Intel Serv Robotics (2009) 2:105–112
DOI 10.1007/s11370-009-0038-7

ORIGINAL RESEARCH PAPER

Gait planning for quadruped robot based on dynamic stability: landing accordance ratio

Minkyu Won · Tae Hun Kang · Wan Kyun Chung

Received: 27 February 2009 / Accepted: 3 March 2009 / Published online: 21 March 2009

Abstract In this article, the method for increasing dynamic stability of quadruped robot is proposed. Previous researches on dynamic walking of quadruped robots have used only walking pattern called central pattern generator (CPG). In this research, different from walking generation with only CPG, a instinctive stability measure called landing accordance ratio, is proposed and used for increasing dynamic stability. In addition, dynamic balance control and control to adjust walking trajectory for increasing dynamic stability measure is also proposed. Proposed methods are verified with dynamic simulation and a large number of experiments with quadruped robot platform.

Keywords Gait planning · Dynamic stability · Quadruped robot

1 Introduction

There are two types of walking in a quadruped robot with respect to the robot's speed as follows: static walking and dynamic walking. Usually static walking is generated by the static stability criteria, stability margin [1]. Hirose [2,3] proposed energy stability margin (ESM) to generate static walking pattern on a slope, and verified with an insect type platform, TITAN. In addition, Garicia and de Santos [4] considered inertial effects with ESM and experimented with another insect type platform, SILO. Recently, the active locomotion with vision and range sensor information is researched [5–7].

On the other hand, the mobility of dynamic walking is not guaranteed because there is no method to measure dynamic stability of locomotion. Most of the groups actualize dynamic walking using only walking patterns. However, these types of walking cannot be preserved against external forces or velocity changes. Kimura et al. [8] generate walking patterns with a mathematical modeling of central pattern generator (CPG). They realized CPG with the neuron model of Matsuoka [9], and implemented it into the small quadruped platform, Tekken, 20 cm height and 4.3 kg weight.

Recently, dynamic walking of quadruped robot is demanded because of its advantageous high speed. High-speed walking and walking stability are contradictory to each other, thus further research on generating stable dynamic walking is required. At a high speed, there are less than two legs on the ground at the stance phase. In addition, considering the inertial effect, stable dynamic walking becomes harder to generate than static walking. However, if we have an index that can measure the stability of dynamic walking, it is easier to realize high speed walking.

In this research, trot gait is chosen as the dynamic gait. Generally, gait types are defined with their order of stance phase. At the trot gait, diagonal limbs are in harmony with each other, and the duty factor is around 0.5 [10], as shown in the timing diagram of Fig. 1.

We can categorize this paper in two subjects as shown below:

- An index that can measure dynamic walking stability is proposed. Because the proposed measure is instinctive and simple to calculate, it can be implemented easily.

M. Won · W. K. Chung (✉)
Robotics Laboratory, Pohang University of Science
and Technology (POSTECH), Pohang, Korea
e-mail: wkchung@postech.ac.kr

M. Won
e-mail: skycow01@postech.ac.kr

T. H. Kang
Pohang Institute of Intelligent Robotics, Pohang, Korea
e-mail: bigxinh.kang@gmail.com

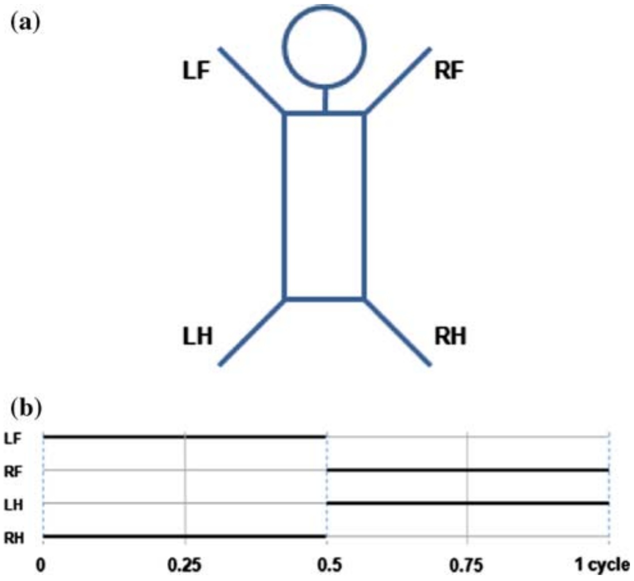

Fig. 1 Trot gait of quadruped robot. **a** Upside view. **b** Ideal timing diagram of trot gait

In addition, it reflects the dynamic walking properties, related to the robot balance. We verified the validity of this measure with dynamic simulations and experiments.

