RESEARCH ARTICLE

Municipal food waste recycling as an environmental strategy: a game‑theoretical approach

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Received: 27 May 2022 / Accepted: 23 October 2022 / Published online: 23 December 2022 © The Author(s), under exclusive licence to Springer-Verlag GmbH Germany, part of Springer Nature 2022

Abstract

With the expansion of urbanization, we are witnessing the growing uncertainty in municipal food demand leading to an increase in urban waste. With the motive of producing organic fertilizers and conserving the environment, expired food can be collected and recycled. This study examines the hypothesis that leasing recycling facilities from peri-urban areas, due to the ban on reproduction operations in the city centers, can manage the recycling system participants' relationship and enhance sustainability in urban communities. The problem has been investigated under two separate sources of uncertainty, namely, quality and capacity. In the frst scenario, a recycling system consisting of a commercial food service located in urban areas, a food waste collection agency, and a suburban fertilizer factory is optimized, in which the commercial food service leases the fertilizer factory's facilities for recycling operations. In the second scenario, the two factories' relationship, in which the frst factory can rent the second factory's facilities in case of capacity shortage, is managed through hybrid contracts and mathematical programming models. The results show that the whole system optimization and Pareto Improvement results for all members are guaranteed under proposed hybrid contracts. These conclusions can help food recycling system managers have a better relationship with other players in their supply chains and enhance their credibility for caring about the environment, social concerns, and government compliance.

Keywords Urban solid waste · Sustainable development · Hybrid contracts · Facility renting · Food recycling · Option contract · Quantity fexibility

Introduction

World's population in urban areas has dramatically increased over the last 50 years (Saborido and Alba [2020](#page-24-0)). Although urban life is an important subject in economic growth and social modernization (Al-Mulali et al. [2012\)](#page-23-0), it is also a major factor in producing several kinds of pollutants which may arouse a lot of environmental problems such as resource scarcity (Wu and Tan [2012](#page-24-1)), excessive energy consumption (Lin and Zhu [2021;](#page-24-2) Chen and Zhou [2021](#page-23-1)), greenhouse gas emissions (Muhammad et al. [2020\)](#page-24-3), global warming, and climate change. For instance, the increase in the consumption of clean water (Liu et al. [2020](#page-24-4); Ma et al. [2022](#page-24-5)), the

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increase in emission of carbon dioxide in urban areas, and the increase in the generation of household waste are all considered as some results of dramatic growth in the urbanization (Zhang and Chen [2021\)](#page-24-6).

The subject of municipal household waste, especially food waste, is one of the most signifcant outcomes of civilization (Sheng et al. [2021](#page-24-7)), which has caused various environmental problems today (Howe and Wheeler [1999](#page-23-2)). At present, household waste and especially food waste constitute a major and signifcant part of solid pollutants and its percentage is increasing annually (Wang et al. [2018b\)](#page-24-8). For example, food is the largest source of solid pollutant waste in the United States, and failure to recycle it may have devastating efects on the environment and social satisfaction (Pai et al. [2019](#page-24-9); Balsalobre-Lorente et al. [2022\)](#page-23-3). Unrecycled food waste is a major cause of methane emissions from the surface of landflls, which will eventually lead to contamination of surface water as well as groundwater (Bulak and Kucukvar [2022](#page-23-4); Ikhlayel and Nguyen [2017](#page-23-5)).

Regarding the truth that reducing food waste is the 3rd of 100 solutions to prevent climate change, it seems necessary for city managers and planners to think about municipal waste and ways of recycling it such as producing animal feed or organic fertilizers (Cheng et al. [2021\)](#page-23-6).

Another issue that needs to be addressed is the fact that chemical fertilizers are currently monopolized in the feld of agriculture. Although about 50% of agricultural materials use chemical fertilizers, there is a general concern in the agricultural industry to reduce the use of excessive and inappropriate amounts of chemicals in non-organic fertilizers (Fernández-Delgado et al. [2022](#page-23-7); Chehade and Dincer [2021](#page-23-8)). One of the benefts of using natural substances is to regulate the carbon level of the soil which prevents harming plants (Sharma et al. [2019](#page-24-10)), so it seems necessary to replace chemical and harmful fertilizers with natural and organic substances (Li et al. [2022;](#page-23-9) Aryal et al., [2021](#page-23-10)).

Numerous studies (Campuzano and González-Martínez [2017;](#page-23-11) Fernández-Delgado et al. [2020](#page-23-12)) can be found in the literature that emphasizes the possibility of producing natural and organic fertilizers from urban food waste. They believe that this urban waste should not be injected directly into the soil due to the toxic particles it may contain, so it must be detoxifed through modern technologies so as not to cause any damage to the soil and plants (Bloem et al. [2017\)](#page-23-13). There are also various articles in the literature, such as (Levis et al. [2010;](#page-23-14) Pai et al. [2019](#page-24-9)), that discuss appropriate technologies, trained personnel, distinct collection or transportation equipment, modern recycling halls, and up-to-date reproduction facilities with sufficient capacity to produce natural fertilizers from municipal food waste.

