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REGULAR ARTICLE

Self-organized criticality in a herd behavior model of financial markets

Makoto Nirei

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Abstract This paper explains the fat-tail distribution of asset transaction volumes and prices by a model of rational herd behavior of traders. Each trader decides whether to buy an asset by observing private information and other traders' actions. A trader's buying action reveals his positive private information and affects the other traders' beliefs in favor of buying, leading to strategic complementarity. A power-law distribution emerges for the number of buying actions in a static Nash equilibrium. This model provides an economic reason as to why the stock market has to exhibit a criticality in the connectivity of the traders' actions.

Keywords Power law · Trading volume · Stock return · Herd behavior · Self-organized criticality

JEL Classification G14

1 Introduction

Price of risky asset reflects collective belief of traders on the value of assets. The efficiency of market obtains when the traders inadvertently reveal their private information through their self-interested transactions. This Hayekian view of market as an

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M. Nirei (✉)
Department of Economics, Carleton University, 1125 Colonel By Drive,
Ottawa, ON K1S 5B6, Canada
e-mail: makoto_nirei@carleton.ca

information processor inevitably involves a process of the price gradually converging to the true value. During this process, traders' actions and revealed beliefs influence with each other. This kind of interdependence across traders does not warrant simple aggregation of information which would be characterized by the central limit theorem. The present paper considers the deviation from the normal distribution as a significant source of fluctuations in financial markets.

Short-run stock returns show a fat-tailed distribution as first noted by Mandelbrot (1963) and Fama (1965). The power-law tail index was estimated to be in the range of 3–5 by Jansen and de Vries (1991), which indicates a large deviation from the normal distribution in the kurtosis. The estimate is consistent with more recent reports from statistical physicists who attempt to characterize the distribution in more precision and broader dimensions. Bouchaud and Potters (2000) and Mantegna and Stanley (2000) provide a review of this literature. Aoki and Yoshikawa (2006) argue that the power-law distribution of the stock returns is the key to understand the risk premium puzzle in the consumption-based asset pricing models.

The empirical regularities of the fat tail and the leptokurtosis of the stock returns have been successfully accounted for by an array of statistical models such as a subordinated process (Clark 1973), ARCH (Stein and Stein 1991), and a truncated Levy process (Mantegna and Stanley 2000). However, it is relatively unknown what kind of economic mechanism generates such regularities. The quest for such mechanism has motivated the growing literature of agent-based simulations in finance (LeBaron et al. 1999; Lux and Marchesi 1999). A recent paper by Gabaix et al. (2003, 2006) pursued a different avenue in which the power-law tail of price fluctuations reflects another power law of the trader size, and successfully matched a broad set of fat-tail regularities in the price, volume, and trader size.

More traditional economic explanations rely on the imitative behavior of traders. The models of herd behavior laid out theoretical foundation of the rational imitative behavior (Scharfstein and Stein 1990; Banerjee 1992; Bikhchandani et al. 1992). The herd behavior model has inspired attempts to reproduce the power law in price fluctuations by employing the methodology used for critical phenomena (Bak et al. 1997; Cont and Bouchaud 2000; Stauffer and Sornette 1999). These models provide stimulus to the existing studies of financial fluctuations by showing how a generic model of critical phenomena can robustly produce a sharp structure such as the power law.

The new models, however, lack the rich structure of traders' purposeful behavior and rational learning which the original herding models had. The lack of purposeful behavior per se may not be a drawback: at the expense of hindering the integration with the existing body of financial studies, the new models illuminate the role of interaction rather than individual behavior behind the market fluctuations. The more important question on the critical phenomena models is as to why at all the market has to exhibit criticality. The power-law fluctuation occurs typically only at the critical point of a parameter which governs the connectivity of the networked traders. In this paper, we propose that the market necessarily converges to this critical point as a result of the purposeful behavior of individual traders who learn information from each other.

