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REGULAR ARTICLE

Competition and strategic alliance in R&D investments: a real option game approach with multiple experiments

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Abstract
In this paper, we analyse the effects that the number and outcomes of R&D experiments have on the strategic equilibria between two firms that can both compete and cooperate in a context of uncertainty. As is well known, R&D projects are characterised by the sequentiality of investments and by the outcomes obtained from the success or failure of their experiments. Furthermore, the positive results and the number of tests carried out in R&D increase the market value of the innovative product. The Real Option Approach evaluates the flexibility of R&D investments and the strategic scenarios. According to Nash equilibria, we show how the market value threshold, for which the investment policy is optimal, depends on the number of experiments and on the information revelation.

Keywords Real options · Game theory · Information revelation · R&D investments

JEL Classification G13 · C70 · D80 · O32

1 Introduction

Real Options Theory analyses the financial instruments applied to real assets, while game theory introduces strategic interactions between firms. R&D investment generates new opportunities to promote economic development, to change market structure and to potentially remove rivals from a given field. This particularly applies to some

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high-tech industries, like the pharmaceutical, software and semiconductor industries, where monitoring the R&D investment, rather than price competition, is prudent. Unfortunately, a R&D project is not often intended to yield immediate profits and generally is characterised by high investment uncertainty. These characteristics and the ability of delayed entry are not all taken into account by traditional net present value (NPV) and internal rate of return (IRR) methods. In this context, the Real Option Analysis (ROA) meets the criteria required to support managers decisions.[1] In the following literature, various ways of evaluating projects have been extensively studied. In Shackleton and Wojakowski (2003), Lee (1997), Trigeorgis (1991) and Majd and Pindyck (1987), it is assumed that the option exercise price and investment cost are fixed. However, it is important to consider the option exercise as a stochastic variable. The exchange option can be employed to value R&D investments in which both the gross project value and the investment cost are uncertain. For this purpose, as described in McDonald and Siegel (1985), a European exchange option is used to value the assets that distribute dividends. Further, in Carr (1988) and Carr (1995), an American exchange option through approximating American put is developed and a model to evaluate European compound exchange options is analysed. Moreover, in Armada et al. (2007), exchange options are also employed to value R&D investments. In the above models, assets distribute "dividends" that, in a real options context, are the opportunity costs if an investment project is postponed (Majd and Pindyck 1987).

R&D investments are characterised by different aspects. First of all, they are obtained from several irreversible and expensive experiments whose successes improve the market value of innovative products (see Kellogg and Charnes 2000; Hauschild and Reimsbach 2015; Myers and Howe 1997; Cassimon et al. 2011 and so on).

Second, the information revelation gained in research experiments influences the strategic choices of other rivals. This information revelation can be acquired in a legal or illegal way, such as through industrial espionage. These phenomena can influence a firm to delay its R&D investment in order to obtain additional information (see Dias 2004; Lewis et al. 2004; Huchzermeier and Loch 2001 and so on).

Real option game theory is an important tool for solving R&D project evaluations. Williams (1993) combines the real options theory with the game theory studying a duopoly market with a continuous-time model under product market competition and determines the Nash equilibrium in a real options framework. Weeds (2002) considers an irreversible investment in competing research projects with uncertain returns under a winner-takes-all patent system, which may shed light on strategic delay in patent races and explain the role of first movers. Furthermore, Lambrecht (2000) considers innovation with uncertainty over completion and time delays, which can explain some phenomena like the faster exit and delayed commercialisation. In addition, Arasteh

[1] A critical aspect in the real option pricing approach is given by the impossibility to construct a replicating portfolio, as the assets are non-tradable. In general, investment problems are much too complex to be modelled as a standard option; hence, the option model must be tailor-made, with standard assumptions no longer applicable. Classic ROA is based on the assumption that the project can be replicated by a portfolio of market-driven instruments that are all exactly equivalent (Brennan and Schwartz 1985; Amram and Kulatilaka 1999). To solve this shortcoming, one issue can be to link the evaluation of a real project with quoted assets that have the same level of risk (see Borison 2005; Smith and Nau 1995).

(2016) shows how investment strategies rely on competitive interactions. Under competition, firms hurry to exercise their options early and this involves an immediate investment behaviour.

Our paper uses real option game approach in order to study the strategic interaction between two firms investing in R&D within a competitive and cooperative context, such as in Trigeorgis (1991), Smit and Trigeorgis (2004), Zandi and Tavana (2010).

Following Kogut (1991), Savva and Sholtes (2005) and Villani (2008), we assume that when firms sign an alliance, they internalise maximum information revelation and share the joint payoff according to their success probability. On the other hand, based on Chang et al. (2016), Smit and Ankum (1993), Kim and Sanders (2002), when firms move in a non-cooperative context, the Nash equilibria are computed in order to determine their strategies according to market value.

In the aforementioned literature, the link between the size of the market and the number of tests carried out by companies is absent.

The novelty of our paper is the analysis of the role that the number of experiments plays in firms' strategic behaviour. In particular, the number of experiments carried out in R&D and their positive results increases the market value of the innovative product. In addition, we show that firms realise the best investment policies based on critical investment thresholds obtained by the number of tests and the information revelation process. We can affirm that the model creates some new insights that shows market behaviour based on information revelation from experimentations carried out. Our paper can model scenarios with only one expensive experiment, such as oil drilling or nuclear test, or with multiple experiments, such as in the pharmaceutical sector or automotive industry. For instance, the self-driving car requires more tests before it is launched onto the market and continued successful experiments will increase the growth market value of this new product. Firms can act singly or in partnership to advance development, such as Nvidia with Audi, Google with FCA to develop the Waymo project, and so on.

The paper is organised as follows. Section 2 illustrates the main results to evaluate simple and compound European exchange options, while Sect. 3 presents the basic model highlighting the information revelation process, the growth market coefficients and the strategic payoffs of two players. Section 4 analyses the Nash equilibria in the case of competition and cooperation behaviour. Section 5 presents some numerical analysis while Sect. 6 shows a sensitivity study. Finally, Sect. 7 concludes.

2 Exchange options methodology

2.1 Simple European exchange option (SEEO)

The model of McDonald and Siegel (1985) gives the value of a SEEO to exchange asset D for asset V at time T, where $s(V, D, T-t)$ denotes the value of SEEO at time t and the payoff at maturity T is $s(V, D, 0) = \max[0, V_T - D_T]$. So, assuming that V and D follow a geometric Brownian motion process given by:

$$\frac{dV}{V} = (\mu_v - \delta_v)dt + \sigma_v dZ_v \tag{1}$$

$$\frac{dD}{D} = (\mu_d - \delta_d)dt + \sigma_d dZ_d \tag{2}$$

$$\text{cov}\left(\frac{dV}{V}, \frac{dD}{D}\right) = \rho_{vd}\sigma_v\sigma_d\, dt \tag{3}$$

where V and D are the gross project value and the investment cost, respectively, μ_v and μ_d are the expected return equilibrium rates, δ_v and δ_d are the "dividend yields", Z_v and Z_d are the Brownian standard motions and σ_d are the volatilities of two assets, ρ_{vd} is the correlation between changes in V and D. McDonald and Siegel (1985) determines the value of a SEEO at time $t = 0$ as:

$$s(V, D, T) = Ve^{-\delta_v T} N(d_1(P, T)) - De^{-\delta_d T} N(d_2(P, T)) \tag{4}$$

in which

$$P = \frac{V}{D}, \quad \sigma = \sqrt{\sigma_v^2 - 2\rho_{v,d}\sigma_v\sigma_d + \sigma_d^2}, \quad \delta = \delta_v - \delta_d$$

$$d_1(P, T) = \frac{\log P + \left(\frac{\sigma^2}{2} - \delta\right) T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

and $N(d)$ is the cumulative standard normal distribution.

