Entrepreneurship and business cycles: technological innovations and unemployment

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Abstract Koellinger and Thurik (2012) find that entrepreneurship Granger-causes the cycles of the world economy, and that entrepreneurial cycles are positively affected by the national unemployment cycles. However they do not present a theoretical model to explain the empirical findings. This paper provides a theoretical explanation through an extended Ramsey model in which a differential equation describing technological innovations led by entrepreneurs, which relates entrepreneurship dynamics to unemployment and output dynamics, is considered as an additional dynamic restriction. The model generates limit cycles through the Hopf bifurcation theorem. The necessary condition for the existence of a limit cycle is that the entrepreneurial economy accumulates more capital than the Ramsey model, yielding that entrepreneurial and unemployment cycles cause business cycles.

Keywords Entrepreneurship · Unemployment · Business cycles · Technological innovations

Introduction

In a recent pioneering and path breaking empirical paper Koellinger and Thurik (2012) find that entrepreneurship Granger-causes the cycles of the world economy, and that entrepreneurial cycles are positively affected by national unemployment cycles. [1] Koellinger and Thurik (2012) let the data speak freely and do not present a theoretical model to explain their empirical findings. This paper fills this gap by analyzing a dynamic micro founded model able to explain how entrepreneurial cycles are affected by unemployment dynamics and, as a consequence, affect output cycles.

The modern knowledge economy is utterly affected by the role of entrepreneurs. Recently small businesses have become more important, and entrepreneurship has been

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[1] Related empirical works are Congregado et al. (2009) and Golpe (2009)

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recognized as one of the main engines of growth of the modern knowledge economy (e.g., Thurik 2009). This is why the knowledge economy is also called entrepreneurial economy. The knowledge economy is characterized by creative destruction, economic instability, technological innovations and globalization (Brock and Evans 1989; Carlsson 1992; Acs and Audretsch 1993; Audretsch and Thurik 2000; Audretsch and Thurik 2001; Audretsch and Thurik 2004). These characteristics explain its flexibility, turbulence, diversity, creativity and novelty (Audretsch and Thurik 2007).

One important issue regarding the modern knowledge economy that remains to be studied in detail is whether entrepreneurship plays a role in the business cycle. Micro founded macroeconomic models usually do not consider entrepreneurship explicitly as a source of business cycles. Among the exceptions are Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Rampini (2004), and more recently Ouyang (2011) and Aghion et al. (2012a,b)

Bernanke and Gertler (1989) in a neoclassical model of business cycle study the principal-agent problem between entrepreneurs (borrowers) and lenders. The model's asymmetry of information referrers to the entrepreneurs' knowledge of the returns of their individual projects, while lenders do not observe them without a cost; they must incur in fixed costs to observe these returns. Entrepreneurs' net worth serves as collateral that help lower the agency costs, however, net worth is generally pro-cyclical, which generates the result that the fraction of entrepreneurs who get funding and produce is pro-cyclical.[2]

Similarly to Bernanke and Gertler (1989) model, Carlstrom and Fuerst (1997) also consider the share of entrepreneurs in the population constant. However, differently from Bernanke and Gertler (1989) pro-cyclical results, they show that the number of solvent entrepreneurs is counter-cyclical, because bankrupt rates and risk premia increase during boom periods as a result of higher capital prices and positive technological shocks.

Rampini (2004) real business cycle model also considers a pro-cyclical entrepreneurship. Entrepreneurship is limited by a financial intermediary that designs a contract allowing entrepreneurs to insure part of their risk through leverage. Given that a positive productivity shock increases all agents wealth, which reduces risk aversion, the number of entrepreneurs during economic booms increase, and in case of an economic recession decrease.

Ouyang (2011) examines and tests the theories on the cyclicality of R&D. On one hand, the opportunity costs hypothesis developed by Aghion and Saint-Paul (1998) says that business cycle downturns reduce the opportunity cost for firms to invest in activities that improve their long-run growth. On the other hand, there is empirical evidence based on aggregate data that R&D is procyclical (e.g. Fatas 2000). Ouyang (2011) studies R&D cycles at the industry level rather than in the aggregate, and finds that R&D is procyclical at the industry level and that short-run demand fluctuations reduce R&D, which is consistent with the opportunity costs hypothesis with liquidity constraints.

