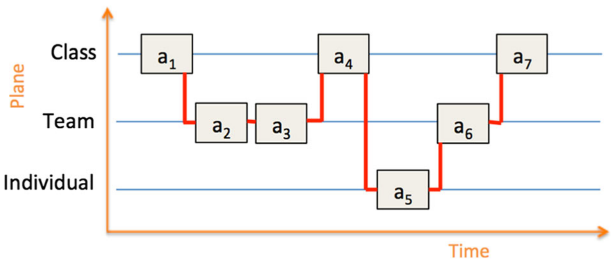

Fig. 1 Sample Orchestration Graph

locus of points, or a class of equivalent expressions to an aggregation on a shared display (Hegedus & Penuel, 2008; Stroup, Ares, Hurford & Lesh, 2007). In each case, the collective construct illustrates a map between the mathematical set and its elements that mirrors the relations between the classroom group and each individual student member. That individual-collective mapping then serves as a resource for directing student attention to the relations among these mathematical objects and guiding classroom discussions about patterns within and generalizations across the array (Stroup, Ares & Hurford, 2005).

This dialectical interplay between mathematical and social structure can also be used to support mathematical conversations and interactions among pairs and small groups of students. While the number of students (often 30 or more) in a typical classroom group can effectively illustrate the variation within sets of mathematical objects that may be infinitely large, many core mathematical objects under study in the k-12 curriculum are composed of sets of just a few coordinated sub-objects: lines are uniquely determined by two points, quadratic expressions are comprised of three distinct monomial terms, equations compare two distinct algebraic expressions, functions are commonly represented through one of three modes—symbols, graphs or tables. Additionally, many of the conceptual challenges that face students as they grapple with these concepts consist of difficulties with the coordinated significance of these sub-objects: how to determine the slope of a line (Leinhardt, Zaslavsky, & Stein, 1990), to combine polynomial terms and simplify expressions (Linchevski & Herscovics, 1996), to solve equations (Kieran, 1992), to interpret relations among representations (Schoenfeld, Smith, & Arcavi, 1993).

These small sets of mathematical objects can be aligned with correspondingly small groups of students through classroom network designs for collaborative learning tasks. In these designs, each learner manipulates a linked point to collectively form a curve in a Cartesian plane (White, Wallace & Lai, 2012), or transforms a respective side of an algebraic equation (Sutherland & White, 2016), or combines different-ordered terms or enacts different binomial operations among polynomial expressions (White, Sutherland & Lai, 2010), or examines a different representation of the same function (White & Pea, 2011). Broadly, problem-solving tasks are structured around these shared mathematical objects in order to make successful solutions dependent on contributions from and coordination between all participants in a small group.

This emphasis on designs for small groups reflects a hypothesis that networked personal devices might be powerful resources for supporting collaboration at the small group level. At the same time, however, the small groups in these classroom network designs are situated within a larger classroom group, and often feature public classroom displays that make the work of individual students and/or small groups visible to all students and the teacher. Consequently, classroom networking designs may hold potential for reorganizing or reshaping conventional classroom activity structures—for blurring the boundaries between forms of instruction oriented toward individual students, small groups, or the whole class. Below, I describe two such designs in detail, and present an analysis of the ways and the extent to which they might leverage intersecting mathematical and social structure to blend different levels of classroom activity.

Learning environment designs for collaborative learning in classroom networks

The data presented in this paper feature two different variations on this approach to classroom network learning activity design: *Terms and Operations*, focused on students' collaborative efforts to construct polynomial expressions, and *Graphing in Groups*, centered on pairs' joint manipulation of linear functions. Both designs were created by my research group at UC Davis using the NetLogo modeling platform (Wilensky, 1999) and HubNet architecture (Wilensky & Stroup, 1999b) in concert with a classroom set of Texas Instruments graphing calculators connected through a TI-Navigator™ network. Students participated in learning activities by using a client application on their calculators, which they used to contribute and operate on mathematical objects collectively displayed through a server application running on the teacher's computer and projected on a whiteboard or screen at the front of the room. Each design featured some mathematical objects belonging to individual student participants and others shared by both students in a pair, and interface elements variously associated with individual users, small groups, and the whole class. Both environments were designed to build opportunities for peer collaboration and classroom discussion around topics from a traditional introductory Algebra course. Below, I describe each of these designs in turn.

The *Terms and Operations* activity

Terms and Operations was designed to make the traditional Algebra topic of simplifying algebraic expressions more interactive and engaging for learners. In this learning activity, student pairs share responsibility for constructing a polynomial expression. Each student uses the directional arrow keys on a calculator to move an icon in a whole-class shared display (left side of Fig. 2) populated with a variety of floating monomial terms (of order, sign, scale and number set by the teacher prior to each activity). The first time a student's icon comes into contact with one of these terms, the student can "capture" the term to her group's expression window (shown in the right side of Fig. 2). Thereafter, each time one of the students in the pair encounters another floating term, she has the option to select a binary operation (add, subtract, multiply or divide) by which to combine the new term with the group's current expression (Fig. 3a). That student is then prompted to enter a new

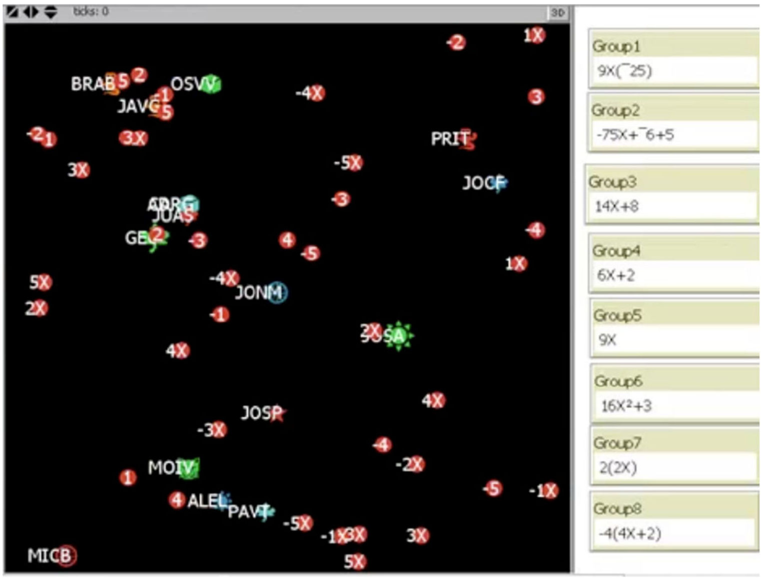

Fig. 2 *Terms and Operations* shared display

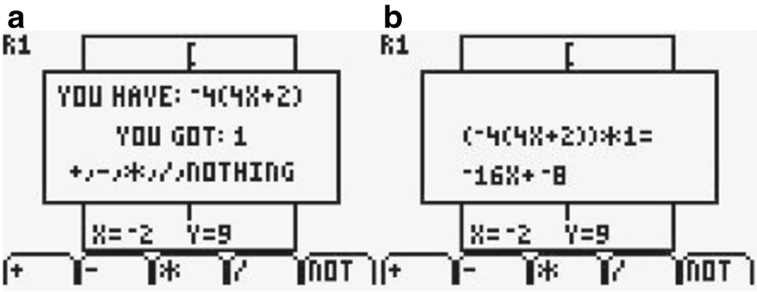

Fig. 3 **a** and **b**. Student calculator screens.

collective polynomial expression representing the result of the chosen operation on the captured term (Fig. 3b). If (and only if) the expression entered by the student is equivalent to the result of the operation, the system updates the collective expression.

In *Terms and Operations*, there are screen elements (icons) and mathematical objects (captured terms and selected operations) associated with individual students, and some segments of activity center on those individuals' interactions with keys, prompts, and symbol entry on the graphing calculator. Other aspects of the design, however, center on the student pair. The polynomial expressions cumulatively constructed over a series of these individual transactions are jointly owned by two students, and the system includes a restriction that the same student icon cannot perform consecutive captures. Moreover, tasks always involved combining several capture steps to construct an expression with particular characteristics (e.g., form a quadratic expression by capturing and operating on only linear and constant terms). These elements were intended to foster collaborative interactions by necessitating contributions from both partners and encouraging conversation about which terms and operations to choose and what new expression to enter. Finally, the public display of all icons, terms and group expressions dynamically updated throughout the activity, provided a resource for collective discussion and interaction at the level of the whole class.