2.2 Compound European exchange option (CEEO)

An option is called compound when the underlying asset is another option. Denoted by $c(s, \varphi D, t_1)$ a CEEO whose payoff at maturity t_1 is $c(s, \varphi D, 0) = \max[0, s - \varphi D]$. Following Carr (1988), the value of a CEEO at initial time $t = 0$ is:

$$\begin{aligned} c(s(V, D, T), \varphi D, t_1) = {} & Ve^{-\delta_v T} N_2\left(d_1\left(\frac{P}{P^*}, t_1\right), d_1\left(P, T\right); \rho\right) \\ & - De^{-\delta_d T} N_2\left(d_2\left(\frac{P}{P^*}, t_1\right), d_2\left(P, T\right); \rho\right) \\ & - \varphi De^{-\delta_d t_1} N\left(d_2\left(\frac{P}{P^*}, t_1\right)\right) \end{aligned} \tag{5}$$

where φ is the exchange ratio of CEEO, t_1 is the expiration date of the CEEO, T is the deadline of the SEEO with $T > t_1$, $\tau = T - t_1$ is the time to maturity of the SEEO, $\rho = \sqrt{t_1/T}$,

$$d_1\left(\frac{P}{P^*}, t_1\right) = \frac{\log\left(\frac{P}{P^*}\right) + \left(-\delta + \frac{\sigma^2}{2}\right) t_1}{\sigma\sqrt{t_1}}, \quad d_2\left(\frac{P}{P^*}, t_1\right) = d_1\left(\frac{P}{P^*}, t_1\right) - \sigma\sqrt{t_1}$$

and $N_2(a, b, \rho)$ is the standard bivariate normal distribution function evaluated at a and b with correlation ρ. Moreover, P^* is the critical price ratio that makes it indifferent whether the SEEO at time t_1 is exercised or not, and solves the following equation:

$$P^* e^{-\delta_v \tau} N(d_1(P^*, \tau)) - e^{-\delta_d \tau} N(d_2(P^*, \tau)) = \varphi$$

3 Basic model

3.1 Information revelation process

Following Dias and Texeira (2004), Dias (2004) and Villani (2008), we assume that two firms A and B invest in R&D at time t_0 or delay their decision at time t_1. Here, the success probabilities of firms A and B are denoted with q and p, respectively. In addition, Ψ_i is the basic research investment realised by firm i =A, B and $\beta_i > 0$ is the level of efficiency. The R&D success probabilities of both firms are:[2]

$$q = \frac{1 - e^{-\beta_A \Psi_A}}{1 + e^{-\beta_A \Psi_A}}; \qquad p = \frac{1 - e^{-\beta_B \Psi_B}}{1 + e^{-\beta_B \Psi_B}} \tag{6}$$

We assume that a firm performs n experiments to both confirm the quality of the new product resulting from the R&D and to increase the quality of its results. Assuming that the cost of each experiment is ω, we can state that the overall R&D investment is $R_i = \Psi_i + n\omega$. We introduce two Bernoulli random variables that describe the initial situation of the R&D success of both firms:

$$X : \begin{cases} 1 & q \\ 0 & 1-q \end{cases} \qquad Y : \begin{cases} 1 & p \\ 0 & 1-p \end{cases}$$

Let us consider the case in which $n = 1$. The R&D success or failure of a firm generates an information revelation that influences the investment decision of the other firm. So, firm A's success probability q changes in positive information revelation q^+ in the case of firm B's success; otherwise, it changes in negative information revelation q^- in the case of firm B's failure. Symmetrically, firm B's R&D success probability modifies in p^+ or in p^- in case of firm A's success or failure, respectively. Using Dias (2004)'s model, it results that:

$$q^+ = \text{Prob}[X = 1 | Y = 1] = q + \sqrt{\frac{1-p}{p}} \cdot \sqrt{p(1-p)} \cdot \rho_1$$
$$q^- = \text{Prob}[X = 1 | Y = 0] = q - \sqrt{\frac{p}{1-p}} \cdot \sqrt{p(1-p)} \cdot \rho_1$$

[2] Probabilities (6) are distributed as a sigmoid function in the interval [0,1[and we assume that when the research investments Ψ_i tend to infinity, then the probabilities p and q tend to 1. On the other hand, when Ψ_i approaches zero, then the success probabilities reach zero. A similar application is given in Petrohilos-Andrianos and Xepapadeas (2017).

where ρ_1 is the intensity of the information revelation between the two firms. In the same manner, we can write p^+ and p^-. So that $q^\pm$ and $p^\pm$ are in $[0, 1[$, the condition $0 \leq \rho_1 \leq \rho_{\max}$ must be satisfied, where

$$\rho_{\max} = \min\left\{\sqrt{\frac{q(1-p)}{p(1-q)}}, \sqrt{\frac{p(1-q)}{q(1-p)}}\right\} \tag{7}$$

is the intensity of the information when firms form an alliance by setting up a joint venture. The information revelation process exists only when the R&D investment is not realised at the same time by both firms.

Let us consider the case in which the experiments are $n = 2$. In this scenario, we have a revelation process composed of two outcomes, which can be positive or negative. Using a recombining revelation process, firm A's success becomes:

$$\begin{aligned}
q^{++} &= q^+ + \sqrt{\frac{1-p^+}{p^+}} \cdot \sqrt{q^+(1-q^+)} \cdot \rho_2 \\
q^{+-} &= q^+ - \sqrt{\frac{p^+}{1-p^+}} \cdot \sqrt{q^+(1-q^+)} \cdot \rho_2 \\
q^{-+} &= q^- + \sqrt{\frac{1-p^-}{p^-}} \cdot \sqrt{q^-(1-q^-)} \cdot \rho_2 \\
q^{--} &= q^- - \sqrt{\frac{1-p^-}{p^-}} \cdot \sqrt{q^-(1-q^-)} \cdot \rho_2
\end{aligned}$$

where $q^{+-} = q^{-+}$ if and only if $\rho_2 = \dfrac{q^+ - q^-}{\sqrt{\frac{p^+q^+(1-q^+)}{1-p^+}} + \sqrt{\frac{(1-p^-)q^-(1-q^-)}{p^-}}}$.

Symmetrically, firm B's success probability changes in p^{++}, $p^{+-} = p^{-+}$ and p^{--}.

Generalising, let us assume that the R&D process is composed of n experiments. If firm B realises h successes and $n - h$ failures, the success probability of firm A becomes:

$$\begin{aligned}
q^{j,y} &= q^{j-1,y} + \sqrt{\frac{1-p^{j-1,y}}{p^{j-1,y}}} \cdot \sqrt{q^{j-1,y}(1-q^{j-1,y})} \cdot \rho_n \\
&= q^{j,y-1} - \sqrt{\frac{p^{j,y-1}}{1-p^{j,y-1}}} \cdot \sqrt{q^{j,y-1}(1-q^{j,y-1})} \cdot \rho_n
\end{aligned}$$

where $j = \overbrace{++\,+}^{\text{h}}$, $y = \overbrace{-\,-\,-}^{\text{n-h}}$ and

$$\rho_n = \frac{q^{j-1,y} - q^{j-2,y+1}}{\sqrt{\frac{p^{j,y-1}q^{j,y-1}(1-q^{j,y-1})}{1-p^{j,y-1}}} + \sqrt{\frac{(1-p^{j-1,y})q^{j-1,y}(1-q^{j-1,y})}{p^{j-1,y}}}}$$

Symmetrically for firm B we have $p^{\varepsilon,\theta}$ where $\varepsilon = \overbrace{++\,+}^{\text{k}}$ and $\theta = \overbrace{-\,-\,-}^{\text{n-k}}$.

3.2 Growth market coefficients (GMC)

An important element that influences the evaluation of R&D investments is the set of externalities that determine a growth in the market size. For these reasons, we assume that, in the case of mutual R&D successes, the market size enlarges. Let us denote $K^i_{t_{0_k}t_{0_h}}$, $K^i_{t_{0_k}t_{1_h}}$, $K^i_{t_{1_k}t_{0_h}}$, $K^i_{t_{1_k}t_{1_h}}$ for $i = A, B$, the GMC of firm i in the case of k successes of firm A and h successes realised by firm B. The subscripts t_0 and t_1 denote the instant in which the research investment is realised. If A and B invest in R&D at time t_0 and t_1 with k and h successes, respectively, firm A takes $K^A_{t_{0_k}t_{1_h}}$ while firm B obtains $K^B_{t_{1_h}t_{0_k}}$. The full failure of the other player does not produce externalities, and therefore,

$$K^i_{t_{0_k}t_{0_0}} = K^i_{t_{0_k}t_{1_0}} = K^i_{t_{0_k}}; \quad K^i_{t_{1_k}t_{0_0}} = K^i_{t_{1_k}t_{1_0}} = K^i_{t_{1_k}}; \quad \text{for} \quad i = A, B.$$

However, in the case of that all experiments fail, the GMC will be equal to zero, i.e. $K^i_{t_{0_0}t_{0_h}} = K^i_{t_{0_0}t_{1_h}} = 0$ and $K^i_{t_{1_0}t_{0_h}} = K^i_{t_{1_0}t_{1_h}} = 0$. In addition, we can set the relations among the GMC with these assumptions:

- Positive network externality. As is shown in Huisman (2001), the GMC in the case of both players' success will be larger than that in which only one firm invests successfully:
$$K^i_{t_{0_k}t_{0_h}} > K^i_{t_{0_k}} \tag{8}$$
- Time of R&D success. The GMC increases if the mutual R&D success is realised at time t_0 rather than t_1:
$$K^i_{t_{0_k}t_{0_h}} > K^i_{t_{1_k}t_{1_h}} \tag{9}$$
- First mover's advantage. If $k = h$, the firm performing the experiments in t_0 will receive a higher GMC than the other player that postpones the realisation at time t_1:
$$K^i_{t_{0_k}t_{1_k}} > K^i_{t_{1_k}t_{0_k}} \tag{10}$$

To analytically determine the GMC, we assume that they depend on the parameters α_{i1} and α_{i2} with $\alpha_{i2} < \alpha_{i1}$ multiplied by the square root of successful experiments realised by both players and by the extend of R&D benefits until the deadline T.