- Based on the proposed stability measure, the dynamic walking is actualized. To improve this measure, dynamic balance control is proposed and the walking trajectory parameter is adjusted for maintaining stable walking. The validity of the control strategy is also proved with simulations and experiments.

First, the dynamic stability measure will be proposed in Sect. 2. In this section, the definition of stable walking and properties of trot gait will be introduced. Furthermore, the gait planning method will be introduced in Sect. 3. Gait pattern generation and control strategy will be proposed in this section. Simulation and experimental results will follow in Sect. 4. Then the conclusion and the future works will be stated in the last section.

2 Dynamic walking stability

Preliminary to make an index of stability, the definition of stable walking has to be considered. In the case of static walking, stable walking is defined as an ability to maintain walking against any external disturbances without tumbling down. To realize stable walking, the robot locates its center of mass in the inner side of the support polygon. On the other hand, in the case of dynamic walking, it is difficult to guarantee stability because there is no definition of dynamic stability. Therefore the index for measuring stability of dynamic walking needs to be defined for self stabilization.

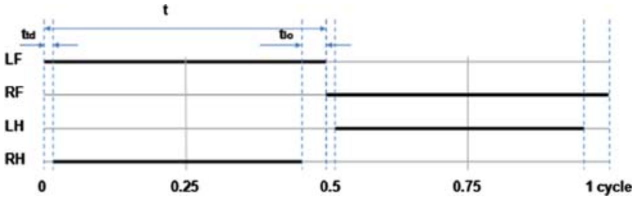

Fig. 2 Real timing diagram of trot gait

If there is no external disturbance exerted on the robot, the ground reaction force is the unique external force. So, it is important to adjust the ground reaction force to realize a stable dynamic walking. At the trot gait, diagonal limbs have to move simultaneously, and to touch down on the ground at the same time. However, there is a discordance of touch down of the diagonal limbs generated while the robot's real action, depicted in Fig. 2. This discordance makes an unexpected moment, and this moment makes the robot to be unstable. We can confirm this phenomenon in the case of an animal that walks with trot gait [11]. In Fig. 2, t_{td} and t_{lo} are the timings of touchdown and lift-off, respectively. Weishaupt says that the hind legs tend to lift-off ahead of the fore legs, but they touch down at the same time in almost stable trot gaits.

From this property of trot, we generated an index for measuring the stability of trotting using the accordance of diagonal limbs, named as landing accordance ration(LAR). This measure is expressed as a numerical value between 0 and 1, and it is written as shown in (1).

$$\lambda = \frac{t - t_{\mathrm{td}}}{t} \tag{1}$$

In this equation, λ means LAR, t is the stance period of the diagonal limbs, and t_{td} is the discordance amount between fore and hind legs at touchdown. That is to say, there is no unexpected moment when the measure is 1 and this indicates that the robot walks stably.

3 Gait planning with LAR

The basic control method for this quadruped robot research is a force control using the reference forces of each foot. Reference force consists of two forces. The first one is the force for generating the walking trajectory, and the other one is the force for dynamic balance. Overall control strategy is shown in the block diagram of Fig. 3. The rhythmic pattern generator generates a control force, $\mathbf{f}_{\mathrm{vsd}}$, as the open loop control without recognition of outer environments. In addition, the dynamic balance controller creates the balance force, f_{bal}. These forces are added together and then inserted into the robot as a form of torque

$$\tau = \mathbf{J}^{\mathrm{T}}\left(\mathbf{f}_{\mathrm{vsd}} + f_{\mathrm{bal}}\hat{\mathbf{k}}\right). \tag{2}$$