The *Graphing in Groups* environment

Graphing in Groups was designed to reinforce students' understanding of the relations between the equation and the graph of a linear function through exploration and peer cooperation. In *Graphing in Groups*, each student in a pair uses the four directional arrow keys on a graphing calculator to move a coordinate point one unit up, down, left or right in a Cartesian plane (Fig. 4a). Those movements of both students' points on their respective devices respective are also mirrored in a shared graphing window in the public display. Students also have the option to "mark" a point at their current coordinate location; each time a student makes a new mark, the line between that point and the one most recently marked by the student's partner is automatically updated in the group's shared graphing window. The slope-intercept form equation of the line simultaneously appears just above or below the group graphing window (Fig. 4b). This linear function serves as a collective object which groups must jointly manipulate, through dialogue and coordinated action, in order to accomplish shared tasks. Likewise, the organization of two students in the small group reflects the relationship between that line and a set of points that graphically determine it, prompting students to reason about that relationship as they construct lines with particular characteristics (e.g., a slope of 2/3, or a negative y-intercept).

Up to two groups' points, lines and equations, differentiated by color, can be displayed in the same group window; this configuration can enable four students to collaboratively construct or investigate relationships (e.g. parallelism, perpendicularity, a particular intersection) between their lines. A grid composed of several groups' graphing windows and projected from the teacher's computer at the front of the classroom provides a collective display of all these small group-level graphs (Fig. 4c). An additional feature of the application allows the teacher to merge all groups' points and lines into a larger single graphing window. Thus, as with *Terms & Operations*, *Graphing in Groups* features mathematical objects associated with both individuals (coordinate points) and small groups (linear functions). Likewise, the environment includes display elements oriented toward individuals (students' calculator screens), small groups (the group graphing window and equation), and the whole class (the grid and merged graphing views).

Method

The *Terms and Operations* and *Graphing in Groups* designs were implemented in successive classroom-based design experiments, each with two groups of 16 9th grade Algebra I students (ages 14–15). The author served as the teacher for all class sessions. Six days of *Graphing in Groups* and four days of *Terms and Operations* activities with each of these groups were part of a year-long project in which students participated in classroom network activities for a one-hour session each week as a supplement to their regular mathematics program. Two to three student pairs in each class were selected as focus groups and videotaped during all activities. An additional camera with a wide zoom setting captured this projected display along with the whiteboard at the front of the room, as well as whole-class discussions and other teacher moves. All states of the classroom network computer server were also recorded as an additional video file and time-synched with camera-recorded sessions. Server logs recorded all terms and operations selected and expressions entered on student calculators.

In order to develop an analysis of interactions across multiple levels of nested activity mediated by these classroom network tools, video files from each pair, the whole class, and the server computer were synchronized and watched by the author and two other researchers to build an annotated, minute-by-minute timeline of events and interactions at the small-group

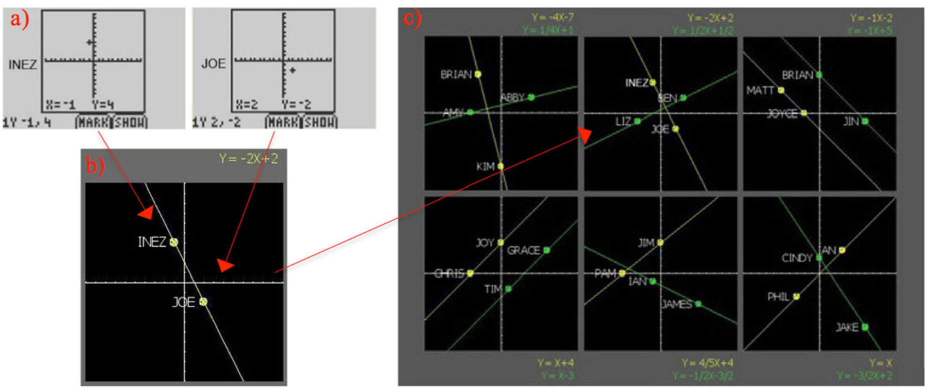

Fig. 4 **a**, **b** and **c**. *Graphing in Groups* individual, small group and whole-class displays.

and whole-class levels for each session. For the purposes of the present study, the last day's session from each of the *Terms and Operations* and *Graphing in Groups* units was purposively selected for more detailed analysis. These final sessions were not selected to be representative of "typical" instances within the data corpus, but rather as instances that came closest to illustrating the kinds of classroom interaction these environments were designed to enable—in general the students appeared to spend less time developing familiarity with the technology interface and more time engaging the mathematics over successive days in each unit. The timelines for these final sessions were subdivided into the four to six mathematical tasks (Stein, Grover & Henningsen, 1996) governing classroom activity on each day, and further reviewed in order to identify task segments in which classroom activity shifted between the small group and whole class levels. Episode selection was also informed by the author's retrospective reflection, as researcher-teacher (Ball, 2000), on what happened in those particular class days—each stood out to me as memorable or interesting in the ways it represented what I perceived as instances of network-supported classroom activity worthy of closer examination. One episode spanning the duration of one mathematical task (several minutes, in each case) was selected from each day for finer-grained examination of the interactions between students, among small groups, between teacher and students, and across levels of activity.

For each episode, each of the three small group videos and the video of the projected display were transcribed, and subtitles of all audible utterances were added to the video files. All four video files were precisely synchronized, and then imported into video analysis software that allowed them to be played simultaneously and layered with analytic codes. Drawing on techniques from interaction analysis (Jordan & Henderson, 1995) and related video-based education research methods (Derry et al., 2010), the episodes were repeatedly reviewed, often over very short intervals, to explore the following questions: When did classroom activity shift between levels? What mechanisms enabled or supported those shifts?

To address the first question, I began by expanding the annotated timeline segments for each of these episodes into tables that show fully transcribed simultaneous dialogue and activity from the teacher and each student group on camera. As a transcription convention, these tables show boxed cells in a single column around segments of dialogue within a group, merged cells spanning two columns when groups on camera (or one group and the teacher) interacted with one another, and merged cells across all columns when the discussion spanned the whole class. Segments of dialogue and activity in horizontally aligned cells spanned the same interval of time, though utterances in different columns on the same horizontal row were not always simultaneous. To complement this detailed view, I also incorporated Dillenbourg's (2015) orchestration graphs to summarize the transitions between levels of activity during these segments. While some segments of activity were clearly at the small group (students interacting with their assigned partners) or whole class (teacher facilitating dialogue with all students), others appeared to merge or feature simultaneous activity across levels (such as interactions between multiple groups); these instances are represented as overlapping levels of activity in the orchestration graphs and elaborated in the analysis.

To address the second research question, I examined each of the transitions or overlaps between levels through the framework of appropriation, mediation, emergence and orchestration outlined above. When one or more of these mechanisms contributed to a shift between levels of activity, or to interactions among participants or groups within that level, those contributions are noted with directional arrows and a notation (A, M, E, O) on the orchestration graph, and further described in the written analysis.

Analysis

This section presents detailed descriptions of the two selected episodes, each of which is further subdivided into two segments to illustrate successive analytic themes. The first segment of episode one provides some examples of transitions between levels of activity, as well an overview of how the different mechanisms of appropriation, mediation, emergence and orchestration each contributed to these shifts. A subsequent portion of this episode is then presented as an opportunity to more deeply explore intersecting mechanisms of emergence and appropriation. In the same fashion, Episode Two is likewise presented in two parts, the first particularly highlighting orchestration and mediation, and the second examining the ways several of these mechanisms interact to support shifts of not only social interaction, but also mathematical structure, across levels.

Episode one: interactions across levels in *Terms & Operations*

Below, I present a classroom episode from the implementation of one of these classroom network designs, selected from the fourth day in a series of lessons and activities featuring the *Terms & Operations* environment. After beginning the class with a brief review of the distributive property from the previous session, the teacher asked students to log in to *Terms & Operations* on their calculators, and then instructed each group to make a collective expression that featured parentheses. As groups began generating expressions that met this requirement, the teacher wrote each one on the board, and asked those groups to write their next expressions without parentheses.