To translate the aggregate action of traders to stock returns, we use an empirical volume–price impact function estimated by Lillo et al. (2003). We show that our model can account for the fat-tail distribution of the stock returns. The fat-tail distribution of returns implies that the process of asset price toward its true value can deviate greatly from the path in which the private information is fully revealed. Consider a situation where traders receive the private information repeatedly over time. The price will reflect the true value in the long run, since the private information accumulates indefinitely. During the process however, the path of actions of the informed traders experiences large volatility. With the fat-tail distribution, a sizable portion of the price adjustment to the true value is accounted for by events of synchronized actions of a large number of traders.

2 Model

The model draws on Nirei (2007). Let us consider a game with N traders of an asset which is worth 1 in the state of economy H (High) and worth 0 in L (Low). Trader i receives a private information x_i on the value of the asset. x_i is an i.i.d. random variable drawn from distribution F in state H and from G in state L. Trader i can choose to buy the asset ($a_i = 1$) for a constant cost $\bar{b}$ or not to buy ($a_i = 0$). The action profile is denoted as $a = (a_1, a_2, \ldots, a_N)$.

Traders can observe the action profile a but do not observe the other traders' private information.[1] Trader i rationally updates their belief according to the Bayes' rule upon the observation of (x_i, a) and obtains the posterior belief $b_{i,1}$, where the belief $b_{i,t}$, $t = 0, 1$, is defined as the subjective probability for state H to realize. Traders are risk-neutral and maximize their subjective expected payoff. Hence, trader i buys the asset if $b_{i,1} \geq \bar{b}$ and does not buy otherwise.

We assume that the prior belief is common across traders: $b_{i,0} = b_0$. We also assume the monotone likelihood ratio property (MLRP): the odds function $\delta(x_i) = g(x_i)/f(x_i)$ is strictly decreasing where f and g are density functions of F and G respectively. The decreasing δ implies that a higher x_i results in a lower posterior probability for state L (see Milgrom 1981). We study a static Nash equilibrium a upon the arrival of $x = (x_1, x_2, \ldots, x_N)$.

The Bayes' rule is equivalently expressed by the update of a likelihood ratio as follows:

$$\theta_{i,1} = \frac{\Pr(x_i, a_{-i} \mid L)}{\Pr(x_i, a_{-i} \mid H)} \theta_{i,0}. \tag{1}$$

Here, a_{-i} is the action profile other than a_i. Since x_i delivers a finer information than a_i, the information revealed by a_i is omitted by trader i. Note that the belief of i can be dependent on a_i indirectly, because the information revealed by a_j, $j \neq i$, depends on j's observation on i's action. However, the only relevant belief for the decision is the one when i is buying, because trader i's expected payoff of buying is $b_{i,1} - \bar{b}$ where the belief is evaluated at $a_i = 1$ while the payoff of not buying is 0 regardless of the belief. Thus, we define $b_{i,1}$ to be the belief evaluated when the other traders observe $a_i = 1$.

Let m denote the number of traders who buy, $m \equiv \sum_{i=1}^{N} a_i$. Let m_{-i} denote the number of traders who buy other than i. In our model of ex-ante homogeneous

[1] Nirei (2007) considers the case where the traders observe $\sum_i a_i$ instead.

traders, m_{-i} is the sufficient statistic for the public information a_{-i}. Suppose that the best response of trader i obeys a threshold rule:

$$a_i = \begin{cases} 1 & \text{if } x_i \geq \bar{x}(m_{-i}) \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Then we can define the likelihood ratio revealed by a "bear" action ($a_i = 0$) and by a "bull" action ($a_j = 1$) under the threshold rule respectively as:

$$A(\bar{x}(m)) \equiv \frac{\Pr(x_i < \bar{x}(m_{-i}) \mid L)}{\Pr(x_i < \bar{x}(m_{-i}) \mid H)} = \frac{\Pr(x_i < \bar{x}(m) \mid L)}{\Pr(x_i < \bar{x}(m) \mid H)} = \frac{G(\bar{x}(m))}{F(\bar{x}(m))}, \tag{3}$$

$$\begin{aligned} B(\bar{x}(m-1)) &\equiv \frac{\Pr(x_j \geq \bar{x}(m_{-j}) \mid L)}{\Pr(x_j \geq \bar{x}(m_{-j}) \mid H)} \\ &= \frac{\Pr(x_j \geq \bar{x}(m-1) \mid L)}{\Pr(x_j \geq \bar{x}(m-1) \mid H)} = \frac{1 - G(\bar{x}(m-1))}{1 - F(\bar{x}(m-1))}. \end{aligned} \tag{4}$$