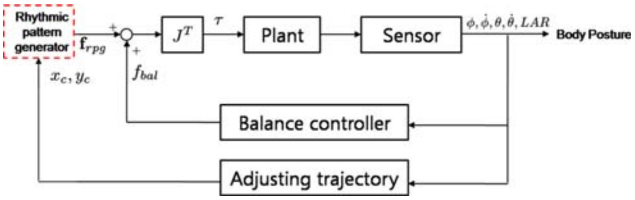

Fig. 3 Overall control design

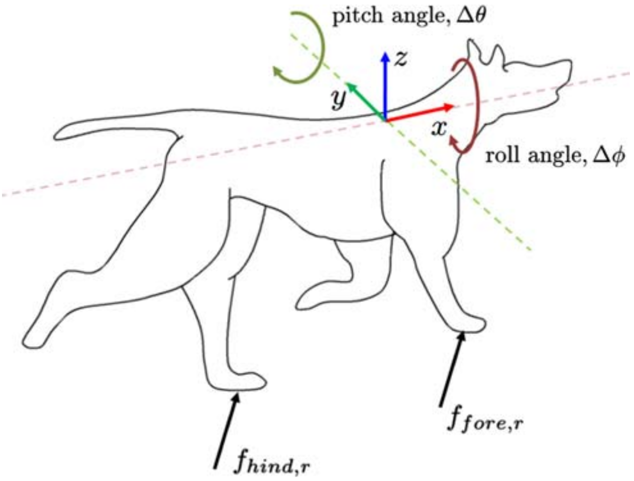

Fig. 4 Reference coordinate of walking

Table 1 Foot trajectory of trotting

	x_d	z_d
Left fore	$-m \cdot \cos \omega t$	$-SH + m \cdot s \cdot \sin \omega t$
Right fore	$m \cdot \cos \omega t$	$-SH - m \cdot s \cdot \sin \omega t$
Left hind	$m \cdot \cos \omega t$	$-SH - m \cdot s \cdot \sin \omega t$
Right hind	$-m \cdot \cos \omega t$	$-SH + m \cdot s \cdot \sin \omega t$

In this equation, $\mathbf{J}^{\mathrm{T}}$ is transpose of the Jacobian matrix of the end points of each foot with respect to the joint angles. $\hat{\mathbf{k}}$ is the unit vector in the vertical direction of the ground.

3.1 Walking pattern generation

As shown in Fig. 4, the moving direction of the quadruped robot is defined as the x axis, and the vertical direction of the ground is the z axis. The ellipsoid type of foot trajectory is selected as a basic walking trajectory with a rhythmic pattern signal, and the input signal equation is depicted in Table 1. Magnitude (m), frequency (ω), and scale factor (s) have to be defined for making the ellipsoidal walking trajectory.

For trot gait, the diagonal limbs have to be shifted simultaneously. The phase of the diagonal limbs has to be controlled for the limbs to coincide with each other. Rhythmic pattern for walking is repeated, irrespective of the external condition, and it is identical to the animals' rhythmic pattern generation system. Usually signals for repeated motions, like scratching or walking of the animal, are generated with iterative signals from the mesencephalon [12]. In this research, this property is inspired.

3.1.1 Control for tracking the generated walking pattern

The virtual spring and damper (VSD) hypothesis, suggested by Arimoto [13], is employed for tracking the generated walking pattern. This is the basic equation of VSD.

$$\mathbf{f}_{\mathrm{vsd}} = k\Delta\mathbf{q} + \zeta\sqrt{k}\Delta\dot{\mathbf{q}} \tag{3}$$

In (3), $\Delta\mathbf{q}$ means the position error of each foot, $\Delta\mathbf{q} = \mathbf{q}_d - \mathbf{q}$, and $\mathbf{q}_d$ is the desired position of each foot. k and ζ are stiffness parameter and damping ratio. For tracking the reference torque at each joint from a given trajectory, VSD hypothesis is used for compensating for the errors. However, dynamic walking cannot be accomplished by only the force of rhythmic pattern $\mathbf{f}_{\mathrm{vsd}}$, because of some exceptional effects. Another control for the measured state is also needed for stable walking.

3.2 Walking control with LAR

In this section, the control method for increasing LAR mentioned before, is considered. At first, to make the diagonal limbs touch down on the ground at the same time, robot body has to be controlled for maintaining horizontality. Because the foot trajectory is defined identically in each foot in the rhythmic pattern generator, if the body level is controlled precisely, touchdown timing is assumed to be identical to each other. When the robot loses its balance because of external conditions, the control for compensating for robot's balance is needed. Therefore in this section, a control method for increasing the dynamic stability will be introduced.

3.2.1 Dynamic balance control

When animals lose their balance, the reaction forces of each foot are redistributed to recover the body posture [14]. In Fig. 5, a counter-clockwise moment is generated with an acceleration, because the horizontal reaction forces are exerted on the stance feet. To compensate this body moment, animals add to the reaction forces of their hind limbs and reduce those of the fore limbs. Similar to this behavior of the animals, the controller of quadruped robot should be developed to generate the torques for compensating for the moment.