Table 1 presents a set of transcripts that span the simultaneous dialogue of three student groups, as well as comments made by the teacher, over a 90 s segment of this session. This segment joins the class 11 min into the activity, as groups were continuing to operate on new terms and write resulting expressions with and without parentheses, and just as Group 6 (not on video) became the first to create an expression with an x^2 term. Taking note of this expression as it appeared in the public display (line 1), the teacher pointed it out to the whole class and noted the absence of x^2's among the floating terms (lines 7–8).

This episode found all three of the student pairs on camera actively shifting between small-, inter- and whole group activity (Fig. 5). When the teacher first directed attention to Group 6's expression (line 1), Group 2 clearly followed along, and noted the novelty of the quadratic term (Jose, line 2) well before the teacher explicitly identified it, discussing at the level of their small group how they would have needed to pick up (and presumably multiply together) two different linear terms (lines 2–6). Meanwhile, the students in Group 8 turned around to identify and interact first with Group 7 and then Group 6, jokingly suggesting to the latter pair that the teacher's singling them out might portend trouble (lines 2–6). While the respective dialogue of Groups 2 and 8 both centered on the activity of Group 6, one pair attended to the quadratic collective expression while the other focused on the students who had produced it. In the following moments, all four students in these two groups turned away from their inter- and intra-group dialogue to participate in a segment of whole-class conversation, taking turns actively responding to questions posed by the teacher (lines 7–9).

Figure 6 captures the sequence of classroom activity from Table 1 in a modified orchestration graph. Note that in line 1 the teacher addressed a single group during a period of small group activity, and Group 2 continued pair-level dialogue, so this interval is represented as featuring activity at on the Team plane. Yet each group's dialogue clearly attended to the

Table 1 Overlapping segments of teacher talk and peer dialogue during episode one

Line	Teacher	Group 2	Group 1	Group 8
1	Tobin: Whoa. [Taps marker on whiteboard projection of Group 6's expression (Figure 5)] Group 6, what happened there? Nice.			
2		Jose: How'd they get a squared?		David: [Turns around to look at groups seated
3				behind him] Oh! You guys are gonna be…
4				Student from Group 7: We're 7.
5		Brian: Cause they got two X's.		David: Who's group 6?
6		Cause they probably picked up, like, X and… 4X and 4X again. Jose: Oh, two X's.		Anita: [Turns and points to Group 6]
7	Tobin: [pointing at Group 6 expression] Group 6… Everybody see what Group 6 has made up here?			David: Yeah.
8	Tobin: You guys have got an X^2 term, are there any X^2's out there?			
9		Jose & Brian: No.	Miguel: (Shakes head)	Anita: Un-hnn.
10	Tobin: No, so you must have done something clever to make that. So, Group 6, now I want to see if you can rewrite this with			
11	parentheses. A new expression with parentheses…		David: [Calls to Miguel, points up at Group 1's current expression, 9X(-	
12		Brian: What do we do?	25), displayed on the screen] What the…?	
13	Tobin: that keeps your X-squared somewhere.		Miguel: [holds his hands up and shrugs in response to David's	
14		Jose: Yeah, what do we do?	question.]	
15	Tobin: That's a good	Brian: What are you doing?	Grace: I guess you can	Anita: Where am I, where am I, where am I?
16	challenge actually.	Jose: I don't know. Just moving	try, um…. Try 9X. And	Oh my God. Go straight down, go straight
17		around.	then, in the	down, go straight down.
18			parentheses, put	David: Where'd you go to? Oh, I see you.
19			negative 22.	Anita: [picks up a 1] Hmmm….
20	Tobin: The rest of you who are looking for some ways to challenge yourselves, see if you can get an X^2 [taps on Group 6's quadratic			
21	term] into your expression.			
22		Jose: There's probably a negative	Miguel: Nope.	Anita: I'm hecka lost. Hecka hecka lost.
23		one X or something.	Grace: No?	David: I know! Let's ask him for help.
24		Brian: Yeah.		

teacher's hailing of Group 6, including Group 8's efforts to interact with other groups, suggesting some overlapping activity at the class level. Likewise, even as the activity moved to a whole-class level exchange facilitated by the teacher in lines 7–14, there was some overlapping interaction within and between groups. Beginning in line 15, all three groups on camera had resumed small group-level activity, even as the teacher reflected further on Group 6's expression (extending the whole-class discussion topic) and then challenged other groups to make something similar (signaling the return to small group activity which had already commenced).

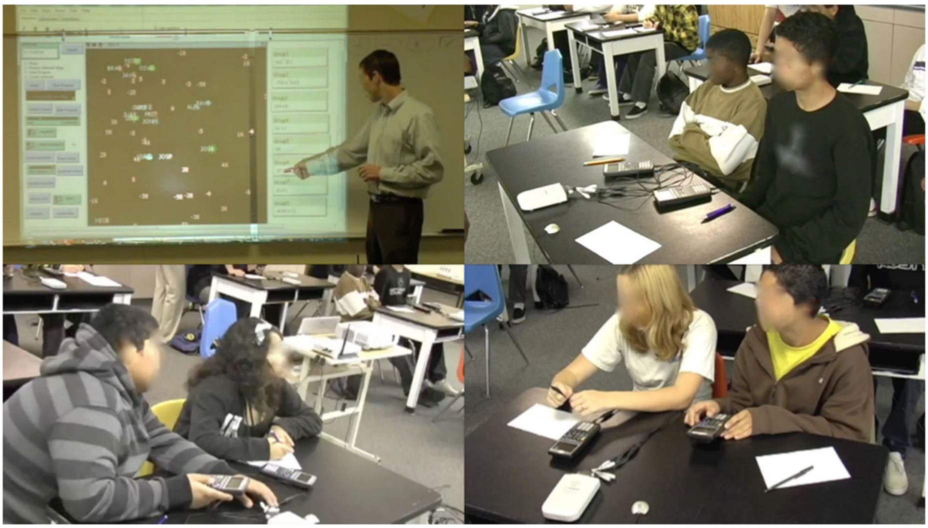

Fig. 5 Teacher with public display and three student groups during *Terms & Operations* activity

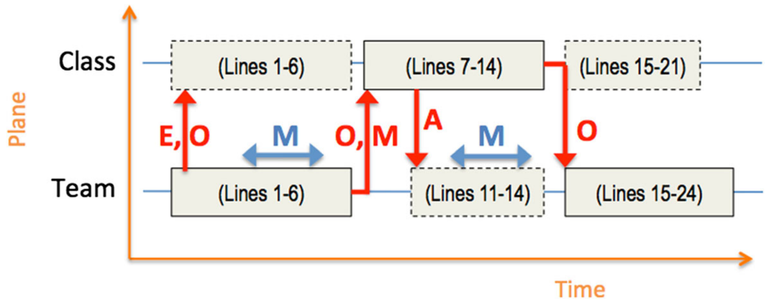

Fig. 6 Orchestration graph showing transitions and overlaps between activity on Team and Class planes in Table 1. Arrows indicate when movement between levels (unidirectional vertical arrow) or interactions within levels (bidirectional horizontal arrow) were supported by mechanisms of (A)ppropriation, (M)ediation, (E)mergence or (O)rchestration

The teacher used several orchestration moves to shift repeatedly between interacting with small groups and with the whole class. In line 1, he spoke directly to Group 6, but clearly drew the attention of students in other groups to an emergent mathematical artifact. In line 7, he again began as if addressing Group 6 directly, but then shifted to calling the attention of “everybody” in the class to Group 6’s quadratic expression displayed on the projected screen. In line 8, he again began by speaking directly to Group 6 (“you guys have got an X^2 term”), but immediately followed with a question to which at least three students from other groups volunteered responses (line 9). And in line 10, the teacher began by revoicing those responses from other groups (“No”), but then returned to addressing Group 6 directly, tasking them with rewriting their quadratic expression using parentheses (lines 10–16).

In each case, these overlaps between interactions with a small group and with the whole class reflected not only orchestration moves by the teacher, but also the mediating role played by dynamic small group-level math objects in the public display. Each time he addressed Group 6, the teacher repeatedly tapped or pointed at their collective expression, usually accompanied by deictic references to that portion of the screen (“there,” line 1; “here,” line 7; “this,” line 10), and alternated the direction of his gaze between the screen at the front of the room and their group at the back of the room. In these ways, he repeatedly reinforced the association between the two students in Group 6 and the polynomial expression they had jointly constructed, and called the attention of the class to the latter. And using this publicly displayed artifact to ask Group 6 about their work provided a ready means of inviting the rest of that class to reflect on the same questions.

