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REGULAR ARTICLE

Dynamic effects of social influence on asset prices

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Abstract

This paper examines the dynamic effects of *Social Influence* on asset prices in the presence of heterogeneous expectations among investors. In our model, the choices of investors' trading strategies are influenced not only by past payoffs but also by their neighbors' choices in the social network. To obtain tractable results with generic implications for social structure, we use a mean-field approximation approach rather than specifying the exact structure of social network. Analytical conditions for the existence and local stability of equilibria of price dynamics are established and validated through numerical simulations. Our analysis shows that social influence increases the dimension of the dynamical system and that equilibria can only be expressed implicitly as solutions of certain equations. We also investigate the long-run behavior of price and fraction of trading strategies using numerical simulation under a scale-free network and a power function type social influence factor. Our results suggest that the system tends to be stable when social influence is small but exhibit complex periodic orbits and even chaos when social influence is large. These findings yield valuable insights into the role of social influence in financial markets.

Keywords Social influence · Asset pricing dynamics · Heterogeneous expectation · Mean-field approximation · Agent-based model

JEL classification G12 · G41

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1 Introduction

In the conventional rational asset pricing models (such as Sharpe 1964 and Ross 2013), the capital market is assumed to be efficient. Asset prices reflect all available information and fluctuate around a general equilibrium (Fama 1970). Investors are rational and they incorporate all available information into their investment decisions. While these conventional models have been proven to be useful in predicting asset prices in regular time periods, real-world capital markets often exhibit excess volatility and returns that such models are unable to account for (Shiller 1992). This led to the development of alternative theories, such as behavioral finance, which attempt to explain the irrational behavior of investors that can lead to bubbles and crashes in financial markets. Shiller (1984) and West (1988) have found early empirical evidences that support the use of behavior model in the financial market as early as the 1980s.

A large number of nonlinear dynamic asset pricing models have been developed ever since to incorporate behavior elements into the financial market. They emphasize the role of heterogeneous expectations in leading to the complexity of asset price dynamics. Some examples include De Long et al. (1990); Day and Huang (1990); Chiarella (1992); Kirman (1993); Lux (1995); Chiarella and He (2003); Chiarella et al. (2009), and zhu et al. (2009), among others. The literature that is most relevant to our work are Brock and Hommes (1997, 1998). They explored the possible routes that lead to the complex behavior of asset prices analytically and numerically, where chaotic attractors are found for a wide range of parameters. Nevertheless, these models focus on whether "irrational" traders could survive in the market but do not provide a "real" framework to capture how investors switch between expectations. In Brock and Hommes (1998)' framework, investors choose from a finite set of heterogeneous expectations based on a simple discrete choice probability model.[1] It assumes that agents make decisions individually and information is freely accessible to everyone. This is far from reality where investors are influenced by their peers and information is not free. In fact, the financial market is well known for its information asymmetry. Less informed investors may heavily rely on their social network to obtain information and make decisions.

The way this paper contributes to this body of literature is to bring in ideas from social network literature and introduce *social influence* elements into a heterogeneous agent-based asset pricing model. The heterogenous expectations are defined as in Brock and Hommes (1998) and Chiarella and He (2003). The way in which interacting investors update their expectations is determined by both the social influence and the characteristics of investors' expectation types. In other words, investors are not only influenced by their beliefs but also by the beliefs of others in their social network. By incorporating social influence elements into the model, it offers a more realistic representation of how financial markets operate. It attempts to bridge the gap between the asset pricing literature and the social network literature, highlighting the importance of social influence in financial markets.

[1] Anderson et al. (1992); Manski and McFadden (1981) provide extensive information on discrete choice modelling.

Shiller (1984,1988) can be seen as the earliest literature that challenge the efficient market hypothesis and bring the concept of social influence into finance research under the name of fashions, or fads. However, at that time, the model of social influence was rarely explicit. Many other early studies tend to represent social influence as the herding behavior of investors. That is, the decision of an investor is influenced by the fractions of different types of investors in the market. This line of literature includes Lux (1995, 1998); Diks and van der Weide (2005); Satchell and Yang (2007); Chang (2007); Alfarano et al. (2008); Lux (2009); Franke and Westerhoff (2012); Di Guilm et al. (2014), among others. In particular, in Lux (1995) and its followers, the probabilities that investors change their types from one to another are explicitly formulated, where the social influence appears as a single variable which affects the transition probabilities.

Modelling social influence with exact network structures and full information on local interactions is challenging since the exact social structure makes the model intractable quickly when the population becomes large. It is difficult to obtain analytical results unless the communication network exhibits very special characteristics (e.g. regular networks or stars, as in Panchenko et al. (2013)). For example, Alfarano and Milakovic (2009) provides a microscopic interpretation of the herding factor in Lux (1995) transition probability approach using an exact social network structure. It brings more information to the theory but at the same time, it causes a heavy burden to analysis and evaluation. Therefore, the authors employ a mean-field approximation approach in their general analysis and only discuss several typical network structures as special cases. Yang (2009) incorporates social networks into opinion formation in the study of price stability. The forecast of the future price of an investor is influenced by the opinion of neighbors in the social network, where the way of influence is captured by a weight matrix. Since the exact network structure is itself difficult to work with, these papers only consider a very simple framework, such as a single-asset market, simple price formation mechanism, and homogeneous investors. Similarily, Panchenko et al. (2013) analyse local interactions in a two-type heterogeneous expectation model and investigate four commonly considered network topologies: a fully connected network, a regular lattice, a small world, and a random graph. They find that these network structures affect the stability of asset price dynamics and the amplitude of fluctuations and statistical properties. Makarewicz (2017) puts local peer effects into price expectation and studies the emergence of contrarian strategy with agent-based simulation using small and large networks.

In those studies, when exact social structure is used, most practical implications will be restrictive to that particular structure and therefore, very little can say about generality. To overcome this weakness, in this paper, we employ a mean-field approximation to embed the social network structure through a function of individual degrees into the discrete choice model. The degree is the number of neighbors that an individual has in their social network, which is used to measure to what extent a person can be influenced by others around her. By focusing on the degree and employing mean-field approximation, we are able to derive general analytical results with much richer structural settings compared to exact social structure, but still maintain the tractability of the model.

We follow the line of research on the discrete choice of strategies with social interactions started by Blume (1993) and Brock (1993), and followed by Brock and Durlauf (2001), Brock and Durlauf (2002). In these models, the individual utility of choosing a strategy is expressed by the sum of the individual's private utility determined by the choice of the individual, the social utility determined by the choices of all individuals in the society, and an idiosyncratic random utility. A basic functional form for social utility considers that the quantity is proportional to the sum of the expected utilities of the individual given the choices of others. This formulation leads to a discrete choice probability. This approach carries the idea that individuals are influenced by the information aggregated from the whole society. The ability of individuals to reach information, as well as their intensity of choice, is assumed to be identical among all individuals. In our model, to utilise mean field approximation, we keep the assumption that each individual has the same ability to gather information but assume that the intensity of choice may differ with different positions in the social network. The characteristic of an individuals' network position is the degree, which is a measure of sociability partially reflects what extent a person can be influenced by others around her. The modelling of heterogeneous expectations formation is in line with Chiarella and He (2003), with two classes of expectations, fundamentalists and trend chasers, and a market maker.

The remainder paper is organised as follows. An asset pricing model with heterogeneous agents and social influence are developed in Sect. 2. The equilibrium solution is obtained in Sect. 3. Section 4 studies the dynamic behavior of the system, and Sect. 5 provides numerical simulations. We conclude the paper in Sect. 6. All the proofs are collected in Appendix.

2 The model

We build our model on a standard heterogeneous agent asset pricing model in discrete time based on Chiarella and He (2003). There are three classes of participants in the market: investors with heterogenous expectations, the fundamentalists and the trend chasers,[2] and a market maker. The investors are assumed to be boundedly rational. Intuitively, the fundamentalists make forecasts on future fundamentals only using the information that they privately observed, while the trend chasers rely on simple measures about price changes in the past but not on fundamentals. The market maker acts as both a liquidity provider and an active investor in a market.[3]

Additionally, investors are influenced by their social connections in the decision of switching strategies, i.e., from a fundamentalist to a trend chaser or vice versa. Social connections are aggregatively characterised by a degree distribution where the degree of an agent represents the number of agents to whom she is connected. The magnitude of influence depends on the degree distribution as well as the degree itself. This is a

[2] Hong and Stein (1999) consider similar settings on two groups of agents - newswatchers (fundamentalists) and momentum traders (trend chasers).

[3] See Chiarella and He (2003), Farmer and Joshi (2002), also refer to Zhu et al. (2009) for the influence of market maker to the stability of market.

mean-field approximation to the model with exact social network structures. [4] In the following we describe the building blocks of the model in detail.