The moment during acceleration induces errors of the robot's body angle and angular velocity. Hence, dynamic balance control is designed as virtual torsional spring and

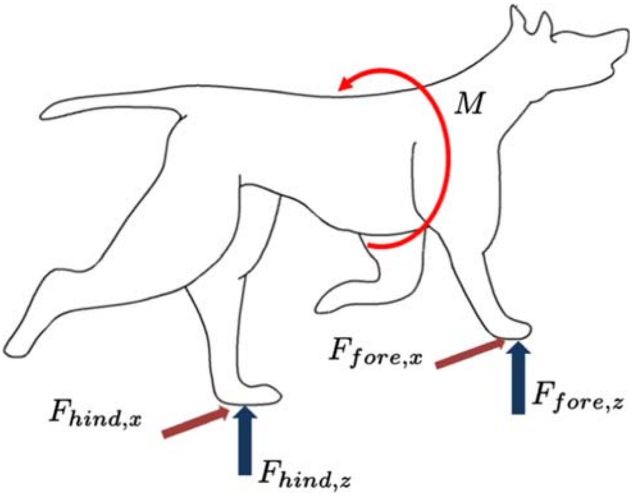

Fig. 5 Generated body moment at trotting

damper against the errors as follows

$$\tau_\phi = k_{p,x}\Delta\phi + k_{d,x}\Delta\dot{\phi} \tag{4}$$

$$\tau_\theta = -(k_{p,y}\Delta\theta + k_{d,y}\Delta\dot{\theta}) \tag{5}$$

$$\begin{aligned} f_{\text{bal}} &= f_{\phi,l} + f_{\theta,l} \\ &= \frac{k_{p,x}\Delta\phi + k_{d,x}\Delta\dot{\phi}}{r_{y_l}} - \frac{k_{p,y}\Delta\theta + k_{d,y}\Delta\dot{\theta}}{r_{x_l}}, \end{aligned} \tag{6}$$

where $f_{\phi,l}$, $f_{\theta,l}$ are the reaction forces for compensating for the roll and pitch angles, respectively. In this equation, τ_ϕ and τ_θ are the generated moments, and r is the vector from the center of mass (COM) to each foot. l means the index of each limb, and x and y are the axes of COM, depicted at Fig. 4.

The sum of reaction forces of each axis, f_{bal}, is always added into the main reference force at the stance phase, and then transformed into the joint torques as shown in (2).

3.2.2 Adjusting foot trajectory parameter

Without a vision or a range sensor, the robot cannot obtain the information on the external environment. For example, when a robot walks on the slope or there is a change in the level, existing walking algorithms cannot maintain stability. In this paper, the robot estimates the walking stability with LAR, and adjusts the walking trajectory to compensate for the robot's balance. This procedure is described in Fig. 6. If LAR is dropped rapidly, the robot finds the abnormal foot, and adjusts the foot trajectory parameter for stable walking. Magnitude, frequency or the scale factor can be adjusted in this situation. Adjusting the foot trajectory means that the robot adjusts a foothold of the next step with gyro sensor information. This control makes the robot step forward to the inclining direction, and the robot can be returned to the stable state.

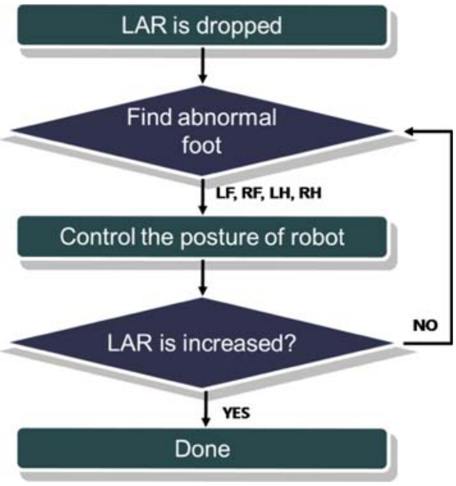

Fig. 6 Procedure for adjusting foot trajectory

4 Simulation and experimental results

Several dynamic simulations and experiments with various situations are done to verify the suggested idea. In the simulations, the robot platform, pQ1 is used as a robot model, depicted in Fig. 7. pQ1 has three links and three joints at each limb, and all of the joints are simplified as hinge joints of 1 degree of freedom (DOF). In reality, there are more than 12 DOFs in a four-legged animal, but this model is simplified for easy control. The length of each link, and other parameters are arranged in Table. 2. This robot model mimics the skeleton of a dog [15]. Scapula and humerus are simplified as one link (l_1), and forearm (l_2) and forefoot (l_3) are also designed as links, respectively. These three links compose a forelimb. Hind limbs are similar to the fore limbs. Pelvis and thigh are simplified as one link, and tibia and hind foot are designed as second and third links, respectively. The length of fore and hind limbs are assumed to be the same as each other, and the mass of the body is assumed to be a point mass at the COM. Open Dynamic Engine is used for dynamic simulation. The ground condition is described as a spring-damper model, whose parameter values are 500 kN/m