In the same moments, David (in Group 8) called across the room to Miguel (in Group 1), directing his attention to the screen in an apparent effort to ask (“What the…?) about his group’s current expression, 9X(25) (lines 11–14). Though David’s particular interest in the expression is unclear, the exchange illustrates how group-level mathematical objects in the public display mediated interactions between groups, as well as between group and teacher.

Indeed, following David throughout this episode also highlights instances of appropriation, as the focus of his own attention appears to parallel the teacher’s efforts to track class activity displayed on the screen, and to interact accordingly with others in his own and other groups. After engaging with Groups 6 and 7 about the former’s expression highlighted by the teacher (lines 2–6), and then with Group 1 about their expression (lines 11–14), David then shifted attention to his own groupmate, but again focused on the public display, this time joining Anita in an effort to locate her own icon in the floating terms field, and then discussing how to proceed after she picked up a new term (lines 15–19, 22–23).

Meanwhile, perhaps chafing at the teacher's attention to Group 6, Brian and Jose in Group 2 asked what they should do next (lines 12, 14). A moment later, the teacher addressed an apparent response to the whole class, inviting other groups to duplicate Group 6's feat of constructing a quadratic expression (line 20). By this time, all three groups on camera had resumed pair-level activity (lines 15–24). Nonetheless, Group 2 appeared to take up the latest direction from the teacher, quickly adjusting from "just moving around" (lines 16–17) to trying to find a "negative one X or something" (lines 22–23) in the floating terms field that they could multiply with their current linear expression. So, a novel construction by one group, highlighted by the teacher, had now emerged as a focal task for other groups. Meanwhile Group 8, struggling with how to simplify their expression, decided to "ask [the teacher] for help" (line 23). The next session analyzes the resulting interaction between these small groups and the teacher as they further illustrate both emergence and appropriation of tasks and solution strategies.

Emergence and appropriation

Prior to the excerpt above, Group 8 had constructed the expression $-4(4x + 2)$. Anita picked up a 1 and chose to multiply it, but she and David both expressed uncertainty about how to proceed with rewriting the resulting expression, $(-4(4x + 2))*1$, without parentheses (lines 19–23). The teacher, still standing at the front of the room near Group 8's table, overheard and followed up:

25. Tobin: Group 8, can you guys get an x^2 into your expression?
26. David: No, we already tried it like three times to get off the parentheses, but we can't.
27. Tobin: Oh, you can't make them go away. Ok.
28. David: No. Somehow, well, she just got a number.
29. Tobin: [approaching Group 8's table] Let's see what you've got. [Looks at A's calculator] Ok,
30. so, here's a suggestion. What do you think that that [(-4(4x+2))*1] is equal to? [David
31. looks at Anita, neither answers] Well, this is really interesting actually, because you
32. picked up a one, right, and you multiplied times one? So, what happens when you
33. multiply, do you know what happens when you multiply something by one?
34. David: It stays the same.
35. Tobin: It stays the same. [Anita nods] So, your expression won't change, right?
36. David: Sure.
37. Tobin: What it's equal to won't change, but you can write it as something else that's
38. equivalent. So how could you rewrite that expression without parentheses?
39. Anita: Ohhhh, ok, I think I get it. [So you multiply…[points to expression on calculator]
40. David: [It could be negative four multiplied by 4x+2, right?
41. Tobin: Sounds like it's worth trying. What do you think that would be? Talk about it. [Walks
42. away]

When the teacher asked if they had an idea about how they would go about the new challenge of forming a quadratic expression, David explained that they were still struggling to complete the previous task of rewriting their expression without parentheses (lines 25–27). On arriving at their table, the teacher looked at Anita's calculator and noted that she had just picked up a 1 and selected multiply, but not yet entered a new expression (line 29). The teacher suggested that it was "really interesting" that they had chosen to pick up and multiply by a one, and asked them about the effect of multiplying by 1 (lines 31–33). When David quickly replied that the resulting expression "stays the same" and Anita indicated agreement (lines 34–36), the teacher encouraged them to use this circumstance as an opportunity to simplify their current expression without also having to incorporate a new term and operation (lines 37–38). Anita voiced understanding of this approach, and she and David simultaneously articulated their realization that they were faced only with the problem of simplifying "negative four multiplied by 4x + 2" (lines 39–40). As the teacher moved away, the students discussed how to multiply both 4x and 2 by −4, soon correctly entering -16×-8 and celebrating with a high-five as their collective expression updated to this new form.

Thus while other groups were taking up the new task of constructing a squared term, the teacher and the students in Group 8 were using their coincidental choices—to pick up a 1 and multiply—to devise a strategy for rewriting expressions in simplified form within the constraints of the *Terms & Operations* environment. This strategy was suggested by the teacher rather than discovered independently by the students, but that suggestion came only as a result of the teacher's noticing an opportunity created through the apparent happenstance of Anita's having selected an identity term and operation. In other words, the strategy was not a planned instructional outcome, but rather an emergent opportunity arising from a complex sequence of interactions among students, teacher, tools, and tasks—the students' movements within a virtual field of randomly floating terms, their earlier efforts at symbol manipulation to produce the current expression, and their interactions with the teacher resulting from seeking help after a period of struggle.

Thirteen minutes later in the session, the students continued to engage in the same activity, but several groups had successfully constructed quadratic expressions, and were subsequently prompted by the teacher to try factoring these expressions—to rewrite them, using parentheses, so that the x^2 term was no longer visible. When the groups struggled with how to simultaneously factor their current expression and pick up a new term, the teacher told the whole class about Group 8's discovery that they could keep their expressions the same by choosing a 1 as their new term and multiplication as their operation.

Once again, this segment blends small and whole-group dialogue and activity in quick succession (Table 2 and Fig. 7). The teacher began by announcing to the class that he wanted

Table 2 Overlapping segments of teacher talk and peer dialogue

Line	Teacher	Group 2	Group 8
43	Tobin: I want to tell you guys about something that David and Anita did a little while ago.		Anita: That was me.
44	Tobin: They were multiplying by one, and we were talking about what happens when you do that. What happens when you multiply		
45	something by one?		
46		Brian: It changes.	David: It stays the same.
47		Jose: It's the same.	
48		Brian: It's… the same.	
49	Tobin: Yeah, multiply something by one? It stays the same? So if you… [makes a side comment about the software interface] If you		
50	multiply something by one, or divide by one, it stays the same.		
51		Jose: Yeah.	
52	Tobin: So if you ever have an expression up here [gestures toward screen], and you want to rewrite it without changing it at all,		
53	[gestures at floating terms] go and find a one…	Jose: Get… get a one [turns head from facing board to facing Brian as he speaks].	
54	and multiply by that one. And then when you	Brian: Wait, I can factor…	
55	write in your answer you can write a new	Jose: Just get a one.	
56	expression that has the same form.	Brian: Did he say factor out?	
57		Jose: Cause the thing is if we can just get a one then we can factor out X, both Xs.	
58		Brian: Well, we could do it now, I just don't know how to do it on the calculator.	
59		Jose: I know, that's why you can do it if you multiply it by one, cause it won't change	
60		it. So then the X would be outside of the parentheses. So if you get 1, it would be	
61		X, and then parentheses 75X+4.	
62		Brian: [After successfully multiplying by one and entering X(75X+4) a little over a	
63		minute later] Oh, ok, I get it! I get it.	