2.1 Market price under a market maker scenario

Suppose that there are two assets in the market, one risky asset whose supply is fixed and one risk-free asset whose supply is perfectly elastic. Let the gross return of risk-free asset be $R = 1 + r > 1$ for each period, where r is the risk-free rate. Denote p_t the (ex-dividend) price per share of the risky asset at period t, and $\{y_t\}$ the stochastic dividend process of the risky asset. Let W_t be the wealth of a typical investor at period t, then

$$W_{t+1} = RW_t + \big(p_{t+1} + y_{t+1} - Rp_t\big)z_t, \tag{1}$$

where z_t is the number of shares of the risky asset purchased by the investor at period t.

The information set at t is denoted by $F_t = \{p_t, p_{t-1}, \ldots; y_t, y_{t-1}, \ldots\}$. Let E_t and V_t be the conditional expectation and conditional variance functions, respectively, based on F_t. Let $h \in \{1, 2\}$ denote the type of investors, where $h = 1$ indicates a fundamentalist and $h = 2$ represents a trend chaser. We use $E_{h,t}$ and $V_{h,t}$ to express the beliefs of type h investors at period t about the conditional expectation and conditional variance, respectively, of a random variable at period $t + 1$. Denote R_{t+1} the excess return at $t + 1$, that is

$$R_{t+1} = p_{t+1} + y_{t+1} - Rp_t. \tag{2}$$

Then, one has

$$E_{h,t}(W_{t+1}) = RW_t + E_{h,t}(R_{t+1})z_{h,t}, \tag{3}$$

$$V_{h,t}(W_{t+1}) = RW_t + V_{h,t}(R_{t+1})z_{h,t}, \tag{4}$$

where $z_{h,t}$ is the number of shares of the risky asset purchased by type h investors.

Both types of investors are assumed to be expected-utility maximisers, i.e., assets with high expected return and low expected risk are preferred. However, different types of investors have different risk attitudes. The risk attitude of type h investor is characterised by the risk-aversion coefficient a_h. Then, the demand of risky asset $z_{h,t}$ for type h investors is given by

$$z_{h,t} = \frac{E_{h,t}[R_{t+1}]}{a_h V_{h,t}[R_{t+1}]}. \tag{5}$$

Let $n_{h,t}$ be the fraction of type h investors at period t so that $\sum_h n_{h,t} = 1$. In the subsequent subsections we will specify a way of approximating these fractions using

[4] See, among other, Jackson and Yariv (2005), Jackson (2007), Bramoullé et al. (2012), Huang et al. (2016), Giovannetti (2021), etc.

a modification of the discrete choice model with social influence. By assuming a zero supply of outside shares, the excess demand for the risky asset is given by

$$z_{e,t} \equiv \sum_h n_{h,t} \cdot z_{h,t} = \sum_h n_{h,t} \cdot \frac{E_{h,t}[R_{t+1}]}{a_h V_{h,t}[R_{t+1}]}. \tag{6}$$

The market maker takes a long position when the excess demand of the risky asset is negative ($z_{e,t} < 0$), and a short position when it is positive ($z_{e,t} > 0$), in order to clear the market. Furthermore, at the end of period t, the marker maker sets the price for the next period that moves in the direction of reducing the excess demand. By considering a simple linear price adjustment function, the price in period $t + 1$ is given by

$$p_{t+1} = p_t + \mu z_{e,t} = p_t + \mu \sum_h n_{h,t} \cdot z_{h,t}, \tag{7}$$

where μ is the market friction coefficient describing the speed of price adjustment of the market maker. When $\mu = 0$, $p_{t+1} = p_t$, the market maker does not adjust actively, while when $\mu > 0$, the market maker adjusts the price in response to excess demand and clears the market. Here we use the same setting as in Chiarella and He (2003) to maintain simplicity since our main focus is on social influence.[5]

2.2 Heterogeneous beliefs

The fundamental solution of price p_t^* under rational expectation satisfies the equation

$$R p_t^* = E_t[p_{t+1}^* + y_{t+1}]. \tag{8}$$

Let the dividend process $\{y_t\}$ be i.i.d., then $E_t[y_{t+1}] = \bar{y}$ is constant. Under the no-bubble condition $\lim_{t\to\infty} E_{t-1} p_t / R = 0$, Equation (8) has a unique solution $p^* = \bar{y}/r$.[6]

The conditional expectations and conditional variances of the two types of investors are given by

$$E_{1,t}[p_{t+1} + y_{t+1}] = E_t[p_{t+1}^* + y_{t+1}], \tag{9}$$

$$E_{2,t}[p_{t+1} + y_{t+1}] = E_t[p_{t+1}^* + y_{t+1}] + \theta f_t, \tag{10}$$

and

$$V_{1,t}[p_{t+1} + y_{t+1}] = V_{2,t}[p_{t+1} + y_{t+1}] = \sigma^2, \tag{11}$$

where $\theta > 0$ and σ^2 are constant. Equation (9) describes that the fundamentalists believe the prices will move towards their fundamental values, while Equation (10)

[5] The micro-foundations of the coefficient μ could be explored in future research.

[6] See Hommes (2001) for further discussions about rational expectation fundamental price under various conditions.

shows that the trend chasers expect that the prices move away from the fundamental values by the deviation of weighted moving-average prices f_t multiplied by the extrapolation rate θ. It is usually considered that the risk attitudes of the two types of investors are different. As pointed out by Campbell and Kyle (1993), fundamentalists are considered to be more risk-averse than trend chasers. It follows that we set the risk-aversion coefficient of fundamentalist higher than the one of trend chasers, i.e., $a_1 > a_2 > 0$.

We consider a geometric decay process (GDP) for the extrapolating term f_t:

$$f_t = b \sum_{i=0}^{L-1} \omega^i p_{t-i} - p_t^*, \tag{12}$$

where b measures the decay rate of the memory, with $b = 1/\sum_{i=0}^{L-1} \omega^i$, $\omega \in [0, 1]$, and $0^0 = 1$ for convenience.

2.3 Adaptive belief and social influence

Let $x_t = p_t - p_t^*$ denotes the deviation of price p_t from the fundamental price p_t^*. Then, the excess return defined in (2) can be rewritten as

$$R_{t+1} = x_{t+1} - Rx_t + \delta_{t+1}, \tag{13}$$

where

$$\delta_{t+1} = p_{t+1}^* + y_{t+1} - E_t[p_{t+1}^* + y_{t+1}]. \tag{14}$$

$\{\delta_{t+1}\}$ is a martingale difference sequence with respect to F_t, i.e., $E_t[\delta_{t+1}] = 0$ for all t. One then has

$$E_{1,t}[R_{t+1}] = -Rx_t, \quad E_{2,t}[R_{t+1}] = \theta g_t - Rx_t, \tag{15}$$

where

$$g_t = b \sum_{i=0}^{L-1} \omega^i x_{t-i}, \quad b = \frac{1}{\sum_{i=0}^{L-1} \omega^i}, \quad \omega \in [0, 1]. \tag{16}$$

Let $\pi_{h,t}$ be the realized profits of investor of type h at period t. Therefore,

$$\pi_{h,t} = R_t \cdot z_{h,t-1} = \big(x_t - Rx_{t-1} + \delta_t\big) z_{h,t-1}, \tag{17}$$

with

$$z_{1,t} = \frac{-Rx_t}{a_1 \sigma^2}, \quad z_{2,t} = \frac{\theta g_t - Rx_t}{a_2 \sigma^2}. \tag{18}$$

In the Brock and Hommes (1997) model, agents switch their types (trading strategies) in each period according to discrete choice probabilities. That is to say, although the beliefs are adaptive, agents make their decisions independently and are not influenced by the information from their connections, such as friends, families or colleagues. However, both theoretical and empirical studies suggest that communication connection is a very important piece in investors' decision-making process (Shiller and Pound 1989; Arnswald 2001; Özsöylev 2005; Özsöylev and Walden 2011; Steiger and Pelster 2020). Therefore, we extend the discrete choice probabilities by introducing social influence into the choice of strategies. We assume that the investors are connected so that they form a social network to receive information. In practice, it is intractable to model the whole exact network when the number of agents is large. Instead, as proposed by Jackson and Yariv (2005), a mean-field approximation approach can be used to express the average behavior of investors where only the degree distribution of the social connections is used. This approximation makes sense if the social network is randomly re-formed in each period rather than fixed over time.[7] Indeed, our social connections are evolving all the time in the sense that new friends are made and old ones are alienated if not contacted for a long time. Formally, let $P(d)$ denote the fraction of investors with degree $d \in \{0, 1, 2, \dots\}$, where the degree of an investor is the number of neighbors she has in the network. The degree of an investor (and therefore the degree distribution) is fixed for each period but to whom an investor will connect can be random. It is known that a random neighbor of some investor (either randomly chosen or not) has a probability $\tilde{P}(d) = P(d)d/\overline{d}$ to have degree d, where $\overline{d} = \sum_d P(d)d$ is the average degree of investors in the network (see Newman, 2010, Chapter 13 for details). Let $n_{h,t}^d$ be the fraction of type h investors within the investors of degree d at period t, and define

$$\tilde{n}_{h,t} = \sum_d \tilde{P}(d) \cdot n_{h,t}^d \tag{19}$$

to be the link-weighted average of the fraction of type h investors. $\tilde{n}_{h,t}$ is then the fraction of type h investors that an investor is expected to connect.