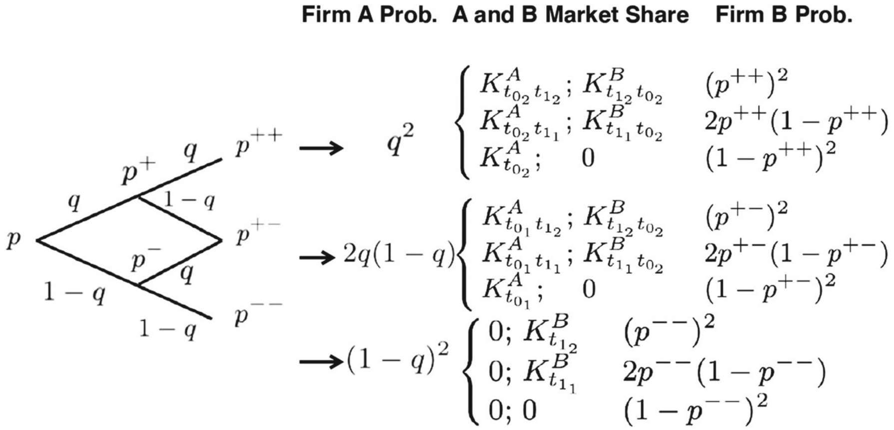

Fig. 1 GMC and success probabilities for A and B assuming $n = 2$ tests when A realises its R&D investment at time t_0 while B postpones until time t_1

In particular, α_{i1} denotes the direct effect that the R&D innovation produces on the market share of firm i, while α_{i2} the indirect effect due to the R&D investment of the second player. In the case of an asymmetric investment, the firm i that invests in t_0 obtains the first mover advantage and so its market share will be α_{i1} from t_0 to T and α_{i2} from t_1 to T. On the other hand, if firm i realises its investment in t_1, it loses the market share from t_0 and t_1. We can describe the GMC of firms A and B as:

$$
\begin{aligned}
K^A_{t_{0_k} t_{0_h}} &= \left(\alpha_{A1}\sqrt{k} + \alpha_{A2}\sqrt{h}\right)\sqrt{T}; \quad & K^B_{t_{0_h} t_{0_k}} &= \left(\alpha_{B1}\sqrt{h} + \alpha_{B2}\sqrt{k}\right)\sqrt{T} \\
K^A_{t_{1_k} t_{1_h}} &= \left(\alpha_{A1}\sqrt{k} + \alpha_{A2}\sqrt{h}\right)\sqrt{T - t_1}; \quad & K^B_{t_{1_h} t_{1_k}} &= \left(\alpha_{B1}\sqrt{h} + \alpha_{B2}\sqrt{k}\right)\sqrt{T - t_1} \\
K^A_{t_{0_k} t_{1_h}} &= \alpha_{A1}\sqrt{kT} + \alpha_{A2}\sqrt{h(T - t_1)}; \quad & K^B_{t_{1_h} t_{0_k}} &= \alpha_{B1}\sqrt{h(T - t_1)} - \alpha_{B2}\sqrt{kt_1} \\
K^A_{t_{1_k} t_{0_h}} &= \alpha_{A1}\sqrt{k(T - t_1)} - \alpha_{A2}\sqrt{ht_1}; \quad & K^B_{t_{0_h} t_{1_k}} &= \alpha_{B1}\sqrt{hT} + \alpha_{B2}\sqrt{k(T - t_1)}
\end{aligned}
$$

with $k \neq 0$ for K^A and $h \neq 0$ for K^B.

Figure 1 graphically depicts the success probability flow after the information revelation and related to GMC of firms A and B assuming $n = 2$ tests.

3.3 The Leader's payoff

We analyse the Leader's payoff assuming that firm A (Leader) invests in R&D at time t_0, while firm B (Follower) decides to wait to invest. We consider the scenario with just one experiment, i.e. $n = 1$. The Leader invests R_A at time t_0 and obtains, in the case of its R&D success with probability q, the development option. If the Follower's R&D investment is successful at time t_1 with probability p^+, the GMC will be $K^A_{t_{0_1} t_{1_1}}$ and the Leader holds the development option $s\left(K^A_{t_{0_1} t_{1_1}} V, K^A_{t_{0_1} t_{1_1}} D, T\right)$. If the Follower's R&D fails with probability $(1 - p^+)$, the Leader's market coefficient will be $K^A_{t_{0_1}}$

and it will receive the development option $s\left(K^A_{t_{0_1}} V, K^A_{t_{0_1}} D, T\right)$. We remark that the development investment is proportional to the market share. Obviously, in the case of the Leader's failure with probability $(1-q)$, its result will be equal to zero. We summarise the Leader's payoff with $n=1$ as the expected value:

$$L_A(V,1) = -R_A + q\left[p^+ s\left(K^A_{t_{0_1}t_{1_1}} V, K^A_{t_{0_1}t_{1_1}} D, T\right) + (1-p^+)\, s\left(K^A_{t_{0_1}} V, K^A_{t_{0_1}} D, T\right)\right]$$

Symmetrically, we derive firm B's payoff as Leader:

$$L_B(V,1) = -R_B + p\left[q^+ s\left(K^B_{t_{0_1}t_{1_1}} V, K^B_{t_{0_1}t_{1_1}} D, T\right) + (1-q^+)\, s\left(K^B_{t_{0_1}} V, K^B_{t_{0_1}} D, T\right)\right]$$

Now we consider the case with $n=2$ experiments. The success probability of Leader A will be q^2 in the case of two successes and $2q(1-q)$ in case of one success and one failure. So, the Follower's success probability is p^{++} or p^{+-}, respectively. The Leader's payoff with two successes is denoted by L^2_A:

$$\begin{aligned} L^2_A = &(p^{++})^2\, s(K^A_{t_{0_2},t_{1_2}} V, K^A_{t_{0_2},t_{1_2}} D, T)\\ &+2p^{++}(1-p^{++})\, s(K^A_{t_{0_2},t_{1_1}} V, K^A_{t_{0_2},t_{1_1}} D, T)\\ &+(1-p^{++})^2\, s(K^A_{t_{0_2}} V, K_{t_{0_2}} D, T)] \end{aligned}$$

The Leader's payoff with one success and one failure denoted by L^1 is:

$$\begin{aligned} L^1_A = &(p^{+-})^2\, s(K^A_{t_{0_1},t_{1_2}} V, K^A_{t_{0_1},t_{1_2}} D, T)\\ &+2p^{+-}(1-p^{+-})\, s(K^A_{t_{0_1},t_{1_1}} V, K^A_{t_{0_1},t_{1_1}} D, T)\\ &+(1-p^{+-})^2\, s(K^A_{t_{0_1}} V, K^A_{t_{0_1}} D, T)] \end{aligned}$$

Obviously, in the case of two failures for the Leader, its payoff result is $L^0 = 0$. Summarising, the Leader's payoff is the expected value

$$L_A(V,2) = -R_A + q^2 L^2_A + 2q(1-q)L^1_A$$

where $R_A = \Psi_A + 2\omega$. Generalising, the Leader's payoff assuming n experiments is:

$$\begin{aligned} L_A(V,n) = &-R_A + \sum_{k=0}^{n} \binom{n}{k} q^k (1-q)^{n-k}\\ &\times \left[\sum_{h=0}^{n} \binom{n}{h} \left(p^{\varepsilon,\theta}\right)^h \left(1-p^{\varepsilon,\theta}\right)^{n-h} s(K^A_{t_{0_k},t_{1_h}} V, K^A_{t_{0_k},t_{1_h}} D, T)\right] \end{aligned} \quad (11)$$