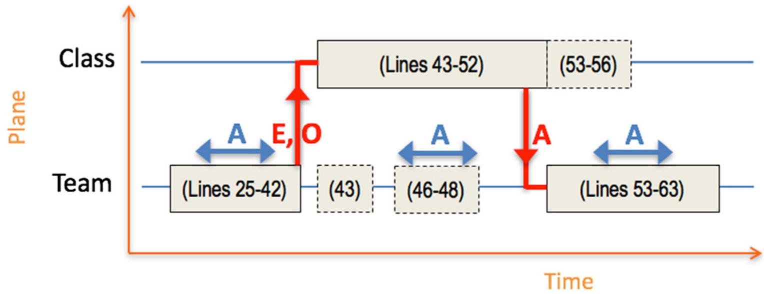

Fig. 7 Orchestration graph showing transitions and overlaps between activity on Team and Class planes in Table 2

to share something David and Anita had done, and Anita responded to the attention by making a joking remark to David about her sole responsibility for their work (line 43). As the teacher went on to ask the class about the consequence of multiplying a quantity by one, Brian, David and Jose all voiced responses (lines 44–48, 51). This sequence follows a well-documented and widespread pattern of classroom discourse known as Initiation-Response-Evaluation (IRE) (Mehan, 1979), in which a teacher poses a question or prompt (lines 44–45), students offer responses (lines 46–48), and the teacher assesses the student contribution(s) (lines 49–50). However, the very same segment also found Brian making a quick self-repair—a similarly well-documented mechanism of small-group conversation (Schegloff et al., 1977)—as he initially replied that the quantity would change (line 46), then quickly adjusted his response (line 48) after hearing and then appropriating that of his groupmate (line 47). This brief exchange illustrates how mechanisms of whole-group discussion and dyadic interaction are hybridized in this segment of classroom network-supported discourse. This hybridity of levels was again in evidence moments later, when, just as the teacher went on to elaborate instructions for using an identity operation to rewrite an expression, Jose and Brian returned to pair-level dialogue and began carrying out those instructions in parallel with the teacher's continuing elaboration of them. Jose's direction to Brian to "get a one" was spoken simultaneously with the teacher's recommendation to "go and find a one," and coupled with a shift in gaze from the teacher and screen to his partner (line 53), as though he shifted mid-utterance from participating in whole- to small-group activity.

The simultaneity of these student and teacher utterances in line 53 also indicates that Jose had already recognized and begun to implement the strategy the teacher would continue to elaborate moments later (lines 54–57). Though Jose had already taken up this approach, Brian remained confused about how they could factor their expression "on the calculator" (line 58)—how they could factor an X out of their current quadratic expression ($75X^2 + 4X$) while simultaneously adhering to the *Terms & Operations* requirement that they incorporate a new term. Jose reiterated that multiplying by one effectively eliminated this constraint, allowing them to draw on prior knowledge about factoring in more conventional Algebra classroom contexts (lines 59–61). In the final remaining minutes of the session, and with scaffolding from Jose, Brian was able to use this approach to rewrite their expression on his calculator in a factored form, $X(75X + 4)$, and voice understanding of the approach (lines 62–63). Thus we see how a strategy that emerged from the activity of one group was introduced by the teacher as a resource at the whole class level, and then successfully incorporated into the work of another group. Moreover, that latter success relied on mechanisms of guided participation and participatory appropriation at two levels—the teacher's uptake and elaboration of Group 8's strategy in the context of a whole-group discussion, which enabled that strategy to be appropriated by Jose, who then in turn guided Brian through his own subsequent appropriation.

Episode two: interactions across levels in *Graphing in Groups*

In this section, I present an episode from the sixth day of activities with the Graphing in Groups environment. This class session began with a teacher-led review of the formula $m = (y_2 - y_1)/(x_2 - x_1)$ for the slope of a line. Students had also participated in activities during prior sessions in which they examined the slope m for each new line they constructed. On this day, the teacher asked student pairs to construct a series of lines with particular characteristics, including one with a slope of three, and then another with a slope of negative one half. The following segment (Table 3) opens as the teacher has just assigned the next such task, namely to make a line with "the biggest slope that you can" (with the constraint that students' points must remain within the 20 by 20 graphing window):

In the period of classroom activity spanned by this table, lasting 36 s, interactions among participants occurred primarily at the level of the small group. After the teacher described the next task (line 1), the three pairs of students on camera initially interacted only within their respective dyads (lines 2–15). Collectively, these three groups exhibited a range of different problem-solving strategies and peer interactions consistent with those observed during other classroom implementations of the *Graphing in Groups* environment (for a detailed analysis, see White, Wallace & Lai, 2012).

Table 3 Overlapping segments of teacher talk and peer dialogue during episode two

Line	Teacher	Group 3	Group 1	Group 4
1	Tobin: Now, I want you to make the biggest slope that you can.			
2		Grace: The biggest slope that we can?		
3		Jamal: Oh, ok, I know how to do this.	Felipe: Uh-oh.	Earl: Hmm? [begins moving right,
4		Grace: You know how to do this?		then down and left toward Ramon's]
5		Jamal: Yeah. Alright, watch.		Ramon: Your biggest slope that we
6		Grace: Where am I going?	Jose: I don't even	can?
7			know.	Earl: Hold on, don't move. [Stops 1
8		Jamal: Alright, hold on, go back, go back.	[Begins moving left]	unit up and 1 right from Ramon]
9		Closer, closer... to the left. Go left...		Don't move.
10		alright, go back. Yeah, right there. Stay		Earl: [Marks to form line with slope
11		right there!	[Jose marks at (0,-10)]	1] The biggest slope...
12		Grace: Right there?	Felipe: Whoa. [Begins	Earl: [Marks 4 units directly above
13		Jamal: Mark it. Yeah.	moving up and right]	Ramon] Mark. [forms Y=N/A]
14		Grace: 7? I think I can make it bigger than	Jose: [Moving his own	Earl: [Moves down 2 units and
15		that. [Moves left and marks at (0,4) to make	point down] Just keep	markes again] It won't let us. [Moves
16	Tobin: Group 3	y=14x+4] Fourteen!	going up.	right and up, marks to form line with
17	has a slope of 14.			slope 6. Continues moving up, then
18	Tobin: That's pretty big.			reaches over to Ramon's calculator
19	Jamal: Yeah.			and begins moving his point down]
20	Grace: Cause we're just that awesome!		[Jose marks at (-10,-	
21	Jamal: I think that's the biggest, cause...		10), begins moving	
22	Tobin: You think that's the biggest you can make? Let's see if		right]	[Earl marks Ramon's point at
23	anybody can make it bigger.			(-2,-10) to from line with slope 12,
24	Tobin: Group 4's got a slope of 12.			begins moving own point up again]

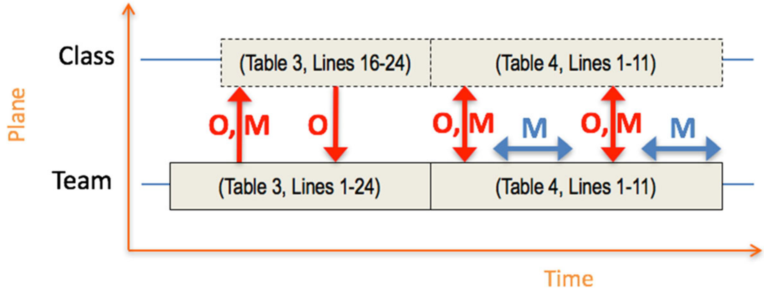

Fig. 8 Orchestration graph showing transitions and overlaps between activity on Team and Class planes in Tables 3 and 4

Orchestration and mediation

Beginning in line 16, the teacher made a series of orchestration moves that bridged this group-level activity with some whole-class level interactions (Fig. 8). When Group 3 constructed a line with a slope of 14 and it appeared in the public display, the teacher announced it to the class (lines 16–17). When the teacher went on to comment that this slope was "pretty big", Jamal and Grace both responded directly (lines 18–20), opening a segment of dialogue with the teacher even as the other groups continued working on their own. When Jamal proposed that their slope was "the biggest, cause…" (line 21), he appeared to be preparing to explain his reasoning. However, the teacher responded before Jamal elaborated further, inviting all groups to try to establish a larger slope (lines 22–23), and then noting the line with slope 12 established by Group 4 (line 24). The moment could be viewed as a missed pedagogical opportunity; encouraging Jamal to explain why 14 might be the largest possible slope may have allowed him to reflect on his own reasoning, or fostered a discussion with his partner and perhaps the rest of the class. But probing Jamal's thinking would have required the teacher to either ignore or interrupt the concurrent work taking place in other groups, and in this moment he opted instead to sustain the current flow of activity.

Perhaps as a result of this orchestration choice, the next sequence of events found the class in a rapid series of interactions between groups, and between the teacher and different groups. In order to illustrate this sequence of events, Table 4 presents the episode in finer grain than previous segments, dividing lines of transcription into co-occurring utterances across groups within one-second increments by row rather than full turns of simultaneous talk. Beginning at time zero with the final comment from the teacher in line 24 of the previous excerpt, the table captures 11 s of dialogue including and immediately following that utterance. Once again, columns are again respectively associated with the teacher and the three student groups on camera. Solid lines between columns demarcate fragments during which pair dialogue remained at the pair level—addressing a groupmate or referring to their respective points or shared line. Instances in which students interacted with members of different groups or the teacher, or referenced other groups' points or lines, are indicated by dotted lines between relevant columns. Arrows drawn from each utterance indicate the group and/or event referenced or responded to by that utterance.