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[2] It is well-known that imperfect information in credit markets such as agency costs, adverse selection and moral hazard affect the provision of credit, making it scarce and costly (e.g., Walsh 1998). There is a branch of the literature that examines how credit restriction generates cycles, Faria and Andrade (1998), and Novak (2000) examine a model with borrowers and lenders that generates stable limit cycles.

Aghion et al. (2012a) discusses the Schumpeterian view that long-term investments are countercyclical. This theory assumes that firms can always borrow enough funds to finance long-term R&D investment. They argue that in case firms are credit-constrained their ability to survive short-term shocks depend on their earnings plus borrowing, and therefore its ability to borrow in order to innovate decreases in recessions. They show evidence from France that investment is indeed pro-cyclical for firms facing credit constraints.

Aghion et al. (2012b) relates two strands of the literature on the effects of counter-cyclical monetary policies on firms' behavior. Countercyclical monetary policies set the short term interest rates lower in recessions and higher in booms. They show a significant growth effect of these monetary policies on industries in OECD countries, evidence in line with the negative correlation in cross-country regressions between volatility and long-run growth (e.g., Ramey and Ramey 1995). On the literature on monetary policy design, they model liquidity provision following Holmstrom and Tirole (1998). In the model if the economy as a whole is experiencing a boom, then short-run profits are sufficient for entrepreneurs to finance their investments, however when the economy is in a slump, reinvestment demands project downsizing, which can be avoided in case entrepreneurs hoard liquidity. A countercyclical monetary policy facilitates entrepreneurial ex ante hoarding, which enhances investment.

The theoretical model presented in this paper differs from the above literature since it does not rely upon exogenous shocks, informational asymmetry, uncertainty and credit constraint to generate cycles between output and entrepreneurship. Its structure is much simpler since it is an extension of a Ramsey model with technological innovations. We consider an additional dynamic constraint describing the time evolution of entrepreneurship in the economy, which relates entrepreneurship dynamics to unemployment and output dynamics. In the model entrepreneurs are responsible for technological innovations. The additional dynamic constraint associates entrepreneurship to unemployment, following the insight of Knight (1921) on the choices of the unemployed. Taking into account the Okun's law that links variations of the unemployment rate to variations of output, the model bridges the dynamics of entrepreneurship, to the dynamics of unemployment and output in a concise set up.

The relationship between unemployment and entrepreneurship is the subject of a large literature (e.g., Oxenfeldt 1943; Blau 1987; Evans and Jovanovic 1989; Evans and Leighton 1990; Blanchflower and Meyer 1994; Pfeifer and Reize 2000a,b; Audretsch et al. 2001; Thurik 2003; Thurik et al. 2008; Parker 2009; Muñoz-Bullón and Cueto 2011). There are studies that find that entrepreneurship and unemployment are inversely related (e.g., Garofoli 1994; Audretsch and Fritsch 1994), while there are others that reach the opposite conclusion, finding that unemployment is associated with greater entrepreneurial activities (e.g., Highfield and Smiley 1987; Tervo and Niittykangas 1994).

Faria et al. (2009, 2010) and Faria (2013) can be seen as precursors of the model analyzed in this paper. Faria et al. (2009) examine a cyclical model between firm creation and unemployment, which may yield a unique stable limit cycle, or dampen cycles. The model is a dynamic system formed by two differential equations, one equation describes how new firms increase competition and output reducing unemployment; the second equation describes how existing firms inhibit the creation of new firms, because business creation is smaller in environments with greater

competition and smaller profitability. Faria et al. (2010) confirm the empirical findings of Faria et al. (2009) showing that the relationship between unemployment and entrepreneurship is dynamic, nonlinear and possibly cyclical.

Faria et al. (2009) however do not examine a micro funded optimizing model and in this sense can be regarded as ad hoc cyclical model. This shortcoming is fixed in the present paper since the representative agent solves an intertemporal optimization problem. The model is a concave two-state-variable optimal control problem that may generate limit cycles between entrepreneurship and output through the Hopf bifurcation theorem. The paper derives and examines the necessary conditions for the existence of a limit cycle between capital, entrepreneurship, and output. It provides a clear explanation of the cyclical mechanism that relates entrepreneurship, technological innovations, consumption and output.