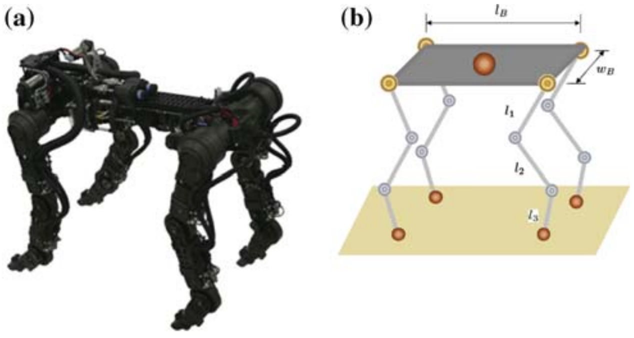

Fig. 7 Robot model. **a** Piro Quadruped ver. 1 (pQ1). **b** Simulation model

Table 2 Parameters in the simulation

	Name	Mark	Value	Unit
Mass	Body	M	30	kg
	Link1	m_1	1	kg
	Link2	m_2	1	kg
	Link3	m_3	1	kg
Length	Body	l_B	0.55	m
	Link1	l_1	0.23	m
	Link2	l_2	0.23	m
	Link3	l_3	0.17	m
Width	Body	w_B	0.30	m

and 100 kNs/m, respectively. The sampling time of control is 0.02 s.

4.1 Simulation results

4.1.1 Verifying LAR as stability measure

At first, verification of the proposed dynamic stability measure, LAR, is performed. Stable walking means well-balanced walking, and is closely related to the robot posture. Therefore the robot posture, especially roll and pitch angles are measured to verify the validity of LAR as dynamic stability index.

Figures 8 and 9 show the results of controlling without LAR and with LAR, respectively. As they show, the robot is well balanced when the LAR is used for a measure of the dynamic stability to generate the trot gait. The center of oscillation is converged to 0, and it means that the robot body oscillates with balance around the center of the robot. In this result, LAR is confirmed to be acceptable as a dynamic stability measure.

4.1.2 Dynamic walking

As mentioned before, dynamic balance control and foot trajectory adjustment are used to increase the walking stability. In this section, the simulations on dynamic walking based on LAR are performed in several situations. At first, the robot state while walking on a slope is simulated. Walking under external forces and over unexpected sunken places are also simulated.

There are a lot of inclined places in a real environment. Therefore, the ability to maintain the body posture on a slope is fundamental. Figure 10 shows results of trotting on a slope using proposed dynamic balance control, where inclined angle is 0.1 rad. The simulations are performed for both cases as follows: generating trot gait without LAR and with LAR. The magnitude of oscillation about the pitch angle is reduced after generating trot with LAR, and this means the robot walks more stable.

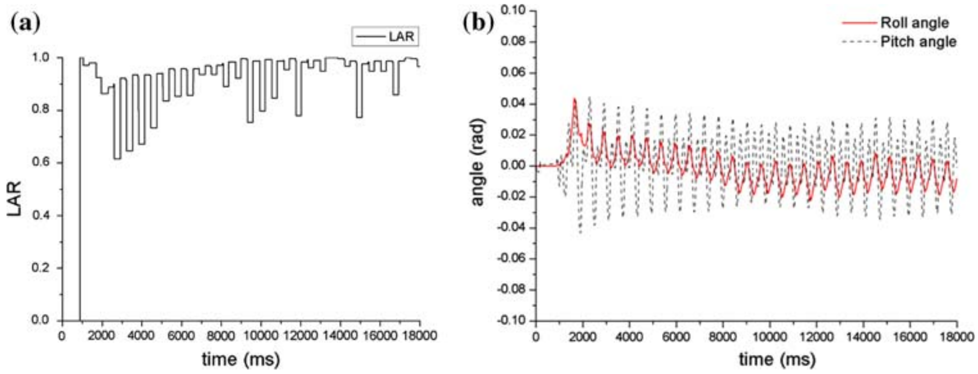

Fig. 8 Generating trot gait without LAR. **a** Dynamic stability, LAR. **b** Roll and pitch angle