Table 4 Overlapping interactions within and across groups

Time	Teacher	Group 3	Group 1	Group 4
0	T: Group 4's got a slope of 12.	Gr: Let me see, hold on, don't move.		
1	Tobin: Oh,	That ought to be		
2	group 4's got a slope of	a little bit better.		
3	20!			Earl: [punches fist in the air]
4	Tobin: Yeah!	Grace: Really then.		
5			Felipe: Awesome!	
6		Grace: Go over to the,		
7	Tobin: Group 1,	10...		
8	negative 14.			
9				Earl: That's...
10	Tobin: Group 3, negative 14.			
11				the lowest you can get.

As shown in the right side of Fig. 8, this brief sequence hybridized small- and whole-group activity, featuring overlapping orchestration and mediation moves between levels (illustrated by bidirectional vertical arrows) as well as mediated interactions between groups. Apart from Grace's comments to her partner (times 0–2 and 6–7), all other utterances in this segment referenced or responded to other groups. The teacher called out four distinct reports of new slopes established by the groups as they appeared in the collective display (column 3, time 0; 1–3; 7–8; 10). When one of these reports was an announcement that his group had achieved a slope of 20, the highest yet by any pair, Earl punched his fist in the air in celebration (column 5, time 3). This action in turn prompted audible responses from the teacher (column 2, time 4) and from a member of each other group (column 3, time 4; column 4, time 5). And finally, as the teacher reported large negative slopes for Group 1 and then Group 4, Earl observed that those slopes were among "the lowest" rather than the highest values they could achieve.

In most cases, these instances of cross-group activity were initiated by or in response to the teacher, whose play-by-play updates on each group's progress based on observations of the public display sparked chains of reactions from students. In this way, the collocation of each group's graphing window in a single shared display mediated whole-class interactions facilitated by the teacher, which in turn mediated small-group interactions within and between pairs as they worked to construct lines with the required characteristics. In the following segment, we see how mathematical objects in that shared display also mediated direct interactions between groups.

Appropriating and Orchestrating Emergent Mathematical Structure.

Table 5 continues to track Groups 1 and 4 in the moments immediately following Table 4:

Table 5 Successive interactions within and across groups

Time	Teacher	Group 1	Group 4
1		Jose: Go up. Go to the same one, just go a little higher. Same one you had.	Ra: [moves left and marks]
2		Felipe: [moving his point from (7, 9) to (1,9)] Oh like, like right here but higher?	Earl: Just stop moving!
3		Jose: Yeah, right there. There. Wait, hold on, hold on. Yeah right there.	Ramon: [moving back to
4		Felipe: [moving up to (1,10)] Oh, do I mark it?	prior location to again form
5		Jose: Yeah, mark it. [M marks to form y=20x-10] Yeah, you can't get higher than 20.	y=20x+30] Never mind!
6	T: Group 1's got 20.	Ramon: Copycatters! [Jose and Felipe both turn around to look at Ramon]	
7		Jose: You can't even go, you can't go higher.	
8		Ramon: Oh no, never mind. [laughs]	
9		Ramon: [reaches for his calculator, moves point to the right, then back again] Make that straight.	
10		Earl: It won't let us go straight. I've tried.	
11		[reaches for Ramon's calc, moves Ramon's point directly under his own, marks to form y=N/A] It does the NA.	
12		Ramon: Nope, it won't let us. [reaches back to his own calc and moves back to starting point. Then each student	
13		moves his point one unit to the right to form the appearance of a single line connecting with the one belonging to	
14		Group 1 in the graphing window directly above (Figure 10).]	
15		Earl: There. [Moves hand up and down in a vertical plane] There, it's one line.	

On the heels of the segment of interaction across groups immediately above, the students on camera briefly returned to interactions at the pair level. Group 1 opened this segment by discussing where Felipe should position his own point, relative to Jose's at (0,-10), in order to form a line with the steepest possible slope (Group 1 column, lines 1–5). Meanwhile, in Group 4, Ramon moved his point to a different location to replace the slope of 20 Earl had celebrated moments earlier with a lower value (Group 4 column, line 1). Earl objected, and Ramon quickly returned to his original position and restored the previous line (lines 2–5).

Beginning in line 6, as the teacher announced that Group 1 had also formed a line with slope 20 (an orchestration move calling for attention at the class level), interactions within the two groups began to intersect (Fig. 9), as Ramon playfully called out that Group 1 had copied their own line. Jose and Felipe, who were seated directly in front of Earl and Ramon, both turned around in their seats to face the other group, and Jose clarified that both groups had achieved the highest possible slope (lines 6–7). Whether or not Group 4 had literally appropriated Group 1's solution in these moments, as Ramon suggested, is not clear, but any event, these two pairs were now closely attending to one another's graphs and how they were related—suggesting two-way interactions across the two groups sustained by both mediation and appropriation mechanisms. Indeed, Ramon relented (line 8), but then shortly afterward began moving his point directly under Earl's as though seeking to confirm whether they could in fact form an even steeper slope by making "that [line] straight" (line 9). "Straight" in this exchange apparently meant *vertical* for both students, as Earl used the same word while himself placing Ramon's point directly below his own and then pressing "mark" to demonstrate that the software would return y = N/A and not draw a line at all under those conditions (lines 10–11). Even as they acknowledged this limitation, both students moved their points one unit to the right and pressed mark, suddenly forming a new line y = 20X + 10 that appeared to extend upward continuously into Group 1's graph of y = 20X-10 directly above their own in the whole-class public display (Fig. 10 and lines 12–15).

So again showcasing the role of orchestration in multilevel activity, the teacher's broadcasting of one group's line drew attention from students in other groups, first in the form of Ramon's playfully competitive jab and Jose's quick defense (lines 6–8), then in Earl and Ramon's efforts to test Jose's assertion that "you can't go higher" than 20 by attempting to

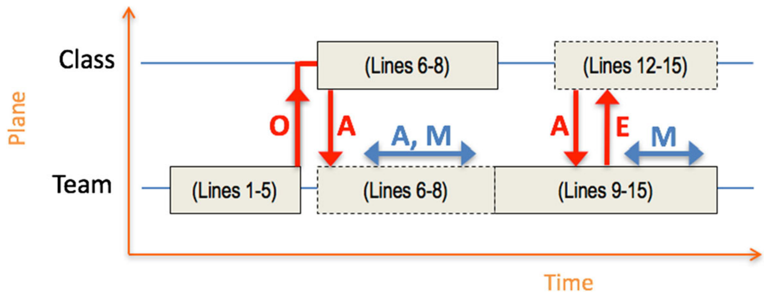

Fig. 9 Orchestration graph showing transitions and overlaps between activity on Team and Class planes in Table 5

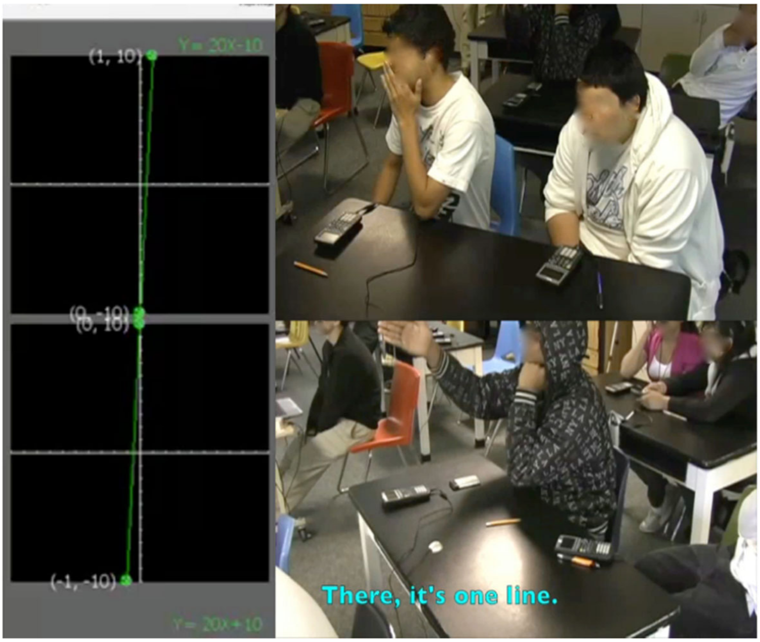

Fig. 10 Group 4 align their points to form the appearance of continuing Group 1's line