Here we assume that $n_{h,t}^d$ is updated according to the following discrete choice probability

$$n_{h,t}^d = \frac{\exp\left[g(d)\tilde{n}_{h,t-1}\left(\pi_{h,t} - C_h\right)\right]}{Z_t^d}, \quad Z_t^d = \sum_h \exp\left[g(d)\tilde{n}_{h,t-1}\left(\pi_{h,t} - C_h\right)\right] \tag{20}$$

where C_h is the cost of type h investors. It is assumed that the cost of fundamentalists is higher than the cost of trend chasers due to the need to collect more information. Furthermore, without loss of generality, we set $C_1 = C > 0$ and $C_2 = 0$. The function

[7] In the same spirit, Giovannetti (2021) employs mean-field approximation in a dynamic model that explains the formation of internal capital markets (a type of decentralized market taking place within business groups). They assume that the participating firms match randomly to exchange loans obtained from an institutional investor. The mean-field approximation is used in the analysis of the distribution of inter-firm loans.

$g(d)$ measures the intensity of choice for investors with degree d. In Sect. 5 we follow Jackson and Yariv (2005) to consider a case $g(d) = \alpha d^{\beta}$ with $\alpha > 0$, but now we just leave it as a general non-negative function of d with $g(0) = 0$. Remark that the value of β in $g(d)$ controls the effect of degree on the speed of switching. When $\beta > 0$, investors with higher degrees have a faster speed of switching, and when $\beta < 0$, investors with higher degrees switch slower. A larger value of $g(d)$ indicates a greater tendency towards switching to the strategy with the highest net profit. The more the value of $g(d)$ is close to zero, the more the investors choose strategies randomly.

Investors are influenced by each other. If one has more connections with a particular type of investor, she is more incline to take a similar strategy of that type. Hence, the switching probability is also affected by $\tilde{n}_{h,t}$ as appeared in the exponential functions in (20). If an investor is isolated from the others, i.e., $d = 0$, equation (20) implies that this investor randomly chooses a strategy. This coincides with our intuition that the isolated investor is unable to collect information from others and therefore has no idea about the differences between strategies.

Note that the update of the fractions of types h for degree d investors ($n^d_{h,t}$) uses the information of the approximated fractions of types of all investors in the previous period ($\tilde{n}_{h,t-1}$). This is fundamentally different from Chiarella and He (2003), in which the fractions are only based on the information of the same period. In other words, even if we let $g(d)$ be constant over degrees, our model will not be simplified to that of Chiarella and He (2003). Furthermore, we assume that $n_{h,t}$ is approximated by $\tilde{n}_{h,t}$ which is based on local information that each agent can obtain.

2.4 The dynamic system of price and fractions of investors

The adaptive belief system under social influence is given by

$$\begin{cases} x_{t+1} = x_t + \mu \sum_h \tilde{n}_{h,t} \cdot z_{h,t}, \\ \tilde{n}_{h,t} = \sum_d \tilde{P}(d) \cdot \dfrac{\exp\left[g(d)\tilde{n}_{h,t-1}\left(\pi_{h,t} - C_h\right)\right]}{Z_t^d}, \end{cases} \tag{21}$$

where

$$\begin{cases} Z_t^d = \sum_h \exp\left[g(d)\tilde{n}_{h,t-1}\left(\pi_{h,t} - C_h\right)\right], \\ \pi_{h,t} = \left(x_t - Rx_{t-1} + \delta_t\right) z_{h,t-1}, \\ z_{1,t} = (-Rx_t)/(a_1\sigma^2), \\ z_{2,t} = (\theta g_t - Rx_t)/(a_2\sigma^2). \end{cases} \tag{22}$$

By defining $m_t = \tilde{n}_{1,t} - \tilde{n}_{2,t}$, $\tilde{n}_{1,t} = (1 + m_t)/2$ and $\tilde{n}_{2,t} = (1 - m_t)/2$, system (21) can be rewritten as

$$\begin{cases} x_{t+1} = x_t + \dfrac{\mu}{2}\big[(1+m_t)z_{1,t} + (1-m_t)z_{2,t}\big], \\ m_t = \sum_d \tilde{P}(d) \cdot \tanh\Big[\dfrac{g(d)}{4}(A_t \cdot m_{t-1} + B_t)\Big], \end{cases} \tag{23}$$

where

$$\begin{cases} A_t = (x_t - Rx_{t-1} + \delta_t)(z_{1,t-1} + z_{2,t-1}) - C, \\ B_t = (x_t - Rx_{t-1} + \delta_t)(z_{1,t-1} - z_{2,t-1}) - C. \end{cases} \tag{24}$$

System (23) is a two-dimensional nonlinear stochastic system of (x, m) because of the random term δ_t. It becomes a deterministic system of order $L + 1$ when $\delta_t = 0$ for all t, which is called the deterministic skeleton of (23). In the next section, we study the equilibrium solutions of this deterministic skeleton.

3 Equilibrium solutions

In this section, we consider the equilibrium state of the deterministic skeleton of the system (23). We first obtain a general result about the existence of equilibria and then discuss some special cases of social influence structures. Define

$$a = \frac{a_2}{a_1}, \quad m^{\#} = \frac{\theta - (1+a)R}{\theta - (1-a)R}, \tag{25}$$

$$M = \sum_d \tilde{P}(d) \cdot \tanh\Big[-\frac{g(d)}{2} \cdot \frac{(\theta - R)C}{\theta - (1-a)R}\Big]. \tag{26}$$

Proposition 3.1 *Assume $\delta_t = 0$ for all t. Let $x^{\#}$ be the positive solution (if exists) of*

$$m^{\#} = \sum_d \tilde{P}(d) \cdot \tanh\Big[\frac{g(d)}{4} \cdot (m^{\#} + 1) \cdot \Big(\frac{2aR(R-1)}{a_2\sigma^2} \cdot x^2 - C\Big)\Big], \tag{27}$$

and m^{eq} be the unique solution of

$$m^{eq} = \sum_d \tilde{P}(d) \cdot \tanh\Big[-\frac{g(d)C}{4}(m+1)\Big]. \tag{28}$$

Then, the deterministic skeleton of (x_t, m_t) in system (23) always has a fundamental equilibrium $(0, m^{eq})$ with $-1 < m^{eq} < 0$. Furthermore, when one of the following conditions holds:

C1. $R < \theta < (1+a)R$ and $m^{\#} > M$,
C2. $\theta \geq (1+a)R$,

system (23) has two other non-fundamental equilibria $(\pm x^{\#}, m^{\#})$.

Proposition 3.1 are consistent with the results of Chiarella and He (2003) but the two differ in detail. System (23) is a real two-dimensional system of (x, m) whereas the model in Chiarella and He (2003) is essentially a one-dimensional one in which m only depends on the values of x up to the current period but not on the previous values of its own. In particular, m^{eq} defined in (28), see Proposition 3.1, cannot be expressed explicitly, and the conditions of having non-fundamental equilibria are slightly more complicated. This difference remains even if when $g(d)$ is constant, i.e., the intensities of choice of investors are not degree dependent. Nevertheless, it follows the same intuition as in Chiarella and He (2003) that the fundamental equilibrium is stable when the extrapolation rate θ is small enough and becomes unstable otherwise. Analysis in Section 4 provides more results on the local stability of the fundamental equilibrium.

The effect of the network connection on the equilibrium analysis is involved in both the expression and the condition of equilibria. Compared with Chiarella and He (2003), one can no longer solve for m^{eq} and $x^{\#}$ analytically because of the weighted averaging of nonlinear terms associated with different degrees, while this averaging was not necessary for Chiarella and He (2003). This could be seen as the price of enriching the structure of the problem. However, it still can be said that, when the network has a relatively uniform structure, i.e., that $g(d)$ does not differ so much for different values of d, it is easier to check the conditions and to find the equilibria. The following special case gives a clear description of this point.

Regular networks

By letting all agents have the same number of neighbors, we are able to simplify the expressions of our model to characterise equilibrium solutions. In the social network literature, this is the case of regular networks, which contains a special case that everyone is connected to everyone, i.e., a complete network, and simply means that all agents are identical with respect to social influence. More precisely, let the set of degrees be $\{k\}$ where k is a constant such that $0 \leq k \leq N-1$, and N denotes the number of agents. Then, one has $P(d) = \tilde{P}(d) = 1$ for $d = k$ and $P(d) = \tilde{P}(d) = 0$ for $d \neq k$. Consequently, $g(d) = g(k) \equiv D$ where D is some non-negative constant, and $\tilde{n}_{h,t} = n^k_{h,t} \equiv n_{h,t}$. We assume that $D = 0$ if and only if $k = 0$. By Proposition 3.1 and with some algebra, one can explicitly express the non-fundamental equilibria with simplified conditions.