Using the notation A and B as before, we express the posterior likelihood ratio $\theta_{i,1}$ when trader i plays $a_i = 1$ and observes $m_{-i} = m - 1$ number of other buying traders as:

$$\theta_{i,1} = A(\bar{x}(m))^{N-m} B(\bar{x}(m-1))^{m-1} \delta(x_i)\theta_0. \tag{5}$$

Note that the bear traders observe m buying actions and the bull traders observe $m - 1$.

A trader chooses to buy if and only if $b_{i,1} \geq \bar{b}$, and this condition is equivalently written as $\theta_{i,1} \leq 1/\bar{b} - 1$. Define $\epsilon \equiv 1/\bar{b} - 1$ as the threshold for the likelihood ratio. The threshold for the private information $\bar{x}(m_{-i})$ is defined as the value of private information at which a bull trader i is indifferent between buying and not buying, given that the trader observes $m_{-i} = m - 1$ buying actions. The threshold function is thus implicitly determined by the following key equations:

$$\epsilon = A(\bar{x}(m))^{N-m} B(\bar{x}(m-1))^{m-1} \delta(\bar{x}(m-1))\theta_0, \quad m = 1, 2, \ldots, N-1, \tag{6}$$

$$\epsilon = B(\bar{x}(N-1))^{N-1} \delta(\bar{x}(N-1))\theta_0. \tag{7}$$

Note that (7) determines $\bar{x}(N-1)$, and the rest of $\bar{x}(m)$ are sequentially determined by (6). Also note that $\bar{x}(m)$ defined by (6,7) is equivalent to the threshold of a bear trader who observes $m_{-i} = m$ for $m = 0, 1, \ldots, N-1$, which must satisfy the following equations:

$$\epsilon = A(\bar{x}(m+1))^{N-m-1} B(\bar{x}(m))^{m} \delta(\bar{x}(m))\theta_0, \tag{8}$$

where the likelihood ratios A and B are evaluated at the hypothetical move $a_i = 1$.

We obtain the optimal threshold rule (2) from Eqs. (6,7), since δ is decreasing. Define $\mathcal{M} = \{0, 1, 2, \ldots, N\}$ as a set of possible equilibrium outcome m. We define a reaction function $\Gamma : \mathcal{M} \mapsto \mathcal{M}$ for each realization of x such that $m' = \Gamma(m)$ is the number of traders who received private information above the threshold: $x_i \geq \bar{x}(m)$. Define $\alpha \equiv m/N$ as a fraction of traders who choose to buy. Define an equivalent threshold for α as $\hat{x}_N(\alpha) = \hat{x}_N(m/N) \equiv \bar{x}(m)$. Also define $\hat{\Gamma}$ as the reaction function in the domain of α. Then we obtain the existence of equilibrium at the limit of N.

Proposition 1 *At the limit $N \to \infty$, for each realization of x the reaction function $\hat{\Gamma}$ is non-decreasing, and there exists an equilibrium outcome α in which the action profile satisfies the optimal threshold rule based on $\hat{x}_\infty$.*

Proof Equation (6) is rewritten as:

$$\frac{\log(\epsilon/\theta_0)}{N} = \left(1-\alpha-\frac{1}{N}\right)\log A\left(\hat{x}_N\left(\alpha+\frac{1}{N}\right)\right) + \alpha \log B\left(\hat{x}_N(\alpha)\right) + \frac{\log \delta\left(\hat{x}_N(\alpha)\right)}{N}. \tag{9}$$

Taking the limit $N \to \infty$, we obtain:

$$(1-\alpha)\log A(\hat{x}_\infty) + \alpha \log B(\hat{x}_\infty) = 0. \tag{10}$$