The model

In this section the Ramsey model is adapted inserting technological innovations led by entrepreneurs, and relating entrepreneurship with unemployment. The new modified Ramsey model has an additional dynamic constraint, which describes how entrepreneurship is associated with unemployment.

In the standard Ramsey model, the representative agent chooses consumption, c, to maximize her lifetime utility subject to her dynamic budget constraint in which output is a function of technology A, and capital per capita, k, $y=f(A,k)$, we have:

$$\underset{c}{Max} \int_{0}^{\infty} U(c)e^{-\rho t}dt$$

$$\text{s.t.} \ \dot{k} = f(A,k) - c - (\delta + n)k \tag{1}$$

Where ρ is the positive subjective rate of time preference and δ is the depreciation rate of capital, and n is the population growth rate.

What happens to this standard Ramsey model when technological innovations, $\dot{A} \equiv dA/dt$, occur? if technological innovations occur, the Ramsey model becomes an optimal control problem with two state variables, since the process of technological innovation that explains the time variation of technology, $\dot{A}$, becomes an additional dynamic constraint, and technology, A, becomes a new state variable.

Technology is associated with entrepreneurship, since entrepreneurs, e, have to find new ways to gain or to create and develop markets to survive, therefore we assume, $A=A(e), A_e > 0$. In order to simplify assume a linear function: $A=A(e)=e$, which says that every new entrepreneur enters the market with one technological innovation. Note that innovation here is not plagued by uncertainty. As a consequence, technological innovations are explained by the time variation in the number of entrepreneurs:

$$\dot{A} = \dot{e} \tag{2}$$

According to Knight ([1921]) individuals choose between unemployment, self-employment and employment. An unemployed may turn to self-employment as the best available alternative. In this sense business creation, entrepreneurship, is linked with unemployment. The greater the actual unemployment rate, u, the greater business creation, $\dot{e}$:

$$\dot{e} = \theta\left(u - \bar{u}\right) \tag{3}$$

Where $\bar{u}$ is the natural rate of unemployment, and θ is a positive parameter. It is important to stress that it is implicit in this formulation that just a share of the unemployed become and succeed as entrepreneurs, either because of practical knowledge (Unger et al. [2009]), human capital (Parker and Belghtar [2006]), discovery of new business opportunities (Shane and Venkatraman [2000]), access to credit (Brush et al. [2001]), or bequests (Faria and Wu [2012]).

In Koellinger and Thurik ([2012]) entrepreneurs at home are not Schumpeterian innovators they only try to escape from unemployment, while at the global level entrepreneurs are innovators. Our model does not make such differentiation, in our minimalist setup unemployment reduces the opportunity cost of entrepreneurship, some of the unemployed become entrepreneurs and a fraction of them enter the market with successful innovations. Braguinsky and Ohyama ([2012]) show that in case of high tech entrepreneurs higher paid workers are more likely to become entrepreneurs than lower paid workers. Hence, according to Braguinsky and Ohyama ([2012]) lower earnings can actually decrease the probability to become high-tech entrepreneur.[3]

One can also relate Eqs. ([2]) and ([3]) to the literature on the R&D cyclicality; here it is assumed that technological innovations are countercyclical as in Aghion and Saint-Paul ([1998]) and as in Aghion et al. ([2012b]) in the absence of credit constraints, rather than procyclical as in Ouyang ([2011]).

Taking into account the Okun's law (e.g., Prachowny [1993]) that states that deviations of income, y, from its potential level, $\bar{y}$, are proportional to the difference between the natural and the actual unemployment rate:

$$y - \bar{y} = \chi\left(\bar{u} - u\right) \tag{4}$$

Where χ is a positive parameter.

From Eqs. ([4]) and ([3]) it follows that the time variation in the number of entrepreneurs is a function of the output variations:

$$\dot{e} = \alpha\left(\bar{y} - y\right) \tag{5}$$

Where $\alpha = \theta/\chi > 0$.

Note that as $y = f(A,k) = f(e,k)$, Eq. ([5]) can be rewritten as:

$$\dot{e} = \alpha\left(\bar{y} - f(e,k)\right) \tag{6}$$

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[3] I thank one referee for remarking this.