3.4 Follower's payoff

We now focus on the Follower's payoff assuming that firm B (Follower) decides to postpone its R&D investment decision at time t_1 and firm A (Leader) invests at time t_0 assuming that $n = 1$. If the Leader's R&D investment is successful, the Follower's probability of success becomes p^+ and its GMC is $K^B_{t_{1_1}t_{0_1}}$. After the investment R_B, the Follower holds with a probability p^+ and the development option is $s(K^B_{t_{1_1}t_{0_1}}V, K^B_{t_{1_1}t_{0_1}}D, \tau)$. So the Follower's payoff at time t_0 is a CEEO with maturity t_1, exercise price equal to R_B and the underlying asset is the development option $s(K^B_{t_{1_1}t_{0_1}}V, K^B_{t_{1_1}t_{0_1}}D, \tau)$. The CEEO payoff at deadline t_1 with positive information revelation is:

$$c(p^+s(K^B_{t_{1_1}t_{0_1}}V, K^B_{t_{1_1}t_{0_1}}D, \tau), R_B, 0) = \max[p^+s(K^B_{t_{1_1}t_{0_1}}V, K^B_{t_{1_1}t_{0_1}}D, \tau) - R_B, 0]$$

According to Carr (1988) model, we assume that $R_B = \varphi_B D$ is a proportion of asset D. Hence, denoting $c(p^+)$ the CEEO at time t_0, namely

$$c(p^+) = c(p^+s(K^B_{t_{1_1}t_{0_1}}V, K^B_{t_{1_1}t_{0_1}}, \varphi_B D, t_1)$$

we write the value of CEEO with positive information using Eq. (5) as:

$$\begin{aligned} c(p^+) = {} & p^+K^B_{t_{1_1}t_{0_1}}Ve^{-\delta_v T}N_2\left(d_1\left(\frac{P}{P^*_{\text{up}}}, t_1\right), d_1(P,T); \rho\right) \\ & -p^+K^B_{t_{1_1}t_{0_1}}De^{-\delta_d T}N_2\left(d_2\left(\frac{P}{P^*_{\text{up}}}, t_1\right), d_2(P,T); \rho\right) \\ & -\varphi_B De^{-\delta_d t_1}N\left(d_2\left(\frac{P}{P^*_{\text{up}}}, t_1\right)\right) \end{aligned} \tag{12}$$

where P^*_{up} is the critical value that makes the underlying asset of $c(p^+)$ equal to the exercise value. Hence, P^*_{up} solves the following equation:

$$p^+s(K^B_{t_{1_1}t_{0_1}}V, K^B_{t_{1_1}t_{0_1}}D, \tau) = \varphi_B D$$

and assuming asset $K^B_{t_{1_1}t_{0_1}}D$ as numeraire, we can rewrite the above equation as:

$$P^*_{\text{up}}e^{-\delta_v\tau}N(d_1(P^*_{\text{up}}, \tau)) - e^{-\delta_d\tau}N(d_2(P^*_{\text{up}}, \tau)) = \frac{\varphi_B}{p^+K^B_{t_{1_1}t_{0_1}}}$$

Alternatively, in the case the Leader fails, the Follower's success probability changes in p^- and its market coefficient is $K^B_{t_{1_1}}$. The Follower's payoff at time t_0 is a CEEO with maturity t_1, exercise price R_B and underlying asset the development option

$s(K^B_{t_{1_1}}V, K^B_{t_{1_1}}D, \tau)$. Hence, the CEEO payoff at expiration date t_1 with negative information revelation is:

$$c(p^- s(K^B_{t_{1_1}}V, K^B_{t_{1_1}}D, \tau), R_B, 0) = \max[p^- s(K^B_{t_{1_1}}V, K^B_{t_{1_1}}D, \tau) - R_B, 0]$$

Denoting $c(p^-)$ as the CEEO at time t_0 with negative information, i.e.

$$c(p^-) = c(p^- s(K^B_{t_{1_1}}V, K^B_{t_{1_1}}D, \tau), \varphi_B D, t_1)$$

we write, using Eq. (5), the value of CEEO with negative information:

$$\begin{aligned} c(p^-) = {} & p^- K^B_{t_{1_1}} V e^{-\delta_v T} N_2\left(d_1\left(\frac{P}{P^*_{dw}}, t_1\right), d_1(P, T); \rho\right) \\ & - p^- K^B_{t_{1_1}} D e^{-\delta_d T} N_2\left(d_2\left(\frac{P}{P^*_{dw}}, t_1\right), d_2(P, T); \rho\right) \\ & - \varphi_B D e^{-\delta_d t_1} N_1\left(d_2\left(\frac{P}{P^*_{dw}}, t_1\right)\right) \end{aligned} \tag{13}$$

where P^*_{dw} is the critical price that solves the following equation:

$$P^*_{dw} e^{-\delta_v \tau} N(d_1(P^*_{dw}, \tau)) - e^{-\delta_d \tau} N(d_2(P^*_{dw}, \tau)) = \frac{\varphi_B}{p^- K^B_{t_{1_1}}}.$$

The Follower obtains the CEEO $c(p^+)$ in the case that the Leader succeeds with probability q or the CEEO $c(p^-)$ in the case that the Leader fails with probability $(1-q)$. Hence, the Follower's payoff at time t_0 is the expected value:

$$F_B(V, 1) = q\, c(p^+) + (1-q)\, c(p^-) \tag{14}$$

Symmetrically, we derive firm A's payoff when it is Follower:

$$F_A(V, 1) = p\, c(q^+) + (1-p)\, c(q^-) \tag{15}$$

Let us assume $n = 2$. The Follower's probability success becomes p^{++} in the case that both Leader's successes with probability q^2, then the Follower's payoff is:

$$\begin{aligned} F^2_B = {} & c\left((p^{++})^2 s(K^B_{t_{1_2}t_{0_2}}V, K^B_{t_{1_2}t_{0_2}}D, \tau)\right) \\ & + c\left(2p^{++}(1-p^{++}) s(K^B_{t_{1_1}t_{0_2}}V, K^B_{t_{1_1}t_{0_2}}D, \tau)\right) \end{aligned}$$

In the case of one success and one failure of the Leader with probability $2q(1-q)$, the Follower's payoff is:

$$F_B^1 = c\left((p^{+-})^2 s(K^B_{t_{1_2}t_{0_1}} V, K^B_{t_{1_2}t_{0_1}} D, \tau)\right) \\ +c\left(2p^{+-}(1-p^{+-}) s(K^B_{t_{1_1}t_{0_1}} V, K^B_{t_{1_1}t_{0_1}} D, \tau)\right)$$

and in the case of the Leader failing both times with probability $(1-q)^2$, the Follower's payoff is:

$$F_B^0 = c\left((p^{--})^2 s(K^B_{t_{1_2}} V, K^B_{t_{1_2}} D, \tau)\right) + c\left(2p^{--}(1-p^{--}) s(K^B_{t_{1_1}} V, K^B_{t_{1_1}} D, \tau)\right)$$

Summarising, the Follower's payoff is the expected value:

$$F_B(V, 2) = q^2 F_B^2 + 2q(1-q)F_B^1 + (1-q)^2 F_B^0 \tag{16}$$

Generalising, if we assume to have n experiments, the Follower's payoff (firm B) results:

$$F_B(V, n) = \sum_{k=0}^{n} \binom{n}{k} q^k (1-q)^{n-k} \\ \times \sum_{h=0}^{n} c\left(\binom{n}{h} \left(p^{\varepsilon,\theta}\right)^h \left(1-p^{\varepsilon,\theta}\right)^{n-h} s(K^B_{t_{1_h}t_{0_k}} V, K^B_{t_{1_h}t_{0_k}} D, \tau), \varphi_B D, t_1\right) \tag{17}$$

3.5 Simultaneous investment payoff

We suppose that both firms invest in R&D at time t_0. There is no information revelation since the investment is simultaneous, but players can benefit from network externalities. Let us set $n = 1$. Assuming the success of firm B, firm A receives the development option with a GMC $K^A_{t_{0_1}t_{0_1}}$ in case of its R&D success; instead, in the case of failure of firm B, firm A receives the development option with a GMC $K^A_{t_{0_1}}$. Hence, firm A's payoff in the case of simultaneous investment is the expected value:

$$S_A(V, 1) = -R_A + q\left[p\, s\left(K^A_{t_{0_1}t_{0_1}} V, K^A_{t_{0_1}t_{0_1}} D, T\right) + (1-p)\, s\left(K^A_{t_{0_1}} V, K^A_{t_{0_1}} D, T\right)\right]$$

Symmetrically, firm B's payoff is:

$$S_B(V, 1) = -R_B + p\left[q\, (s\left(K^B_{t_{0_1}t_{0_1}} V, K^B_{t_{0_1}t_{0_1}} D, T\right) + (1-q)\, s\left(K^B_{t_{0_1}} V, K^B_{t_{0_1}} D, T\right)\right]$$

Generalising, if we assume that the number of experiments is n, it results:

$$S_A(V, n) = -R_A + \sum_{k=0}^{n} \binom{n}{k} q^k (1-q)^{n-k} \\ \times \left[\sum_{h=0}^{n} \binom{n}{h} p^h (1-p)^{n-h}\, s\left(K^A_{t_{0_k}t_{0_h}} V, K^A_{t_{0_k}t_{0_h}} D, T\right)\right] \tag{18}$$

Symmetrically, we can determine firm B's payoff in the case of simultaneous investment.