This study deals with the issue of urban food recycling systems, and the environmentally friendly transformation of municipal organic waste into natural fertilizers through hybrid contracts such as Quantity Flexibility (QF), Single Cost-Sharing (SCS), Two-Way Cost-Sharing (TWCS), and Option Contract as well as mathematical programming models. Besides, to deal with possible uncertainties, the problem has been investigated in two separate scenarios. In the frst scenario, the collected waste quality, and in the second scenario, the recycling capacity is assumed to be uncertain with specific probability distribution functions. In addition, in diferent countries, there are some governmental laws and regulations prohibiting manufacturing activities in urban areas to prevent air, water, and noise pollution in residential zones. Therefore, in this study, the commercial food service, which is normally supposed to be located in the city centers, is forced to rent reproduction facilities from fertilizer factories in the suburbs.

While recycling food waste is technologically possible and proftable, our observations reveal a large amount of disposal of not-recycled food waste especially in megacities in developing countries. In some cases, using/selling spoiled foods is also reported which is illegal and harmful to human health. While the creation of some food waste is inevitable due to urbanization, it is possible to recycle them to produce organic fertilizers. In this way, we are able to avoid using chemical fertilizers, replace them with organic fertilizers, and make the planet greener. We realized that there are some managerial and planning obstacles in recycling food waste in urban areas; the most important is the issue of coordination between players in the reverse logistics system. This problem motivates us to analyze and optimize the reverse operations of a food waste recycling system. This paper seeks to achieve the following aims:

- Collecting and recycling municipal food waste, which is increasingly damaging the environment due to the expansion of urbanization.
- Preventing the sale of expired food in cafes, hotels, fast foods, etc. to control the spread of disease transmission
- Replacing chemical fertilizers which are harmful to soil and agricultural products with natural and organic fertilizers
- Creating social satisfaction and following the governmental laws and environmentally friendly restrictions on not carrying out recycling activities in the city by renting remanufacturing facilities from suburbs
- Helping recycling system managers to create better collaboration between channel participants and increase the whole system's proftability.

The remaining of the research is organized in this way: in the second part, a research background review is conducted, in the third part, the problem under study is described in all its details, the fourth part is related to mathematical modeling and solving them, and the ffth and sixth sections are devoted to the discussion of numerical results and conclusions, respectively.

Literature review

This research is related to three research areas in the literature, namely game theory and waste recycling, channel optimization using supply contracts, and reverse/closed-loop recycling systems, which will be reviewed in the following.

Game theory and waste recycling

Beheshti et al. [\(2022](#page-23-15)) examined a closed-chain system in which a commercial enterprise, through a waste collection agency, collects expired food from agricultural sales centers. In their research, due to governmental restrictions on not carrying out recycling activities in the city, the commercial enterprise is obliged to rent production facilities in suburban areas. They also used a new QF contract to optimize the profts of the entire system and individual components by considering two separate modes for procurement time. Mak et al. [\(2021](#page-24-11)) aimed to gather information that might change corporate recycling behavior. By designing a questionnaire and conducting a survey on food waste management from various industries such as hotels, restaurants, and cafes in Hong Kong and Malaysia, they analyzed the issue of changing recycling behavior after clearing the information. Their experiments showed that in Malaysia, the most important factors that affect the recycling behavior of food waste producers are transportation status, infrastructures, and rewards or managerial motivations, respectively. Siddiqui et al. [\(2021](#page-24-12)) in their research raised the issue of producing liquid fertilizers and poultry feed from food waste. They collected and tested expired food from relevant agencies such as restaurants, hotels, cafes, and leisure centers. Their research showed that feed containers produced through food waste generally contain 19% protein, with an acceptable range of 15 to 23% in the National Research Council. Zhu et al. [\(2020\)](#page-24-13) by using mathematical modeling and theory algorithms based on simulated analysis evaluated the interactions and behaviors between food waste recycling channel members through dynamic systems and game theory approaches. To avoid optimistic behavior and unfavorable rules that may arise due to fxed penalties in the organization, they designed and proposed a penalty scheme with random inspection in the form of a three-player game in the feld of municipal food waste management. They then tested both fxed and random penalty schemes to fnd the impact of important factors on members' decisions. Their results emphasize that a random penalty scheme, compared to a fxed penalty scheme, improves both pragmatism and stability in the system.