Corollary 3.2 *When the social influence structure follows a regular network with* $k > 0$*, under the conditions of Proposition 3.1, the deterministic skeleton of* (x_t, m_t) *in system (23) always has a fundamental equilibrium* $(0, m^{eq})$ *such that*

$$m^{eq} = \tanh\Big[-\frac{DC}{4}(m^{eq}+1)\Big]. \tag{29}$$

This implies $-1 < m^{eq} < 0$*. Furthermore, when one of the following conditions holds,*

C1^{rn}. $R < \theta < (1+a)R$ *and* $\ln\left(\frac{\theta - R}{aR}\right) > -\frac{D(\theta - R)C}{\theta - (1-a)R}$,
C2^{rn}. $\theta \geq (1+a)R$,

system (23) has two other non-fundamental equilibria $(\pm x^{\#}, m^{\#})$ *where*

$$x^{\#} = \sqrt{\frac{a_2\sigma^2\big[\theta - (1-a)R\big]\ln\left(\frac{\theta-R}{aR}\right) + a_2\sigma^2 D(\theta - R)C}{2D(\theta - R)aR(R-1)}}. \tag{30}$$

It is easy to see that when the uniform degree k of agents increases, $D = g(k)$ increases. If $g(d)$ is an increasing function of d, m^{eq} decreases (increases in magnitude) as well as $x^{\#}$. The value of $m^{\#}$ is not affected by k. Intuitively, this means, holding all the other factors fixed, when individuals have more social connections, they are more likely to be influenced by trend chasers and therefore to become a trend chaser, which decreases the value of m^{eq} in the fundamental equilibrium. In the non-fundamental equilibria, only if a small deviated price (a lower $x^{\#}$) can keep the same fractions of types of investors (a fixed $m^{\#}$).

An extreme case of regular networks is for $k = 0$. In this case, every individual is isolated from the rest of society. It leads to $D = 0$ and $n_{1,t} = n_{2,t} = 1/2$, i.e., individuals choose their types randomly. Obviously, the only equilibrium of (23) in such a world is $(x^{\#}, m^{\#}) = (0, 0)$.

Simple core-periphery structure

Now, we consider another special network structure where the set of degrees is $\{d_1, d_2\}$ with $0 < d_1 < d_2$ and $P(d_1) > P(d_2)$. In other words, there are two groups of individuals. Group 1 contains the ones who have fewer connections but make up the majority of society, and group 2 consists of the rest who have more connections. This is a simple example of the core-periphery structure (Borgatti and Everett 2000) that is observed in many real-world networks. It describes the phenomenon that a small part of people (the core) is better connected than the others (the periphery). In this case, the value of m^{eq} in the fundamental equilibrium is bounded in the way stated in the following corollary, which follows from Proposition 3.1.

Corollary 3.3 *When the social influence structure follows a simple core-periphery network with* $0 < d_1 < d_2$ *and* $P(d_1) > P(d_2)$*, and if* $g(d)$ *is increasing in* d*, then the fundamental equilibrium* $(0, m^{eq})$ *of (23) satisfies*

$$-1 < m_2^{eq} < m^{eq} < m_1^{eq} < 0, \tag{31}$$

where m_1^{eq} *and* m_2^{eq} *are the values of* m^{eq} *in the fundamental equilibrium in Corollary 3.2 for* $k = d_1$ *and* $k = d_2$*, respectively.*

4 Nonlinear dynamic behavior

In this section, we examine the dynamic behavior of our nonlinear asset pricing model in the presence of social influence. We focus on the local stability of the fundamental equilibrium and provide two propositions to illustrate the conditions for stability. We will also investigate the time series properties of the model.

4.1 Local stability analysis

In the following analysis we set $\delta_t = 0$. For convenience, we temporarily rewrite system (23) to make the time indices coincide,

$$\begin{cases} x_{t+1} = x_t + \frac{\mu}{2}\big[(1+m_t)z_{1,t} + (1-m_t)z_{2,t}\big], \\ m_{t+1} = \sum_d \tilde{P}(d) \cdot \tanh\Big[\frac{g(d)}{4}(A_{t+1} \cdot m_t + B_{t+1})\Big], \end{cases} \tag{32}$$

where

$$\begin{cases} A_t = (x_t - Rx_{t-1})(z_{1,t-1} + z_{2,t-1}) - C, \\ B_t = (x_t - Rx_{t-1})(z_{1,t-1} - z_{2,t-1}) - C, \\ z_{1,t} = (-Rx_t)/(a_1\sigma^2), \\ z_{2,t} = (\theta g_t - Rx_t)/(a_2\sigma^2). \end{cases} \tag{33}$$

Furthermore, we define

$$\begin{aligned} \gamma &= \frac{\mu}{4a_2\sigma^2}, \\ \theta^* &= R\Big(1 + a\frac{1+m^{eq}}{1-m^{eq}}\Big), \quad \text{and} \\ \tau &= \sum_d \tilde{P}(d)\frac{g(d)C}{4}\Big\{1 - \tanh^2\Big[-\frac{g(d)C}{4}(1+m^{eq})\Big]\Big\}. \end{aligned} \tag{34}$$

As stated in Chiarella and He (2003), it is generally difficult to derive sufficient conditions for the local stability of the fundamental equilibrium for general lag length, however, stability results for lower lag length such as $L = 1, 2, 3$ can be obtained and can provide some insights into the stability for the general case. In order to analyse the local stability of $(L+1)$ dimensional first-order system defined by (32), we follow the usual procedure consisting in studying the characteristic polynomial of the Jacobian matrix and in analysing its eigenvalues. In particular, the following Proposition establishes conditions for the modulus of all eigenvalues λ_i to be lower than one, i.e., $|\lambda_i| < 1$.

Proposition 4.1 *For* $L = 1, 2, 3$*, the fundamental equilibrium* $(0, m^{eq})$ *of (32) is locally asymptotically stable if and only if conditions* C_a*,* C_b *and* C_L *hold true.*

C$_a$. $\tau < 1$.
C$_b$. $\theta < \theta^*$.
C$_L$. $\gamma < \gamma_L^*$ *for* $L = 1, 2, 3$, *where*

$$\gamma_1^* \equiv \frac{1}{(1-m^{eq})(\theta^*-\theta)}, \tag{35}$$

$$\gamma_2^* \equiv \frac{1}{(1-m^{eq})\Big(\theta^*-\theta\frac{1-\omega}{1+\omega}\Big)}, \tag{36}$$

$$\gamma_3^* \equiv \frac{1}{(1-m^{eq})\Big(\theta^*-\theta\frac{1-\omega+\omega^2}{1+\omega+\omega^2}\Big)}. \tag{37}$$

Period-doubling bifurcations occur at $\tau = 1$ *or at* $\gamma = \gamma_L^*(L = 1, 2, 3)$*, and pitchfork bifurcations occur at* $\theta = \theta^*$*. Furthermore, one has* $\gamma_2^* < \gamma_3^* < \gamma_1^*$.

It is not surprising that condition C$_L$ in Proposition 4.1 coincides with the conditions in Proposition 2 of Chiarella and He (2003). Condition C$_a$ is due to the social influence in the decision of switching strategies. Apparently, involving the network effect adds some complexity to the conditions, as it should, but does not change the structure dramatically. In particular, the geometric decay for the extrapolation of past prices for trend chasers to the behavior of the system is not directly affected by the network setting, since the social influence is only embedded into the model through strategy choices. Therefore, the difference made by different values of lag length L and weight parameter ω has the same structure as that is discussed in Chiarella and He (2003), so we will not repeat the analysis again. However, there is an indirect effect through m^{eq}, which depends on social structure $g(d)$ and appears both in conditions C$_a$ and C$_L$. This dependence leads to different values of γ_L^*, but does not change the expression of condition C$_L$. Condition C$_a$ tends to be satisfied when $g(d)$ has small values for all d, or a large $g(d)$ is associated with a small $\tilde{P}(d)$ so that the expected impact is kept small. This is because the value of the expression inside the curly brackets on the right-hand side of equation (34) is bounded by 0 and 1. Intuitively, if the social structure does not result in a big difference among individuals with different degrees, the system will behave much like that of the system without social influence.

The following result on the local stability of the fundamental equilibrium for general $L > 1$ can be derived by applying Rouché's theorem[8]. The proof is similar to that of Proposition 3 of Chiarella and He (2003) and therefore is omitted.

Proposition 4.2 *For* $L > 1$, *it holds that*

(i) *The fundamental equilibrium* $(0, m^{eq})$ *of (32) is locally asymptotically stable if conditions* C$_a$ *and* C$_b$ *hold, and*

$$\gamma < \gamma^* \equiv \frac{1}{(1-m^{eq})(\theta^*+\theta-2b\theta)}. \tag{38}$$

[8] Rouché's theorem states that, for a simple closed path $\mathcal{C}$ on the complex plane, if functions f and g are analytic inside and on $\mathcal{C}$ and if $|g(z)| < |f(z)|$ for all z on $\mathcal{C}$, then $f + g$ and f have the same number of zeros inside $\mathcal{C}$. See, e.g., Asmar and Grafakos (2018).

(ii) *The fundamental equilibrium* $(0, m^{eq})$ *of (32) is unstable if conditions* C_a *and* C_b *hold, and* $\gamma > \gamma_1^*$.