3.6 Waiting investment payoff

We suppose that both firms decide to delay their R&D investment decision at time t_1 and we assume that there is no information revelation. Also in this case, firstly we assume $n = 1$. Considering the R&D success of firm B, the GMC of firm A will be $K^A_{t_{1_1}t_{1_1}}$. So, after the investment R_A at time t_1, firm A holds with probability q the development option $s(K^A_{t_{1_1}t_{1_1}}V, K^A_{t_{1_1}t_{1_1}}D, \tau)$. Then, firm A's payoff at time t_0 is a CEEO with maturity t_1, exercise price R_A and the underlying asset $s(K^A_{t_{1_1}t_{1_1}}V, K^A_{t_{1_1}t_{1_1}}D, \tau)$ with probability q. According to Carr (1988)'s model and assuming that $R_A = \varphi_A D$, the CEEO in the case of firm B's success is $c(q\,s(K^A_{t_{1_1}t_{1_1}})) = c\left(q\,s(K^A_{t_{1_1}t_{1_1}}V, K^A_{t_{1_1}t_{1_1}}D, \tau), \varphi_A D, t_1\right)$ and specifically:

$$
\begin{aligned}
c(q\,s(K^A_{t_{1_1}t_{1_1}})) = K^A_{t_{1_1}t_{1_1}} V e^{-\delta_v T} N_2\left(d_1\left(\frac{P}{P^*_{ws}}, t_1\right), d_1(P, T); \rho\right) \\
-qK^A_{t_{1_1}t_{1_1}} D e^{-\delta_d T} N_2\left(d_2\left(\frac{P}{P^*_{ws}}, t_1\right), d_2(P, T); \rho\right) \\
-\varphi_A D e^{-\delta_d t_1} N\left(d_2\left(\frac{P}{P^*_{ws}}, t_1\right)\right)
\end{aligned}
\tag{19}
$$

where P^*_{ws} is the critical value that solves the following equation:

$$
\begin{aligned}
&q\,s\left(K^A_{t_{1_1}t_{1_1}}V, K^A_{t_{1_1}t_{1_1}}D, \tau\right) = \varphi_A D \iff \\
&P^*_{ws} e^{-\delta_v \tau} N(d_1(P^*_{ws}, \tau)) - e^{-\delta_d \tau} N(d_2(P^*_{ws}, \tau)) = \frac{\varphi_A}{qK_{t_{1_1}t_{1_1}}}
\end{aligned}
$$

In the case of firm B's failure, firm A's GMC will be $K^A_{t_{1_1}}$. After the investment R_A at time t_1, A obtains the development option $s(K^A_{t_{1_1}}V, K^A_{t_{1_1}}D, \tau)$ with probability q. Thus, firm A's payoff at time t_0 is a CEEO where the underlying asset is $s(K^A_{t_{1_1}}V, K^A_{t_{1_1}}D, \tau)$ with probability q that we denote as $c\left(q\,s(K^A_{t_{1_1}}V)\right) = c\left(q\,s(K^A_{t_{1_1}}V, K^A_{t_{1_1}}D, \tau), \varphi_A D, t_1\right)$, i.e.:

$$
\begin{aligned}
c\left(q\,s(K^A_{t_{1_1}}V)\right) = qK^A_{t_{1_1}} V e^{-\delta_v T} N_2\left(d_1\left(\frac{P}{P^*_{wf}}, t_1\right), d_1(P, T); \rho\right) \\
-qK^A_{t_{1_1}} D e^{-\delta_d T} N_2\left(d_2\left(\frac{P}{P^*_{wf}}, t_1\right), d_2(P, T); \rho\right)
\end{aligned}
$$

$$-\varphi_A D e^{-\delta_d t_1} N\left(d_2\left(\frac{P}{P^*_{wf}}, t_1\right)\right) \tag{20}$$

where P^*_{wf} is the critical value that solves the following equation:

$$q \cdot s\left(K^A_{t_{1_1}} V, K^A_{t_{1_1}} D, \tau\right) = \varphi_A D \quad \Longleftrightarrow$$
$$P^*_{wf} e^{-\delta_v \tau} N(d_1(P^*_{wf}, \tau)) - e^{-\delta_d \tau} N(d_2(P^*_{wf}, \tau)) = \frac{\varphi_A}{q K^A_{t_{1_1}}}$$

Hence, firm A's payoff is the expected value:

$$W_A(V, 1) = p\, c\left(q\, s(K^A_{t_{1_1} t_{1_1}} V, K^A_{t_{1_1} t_{1_1}} D, \tau), \varphi_A D, t_1\right)$$
$$+(1-p)\, c\left(q\, s(K^A_{t_{1_1}} V, K^A_{t_{1_1}} D, \tau), \varphi_A D, t_1\right)$$

Similarly, we determine firm B's payoff.

Generalising, the waiting payoff of firm A with n experiments is:

$$W_A(V, n) = \sum_{h=0}^{n} \binom{n}{h} p^h (1-p)^{n-h}$$
$$\times \sum_{k=0}^{n} c\left[\binom{n}{k} q^k (1-q)^{n-k} s(K^A_{t_{1_k} t_{1_h}} V, K^A_{t_{1_k} t_{1_h}} D, \tau), \varphi_A D, t_1\right] \tag{21}$$

4 Competition and cooperative scenario

We investigate the situation in which firms decide to compete using the information revelation or to sign a strategic alliance that allows them to use the information $\rho_{\max}$. Figure 2 shows the bimatrix of firm A and B's payoffs with respect to the immediate or waiting investment strategies in the case of competition. We solve the game computing the Nash equilibria. In the competitive scenario, payoff values were computed in the previous section. The following propositions help us to determine the Nash equilibria N_A and N_B.

Proposition 1 *There exists a unique critical market value V^*_{Wi} such that $L_i(V^*_{Wi}, n) = W_i(V^*_{Wi}, n)$. It results that $L_i(V, n) < W_i(V, n)$ when $V < V^*_{Wi}$ and $L_i(V, n) > W_i(V, n)$ when $V > V^*_{Wi}$ for $i = A, B$, respectively.*

Proposition 2 *If $\frac{\partial S_i}{\partial V} > \frac{\partial F_i}{\partial V}$, then there exists a unique critical market value V^*_{Fi} such that $S_i(V^*_{Fi}, n) = F_i(V^*_{Fi}, n)$. It results that $S_i(V, n) < F_i(V, n)$ when $V < V^*_{Fi}$ and $S_i(V, n) > F_i(V, n)$ when $V > V^*_{Fi}$ for $i = A, B$, respectively. Otherwise, if $\frac{\partial S_i}{\partial V} < \frac{\partial F_i}{\partial V}$, then $S_i(V, n) < F_i(V, n)$ for each value of V.*

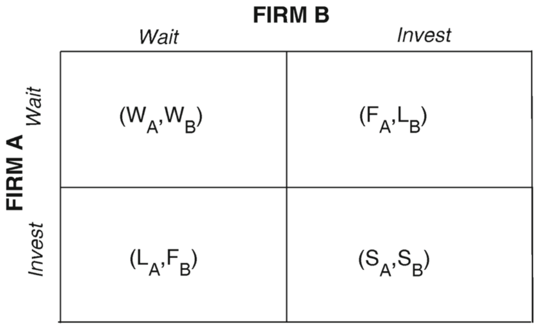

Fig. 2 Bimatrix payoffs in the competitive case

Proofs are illustrated in "Appendix".