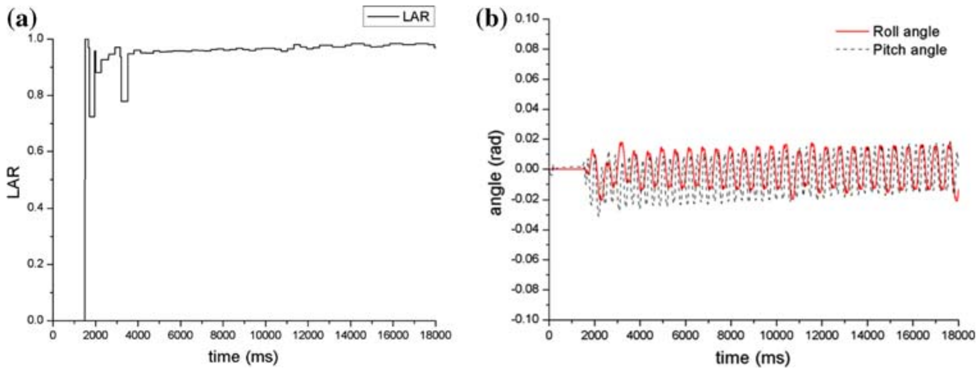

Fig. 9 Generating trot gait with LAR. **a** Dynamic stability, LAR. **b** Roll and pitch angle

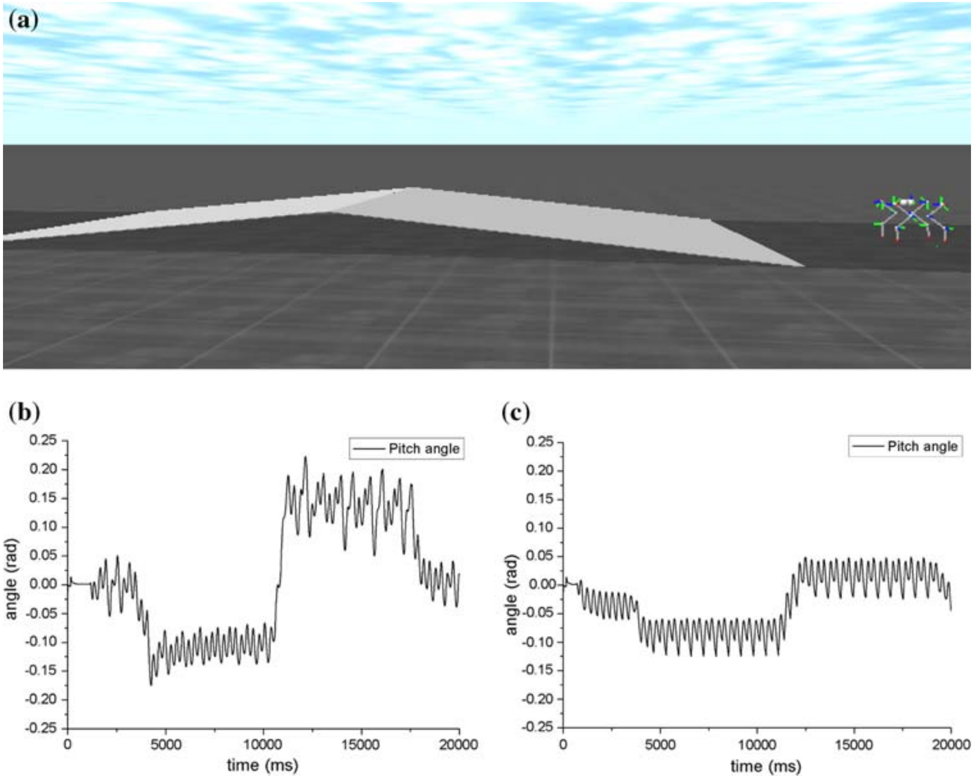

Fig. 10 Trotting on the slope. **a** Stimulation environment. **b** Without LAR. (**c**) With LAR