According to Eq. (6) time evolution of the number of entrepreneurs is a function of the variation in output that depends on the levels of capital and entrepreneurship in the economy.

Taking into account the role of entrepreneurs in creating technological innovations, and the link between variations in entrepreneurship, unemployment and output, the modified Ramsey model is:

$$Max_c \int_0^\infty U(c)\, e^{-\rho t} dt$$

Subject to $\dot{k} = f(e,k) - c - (\delta + n)k$

$$\dot{e} = \alpha\left(\bar{y} - f(e,k)\right)$$

Note that the representative agent in this model is an employed worker that supplies labor inelastically and chooses a consumption path to solve the above problem. In addition it is important to stress that the above theoretical model does not rely upon exogenous shocks, informational asymmetry, uncertainty and credit constraint to generate cycles between output and entrepreneurship.

The Hamiltonian of this problem is:

$$H = U(c) + \lambda[f(e,k) - c - (\delta + n)k] + \mu\alpha\left(\bar{y} - f(e,k)\right)$$

Where the variables λ and μ are the co-state variables of capital stock, k, and the number of entrepreneurs, e, respectively. The first order conditions are given by:

$$U_c(c) - \lambda = 0 \Rightarrow U_c(c) = \lambda \tag{7}$$

$$\dot{\lambda} - \rho\lambda = -\{\lambda[f_k(e,k) - (\delta + n)] - \mu\alpha f_k(e,k)\} \tag{8}$$

$$\dot{\mu} - \rho\mu = -[\lambda f_e(e,k) - \mu\alpha f_e(e,k)] \tag{9}$$

$$\dot{k} = f(e,k) - c - (\delta + n)k \tag{10}$$

$$\dot{e} = \alpha\left(\bar{y} - f(e,k)\right) \tag{11}$$

Plus the transversality conditions.

From Eq. (7) one can express consumption, c, as a function of λ, $c = U_c^{-1}(\lambda) = g(\lambda)$, and substitute it into Eq.(10) yielding:

$$\dot{k} = f(e,k) - g(\lambda) - (\delta + n)k \tag{10'}$$

The canonical equations (8), (9), (10') and (11) form the dynamic system of our interest. The local stability properties of this hyperbolic system follow from the analysis of the linearized system calculated at the steady state equilibrium. In order to find the steady state equilibrium we have to solve the equations (8), (9), (10') and (11) for λ, μ, k, and e:

$$\dot{\lambda} = 0 \Rightarrow \lambda[f_k(e,k) - (\rho + \delta + n)] = \mu \alpha f_k(e,k) \tag{12}$$

$$\dot{\mu} = 0 \Rightarrow \lambda f_e(e,k) = \mu[\rho + \alpha f_e(e,k)] \tag{13}$$

$$\dot{k} = 0 \Rightarrow f(e,k) = g(\lambda) + (\delta + n)k \tag{14}$$

$$\dot{e} = 0 \Rightarrow f(e,k) = \bar{y} \tag{15}$$

The steady state equilibrium is denoted by an asterisk over the endogenous variable, λ^*, μ^*, k^*, and e^* .

The Jacobian of the dynamic system (8), (9), (10') and (11) is:

$$J = \begin{vmatrix} \dfrac{\partial \dot{k}}{\partial k} & \dfrac{\partial \dot{k}}{\partial e} & \dfrac{\partial \dot{k}}{\partial \lambda} & \dfrac{\partial \dot{k}}{\partial \mu} \\[2mm] \dfrac{\partial \dot{e}}{\partial k} & \dfrac{\partial \dot{e}}{\partial e} & \dfrac{\partial \dot{e}}{\partial \lambda} & \dfrac{\partial \dot{e}}{\partial \mu} \\[2mm] \dfrac{\partial \dot{\lambda}}{\partial k} & \dfrac{\partial \dot{\lambda}}{\partial e} & \dfrac{\partial \dot{\lambda}}{\partial \lambda} & \dfrac{\partial \dot{\lambda}}{\partial \mu} \\[2mm] \dfrac{\partial \dot{\mu}}{\partial k} & \dfrac{\partial \dot{\mu}}{\partial e} & \dfrac{\partial \dot{\mu}}{\partial \lambda} & \dfrac{\partial \dot{\mu}}{\partial \mu} \end{vmatrix} \tag{16}$$