In the case of cooperation, the surplus is provided by the difference between the cooperation value $C(A \cup B) = \max[W_C, LF_C, FL_C, S_C]$ and the Nash equilibrium when A and B compete. There are four cooperation strategies: both players decide to delay their investment at time t_1, then $W_C(V, n) = W_A^C(V, n) + W_B^C(V, n)$; firm A invests at time t_0 while firm B postpones its decision at time t_1, then $LF_C(V, n) = L_A^C(V, n) + F_B^C(V, n)$; symmetrically, firm B invests at time t_0 and A delays its decision at time t_1, then $FL_C(V, n) = F_A^C(V, n) + L_B^C(V, n)$; finally both firms decide to invest simultaneously at time t_0, then $S_C(V, n) = S_A^C(V, n) + S_B^C(V, n)$. The surplus of cooperation is split according to success probabilities, and so the cooperative payoffs are:

$$\text{Coop}_A = N_A + \frac{q}{p+q}\left[C(A \cup B) - (N_A + N_B)\right]; \tag{22}$$

$$\text{Coop}_B = N_B + \frac{p}{p+q}\left[C(A \cup B) - (N_A + N_B)\right]; \tag{23}$$

5 Numerical applications

In order to support our approach, we present a case study, i.e. a collaboration between Volkswagen (firm A) and Ford (firm B) in order to form Argo, i.e. the joint venture R&D that deals with self-driving cars.[3] The agreement provides a research investment of approximately $\Psi_A = 600$ million dollars for Volkswagen and $\Psi_B = 1.1$ billion for Ford, $\omega = 800$ million dollars for tests related to self-driving cars, and the development cost is quantified at $D = 7$ billion dollars. Moreover, the deadline to commercialise these innovations is $T = 7$ years and the delayed time to realise R&D investment is $t_1 = 3$ years. To value σ_v and σ_d, we assume as proxy the volatilities of the automotive and technology-sensorial sector, respectively. So we have $\sigma_v = 0.67$, $\sigma_d = 0.54$ with a

[3] This is an illustrative example of our model. The data have been taken from the Ford and Volkswagen website and refer to the year 2018 (for more details https://media.ford.com and https://www.volkswagengroup.it/eng/media). The growth market innovation coefficients $\alpha_{i,1}$ and $\alpha_{i,2}$ are provided by Allied Market Research. The coefficients β_i were determined by the ratio between the impacts of R&D investments on profits.

Table 1 Evolution of success probabilities with $\rho_1 = 0.30$, $\rho_2 = 0.2303$ and $\rho_3 = 0.1869$

$q = 0.2091$	$q^{+} = 0.4296$	$q^{++} = 0.5480$	$q^{+++} = 0.6217$
	$q^{-} = 0.1416$	$q^{+-} = 0.3197$	$q^{++-} = 0.4307$
		$q^{--} = 0.1054$	$q^{+--} = 0.2513$
			$q^{---} = 0.0828$
$p = 0.2343$	$p^{+} = 0.4814$	$p^{++} = 0.6141$	$p^{+++} = 0.6967$
	$p^{-} = 0.1690$	$p^{+-} = 0.3815$	$p^{++-} = 0.5139$
		$p^{--} = 0.1339$	$p^{+--} = 0.3193$
			$p^{---} = 0.1120$

correlation coefficient $\rho_{vd} = 0.15$. We assume a level of non-cooperative information revelation $\rho_1 = 0.30$ and dividend yields $\delta_v = 0.15$, $\delta_d = 0$. The efficiency level of R&D investments is $\beta_A = 0.7076$ for Volkswagen and $\beta_B = 0.4342$ for Ford. Finally, the growth market innovation is assumed as $\alpha_{A,1} = 0.40$, $\alpha_{A,2} = 0.05$, $\alpha_{B,1} = 0.40$ and $\alpha_{B,2} = 0.05$.

Based on formula (6), we determine that the initial success probabilities of firm A is $q = 0.2091$, B is $p = 0.2343$ and the maximum information revelation is $\rho_{\max} = 0.9294$. The evolution of success probabilities is illustrated in Table 1.

Let us assume $n = 1$. Then, the non-cooperative critical market values are:

$$V^*_{WA}(1) = 42,770\,\$;\quad V^*_{WB}(1) = 50,470\,\$;$$
$$V^*_{FA}(1) = 40,455\,\$;\quad V^*_{FB}(1) = 47,950\$$$

Using Propositions 1 and 2, if $V < 42,770$ \$ then the waiting policy (W_A, W_B) is the Nash equilibrium; when $42,770\ \$ < V < 47,950\ \$$ the Nash equilibrium is (L_A, F_B), if $V > 47,950\,\$$ we realise the Nash equilibrium (S_A, S_B). In case of a cooperation between firms A and B, Fig. 3 shows that the optimal Pareto strategy is W_C for $V < 34,750\,\$$, LF_C when $34,750\,\$ < V < 62,600\,\$$ and when $V > 62,600\,\$$ the optimal Pareto payoff is S_C.

Table 2 contains the cooperative payoff when the market value changes and the bold values denote the maximum joint payoff. Another important consideration about the optimal cooperation strategy is that the firm with better success probability invests at time t_0 and the other postpones its research investment at time t_1 benefiting from the maximum information revelation, namely LF_C.

Let's assume $n = 2$. For our numerically adapted simulations, we have the non-cooperative critical market values as:

$$V^*_{WA}(2) = 31{,}518\,\$;\quad V^*_{WB}(2) = 34{,}090\,\$;$$
$$V^*_{FA}(2) = 28{,}315\,\$;\quad V^*_{FB}(2) = 31{,}236\,\$$$

From Propositions 1 and 2, we see that the waiting policy (W_A, W_B) is a Nash equilibrium when $V < 31{,}236\,\$$. If $31{,}236\,\$ < V < 31{,}518\$$ two Nash equilibria exist, i.e. (W_A, W_B) and (S_A, S_B). Finally, if $V > 31{,}518\,\$$, we have a simultaneous equi-

Fig. 3 Cooperation payoffs with $n = 1$

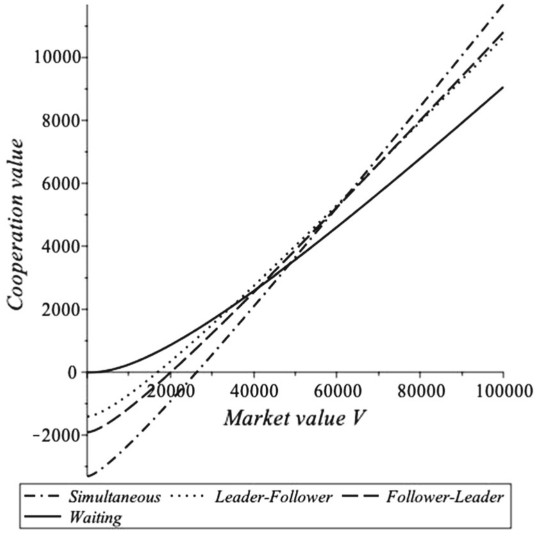

Table 2 Cooperative payoffs with $n = 1$ experiment

Market V	LF_C	FL_C	S_C	W_C	$N(A)$	$N(B)$	Coop_A	Coop_B
20,000	350	−2	−913	**871**	432	439	432	439
40,000	**2748**	2545	2588	2093	1264	1324	**1339**	**1408**
60,000	**5308**	5240	4612	5235	2631	2603	**2665**	**2642**
80,000	7949	8003	**8442**	6791	4146	4296	4146	4296
100,000	10,638	10,804	**11,687**	9063	5678	6008	5678	6008

librium (S_A, S_B). We observe that critical market values decrease with respect to the previous scenario $n = 1$. One explanation is that the increase in information revelation with the two experiments induces firms to invest at a lower market value than in the previous case, i.e. $V > 31{,}236\,\$$. About the cooperative strategy, as depicted in Fig. 4a, it results in $C(A \cup B) = W_C$ if $V < 20{,}340\,\$$, $C(A \cup B) = LF_C$ if $20{,}340\,\$ < V < 94{,}700\,\$$ and $C(A \cup B) = S_C$ when $V > 94{,}700\,\$$. The novelty in this case with only one experiment is that the range in which the strategy LF_C is optimal increases. Table 3 summarises this scenario assuming different market values.