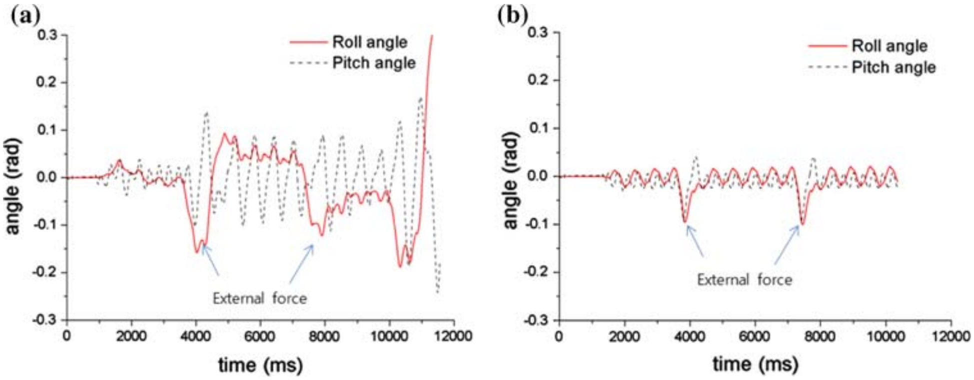

Fig. 11 Trotting under external forces. **a** Without LAR. **b** With LAR

External forces are exerted on the robot with various forms frequently. When the robot bumps into some obstacles; a heavy load is loaded; or some task is performed with the manipulator; the situations are regarded as external forces. Therefore, the ability to keep a stable walk under external forces is important. The simulation results of trotting under external forces are shown in Fig. 11. The robot walks on the flat ground and exerted two external forces of about 5,000 N for 0.02 s, at indicated points. When the robot is controlled with LAR, it maintains walking under external forces.

To show walking performance using LAR on a rough terrain, walking on a sunken place is simulated. The depth of the hollow surface is set about 7 cm. In Fig. 12, we can see that the robot can walk stably on a sunken place without tumbling down.

4.2 Experimental result

Finally, based on the simulation results, the experiment on trotting on uneven grounds is performed. A treadmill is used for the experimental environment, and the obstacles about 4 cm are inserted as depicted in Fig. 13a. The robot is controlled for increasing dynamic stability, LAR.

In Fig. 13b, as a result of the control, the robot can maintain walking on the uneven ground stably. The robot lost balance at a moment when it stepped on the obstacle, however, the balance is recovered immediately and walking is maintained stably.

From these simulations and experimental results, the gait planning with respect to dynamic stability measure, LAR, is verified to be a good approach for generating stable dynamic walking of a quadruped robot.

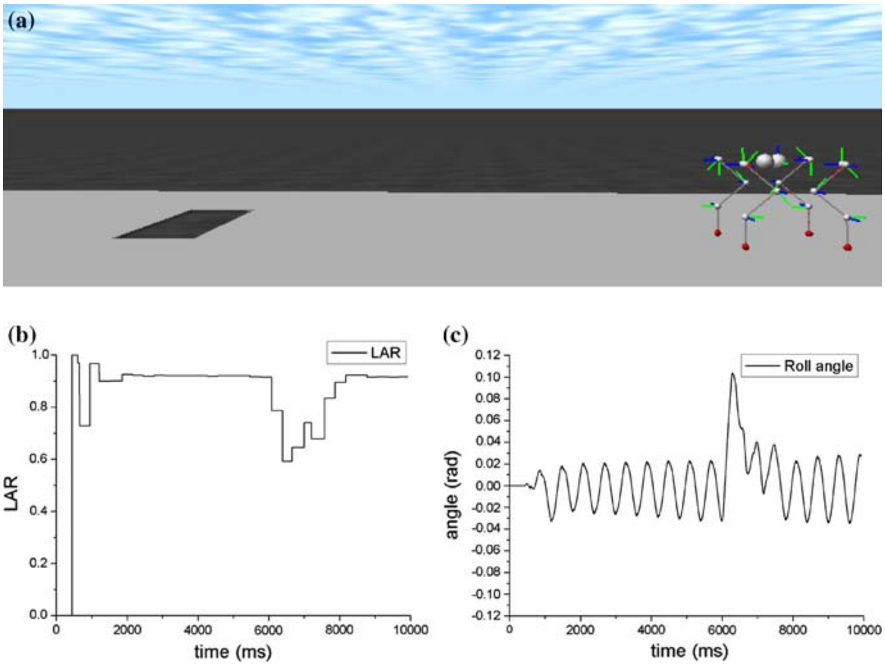

Fig. 12 Trotting on the sunken place. **a** Stimulation environment. **b** Dynamic stability, LAR. **c** Roll angle