The eigenvalues of J evaluated at the steady state equilibrium, λ^*, μ^*, k^*, and e^* determine the local stability properties. The eigenvalues $E(i)$, i=1 to 4, can be calculated using a formula derived by Dockner (1985):

$$E(i) = \rho/2 \pm \sqrt{\frac{\rho^2}{4} - \frac{M}{2} \pm \frac{1}{2}\sqrt{M^2 - 4\det J}}, \; i = 1 \; to \; 4 \tag{17}$$

Where *det J* denotes the determinant of the Jacobian J and the coefficient M is defined as the sum of the principal minors of the Jacobian

$$\mathrm{M} = \begin{vmatrix} \dfrac{\partial \dot{k}}{\partial k} & \dfrac{\partial \dot{k}}{\partial \lambda} \\[2mm] \dfrac{\partial \dot{\lambda}}{\partial k} & \dfrac{\partial \dot{\lambda}}{\partial \lambda} \end{vmatrix} + \begin{vmatrix} \dfrac{\partial \dot{e}}{\partial e} & \dfrac{\partial \dot{e}}{\partial \mu} \\[2mm] \dfrac{\partial \dot{\mu}}{\partial e} & \dfrac{\partial \dot{\mu}}{\partial \mu} \end{vmatrix} + 2 \begin{vmatrix} \dfrac{\partial \dot{k}}{\partial e} & \dfrac{\partial \dot{k}}{\partial \mu} \\[2mm] \dfrac{\partial \dot{\lambda}}{\partial e} & \dfrac{\partial \dot{\lambda}}{\partial \mu} \end{vmatrix} \tag{18}$$

Dockner ([1985](#)) proved that the necessary and sufficient conditions for the local stability in optimal control problems with two state variables are the following:

$$
\begin{aligned}
&1)\, M < 0 \\
&2)\, \det J \in \left(0,\, M^2/4\right)
\end{aligned}
$$

Conditions 1) and 2) guarantee the existence of two eigenvalues which are either negative or have negative real parts.

Feichtinger et al. ([1994](#)) (see also Dockner and Feichtinger [1991](#)) show that in order to generate a limit cycle in two-state-variable optimal control problems as our modified Ramsey model, the necessary and sufficient conditions are that the *detJ* and *M* are positive when calculated with the steady-state levels. In addition, the value of the bifurcation parameter (ρ) given by the following condition

$$
\det J = \left(\frac{M}{2}\right)^2 + \rho^2 \left(\frac{M}{2}\right) \tag{19}
$$

must be positive as well.

For our model, the determinant of the Jacobian calculated at the steady state equilibrium is:

$$
\det J = \begin{vmatrix}
f_k(e*,k*)-(\delta+n) & f_e(e*,k*) & -g_\lambda(\lambda*) & 0 \\
-\alpha f_k(e*,k*) & -\alpha f_e(e*,k*) & 0 & 0 \\
f_{kk}(e*,k*)(\alpha\mu*-\lambda*) & f_{ke}(e*,k*)(\alpha\mu*-\lambda*) & \rho+\delta+n-f_k(e*,k*) & \alpha f_k(e*,k*) \\
f_{ek}(e*,k*)(\alpha\mu*-\lambda*) & f_{ee}(e*,k*)(\alpha\mu*-\lambda*) & -f_e(e*,k*) & \rho+\alpha f_e(e*,k*)
\end{vmatrix} \tag{20}
$$

and the coefficient *M* at the steady state equilibrium is:

$$
M = \begin{vmatrix}
f_k(e*,k*)-(\delta+n) & -g_\lambda(\lambda) \\
f_{kk}(e*,k*)(\alpha\mu*-\lambda*) & \rho+\delta+n-f_k(e*,k*)
\end{vmatrix} + \begin{vmatrix}
-\alpha f_e(e*,k*) & 0 \\
f_{ee}(e*,k*)(\alpha\mu*-\lambda*) & \rho+\alpha f_e(e*,k*)
\end{vmatrix}
$$
$$
+ 2\begin{vmatrix}
f_e(e*,k*) & 0 \\
f_{ke}(e*,k*)(\alpha\mu*-\lambda*) & \alpha f_k(e*,k*)
\end{vmatrix} \tag{21}
$$