4.2 Time series properties

It is usually difficult to draw general conclusions on the motion of a higher-order non-linear system. However, for the system considered in this paper, it is possible to obtain some time series properties. The following proposition states that if two trajectories of system (32) start at initial states that are symmetric about $x_0 = 0$, then the trajectories are symmetric about $x_t = 0$ for all $t > 0$.

Proposition 4.3 *Let* (x_t^1, m_t^1) *and* (x_t^2, m_t^2) *denote two trajectories of system (32). If* $x_0^1 = -x_0^2$, $m_0^1 = m_0^2$, *then* $x_t^1 = -x_t^2$, $m_t^1 = m_t^2$ *for all* $t > 0$.

Corollary 4.4 *The basins of attraction of the two non-fundamental equilibria* $(\pm x^{\#}, m^{\#})$ *of (32) are symmetric about the line* $x = 0$. *More specifically, the point* $(\underline{x}, \underline{m})$ *is in the basin of attraction of* $(x^{\#}, m^{\#})$ *if and only if the point* $(-\underline{x}, \underline{m})$ *is in the basin of attraction of* $(-x^{\#}, m^{\#})$.

5 Numerical simulation

In this section, we conduct numerical simulations of the deterministic skeleton of the system (32) to provide insights on how social influence affects the local stability of the fundamental equilibrium and the bifurcations of price dynamics. From now on we consider $g(d) = \alpha d^{\beta}$. The role of social influence is implied by the effect of parameters α and β. α is considered to be positive and functions as a linear amplifier of the social influence captured by the degree d. β adjust the effect in a non-linear way such that, if $\beta > 0$, social influence shows a positive effect and people with more connections have a larger tendency of following crowds; on the contrary if $\beta < 0$, people exhibit negative feedback to social influence and tend to behave oppositely from the crowds. The underlying social network is assumed to follow a power law distribution $P(d) \propto d^{-\zeta}$ with $d \in \{1, 2, \cdots, 100\}$. Networks with a power law degree distribution are also called scale-free networks. Many real-world networks, including the WWW, the Internet, science collaboration networks, citation networks, and metabolic networks, are proven to exhibit scale-free property with a value of ζ roughly between 2 and 3. [9]

Table 1 summarises the baseline values of the parameters we used in the simulation.[10] Most parameter values are chosen as close as to those used in Brock and Hommes (1998) and Chiarella and He (2003) for comparison purposes. A few important numerical tools are used to describe the long-run behavior of the system such as

[9] See Barabási (2016) for a comprehensive introduction.

[10] When we discuss a particular parameter (α or β), the other parameters take the baseline values.

Table 1 The baseline parameter values used in simulations

Social influence				Price dynamics						
α	β	d	ζ	a	θ	μ	R	C	L	ω
5	0.2	$\{1,\ldots,100\}$	2.5	0.4	1.2	1.2	1.1	1	3	0.5

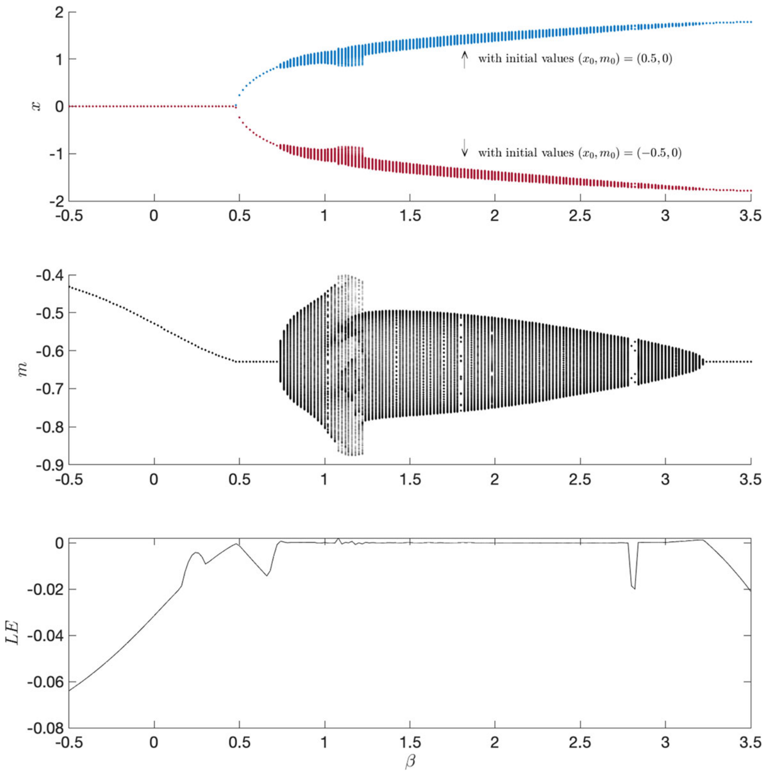

Fig. 1 Bifurcation diagrams about β and the corresponding Lyapunov exponents

the bifurcation diagram, the Lyapunov exponent, time series plot, and phase plot.[11] If not mentioned otherwise, the initial values are set as $(x_0, m_0) = (0.5, 0)$.

Social influence parameter β (Degree-dependent)

Figure 1 depicts bifurcation diagrams of variables x and m with respect to the social influence parameter β and the corresponding values of the Lyapunov exponents. β adjusts the social influence in a nonlinear way that depends on the degree of the social network. x is the price deviation from the fundamental price. m is the difference between the fractions of heterogeneous expectations. The results suggest the increase of β leads to periodic and quasi-periodic dynamics. The dynamics are examined by using two sets of initial values respectively with $(x_0, m_0) = (0.5, 0)$ and $(x_0, m_0) =$

[11] A brief description of numerical simulation techniques of nonlinear dynamics can be found in Brock and Hommes (1998).

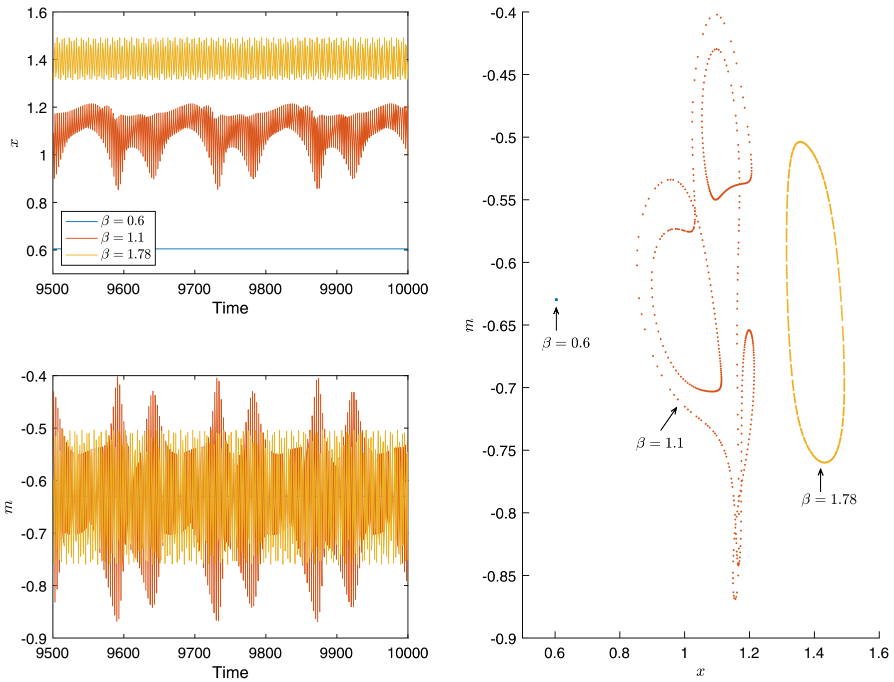

Fig. 2 Time series plots and limiting orbits for $\beta \in \{0.6, 1.1, 1.78\}$

$(-0.5, 0)$, and is computed for 10000 periods with the value of β ranging from -0.5 to 3.5.

It can be observed that the orbits of x for the two sets of initial values are symmetric about $x = 0$, but the orbits of m coincide (thus shown as black points). This has also been proven in Proposition 4.3. A hopf bifurcation occurs at $\beta = 0.72$. The system loses stability and exhibits periodic and quasi-periodic dynamics from there, which can be observed from the Lyapunov exponent plot as well. As β increases, the dynamical instability of the system varies, with the most complexity arising when $\beta \in [1.05, 1.23]$. However, the system converges to a stable regime after a supercritical hopf bifurcation appears at $\beta = 3.24$. The system is stabilised when the strength of social influence is large enough. This can also be seen in Fig. 2 which illustrates the time series and limiting orbits for selected values of β. For $\beta = 0.6$, the system converges to a fixed steady state. The time series and limiting orbit change dramatically at $\beta = 1.1$, where both the trajectory and the frequency of oscillation show a great level of complexity. At $\beta = 1.78$, the trajectory simplifies to a loop, but the frequency is not clear.