Briški et al [\(2007\)](#page-23-16) introduced a mathematical approach to analyze the process of material decomposition related to natural municipal solid waste. They managed to get a good estimate of the process performance using their mathematical and experimental approach. In another study, Edwards et al. [\(2016](#page-23-17)) introduced a novel mathematical programming approach to predict energy and time consumption of waste collection and recycling. They proved that their urban waste recycling model is more efficient than other models in order to estimate the amount of fuel and the number of needed vehicles. In a recent study, Chauhan et al. [\(2022](#page-23-18)) investigated waste recycling concepts in fuzzy environments with the aim of optimizing expected costs and proft for incomplete production processes. They developed sustainable manufacturing systems with fuzzy methods and also showed the credibility of their model with numerical examples. They illustrated that by using their proposed approach and analytical methods, it is possible to increase the proft per unit of time to its highest level.

Regarding the development of game theory, Von Neumann and Morgenstern ([1944\)](#page-24-14) spent many years investigating the issue of human decision-making in logical conditions. Finally, they showed in their classic theory that humans take a series of actions to increase their proft for rational decision-making, which led to the formation of game theory (Von Neumann and Morgenstern [1944\)](#page-24-14). In addition, Aumann and Schelling ([2005\)](#page-23-19) provided a very useful tool for a better understanding of human behavior by using repeated game theoretical methods and presenting two diferent strategies. Gul [\(1997\)](#page-23-20) reviewed John C. Harsanyi, John F. Nash, and Reinhard Selten's work, which was awarded the Nobel Prize in Economics by the Royal Swedish Academy of Sciences in 1994 for their studies in the feld of non-cooperative games. They facilitated science development in the present years by contributing to some felds such as asymmetric information, equilibrium, and credibility, which are considered the most important topics in game theory.

Regarding advances in game theory and its various applications, İzgi and Özkaya ([2019](#page-23-21)) introduced a new method to solve two-player zero-sum matrix games according to the size and norms of the matrix. They also showed how to get a good approximate solution to the problem without solving all the existing equations. In another research, D'Orsogna and Perc ([2015\)](#page-23-22) analyzed various methods available in the literature, including game theoretical approaches, to improve the understanding and recognition of crime. They believed that crime statistics can help to produce a suitable strategy for crime modeling and crime prevention. As another application of game theory, Özkaya and İzgi (2021) (2021) by using the available approaches in game theory, investigated the efects of home quarantine on the prevalence of the frst wave of coronavirus. Similar to other game theoretical research in various areas in the literature, they concentrated on applying game theoretical methods to investigate the efects of home quarantine on the initiation, expansion, and termination of this virus. Perc et al. [\(2017\)](#page-24-16) studied theoretical and practical research that can bring us a better recognition of cooperation and coordination concepts by using spatial patterns and dynamic visions. Madani ([2010\)](#page-24-17) studied the utilization of game theory methods to improve resource preservation such as water. The author proved that the problems related to natural resources such as water always consist of a dynamic structure, by using a set of non-cooperative games. İzgi and Özkaya [\(2020\)](#page-23-23) examined the problem of decision-making on agricultural insurance. They created a zero-sum matrix game against nature using real data. They frst solved this game separately through the max–min and regret criteria in decision theory. Then, by reconsidering this game where mixed strategies are allowed, they solved it with the known methods for solving zero-sum matrix games in the literature.