In order to obtain crisp solutions and easy to interpret results we assume the following explicit functions: $U(c)=ln\ c$, and $Y=F(A,K,L)=AL+K^aL^{1-a}$, $0<a<1$, so output per capita is:

$$
y = \frac{Y}{L} = \frac{F(\bullet)}{L} = A + \frac{K^aL^{1-a}}{L} = A + \left(\frac{K}{L}\right)^a\left(\frac{L}{L}\right)^{1-a} = A + k^a = f(A,k)
$$

As we assume $A=e$, it follows that:

$$
y = f(A,k) = f(e,k) = e + k^a
$$

This production function is characterized by: $f_e(e,k)=1$; $f_k(e,k)=ak^{a-1}$; $f_{ek}(e,k)=0$.

With these explicit functions the steady state values for k^*, e^*, y^*, c^*, λ^*, μ^* are respectively:

$$f_k = \left(\frac{\rho + \alpha}{\rho}\right)(\rho + \delta + n)] \Rightarrow k^* = \left[\left(\frac{\rho + \alpha}{a\rho}\right)(\rho + \delta + n)\right]^{1/(a-1)} \quad (22)$$

$$e^* = \bar{y} - (k^*)^a \Rightarrow e^* = \bar{y} - \left[\left(\frac{\rho + \alpha}{a\rho}\right)(\rho + \delta + n)\right]^{a/(a-1)} \quad (23)$$

$$y^* = e^* + (k^*)^a = \bar{y} \quad (24)$$

$$\dot{k} = 0 \Rightarrow c^* = y^* - (\delta + n)k^* \Rightarrow c^* = \bar{y} - (\delta + n)\left[\left(\frac{\rho + \alpha}{a\rho}\right)(\rho + \delta + n)\right]^{1/(a-1)} \quad (25)$$

$$\lambda^* = 1/c^* = \left(\bar{y} - (\delta + n)\left[\left(\frac{\rho + \alpha}{a\rho}\right)(\rho + \delta + n)\right]^{1/(a-1)}\right)^{-1} \quad (26)$$

$$\dot{\mu} = 0 \Rightarrow \mu^* = \lambda^*/[\rho + \alpha] = \left(\bar{y} - (\delta + n)\left[\left(\frac{\rho + \alpha}{a\rho}\right)(\rho + \delta + n)\right]^{1/(a-1)}\right)^{-1}[\rho + \alpha]^{-1} \quad (27)$$

Taking into account the steady-state values in Eqs.(22)-(27) we can calculate $detJ$, and M, which yields the following Proposition and Corollary that present the main results of this paper.

Proposition 1 There is a limit cycle between entrepreneurship and capital stock, if 1) $\rho + n + \delta > f_k(e,k) = ak^{a-1} > n + \delta > \rho$; and 2) ρ is positive and satisfies: $detJ = M^2/4 + \rho^2(M/2)$.

Proof If inequalities 1) and 2) are verified then $detJ$ and M are positive, since

$$detJ = \alpha(\delta + n)\left[\alpha ak^{a-1} + (\alpha + \rho)(\rho + \delta + n - \alpha ak^{a-1})\right] + (\alpha + \rho)(\lambda^*)^{-2}\left[\alpha a(a-1)k^{a-2}(\alpha\mu^* - \lambda^*)\right]$$

As $\alpha\mu^* - \lambda^* < 0$, if $\rho + n + \delta > f_k(e,k) = ak^{a-1}$ then $detJ^* > 0$. In the same vein, since

$$M = \left[ak^{a-1} - (\delta + n)\right]\left[\rho + \delta + n - \alpha ak^{a-1}\right] - (\lambda^*)^{-2}a(a-1)k^{a-2}(\alpha\mu^* - \lambda^*) - \alpha(\alpha + \rho) + 2a\alpha k^{a-1}$$