A is strictly decreasing from $\delta(-\infty)$ to 1 and B is strictly decreasing from 1 to $\delta(\infty)$ in the support of the random variable x_i. Hence the left hand side of (10) travels from a positive value to a negative value for any fixed $\alpha \in [0, 1]$. Thus the solution $\hat{x}_\infty$ exists. Since F and G are continuous, A and B are continuous and thus the solution $\hat{x}_\infty$ is also continuous. Then $\hat{\Gamma}$ is a continuous function that maps $[0, 1]$ onto itself. Thus a fixed point of $\hat{\Gamma}$ exists. From (10), we establish that an increase in buying actions always reduces the threshold in the limit of $N \to \infty$:

$$\frac{d\hat{x}_\infty}{d\alpha} = \frac{\log A(\hat{x}_\infty) - \log B(\hat{x}_\infty)}{(1-\alpha)A'(\hat{x}_\infty)/A(\hat{x}_\infty) + \alpha B'(\hat{x}_\infty)/B(\hat{x}_\infty)} < 0, \tag{11}$$

where we used the properties $A > B$, $A' < 0$, and $B' < 0$ (see Nirei 2007). □

It follows from the proof that $\alpha = 0, 1$ are always equilibria regardless of the realization of x when $N \to \infty$. Hence, the minimum equilibrium is always 0. The intuition is that there is no good enough draw of private information that can overturn the revealed information when infinitely many traders are not buying.

In this paper, we focus on the equilibrium with a minimum number of buying traders, m^*, in order to demonstrate that even the minimum shift in equilibrium exhibits large fluctuations. As shown above, however, interesting fluctuations of m^* occur only when N is large but finite. Therefore, in the rest of the paper, we concentrate on the case when N is finite and Γ is non-decreasing. We can establish the non-decreasing Γ for a finite N under a different equilibrium notion.[2] When Γ is a non-decreasing mapping of a finite discrete set $\mathcal{M}$ onto itself, there exists a non-empty closed set of fixed points by Tarski's fixed point theorem (Vives 1990). We can show that there exists an equilibrium when Γ is non-decreasing and N is finite as follows. Pick a minimum fixed point m^*. We verify the incentive compatibility of traders at m^*. Note that $N - m^*$ bear traders satisfy the incentive compatibility, $x_i < \bar{x}(m^*)$, by the construction of $\Gamma(m^*)$.

[2] Nirei (2007) shows the existence of a rational expectations equilibrium and a non-decreasing Γ for any finite N. We maintain the Nash formulation in the present paper, however, for its merit in intuitiveness.

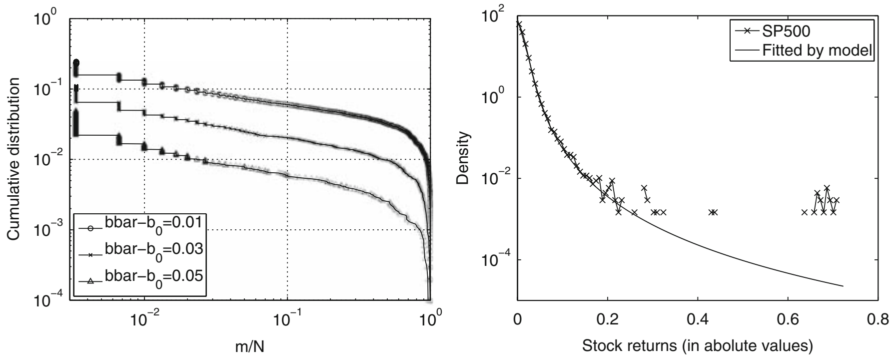

Fig. 1 *Left* cumulative distribution of m^*/N (cumulated from above). *Right* probability density function of S&P500 daily stock returns and the model prediction at the criticality. The price-impact function is specified as $\Delta = 0.025m^{0.38}$ and $\mu = 0.3$ to fit the empirical density

When $m^* = 0$, all the traders are bear and thus the incentive compatibility is satisfied for all of them. When $m^* > 0$, the incentive of bull traders, $x_i \geq \bar{x}(m^* - 1)$, need be examined. We obtain $\Gamma(m^* - 1) > m^* - 1$ from the facts that m^* is the minimum fixed point, that $m^* > 0$, and that Γ is non-decreasing. Also $\Gamma(m^* - 1) \leq \Gamma(m^*) = m^*$ obtains, since Γ is non-decreasing. Hence $\Gamma(m^* - 1) = m^*$. Namely, the traders who satisfy $x_i \geq \bar{x}(m^*)$ must also satisfy $x_i \geq \bar{x}(m^* - 1)$. But the latter is exactly the incentive compatibility condition for the bull trader i.