These results suggest that social influence contributes to complex and unstable nonlinear dynamics in financial markets. As investors become more inclined to imitate each other, the percentage of trend chasers increases. This, in turn, amplifies price deviations from the fundamental level, which can lead to complex price dynamics. However, this analysis also highlights that cyclical behavior is only observed in

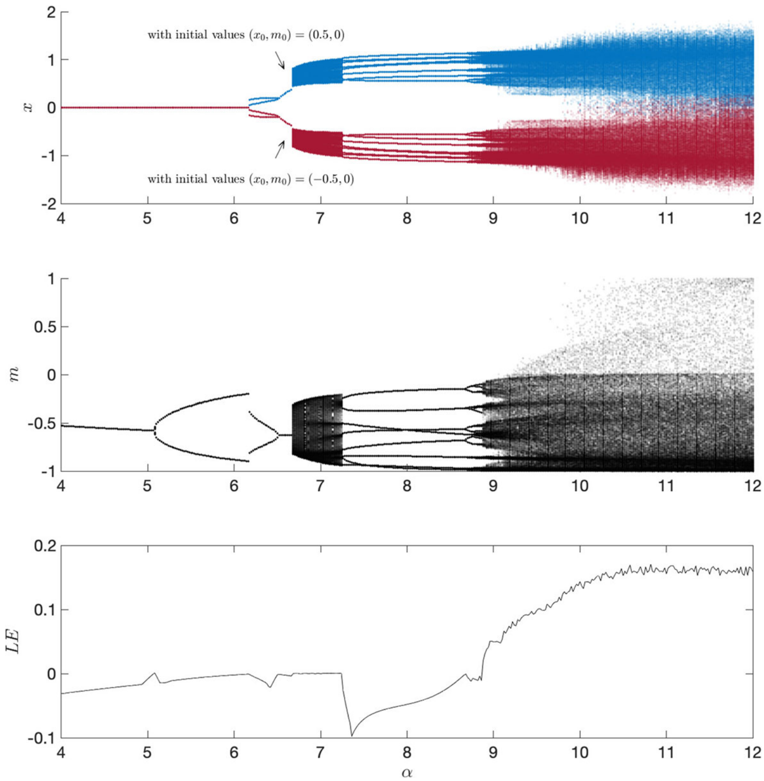

Fig. 3 Bifurcation diagrams on the value of α and the corresponding Lyapunov exponents

situations where the degree-dependent social influence parameter β is positive and moderate. If β is large, it stabilises the system, but also results in a limiting price that is farther away from the fundamental level.

Social influence parameter α (Linear amplifier)

The bifurcation diagrams and the Lyapunov exponents on the value of α are illustrated in Fig. 3, with α varying within the interval [4, 12]. β is fixed at 0.2 in the simulation. α has a linear effect on all agents across social network so it is not influenced by degrees. The bifurcation diagrams show the behavior of the system as the parameter α is varied. The diagrams indicates that there is a primary period-doubling bifurcation, which is followed by several bifurcations before the system exhibits chaos. Specifically, a pitchfork bifurcation occurs at $\alpha = 6.47$, and two hopf bifurcations appear at $\alpha = 6.66$ and $\alpha = 7.25$, with one being supercritical and the other subcritical. With a larger social influence parameter α, the system exhibits chaos at $\alpha = 8.88$, which is consistent with the corresponding Lyapunov exponents and the time series and phase plot in Fig. 4.

The social influence parameter α has a linear effect on all agents across social network and is not influenced by degrees. When the value of α is large, the power of social influence increases across the entire society. This can result in a highly unstable

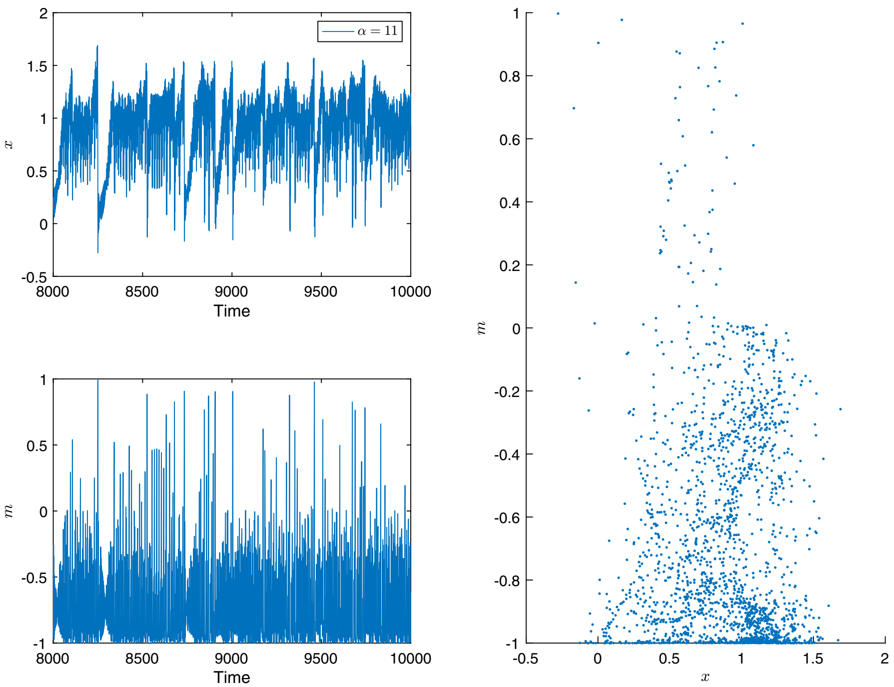

Fig. 4 Time series plots and limiting orbits for $\alpha = 11$

situation, as indicated by the chaotic region in the graph. Note that this is observed with a low level of β, which means degree dependent influence is ignorable.

Summary

We conducted numerical simulations to analyse the impact of social influence parameters β and α on the long-run local instability of the system. The results suggest that the model can exhibit complex dynamics, including periodic, quasi-periodic, and chaotic behavior, depending on the values of β and α. When social influence is small, the system tends to be more stable and converges to the fundamental equilibrium. However, with an increase in social influence parameters, both β and α, the system can become more complex and unstable. The economic rationale behind these bifurcations is as follows. When social influence is low, investors make decisions independently, which can lead to a more stable market. They have less connections and they are less likely to be influenced by their neighbors in the social network. However, when more investors have access to larger number of connections, they are more likely to be influenced by their peers, which can lead to increased complexity and instability. In times of crisis, social influence can amplify small market fluctuations, leading to larger market fluctuations and even market crashes. The study by Brunetti et al. (2019) provides evidence for the impact of social connectedness on market dynamics during the 2008 global financial crisis. Increased connectedness among banks led to greater correlation in bank stock returns and amplified the impact of small fluctuations, contributing to the larger market fluctuations seen during the crisis.

6 Concluding remarks

In this paper we provide a way to incorporate social influence and heterogenous expectations into asset pricing models, allowing for a more nuanced understanding of market dynamics. We focus on the dynamic effects of social influence on asset prices. We examined the existence and local stability of equilibria of the dynamics both analytically and numerically under a range of parameters. To incorporate social influence and obtain analytical results with generic implications for social structure, a mean-field approximation is employed to keep the model tractable. This approach allows for the analysis of large populations, which is a major advantage when studying the impact of social influence in the real world.

We observed that the embedment of social influence in the model fundamentally increased the dimensionality of the system, and thus imposing more restrictions on the existence and stability of equilibria. It implies that the presence of social influence can lead to greater instability in asset price dynamics compared to benchmark models that do not include social influence, as in Brock and Hommes (1998) and Chiarella and He (2003). High levels of social influence creates complexity and chaos universally and degree-dependently.[12] These findings have important implications for understanding the role of social influence in financial markets and the potential for market instability.

The results also yield important insights into policies for monitoring the instability of financial markets. As suggested by the results, high levels of social influence can increase the complexity of the system and potentially lead to market instability and crashes. Policymakers should closely monitor the level of social influence in financial market and take action to prevent the system from reaching dangerous levels of instability. By understanding the role of social influence in financial markets, policy makers can take proactive steps to promote market stability and prevent catastrophic events.

The work in this paper could be extended in several ways. One possible extension is to calibrate the model to real-world data and test the robustness of the findings. The model could also be modified to study the impact of different types of social influence on asset price dynamics, such as the influence of social media or news media. Additionally, more realistic assumptions could be considered by using machine learning algorithms.

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[12] Similar conclusions as in Panchenko et al. (2013); Özsöylev and Walden (2011); Özsöylev et al. (2013).