If $\frac{k^*}{\bar{y}} > \frac{(1-a)\rho}{2\alpha(\alpha+\rho)+(1-a)(\delta+n)}$, and $\rho + n + \delta > f_k(e,k) = ak^{a-1} > n + \delta$, then $M > 0$. Note, however that $\frac{k^*}{\bar{y}} > \frac{(1-a)\rho}{2\alpha(\alpha+\rho)+(1-a)(\delta+n)}$ implies $\frac{2\alpha(\alpha+\rho)}{(1-a)\rho} + \frac{\delta+n}{\rho} > \frac{\bar{y}}{k^*} > 1$, this inequality can be easily fulfilled if either $\rho < n + \delta$ or $2\alpha(\alpha+\rho) > (1-a)\rho$. Therefore if: $\rho +$

$n+\delta>f_k(e,k)>n+\delta>\rho$, $detJ$ and M are positive. In addition note that the necessary condition 2) is satisfied since the value of the bifurcation parameter ρ determined by: $\det J = M^2/4 + \rho^2(M/2)$, is positive.

These three conditions: $detJ>0$, $M>0$, $\rho>0$, are sufficient, according to Feichtinger et al. (1994), to generate a limit cycle, through the Hopf bifurcation theorem, between entrepreneurship and capital stock.

According to Proposition 1 a necessary condition for the existence of the limit cycle imply that the marginal productivity of capital [real interest rate, $f_k(e,k)$] lies in the interval $\rho+n+\delta>f_k(e,k)>n+\delta>\rho$. Note that in the standard Ramsey model the marginal productivity of capital equals the sum of the subjective rate of time preference, population growth rate and rate of capital depreciation $f_k=(\rho+n+\delta)$, therefore in our modified Ramsey model the real interest rate is smaller than in the standard Ramsey model. As a result of this condition, in the present model the optimal capital stock is greater than the optimal capital stock of the standard Ramsey model.

The existence of a limit cycle between capital stock and entrepreneurship leads to the existence of other cyclical relationships. These are the subject of the Corollary below.

Corollary As capital stock and entrepreneurship are cyclical, they make output and consumption cyclical.

Proof Note that output is given by: $y=e+(k)^a$ as e and k are cyclical it is clear that y is also cyclical. In addition, note that consumption is given by: $c=y-(\delta+n)k$, as y and k are cyclical, then consumption c is also cyclical.

The cycle has an unmistakable Schumpeterian (Schumpeter 1934) taste, and its description is as follows: When unemployment is high, the pool of entrepreneurs increase since the unemployed can always opt to become entrepreneurs; new entrepreneurs enter the market with new innovations, increasing the capital stock, employment, output, and consumption; when the economy is booming with high output and low unemployment the number of new entrepreneurs entering the market decrease, so does technological innovations, reducing the capital stock, employment, output, and consumption and the cycle repeats itself.

Concluding remarks

In a recent pioneering empirical paper Koellinger and Thurik (2012) find that entrepreneurship Granger-causes the cycles of the world economy, and that entrepreneurial cycles are positively affected by the national unemployment cycles. However, they do not present a theoretical model to explain the empirical findings. This paper fills this gap providing a theoretical explanation through an extended Ramsey model.

In the extended Ramsey model the differential equation describing technological innovations led by entrepreneurs, which relates entrepreneurship dynamics to unemployment and output dynamics, is considered as an additional dynamic restriction. The model becomes a concave two-state-variable optimal control problem that may generate limit cycles through the Hopf bifurcation theorem. It is shown that one necessary condition for the existence of a limit cycle between capital stock and entrepreneurship

is that the real interest rate is below the equilibrium real rate of interest of the standard Ramsey model and is greater than the sum of population growth rate and capital depreciation rate. As a consequence of the cyclical behavior of entrepreneurship and capital, output and consumption also display cycles. Therefore, if the entrepreneurial economy accumulates more capital than the Ramsey model, it yields as a result that entrepreneurial and unemployment cycles cause business cycles.

The cycle generated by our model is easy to understand and is quite intuitive. When unemployment is high, the unemployed may choose to become entrepreneurs and in this case they enter in the market carrying a new technological innovation. This makes the capital stock to increase leading to the creation of new jobs, raising output and consumption. When the economy is booming and unemployment is low the number of new entrepreneurs is reduced leading to a fall in technological innovations and capital stock, resulting in a reduction of employment, output and consumption, and the cycle repeats itself.

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