Finally, we analyse the scenario with $n = 3$ experiments. Using the same procedure, we obtain:

$$V^*_{WA}(3) = 27{,}440\,\$; \quad V^*_{WB}(3) = 28{,}367\,\$;$$
$$V^*_{FA}(3) = 24{,}760\,\$; \quad V^*_{FB}(3) = 26{,}490\,\$$$

Following Propositions 1 and 2, the waiting policy (W_A, W_B) is a Nash equilibrium when $V < 26{,}490\,\$$. If $26{,}490\,\$ < V < 27{,}440\,\$$, then two Nash equilibria exist, i.e. (W_A, W_B) and (S_A, S_B). Finally, if $V > 27{,}440\,\$$, we have the simultaneous

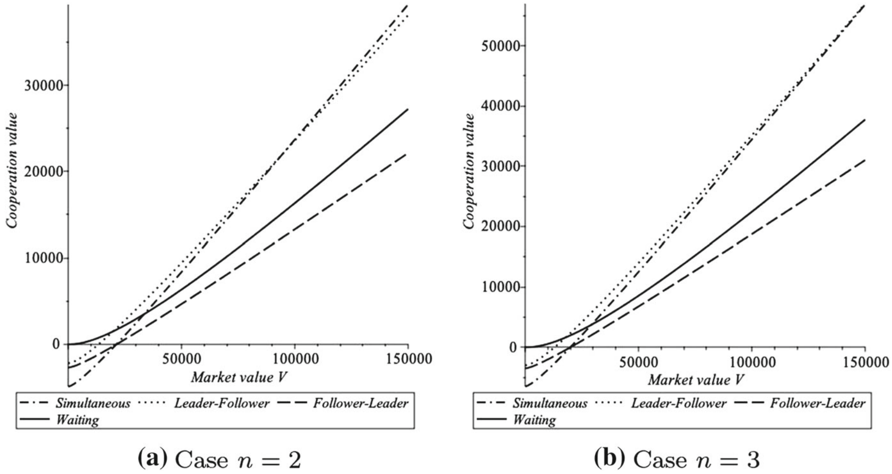

Fig. 4 Cooperative payoffs

Table 3 Cooperative payoffs with $n = 2$ experiments

Market V	LF_C	FL_C	S_C	W_C	$N(a)$	$N(b)$	Coop_A	Coop_B
10,000	− 725	−1660	−2974	**439**	218	220	218	220
20,000	1375	−206	−344	**1522**	749	772	749	772
40,000	**6709**	2999	5394	4572	2688	2706	**3308**	**3400**
60,000	**12,204**	6367	11,392	8215	5536	5856	**5918**	**6285**
80,000	**17,852**	9812	17,513	12,177	8443	9070	**8602**	**9249**
100,000	23,586	13,302	**23,707**	16,337	11,384	12,323	11,384	12,323

Table 4 Cooperative payoffs with $n = 3$ experiments

Market V	LF_C	FL_C	S_C	W_C	$N(a)$	$N(b)$	Coop_A	Coop_B
10,000	−912	−2029	−3744	**551**	274	276	274	276
20,000	**2368**	−2	19	1977	974	1 003	**1158**	**1209**
40,000	**9951**	4446	8231	6109	4026	4204	**4837**	**5113**
80,000	**26,535**	13,895	25,572	16,628	12,297	13,275	**12,751**	**13,783**
100,000	**35,123**	18,738	34,435	22,436	16,524	17,910	**16,848**	**18,274**
180,000	70,272	38,444	**70,404**	47,274	33,680	36,723	33,680	36,723

equilibrium (S_A, S_B). Moreover, as is illustrated in Fig. 4b, it results that if $V < 17{,}865\,\$$, then $C(A \cup B) = W_C$, if $17{,}865\,\$ < V < 164{,}550\,\$$, then $C(A \cup B) = LF_C$, and when $V > 164{,}550\,\$$, then $C(A \cup B) = S_C$. Table 4 lists several strategy values when $n = 3$.

Summarising, we observe that as the number of tests increases, the market value threshold for which investment is profitable decreases. Unlike the competitive case, the

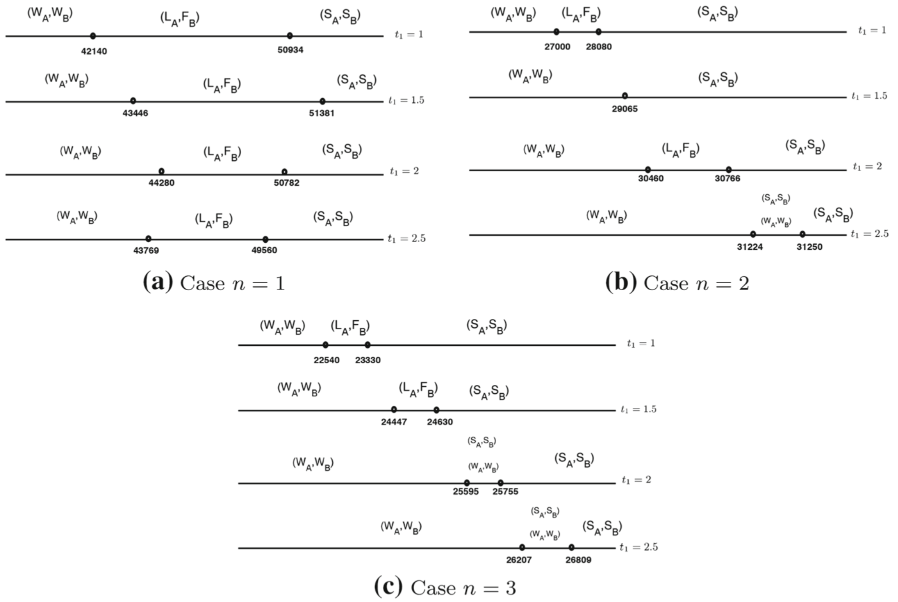

Fig. 5 Competitive Nash Equilibria assuming $\rho = 0.30$ when the delayed time t_1 changes

creation of Argo joint venture resulting from the collaboration between Volkswagen and Ford increases the value of their investment and also suggests a different strategy. While in the competitive case generally the Nash equilibrium is the waiting policy or simultaneous investment, in the cooperative case the LF_C strategy is more convenient, i.e. Volkswagen makes the R&D investment at t_0 and transmits the information to Ford for the realisation of its investment at t_1. Moreover, the range for which the LF_C strategy is optimal increases when the number of tests goes up.

6 Sensitivity analysis

In this section, we study the effects that the delayed time t_1 and the information revelation produced on the equilibria of the game. Assuming the same parameters in the previous section, we change the delayed time assuming several values between $t_1 = 1$ year and $t_1 = 2.5$ years. Figure 5 shows how the Nash equilibria change as the delayed time t_1 varies if the experiments carried out are $n = 1$, $n = 2$ and $n = 3$. In particular, by analysing Fig. 5a, we remark how in the scenario with one test, the market region in which the (L_A, F_B) strategy is a Nash equilibrium is reduced when the postponement time t_1 increases, but is the best replay for medium market values. When n improves, the strategy set and its elements make significant changes. In fact, comparing Fig. 5b, c, when the firm with the lowest probability of success (Volkswagen) invests first, then the other firm (Ford) does not appear if $t_1 = 2.5$ and begins to leave the optimal strategy for medium–high market values. In addition, when t_1 increases, (W_A, W_B) and (S_A, S_B) play more and more crucial roles in the optimal

Table 5 Cooperative value $C(A \cup B)$ when the delayed time t_1 varies

Delayed time	W_C	LF_C	FL_C	S_C
$n = 1$				
$t_1 = 1$	$V < 22{,}105$	$22{,}105 < V < 58{,}450$	$58{,}450 < V < 142{,}800$	$V > 142{,}800$
$t_1 = 1.5$	$V < 26{,}035$	$26{,}035 < V < 62{,}325$	$62{,}325 < V < 121{,}435$	$V > 121{,}435$
$t_1 = 2$	$V < 29{,}765$	$29{,}765 < V < 66{,}470$	$66{,}470 < V < 88{,}035$	$V > 88{,}035$
$t_1 = 2.5$	$V < 32{,}842$	$32{,}842 < V < 70{,}268$	$70{,}268 < V < 70{,}298$	$V > 70{,}298$
$t_1 = 3$	$V < 34{,}750$	$34{,}750 < V < 62{,}600$		$V > 62{,}600$
$n = 2$				
$t_1 = 1$	$V < 16{,}200$	$V > 16{,}200$		
$t_1 = 1.5$	$V < 17{,}526$	$V > 17{,}526$		
$t_1 = 2$	$V < 18{,}875$	$18{,}875 < V < 92{,}745$		$V > 92{,}745$
$t_1 = 2.5$	$V < 20{,}192$	$20{,}192 < V < 93{,}857$		$V > 93{,}857$
$t_1 = 3$	$V < 20{,}340$	$20{,}340 < V < 94{,}700$		$V > 94{,}700$
$n = 3$				
$t_1 = 1$	$V < 14{,}533$	$V > 14{,}533$		
$t_1 = 1.5$	$V < 15{,}267$	$V > 15{,}267$		
$t_1 = 2$	$V < 16{,}103$	$V > 16{,}103$		
$t_1 = 2.5$	$V < 16{,}984$	$V > 16{,}984$		
$t_1 = 3$	$V < 17{,}865$	$17{,}865 < V < 164{,}550$		$V > 164{,}550$

strategy set. This means that the benefits of information revelation are less than the first mover advantages and they decrease over time. Therefore, the increase in the number of tests and the delayed time encourages firms to wait or invest simultaneously.