The minimum equilibrium m^* can be reached by a sequence of the best response which requires the traders to know only the "aggregate" information, m in our case. Following Vives (1995), we call this best response dynamics as informational tatonnement. It is shown that the informational tatonnement converges to the minimum equilibrium (see Vives 1990; Nirei 2007).

Proposition 2 *Suppose that Γ is non-decreasing. Consider an informational tatonnement process m^u, where $m^0 = 0$ and $m^u = \Gamma(m^{u-1})$ for $u = 1, 2, \ldots, T$, where the stopping time T is the smallest u such that $m^{u-1} = m^u$. Then, m^u converges to the minimum equilibrium m^* for each realization of x. Also, the threshold decreases over the informational tatonnement process: $\bar{x}(m^{u+1}) < \bar{x}(m^u)$ for each realization of x.*

The minimum Nash equilibrium outcome m^* depends on the realization of x. Thus, the probability measure for x determines the fluctuation of m^*. The left panel of Fig. 1 shows a simulated cumulative distribution of m^* when F and G are specified as exponential distributions with mean 1.1 and 1, respectively and the threshold belief is set at $\bar{b} = 0.9$. We observe that m^* follows a power-law distribution with exponent about 0.5 with an exponential truncation at the tail.

We can provide an analytical foundation for this distributional shape of m^* as in Nirei (2007). The informational tatonnement process m^u can be regarded as a binomial process unconditional to x. The binomial process allows an approximation as a Poisson branching process when N is large. By the theorem by Otter, we know that the cumulative sum of a branching process follows a power-law distribution with exponent

0.5 with exponential truncation. The distribution of the total sum of a branching process with a Poisson distribution with mean ϕ and with m^1 following a Poisson distribution with mean μ is given as follows (Harris 1989):

$$\Pr(m^* = m) = \frac{\mu e^{-(\phi m+\mu)}}{m!}(\phi m + \mu)^{m-1} \tag{12}$$

$$\propto (\phi e^{1-\phi})^m m^{-1.5}. \tag{13}$$

The distribution converges to a pure power law if $\phi = 1$, i.e., when the branching process is a martingale. Numerical computations in Nirei (2007) show that our Poisson branching process has the martingale property around the equilibrium when the difference between F and G is vanishingly small. The F and G are close to each other if the informativeness of the private information is small. This is the case for a short time horizon, since not much information should arrive in a short time period. This observation is consistent with the fact that the stock returns exhibit fat tails for a shorter time horizon.

What is the intuition for the criticality result in our informational tatonnement? Suppose that a good news induced a trader to buy, whereas the other traders did not buy despite their observation of the action. Then their inactions reveal their private information partially. The total impact of the action on the revealed information is discounted by $N - 1$, if the private information is equally informative across the traders. When this is the case, the chance for the initial action to induce another action is of order $1/N$. Then the tatonnement triggered by the initial action becomes a martingale, in which the distribution of the total number of buying actions during the tatonnement exhibits a power law.

We can further show that, when the static game is repeated over time, the triggering action almost surely occurs and the mean impact of the action in the chain reaction evolves toward the critical level. This implies that the rational learning of traders self-organizes their beliefs to the critical state at which a power-law clustering of actions emerges. Note that in the left panel of Fig. 1 the probability of non-zero propagation is decreasing in the gap between the belief and the threshold belief. Numerical studies also show that the informational tatonnement tends to be subcritical when the gap is large (Nirei 2007). Thus, the market travels from a subcritical regime to a critical regime as the traders learn the true state of the economy. If so, the "sweeping of an instability" by Sornette (2004) applies: the unconditional distribution of trading volume becomes an integral of (12) over ϕ, which leads to the power-law exponent 1.5 rather than 0.5. Indeed, Gopikrishnan et al. (2000) estimated the exponent for the power law of trading volume to be around 1.5 in the stock market data.