Declarations

Conflicts of interest The authors declare that there is no conflict of interest. They have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

A Appendix

A.1 Proof of Proposition 3.1

Denote (x^*, m^*) the equilibrium state of (23) when $\delta_t = 0$. At equilibrium it holds that

$$x^* = x^* + \frac{\mu}{2}\Big[(z_1^* + z_2^*) + m^*(z_1^* - z_2^*)\Big], \tag{39}$$

$$m^* = \sum_d \tilde{P}(d) \cdot \tanh\Big[\frac{g(d)}{4}(A^* m^* + B^*)\Big], \tag{40}$$

where

$$A^* = (1 - R)x^*(z_1^* + z_2^*) - C, \quad B^* = (1 - R)x^*(z_1^* - z_2^*) - C, \tag{41}$$

$$z_1^* + z_2^* = \frac{\theta - (1 + a)R}{a_2\sigma^2}x^*, \quad z_1^* - z_2^* = -\frac{\theta - (1 - a)R}{a_2\sigma^2}x^*. \tag{42}$$

$x^* = 0$ is a solution of Equation (39). When $x^* = 0$, m^* is the unique solution, denoted m^{eq}, of the following equation (i added * in the equation)

$$m^* = \sum_d \tilde{P}(d) \cdot \tanh\Big[-\frac{g(d)C}{4}(m^* + 1)\Big]. \tag{43}$$

Considering the shape and range of hyperbolic tangent function, $g(d)C > 0$, and the fact that $\tilde{P}(d)$ is a probability distribution over the domain of d, one has $m^{eq} \in (-1, 0)$.

When $x^* \neq 0$, (39) implies

$$m^* = \frac{z_1^* + z_2^*}{z_2^* - z_1^*} = \frac{\theta - (1 + a)R}{\theta - (1 - a)R} = m^\#, \tag{44}$$

provided $\theta \neq (1-a)R$. By substituting $m^* = m^\#$ in to (40), one has

$$m^\# = \sum_d \tilde{P}(d) \cdot \tanh\Big[\frac{g(d)}{4} \cdot (m^\# + 1) \cdot \Big(\frac{2aR(R-1)}{a_2\sigma^2}(x^*)^2 - C\Big)\Big]. \tag{45}$$

In order to have $m^\# \in [-1, 1]$ one needs $\theta \geq R$, otherwise there is no meaningful solution. When $\theta = R$ one has $m^\# = -1$. The R.H.S. of (45) is always 0, so there is no solution. When $R < \theta < (1+a)R$, one has $-1 < m^\# < 0$ and the range of the graph of the R.H.S. of (45) is $[M, 1)$. Since $M < 0$, the number of solutions is determined by the values of $m^\#$ and M, that is

$$\text{the number of solutions } = \begin{cases} 2 & \text{if} \quad m^\# > M, \\ 1 & \text{if} \quad m^\# = M \text{ (in this case the solution is } x^* = 0), \\ 0 & \text{if} \quad m^\# < M. \end{cases} \tag{46}$$

When $\theta = (1+a)R$, $m^\# = 0$, and the R.H.S. of (45) equals 0 if and only if

$$\frac{2aR(R-1)}{a_2\sigma^2}(x^*)^2 - C = 0, \tag{47}$$

implying that $x^* = \pm\sqrt{\frac{Ca_2\sigma^2}{2aR(R-1)}}$. When $\theta > (1+a)R$, one has $m^\# > 1$ and the range of the R.H.S. of (45) is $[M, 1)$ with $M < 0$, therefore there are always two solutions.

A.2 Proof of Proposition 4.1

Equation (32) is a two-dimensional Lth-order system with $L+1$ variables. In the following analysis we let

$$\begin{aligned}
x_{t-L+1} &\equiv \xi_{1,t}, \\
x_{t-L+2} &= \xi_{1,t+1} \equiv \xi_{2,t}, \\
x_{t-L+3} &= \xi_{1,t+2} = \xi_{2,t+1} \equiv \xi_{3,t}, \\
&\vdots \\
x_{t-2} &= \xi_{1,t+L-3} = \xi_{2,t+L-4} = \cdots = \xi_{L-3,t+1} \equiv \xi_{L-2,t}, \\
x_{t-1} &= \xi_{1,t+L-2} = \xi_{2,t+L-3} = \cdots = \xi_{L-3,t+2} = \xi_{L-2,t+1} \equiv \xi_{L-1,t}, \\
x_t &\equiv \xi_{L,t}, \\
m_t &\equiv \xi_{L+1,t}.
\end{aligned}$$

Then Equation (32) can be rewritten as

$$\begin{cases} \xi_{1,\,t+1} &= \xi_{2,\,t} \\ \xi_{2,\,t+1} &= \xi_{3,\,t} \\ \quad\vdots & \\ \xi_{L-3,\,t+1} &= \xi_{L-2,\,t} \\ \xi_{L-2,\,t+1} &= \xi_{L-1,\,t} \\ \xi_{L-1,\,t+1} &= \xi_{L,\,t} \\ \xi_{L,\,t+1} &= f(\xi_{1,\,t}, \xi_{2,\,t}, \ldots, \xi_{L,\,t}, \xi_{L+1,\,t}) \\ \xi_{L+1,\,t+1} &= h(\xi_{1,\,t}, \xi_{2,\,t}, \ldots, \xi_{L,\,t}, \xi_{L+1,\,t}) \end{cases} \tag{48}$$

where

$$f(\xi_1, \xi_2, \ldots, \xi_L, \xi_{L+1}) = \xi_L + \frac{\mu}{2}\Big[(1+\xi_{L+1})\frac{-R\xi_L}{a_1\sigma^2} + (1-\xi_{L+1})\frac{\theta G - R\xi_L}{a_2\sigma^2}\Big], \tag{49}$$

$$G = \sum_{i=1}^{L} b_i\,\xi_i\ , \quad b_i = \frac{\omega^{L-i}}{\sum_{j=0}^{L-1}\omega^j} \quad \text{for} \quad i = 1, 2, \ldots, L, \tag{50}$$

and

$$h(\xi_1, \xi_2, \ldots, \xi_L, \xi_{L+1}) = \sum_d \tilde{P}(d)\cdot\tanh\Big[\frac{g(d)}{4}\big(A\cdot\xi_{L+1} + B\big)\Big], \tag{51}$$

$$A = \Big(f(\xi_1, \xi_2, \ldots, \xi_L, \xi_{L+1}) - R\xi_L\Big)\Big(\frac{-R\xi_L}{a_1\sigma^2} + \frac{\theta G - R\xi_L}{a_2\sigma^2}\Big) - C, \tag{52}$$

$$B = \Big(f(\xi_1, \xi_2, \ldots, \xi_L, \xi_{L+1}) - R\xi_L\Big)\Big(\frac{-R\xi_L}{a_1\sigma^2} - \frac{\theta G - R\xi_L}{a_2\sigma^2}\Big) - C. \tag{53}$$

System (48) becomes an $(L+1)$-dimensional first-order system, who has a fundamental equilibrium $(0, 0, \ldots, 0, m^{eq})$. The Jacobian matrix of system (48) evaluated at this fundamental equilibrium is then

$$J = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ J_{L,1} & J_{L,2} & J_{L,3} & \cdots & J_{L,L-1} & J_{L,L} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & J_{L+1,L+1} \end{bmatrix}, \tag{54}$$

where

$$J_{L,i} = \frac{\mu}{2} \cdot \frac{\theta b_i}{a_2\sigma^2}(1 - m^{eq})$$
$$= 2\gamma(1 - m^{eq})\theta b_i, \quad i = 1, \ldots, L-1, \tag{55}$$
$$J_{L,L} = 1 + \frac{\mu}{2}\Big[(1 + m^{eq})\frac{-R}{a_1\sigma^2} + (1 - m^{eq})\frac{\theta b_L - R}{a_2\sigma^2}\Big]$$
$$= 1 + \frac{\mu}{2a_2\sigma^2}(1 - m^{eq})\Big[\theta b_L - R\Big(1 + a\frac{1 + m^{eq}}{1 - m^{eq}}\Big)\Big]$$
$$= 1 + 2\gamma(1 - m^{eq})\big[\theta b_L - \theta^*\big], \tag{56}$$

and

$$J_{L+1,L+1} = \sum_d \tilde{P}(d)\frac{g(d)C}{4}\Big\{\tanh^2\Big[-\frac{g(d)C}{4}(1 + m^{eq})\Big] - 1\Big\}. \tag{57}$$

Let $Q = J - \lambda I$, and denote $Q_{(i,j)}$ the submatrix of Q obtained by deleting the ith row and jth column. The corresponding characteristic polynomial can be expressed as

$$\det(J - \lambda I) = (-1)^{L+1}J_{L,1}\det(Q_{(L,1)}) + (-1)^{L+2}J_{L,2}\det(Q_{(L,2)}) + \cdots + (-1)^{2L-1}J_{L,L-1}\det(Q_{(L,L-1)}) + (-1)^{2L}(J_{L,L} - \lambda)\det(Q_{(L,L)}), \tag{58}$$

which is obtained by applying the Laplace expansion along the Lth row. After evaluating the terms $\det(Q_{L,j})$ for $j = 1, \ldots, L$ of the characteristic polynomial, one has

$$\det(J - \lambda I) = (-1)^{L-1}(J_{L+1,L+1} - \lambda)\Big(\sum_{i=1}^{L} J_{L,i}\lambda^{i-1} - \lambda^L\Big). \tag{59}$$