form a vertical line, (lines 9–14), and then finally in Earl's suggestion that they had formed a single continuous line between the two groups' graphing windows. This appearance of continuity was an artifact of the display configuration, which played an equally critical role in mediating interactions both within and between groups. As evidenced by their distinct equations, the two lines had the same slope but different intercepts, and they only appeared the same because of the way the two groups' windows were stacked so closely on top of one another in the public display. On the other hand, if one imagined either visible line segment to extend beyond the confines of the window provided, its appearance would match that of the other, so that in effect the two groups' segments did indeed form a single line relative to either one group's axes or the other's. Perhaps unfortunately, the teacher did not overhear this exchange until subsequent review of the video, so the orchestration opportunity to sustain this interaction across groups through discussion of whether the lines were the same was missed. Instead, shortly afterward, Group 3 and others in the room also formed lines with slope 20, and the class moved on to a whole-group discussion about whether and why this was the steepest slope they could achieve given the constraints of a 20×20 graphing window and integer coordinates.

In any event, this effort by Group 4 to explore connections between the graph of their own line and that of Group 1, in a separate graphing window, marks a novel extension of the mapping between social and mathematical structure in the *Graphing in Groups* design. In addition to the mapping at the group level between two collaborating students and the two points that uniquely determine a line, Earl and Ramon effectively leveraged the material properties of the public display to explore an additional, emergent layer of overlapping social and mathematical structure. At the level of whole-class activity, mediated by the public display and supported both by attention to another group's solution (appropriation) and unexpected intersections across graphing windows (emergence), these students established a new mapping between their two interacting student groups and the matching slopes of their respective lines in two different graphing windows.

Extending the mapping of social and mathematical structure from small group to whole class can also be accomplished through orchestration, as illustrated by another brief episode earlier in the same class session. As the groups completed a different task of positioning their points to form lines with slope 3, the teacher had used a *Graphing in Groups* display feature to suggest a similarly extended mapping. When four groups had created lines with slopes of 3, but a fifth group was struggling to complete the same task, the teacher switched the display so that the lines of all the groups (Fig. 11a) were redrawn in a single graphing window (Fig. 11b), prompting one student to comment that the lines were “all parallel”. With this relationship foregrounded, the class went on to collectively discuss what the remaining group needed to do in order to form a line parallel to the others and thus complete the task. In this way, the mathematical relationship—parallelism—between objects defined at the level of the small group became a resource for facilitating a teacher-led whole class discussion that then became a resource for scaffolding problem-solving efforts within a small group.

Discussion

Each of these episodes features segments in which instructional activity in the context of these classroom network tools shifted rapidly, and sometimes repeatedly, between small and whole-group levels of interaction. Importantly, the *Terms & Operations* and *Graphing in Groups* environments were both designed primarily with small group collaboration in mind, and the sessions of class activity examined in this study mainly emphasized tasks to be completed by student pairs. But each of these episodes showcases instances in which activity at the pair level was punctuated by sequences of interaction between multiple groups, or brief segments of teacher-led class discussion. In the first excerpt (Table 1), the teacher interrupted a groupwork session to showcase one group’s polynomial expression, which led to a quick succession of interactions between teacher and students, between students within pairs, and between multiple student pairs. In the second episode (Table 4), students simultaneously attended to completing their small-group task and to assessing the performances of classmates in other groups. Both

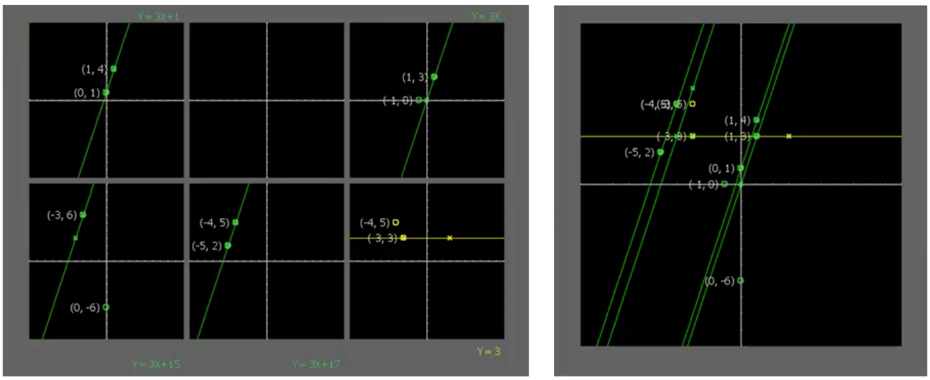

Fig. 11 **a** and **b**. Merging lines from group-level windows into a shared class graph.

segments resist classification at *only* the small-group level, yet neither marks a clear shift to a new, sustained period of activity at the whole class level.

Dillenbourg's orchestration graphs approach offers a useful framework for characterizing these episodes. The above analyses, however, suggests modifications to this approach necessary to represent multilevel activity in this classroom network environment. Indeed, applying orchestration graphs in the above analyses draws attention and gives shape to some distinctive characteristics of multilevel activity in this classroom network environment. Fig. 12a depicts a hypothetical sequence from the teacher's initial setting up of the activity, to a period in which small groups work on the collaborative task, to a period in which the class discusses and compares solutions. While this diagram may be sufficient to illustrate the planned instructional sequence in each episode, it does not adequately capture the array of successive interactions at the whole-class and small group levels depicted in the episodes above. Fig. 12b proposes an alternative illustration of the sequences presented in these episodes, which seeks to better represent the intermingling of intra/inter-group and whole-class interaction on display in each case. This approach acknowledges that segments during which the primary activity was groupwork were sometimes punctuated by brief intervals of whole-class discussion, such as in lines 7–10 of Table 1 or lines 43–52 of Table 2.

Other portions of these same episodes, however, are more difficult to characterize in terms of these rapid shifts between small-group and whole-class levels. For example, in Table 1, even as the teacher continued the thread of a discussion several students had participated in moments before, the students quickly shifted back to interactions across (lines 11–14) and within (lines 15–18) their groups—hence the need for overlapping segments of Team and Class activity in Fig. 6. Similarly, in Table 2, lines 53–56, Group 2 shifted from interaction at the class plane to the small group plane mid-sentence, and even as the teacher finished illustrating a point to others in the class. And in Table 4 and 5, teacher-facilitated whole-class interaction as well as exchanges within and between student pairs are all simultaneously on display even over this very brief segment. Fig. 12c offers a second alternative to representing the interplay between levels in these episodes, which generalizes the results of the orchestration graphing analyses shown in Figs. 6-9, in which relations between small- and whole-group activity are sometimes represented as simultaneous rather than sequential. In other words, some phases of classroom activity supported

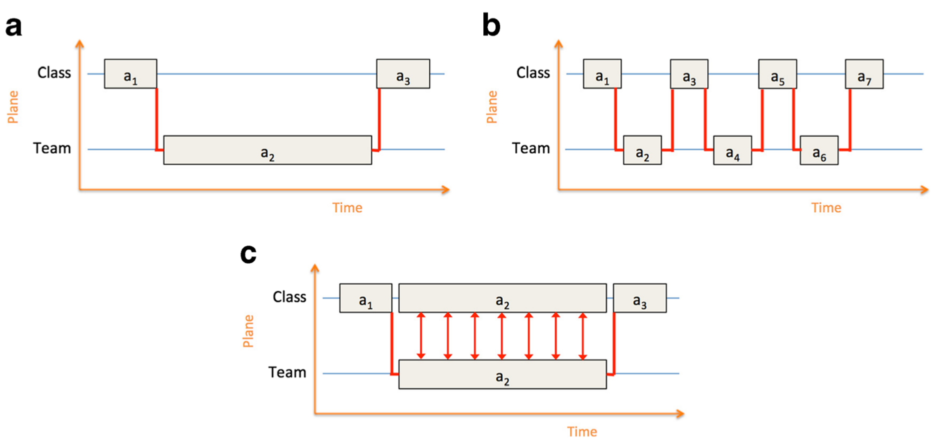

Fig. 12 **a** and **b**. Orchestration Graphs showcasing less and more frequent shifts between Team and Class levels. **c**. Orchestration Graph illustrating hybrid activity spanning Team and Class planes.

by these classroom network tools may best be characterized not simply as occurring on one plane or the other, but rather in terms of hybridity across multiple levels of interaction.