3 Stock prices

Lillo et al. (2003) found a stable relationship between trading volume m and stock returns Δ in wide variety of high-frequency stock data in the form $|\Delta| \propto m^\gamma$. This approach implicitly assumes that uninformed traders absorb any demand (or supply) placed by informed traders with a constant supply (or demand) elasticity $1/\gamma$.

The impact function in the form of Lillo et al. can yield a power law in the stock returns when combined with a power law in the volume size m. Suppose that the volume follows a power law with exponent ξ: $f_m(x) \propto x^{-\xi-1}$. Then the density of the returns follows: $f_\Delta(y) \propto f_m(y^{1/\gamma})y^{1/\gamma-1} \propto y^{-\xi/\gamma-1}$. Hence the exponent of the power law for the returns is determined by ξ/γ.

Gabaix et al. (2006) reports that the exponent of the returns is about three for various stock returns data when fitted by a pure power law. Bouchaud and Potters (2000) estimates the exponent at about 1.5 when the returns data are fitted by a power law with an exponential truncation. Our model of branching process predicts that the volume has an exponent $\xi = 0.5$ at the criticality, and with an exponential truncation off the criticality. Thus, our model is consistent with the returns exponent 1.5 if $\gamma = 1/3$ in the case of criticality, and it is consistent with the exponent 3 if $\gamma = 1/6$ in the case of off-criticality. Lillo et al. estimates that the elasticity of the impact function varies from 0.1 to 0.4. Thus our model is consistent with the existing estimate range of ξ and γ.

The right panel of Fig. 1 shows a density function estimated from an empirical histogram of daily returns of S&P500 stocks. The data covers a year starting from 1 July 2004. The empirical distribution is fitted by a combination of a price-impact function $\Delta = 0.025m^{0.38}$ and Eq. (12) evaluated at criticality $\phi = 1$ and $\mu = 0.3$. Parameter μ affects the density around $m = 0$. In the model, μ is determined by the gap between the prior and threshold belief. The location parameter of the impact function is set at 0.025, which implies that the initial action by an informed trader causes 2.5% impact on the price. The most important parameter is the elasticity of price with respect to volume, $\xi = 0.38$. This parameter determines the curvature of the distribution. The value 0.38 falls within the range of ξ estimated by Lillo et al. Although the parameter values are chosen from a reasonable range, the choice is still quite tentative. Nonetheless, the fitting shows that our simple model is capable of generating a fat-tail distribution that could match in magnitude with the empirical stock returns distribution. The simplicity of our model merits in possible extensions in order to obtain a more accurate exponent of the power law.

4 Conclusion

This paper presents a static game of traders with private information. The minimum Nash equilibrium outcome exhibits fluctuations depending on the realization of the private information profile. An action of a trader reveals the trader's private information partially and induces another trader to choose a similar action. Namely, the traders' actions are strategic complement. The correlation of the traders' actions renders the aggregate fluctuations to follow a non-normal distribution.

We characterize the distribution of the equilibrium number of buying traders. It turns out that the distribution exhibits a power-law with exponential truncation. The power-law behavior emerges from a rational herding of the traders. When traders are symmetric with respect to the information contents of their actions, the *inaction* of traders upon an action of a trader reveal their private information. Thus the impact of a trader's action on subjective belief is *discounted* by other traders' inaction to the level of order $1/N$. This leads to the critical propagation of a buying action. The mechanism is analogous to the Keynes' beauty contest.

The empirical exponent of the power-law for the trading volume can be matched with a dynamic extension of our model in which traders eventually learn the true state of the economy. Moreover, an empirical distribution of daily stock returns can be fitted well by a combination of the power-law of the traders' aggregate action in our model and an empirically admissible price-impact function. The microfoundation of the price-impact function is left as an open question.

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