The characteristic equation $\det(J - \lambda I) = 0$ leads to

$$\lambda_1 = J_{L+1,L+1}, \tag{60}$$

and λ_i $(i = 2, \ldots, L+1)$ being the solutions of

$$\lambda^L - J_{L,L}\lambda^{L-1} - J_{L,L-1}\lambda^{L-2}\cdots - J_{L,2}\lambda - J_{L,1} = 0. \tag{61}$$

The fundamental equilibrium is locally asymptotically stable if and only if $|\lambda_i| < 1$ for $i = 1, \ldots, L+1$. Since $-1 < \tanh\big[-\frac{g(d)C}{4}(1 + m^{eq})\big] < 0$, one has $|\lambda_1| < 1$ if and only if

$$\sum_d \tilde{P}(d)\frac{g(d)C}{4}\Big\{1 - \tanh^2\Big[-\frac{g(d)C}{4}(1 + m^{eq})\Big]\Big\} = \tau < 1. \tag{62}$$

Obviously, $-1 < \lambda_1 < 0$ under this condition. Furthermore, $\lambda_1 = -1$ at $\tau = 1$.
For $L = 1$. In this case, $b_1 = 1$, and

$$\lambda_2 = 1 + 2\gamma(1 - m^{eq})\big[\theta - \theta^*\big]. \tag{63}$$

Given $\gamma > 0$, $\theta > 0$, and $-1 < m^{eq} < 0 \Leftrightarrow 1 < 1 - m^{eq} < 2$, one has $|\lambda_2| < 1$ if and only if

$$\theta < \theta^* \quad \text{and} \quad \gamma < \frac{1}{(1 - m^{eq})(\theta^* - \theta)} = \gamma_1^*. \tag{64}$$

Furthermore, $\lambda_2 = 1$ at $\theta = \theta^*$ and $\lambda_2 = -1$ at $\gamma = \gamma_1^*$.
For $L = 2$. In this case, $b_1 = \omega/(1 + \omega)$ and $b_2 = 1/(1 + \omega)$. λ_2 and λ_3 are solutions of the quadratic equation

$$\lambda^2 - J_{2,2}\lambda - J_{2,1} = 0. \tag{65}$$

Using Jury's stability criterion (Jury, 1971; Jury, 1982, p.35), $|\lambda_i| < 1$ $(i = 2, 3)$ if and only if

$$1 - J_{2,2} - J_{2,1} > 0, \quad 1 + J_{2,2} - J_{2,1} > 0, \quad \text{and} \quad |J_{2,1}| < 1 \tag{66}$$

where

$$J_{2,1} = 2\gamma(1 - m^{eq})\theta\frac{\omega}{1 + \omega}, \quad J_{2,2} = 1 + 2\gamma(1 - m^{eq})\Big[\theta\frac{1}{1 + \omega} - \theta^*\Big]. \tag{67}$$

It can be verified that these conditions are equivalent to

$$\theta < \theta^* \quad \text{and} \quad \gamma < \frac{1}{(1 - m^{eq})\Big(\theta^* - \theta\dfrac{1 - \omega}{1 + \omega}\Big)} = \gamma_2^*. \tag{68}$$

Furthermore, one of λ_2 and λ_3 equals 1 at $\theta = \theta^*$, and one of λ_2 and λ_3 equals -1 at $\gamma = \gamma_2^*$. The discriminant of (65) is $J_{2,2}^2 + 4J_{2,1} > 0$, indicating that both λ_2 and λ_3 are real numbers.
For $L = 3$. In this case, $b_1 = \omega^2/(1 + \omega + \omega^2)$, $b_2 = \omega/(1 + \omega + \omega^2)$, and $b_3 = 1/(1 + \omega + \omega^2)$. λ_2, λ_3 and λ_4 are solutions of the cubic equation

$$\lambda^3 - J_{3,3}\lambda^2 - J_{3,2}\lambda - J_{3,1} = 0. \tag{69}$$

Jury's stability criterion implies that $|\lambda_i| < 1$ $(i = 2, 3, 4)$ if and only if

$$D_1 = 1 - J_{3,3} - J_{3,2} - J_{3,1} > 0, \tag{70}$$
$$D_2 = 1 + J_{3,3} - J_{3,2} + J_{3,1} > 0, \tag{71}$$
$$D_3 = 1 + J_{3,3}J_{3,1} + J_{3,2} - J_{3,1}^2 > 0, \tag{72}$$

$$D_4 = 1 - J_{3,3}J_{3,1} - J_{3,2} - J_{3,1}^2 > 0, \tag{73}$$

where

$$J_{3,1} = 2\gamma(1 - m^{eq})\theta \frac{\omega^2}{1 + \omega + \omega^2}, \tag{74}$$

$$J_{3,2} = 2\gamma(1 - m^{eq})\theta \frac{\omega}{1 + \omega + \omega^2}, \tag{75}$$

$$J_{3,3} = 1 + 2\gamma(1 - m^{eq})\Big[\theta \frac{1}{1 + \omega + \omega^2} - \theta^*\Big]. \tag{76}$$

It is not hard to verify that $D_1 > 0$ is equivalent to $\theta < \theta^*$, $D_2 > 0$ is equivalent to

$$\gamma < \frac{1}{(1 - m^{eq})\Big(\theta^* - \theta \frac{1 - \omega + \omega^2}{1 + \omega + \omega^2}\Big)} = \gamma_3^*, \tag{77}$$

and $D_1 > 0$, $D_2 > 0$ together imply $D_3 > 0$ and $D_4 > 0$. Furthermore, one of λ_2, λ_3 and λ_4 equals 1 at $\theta = \theta^*$, and one of λ_2, λ_3 and λ_4 equals -1 at $\gamma = \gamma_3^*$. In addition, since

$$\frac{1 - \omega}{1 + \omega} < \frac{1 - \omega + \omega^2}{1 + \omega + \omega^2} < 1, \tag{78}$$

it holds that

$$\gamma_2^* < \gamma_3^* < \gamma_1^*. \tag{79}$$

A.3 Proof of Proposition 4.3

Let $x_0^1 = -x_0^2$ and $m_0^1 = m_0^2$. We prove the proposition by induction.
For $t = 1$. From (32) we have

$$x_1^1 = \Big[1 - \frac{\mu(1 + m_0^1)R}{2a_1\sigma^2} + \frac{\mu(1 - m_0^1)(\theta - R)}{2a_2\sigma^2}\Big]x_0^1, \tag{80}$$

$$x_1^2 = \Big[1 - \frac{\mu(1 + m_0^2)R}{2a_1\sigma^2} + \frac{\mu(1 - m_0^2)(\theta - R)}{2a_2\sigma^2}\Big]x_0^2, \tag{81}$$

and

$$m_1^1 = \sum \tilde{P}(d) \cdot \tanh\Big[\frac{g(d)}{4}(A_1^1 m_0^1 + B_1^1)\Big], \tag{82}$$

$$m_1^2 = \sum \tilde{P}(d) \cdot \tanh\Big[\frac{g(d)}{4}(A_1^2 m_0^1 + B_1^2)\Big] \tag{83}$$

where

$$A_1^1 = (x_1^1 - Rx_0^1)\Big[-\frac{Rx_0^1}{a_1\sigma^2} + \frac{(\theta - R)x_0^1}{a_2\sigma^2}\Big] - C, \tag{84}$$

$$A_1^2 = (x_1^2 - Rx_0^2)\Big[-\frac{Rx_0^2}{a_1\sigma^2} + \frac{(\theta - R)x_0^2}{a_2\sigma^2}\Big] - C, \tag{85}$$

$$B_1^1 = (x_1^1 - Rx_0^1)\Big[-\frac{Rx_0^1}{a_1\sigma^2} - \frac{(\theta - R)x_0^1}{a_2\sigma^2}\Big] - C, \tag{86}$$

$$B_1^2 = (x_1^2 - Rx_0^2)\Big[-\frac{Rx_0^2}{a_1\sigma^2} - \frac{(\theta - R)x_0^2}{a_2\sigma^2}\Big] - C. \tag{87}$$

Then it holds that $x_1^1 = -x_1^2$, $A_1^1 = A_1^2$, $B_1^1 = B_1^2$, and thus $m_1^1 = m_1^2$.
For $t > 1$. Suppose that $x_k^1 = -x_k^2$, $m_k^1 = m_k^2$ for all $1 \leq k \leq t$. Then x_{t+1}^1 can be written as a linear function of x_k^1 for $\max(t - L + 1, 1) \leq k \leq t$, and similarly for x_{t+1}^2. Then we have $x_{t+1}^1 = -x_{t+1}^2$. The expressions of m_{t+1}^1 and m_{t+1}^2 contain A_{t+1}^1, A_{t+1}^2, B_{t+1}^1, and B_{t+1}^2. These terms can be also written in a similar manner as in the case of $t = 1$, where the two factors of the product term in each expression only contains linear functions of x_k^1's and x_k^2's for $\max(t - L + 1, 1) \leq k \leq t$, implying $A_{t+1}^1 = A_{t+1}^2$, and $B_{t+1}^1 = B_{t+1}^2$. Therefore we have $m_{t+1}^1 = m_{t+1}^2$.

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