Surprisingly, the set of optimal strategies changes profoundly if we analyse the cooperative aspect. As is illustrated in Table 5, in the case of $n = 1$, the optimal strategy is to wait for low market values, LF_C for low–medium values, FL_C for medium–high values, and finally, the simultaneous investment S_C is optimal for high market values. We observe how the FL_C strategy disappears after $t_1 = 2.5$. We can see how the advantage of waiting to invest W_C increases over time but decreases with increasing number of tests. In this way, when the number of tests increases, the two firms may consider investing at different times for lower market threshold values in order to attain the greatest benefits of information revelation. The best cooperative strategy with a differentiated investment is LF_C, if the tests carried out are greater than one.

Finally, Fig. 6 depicts the equilibria when the information revelation changes. Also in this case, the number of experiments plays an important role. We observe in Fig. 6a that, in the competitive case, when the information revelation ρ_1 increases, then the Nash equilibrium (L_A, F_B) occurs for broader market values. This market range is the largest in the cooperation case when the strategy LF_C occurs for the ρ_{max} value. But, when the number of tests increases, as is illustrated in Fig. 6b, c, the (L_A, F_B) equilibrium disappears for low information revelation intensities. In this case, both players will prefer to wait or to invest simultaneously. The (L_A, F_A) strategy appears

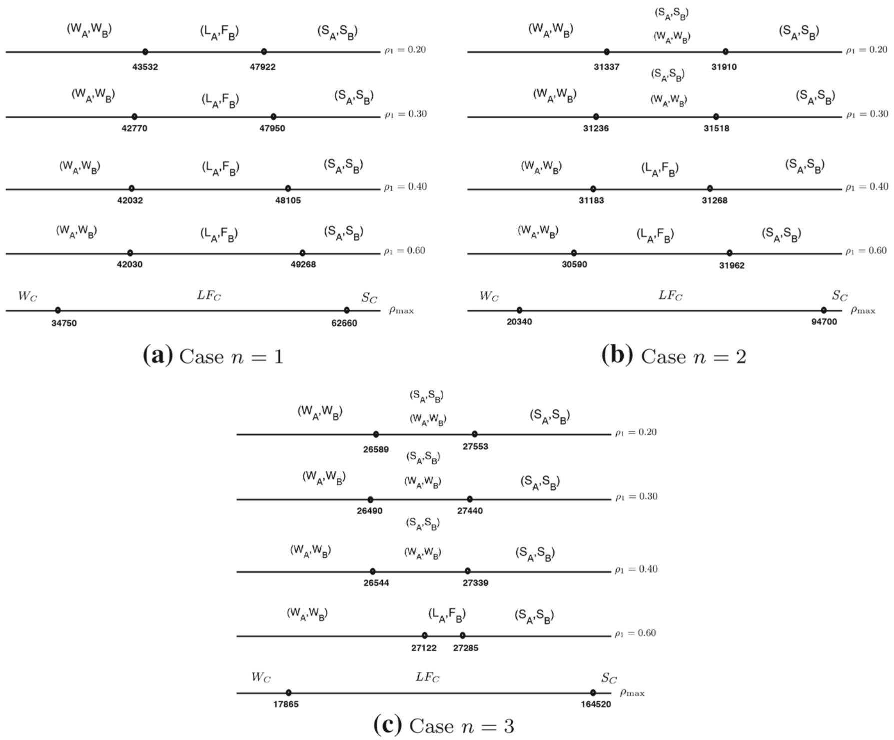

Fig. 6 Competitive Nash equilibria and cooperative value assuming $t_1 = 3$ when the information revelation ρ_1 changes

when $\rho_1 = 0.40$ with $n = 2$, and when $\rho_1 = 0.60$ with $n = 3$. So, this means that the increase in the number of tests makes the strategy with differentiated investments preferable when the information revelation becomes considerable. However, in the cooperation case, the LF_C strategy is always significant and the increase in the number of tests widens the market values for which this strategy fits best.

7 Concluding remarks

In our paper, we proposed a real option game between two firms investing in R&D with multiple experiments. We contemplated two scenarios in which firms can compete or cooperate. In both cases, four strategies can be achieved: the waiting strategy in which each firm postpones its investment at time t_1 waiting for better market conditions; the simultaneous strategy in which both players realise their R&D investments at initial time t_0; the Leader and Follower strategies in which the Leader realises the investment at time t_0 benefiting from a first mover advantage while the Follower delays its investment to time t_1 obtaining an information revelation. In particular, we have considered that, in the case of a cooperation, firms completely internalise the overall information revelation. We have carried out our analysis supposing that firms can realise several experiments and, in the case of success, the growth market coefficients go up and the Follower's success probability improves.

We proposed some numerical simulations starting from a case study that described the agreement between Volkswagen and Ford collaborating to form Argo joint venture. In a particular way, we have underlined the role of the number of tests realised. We observed, in the competitive case, that the Nash equilibria are represented by waiting or simultaneous strategies when the tests number is more than one. Otherwise, in the cooperative case, the strategy with a differentiated investment over time (Leader–Follower) may occur. We completed our analysis with a study of the effects that delayed time and information revelation produce on the equilibria of the game.

Appendix

Proof of Proposition 1 We analyse the strategic payoffs assuming as variable the market value V. We can observe that:

$$L_A(0,n) = -R_A; \quad W_A(0,n) = 0;$$
$$\frac{\partial L_A}{\partial V} = N(d_1(P,T))\, e^{-\delta_v T} \sum_{k=0}^{n} \binom{n}{k} q^k (1-q)^{n-k} \left[\sum_{h=0}^{n} \binom{n}{h} \left(p^{\varepsilon,\theta}\right)^h \left((1-p^{\varepsilon,\theta}\right)^{n-h} K_{t_{0_k} t_{1_h}} \right]$$
$$\frac{\partial W_A}{\partial V} = N_2\left(d_1\left(\frac{P}{P_w^*}, t_1\right), d_1(P,T); \rho\right) e^{-\delta_v T} \times \sum_{h=0}^{n} \binom{n}{h} p^h (1-p)^{n-h} \left[\sum_{k=0}^{n} \binom{n}{k} q^k (1-q)^{n-k} K_{t_{1_k} t_{1_h}} \right]$$

As $N(d_1(P,T)) > N_2\left(d_1\left(\frac{P}{P_w^*}, t_1\right), d_1(P,T); \rho\right)$, moreover, $p^{\varepsilon,\theta} > p$ and $K_{t_{0_k} t_{1_h}} > K_{t_{1_k} t_{1_h}}$, it results that $\frac{\partial L_A}{\partial V} > \frac{\partial W_A}{\partial V} > 0$. This condition assures us a unique critical market value $V^*_{A,W}$. The same results occur for firm B. □

Proof of Proposition 2 We observe that:

$$S_A(0,n) = -R_A; \quad F_A(0,n) = 0;$$
$$\frac{\partial S_A}{\partial V} = N(d_1(P,T))\, e^{-\delta_v T} \sum_{k=0}^{n} \binom{n}{k} q^k (1-q)^{n-k} \left[\sum_{h=0}^{n} \binom{n}{h} p^h (1-p)^{n-h} K^A_{t_{0_k} t_{0_h}} \right]$$
$$\frac{\partial F_A}{\partial V} = N_2\left(d_1\left(\frac{P}{P^*}, t_1\right), d_1(P,T); \rho\right) e^{-\delta_v T}$$

$$\times \sum_{h=0}^{n} \binom{n}{h} p^h (1-p)^{n-h} \left[\sum_{k=0}^{n} \binom{n}{k} \left(q^{j,y}\right)^k \left(1-q^{j,y}\right)^{n-k} K^A_{t_{1_k} t_{0_h}} \right]$$

with $\frac{\partial F_A}{\partial V} > 0$ and $\frac{\partial S_A}{\partial V} > 0$ and $\frac{\partial F_A}{\partial V} \gtrless \frac{\partial S_A}{\partial V}$. The same results occurs for firm B. □

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