These intersections between small-group and whole-class activity sometimes hinged on novel solution strategies and mathematical relationships that emerged from students' and groups' explorations of these virtual objects and environments. In Episode One, the unexpected appearance of a quadratic expression during a group activity focused on linear terms occasioned both a class discussion and a new task for groups. And an incidental choice during Group 8's problem solving emerged as an important resource for other groups. In Episode Two, students exploited material features of the graphical display to extend a group-level graphing task into an emergent exploration of relations across multiple groups' graphs. In each case, dynamic mathematical objects in a shared virtual space opened up unscripted opportunities to consider mathematical relationships as well as fluid interactions across individual, small-group and whole class planes.

These shifts between and hybridization across levels were both facilitated by orchestration moves on the part of the teacher. Sometimes, the teacher clearly signaled the opening of a new task for students to undertake at the group level, as in line 1 of Table 3, or the initiation of a class discussion, as in line 43 of Table 2. Other times, these orchestration moves contributed to the superposition of levels, as in Table 1 when the teacher alternately addressed Group 6 (lines 1, 8, 10) and the whole class (lines 7, 8), prompting interactions both within and between groups as well as class-level utterances, and in Table 4 when the teacher called the attention of the whole class to the successive constructions of different groups.

In each case, these orchestrations were mediated by small-group-level mathematical objects in the public display through the teacher's explicit efforts to call attention to the group expressions in the *Terms & Operations* environment and the equations of the lines in *Graphing in Groups*. Likewise, students actively attended to and made comparisons across these publicly displayed group-level objects as they participated in both team tasks and class-wide discussions. In other words, the salience of objects that were associated with specific student groups appear to have contributed to the overlap of activity on the whole class and small group planes.

Students' and the teacher's attention to these group-level mathematical objects sometimes led to instances of appropriation that supported transactions between levels of activity. In the first episode, one student took up a practice (attending to mathematical objects in the public display) introduced by the teacher in a whole-class segment, and appropriated it to initiate interactions both with another group and with his partner. Later, the teacher used another whole-class segment to introduce a strategy from one group that other groups and students within groups then took up, marking a quick sequence of shifts between class, group and individual activity. And in the second episode, one group's accusation that another had appropriated their solution initiated a segment of between-group interaction in which they compared functions and explored constraints of the graphing window.

Collectively, these mechanisms of emergence, orchestration, mediation and appropriation each contribute to the flexible reconfiguration and intermingling of levels of activity supported by these classroom network designs. The layering of dynamic math representations over multiple interactional planes creates conditions of classroom complexity in which novel and important mathematical artifacts sometimes *emerge*. Such occasions mark opportunities for teacher *orchestrations* that leverage these emergent artifacts in ways that draw more classroom participants into these moments of mathematical insight. Key orchestration moves appear to include identifying and drawing attention at the whole-group level to mathematical representations that *mediate* the production of the new artifacts, and supporting processes of *appropriation* through which individuals or small groups might make that meditational means their own as they likewise produce novel artifacts.

The finding that classroom levels in these episodes were overlapping and simultaneous, rather than successive and distinct as instructional design might suggest, is consistent with the characterization of nested planes or layers of activity anticipated in Rogoff's (1995) and Ludvigsen and Arnseth's (2017) sociocultural accounts. However, prior accounts linking levels of activity in mathematics classrooms have typically described processes through which interactions among individual participants lead to the establishment of collective practices over relatively long timescales, typically multiple days of instruction (Cobb, 1999; Enyedy, 2005; Saxe, 2002). The analysis presented here suggests complementary, micro-dynamic processes through which levels of instructional activity can sometimes intersect, over much briefer intervals of classroom interaction.

In a similar vein, Stahl (2013) has suggested that regular and rapid movement between planes might be fundamentally characteristic of collective mathematics activity:

> Uses of math resources—such as manipulating visual representations, referencing recent findings, expressing relationships symbolically—move fluidly between individual perceptual behavior, group problem-solving sequences and the cultural stockpile of mathematical knowledge. Perhaps the incessant traversal of levels is particularly visible in collaborative math discourse because of its explicit use of multiple layers of reality: a physical drawing, the intended figure, a narrative description, a symbolic expression, the conceptualization, the mathematical object (p. 4).

The analyses presented above suggest that this fluidity and hybridity of levels might be further reinforced when mathematical representations are integrated with infrastructures for classroom communication. Throughout each segment, the students repeatedly shifted attention between their own mathematical objects and those shared with their classmates, and between multiple representations of those objects variously featured in private and public displays. Likewise, the teacher continually monitored dynamic objects associated with each small group, drawing comparisons between them and using them as resources to facilitate both communication with groups and whole-class discussion. The multiple layers of this classroom reality—mathematical objects, dynamic representations, digital devices, interconnected displays—were each also mapped to social actors at multiple scales of collective participation. The resulting dialectics of multiple mathematical and social structures point to novel and emerging opportunities for classroom interaction and learning activity, characterized by hybrid and dynamic rather than discrete planes of activity and pedagogical modes.

While layered and dynamic mathematical representations, and CSCL systems with intentionally designed small- and whole group elements like the one described in this paper, might make this hybridity particularly salient, the boundaries between levels of activity in real collaborative classroom settings may always be blurrier than instructional design anticipates. Indeed, this observation evokes Lave & Wenger's (1991) distinction between a *teaching curriculum,* devised with specific instructional intent, and a *learning curriculum* comprising "a field of learning resources in everyday practice *viewed from the perspective of learners*" (p. 96, emphasis in original). From this latter perspective, which takes learning environments to be "situated opportunities…for the improvisational development of new practice" (p. 96), the orchestration moves of teachers and the mediating role of designed artifacts will always be complemented (and often disrupted, reframed or transformed) by the appropriation moves of learners and the emergent dynamics of complex classroom systems. Indeed, from the standpoint of learners, levels of classroom activity may resemble other institutional norms and practices that students must navigate even as they engage in conceptual sensemaking (Furberg et al., 2013). Along these lines, Damsa and Jornet (2016) have argued that when viewed from a learner's perspective, seemingly distinct levels and contexts of learning activity in fact form continuous ecology. Indeed, particularly in the increasingly digital modern classroom, levels of learning activity might be best understood in ecological terms—not as bounded instructional segments, but as overlapping layers of activity in a complex and shifting landscape of contemporary learning environments (Säljö, 2010).

Conclusion

This paper offers both theoretical and methodological resources for examining the ways CSCL systems intersect with different modes of instruction and levels of activity in a classroom ecosystem. Classrooms routinely find learners engaged in individual, small- and whole-group forms of interaction. While sociocultural theory provides a broad framework for conceptualizing interrelations among these different scales of activity, the particular mechanisms through which these levels might intersect in the real-time flow of classroom instruction, and in the context of digital tools for mathematical representation and peer communication, need further elaboration.

The episodes examined above offer a view of the ways classroom CSCL tools can equip both students and teachers with an array of conceptual and interactional resources for shared mathematical work. They also show that students' construction of shared mathematical objects and the teacher's attention to those group-level objects as they appeared in the collective display led to novel solutions and dynamic resources for blending small group work and whole-class discussion. The results presented in this paper demonstrate the utility of an analytic framework that integrates the mechanisms of appropriation, mediation, emergence and orchestration in the examination of classroom activity across multiple levels.

Of course, the affordances of these CSCL tools for supporting multiple and potentially overlapping levels of classroom activity also face teachers with new choices regarding when, whether and how to attend to, transition between, or merge these levels. Just as students' personal devices in classrooms simultaneously represent the potential risk of distraction and reward of novel forms of interaction, designs for device networks like the ones presented in this paper pose both opportunities and challenges for orchestration amid growing classroom complexity. This study offers an initial exploration of the possibilities and mechanisms for supporting learning activity with these tools; the different ways other teachers might take up these learning environment designs and manage the real-time flow of information and the orchestration resources they provide would be an important area for further investigation.

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