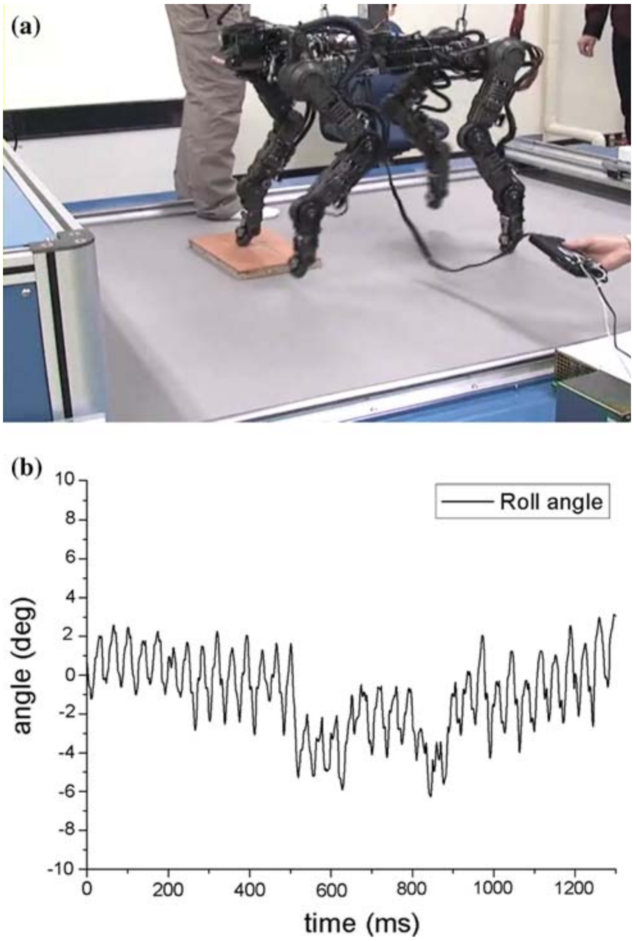

Fig. 13 Trotting on obstacles. **a** Experiment environment. **b** Roll angle

5 Conclusions and future works

5.1 Conclusions

In this article, LAR is proposed as a dynamic stability measure, and the control method for increasing dynamic stability is suggested. Because LAR is a distinct measure using touch down accordance of diagonal foot, implementation is easy and computing burden is reduced. For increasing LAR, dynamic balance control with virtual torsional spring and damper and adjusting the foot trajectory parameter is proposed. These proposed methods are verified with dynamic simulations and experimental results. These results can be used to guarantee the mobility of dynamic walking.

5.2 Future works

Animals use visual information from their eyes, and this information is critical for walking in a real environment. Therefore, active locomotion, with the information of external environment using vision or range sensors, has to be studied for actualizing a real walking, similar to one of an animal's.

References

1. de Santos PG, Garcia E (2006) Quadrupedal locomotion. Springer, Berlin
2. Arikawa K, Hirose S (1996) Development of quadruped walking robot TITAN-VIII. In: Proceedings of the IEEE international conference on intelligent robots and systems

3. Hirose S et al (2001) Normalized energy stability margin and its contour of walking vehicles on rough terrain. In: Proceedings of the IEEE international conference on robotics and automation
4. Garcia E, de Santos PG (2005) An improced energy stability margin for walking machines subject to dynamic effects. Robotica 23:13–20
5. Pongas D, Mistry M (2007) A robust quadruped walking gait for traversing rough terrain. In: Proceedings of the IEEE international conference on robotics and automation
6. Hosoda K (2000) Emergence of quadruped walk by a combination of reflexes. In: Proceedings of the international symposium on adaptive machine control
7. Hiroshi K, Masayoshi K (2003) Local obstacle recognition for a quadruped robot by distance sensors. In: Proceedings of the IEEE international conference on robotics, intelligent systems and signal processing
8. Kimura H et al (2007) Adaptive dynamic walking of a quadruped robot on natural ground based on biological concepts. Int J Robot Res 26:275–290
9. Matsuoka K (1987) Mechanisms of frequency and pattern control in the neural rhythm generators. Biol Cybern 56:345–353
10. Alexander RM (1984) The gaits of bipedal and quadrupedal animals. Int J Robot Res 3:49–59
11. Lee DV et al (2004) Effects on mass distribution on the mechanics of level trotting in dogs. J Exp Biol 207:1715–1728
12. Orlovsky GN, Deliagina TG (2003) Neuronal control of locomotion. Oxford University Press, New York
13. Sekimoto M, Arimoto S (2006) Experimental study on reaching movements of robot arms with redundant DOFs based upon virtual spring-damper hypothesis. In: Proceeding of the IEEE international conference of intelligent robots and systems
14. Lee DV, Bertram JEA (1999) Acceleration and balance in trotting dogs. J Exp Biol 202:3565–3573
15. Goldfinger E (2004) Animal anatomy for artists. Oxford University Press, New York