Optimization mechanisms and supply contracts

Elmaghraby [\(2000](#page-23-24)) conducted a general review in the feld of sourcing methods with the help of optimization and economics tools. In general, the author showed how market characteristics and supply chain contracts can afect the relationship between buyers and suppliers and lead to the creation of diferent supplier selection approaches. Henig et al. ([1997](#page-23-25)) optimized supply chain contract terms in the feld of inventory planning. First, they modeled inventory control policies with a periodic review approach under their suggested contract, which offered the ordering costs by a multi-interval linear function. Then, they used the Markov Chain method for calculating transportation, shortage, and holding costs as well. Wang [\(2002](#page-24-18)) used supply chain contracts as a useful tool for creating coordination and motivating channel members in order to make their decisions similar to an integrated structure. They reviewed a wide range of supply chain structures and provided a general framework for combining different types of supply contracts to offer new hybrid coordination mechanisms. More recently, Li et al. ([2016](#page-24-19)) used a QF contract between a cosmetics manufacturer and a retailer to enhance the system efficiency. They showed that the channel members, under their contract, will achieve the highest possible level of proft. Patra and Jha [\(2022\)](#page-24-20) tested the efectiveness of a bilateral option contract to control potential risks in a two-tier system. Their network includes a supplier and a humanitarian agency, which has been evaluated using a novel option contract. In their research, they assumed the demand function as a uniform and exponential function and addressed the issue of preprepared relief items before the disaster, as well as the exact number of needed relief items after the disaster. Wang et al. [\(2021](#page-24-21)) also used an option contract to optimize a production system as well as its manufacturing, ordering, and inventory decisions. They frst applied a simple wholesale price contract to have a suitable pattern for comparison, and then incorporated other infuential factors such as overconfdence into their model, and re-analyzed the expected proft of the channel members. Finally, by using the option contract, they optimized the whole network by calculating the option and exercise prices. To resolve inconsistencies among a three-tier food network's members, Pourmohammad-Zia et al. [\(2021](#page-24-22)) addressed the issue of channel optimization using customized contracts. Their network consists of three separate members: a farm as a supplier, a food producer as a manufacturer, and a food retailer as a distributor. In their research, to reduce food waste and improve economic, production, and inventory conditions, the manufacturer has used the Vendor Managed Inventory system to control the level of retail orders. In addition, to have more market share and increase the level of sales coverage, the manufacturer has ofered a cost-sharing contract as an incentive to share some of the potential risks of the retailer. Besides, to improve the returning used products conditions, Dutta et al. ([2016](#page-23-26)) implemented a buy-back contract from the manufacturer to the retailer. In their proposed buy-back contract, they considered the demand and capacity to be uncertain in a multi-period closed-loop network structure. Bai et al. ([2017\)](#page-23-27) implemented a Cap and Trade system in a two-tier sustainable channel, including a manufacturer of degradable products and a retailer in which the demand is considered to be time-dependent. They frst analyzed the centralized and decentralized scenarios and then used the revenue-sharing, cost-sharing, and two-part tarif contracts to optimize the member relationships. Their results show that under their proposed contract, the whole channel's proft and the carbon emission level are both improved. Heydari and Ghasemi [\(2018\)](#page-23-28) studied a two-tier reverse network including a producer and a collection agency in a situation where the collection agency persuades customers to return their used products by providing an incentive scheme. In their study, both the quality of the returned products and the recycling capacity of the recycler are assumed to be uncertain, and only products that have the minimum required quality are accepted. Finally, they used a modifed revenue-sharing contract to optimize the entire network relationships and the channel's overall proftability, and share existing risks fairly between both participants.

Reverse/closed‑loop recycling systems

Savaskan et al. [\(2004\)](#page-24-23) investigated the various forms of supply chain network structures in reverse directions to fnd the best methods of collecting and recycling used products. They considered three diferent options for collecting mechanism and calculated the optimal strategies under a Stackelberg game between a manufacturer as the leader and a buyer as the follower. Krikke et al. ([2003\)](#page-23-29) presented a mathematical model to improve the decision-making process that fts both product design and transportation structures. They also investigated the efective factors of improving environmental conditions using linear functions based on energy consumption and waste generation. They fnally applied and tested their proposed model to design a closed-loop supply chain structure for collecting and recycling refrigerators of a Japanese company. More recently, Wang et al. ([2018a](#page-24-24)) by using an evolutionary and meta-heuristic algorithm based on a modifed genetic simulated annealing algorithm investigated the effect of customer satisfaction on the collection of broken bikes in a recycling system in certain areas. Ramezani et al. ([2013](#page-24-25)) studied network design in both forward and reverse directions and modeled and solved the uncertainty of environmental conditions using mathematical multiobjective stochastic models. In the forward direction, they considered three parts: suppliers, factories, and distribution centers, and in the recycling structure, two parts including waste collection agency and disposal centers. Intending to optimize the whole system's proft, improve the customers' situations, and raise the products' quality, they presented their new method for analyzing the transportation status of the channel. They also introduced a risk management plan to create a tradeoff between different goals and showed that eventually a set of optimal Pareto solutions would be obtained. By using two-objective mathematical programming models, Rahimi et al. ([2016](#page-24-26)) analyzed a sustainable recycling system of perishable food waste with a defnite expiration date, in which addressing environmental, social, and economic concerns are the main goals of managers. In their two-objective programming model, the frst objective function is designed to optimize the distribution and maintenance costs of inventories, and the second objective function is related to the issue of reducing vehicle crashes and decreasing the number of corrupted products. In addition, to reduce the computational complexity, they considered other environmental criteria and the issue of social dissatisfaction factors such as vehicle noise pollution as constraints in their model. By presenting a new method based on mathematical programming models for analyzing the issue of locationallocation in an urban freight recycling system, Ndhaief et al. [\(2017](#page-24-27)) examined the status of both forward and reverse logistics structures. With their proposed model, they were fnally able to improve the city's logistics situation and save on network operating costs. Chen et al. ([2021](#page-23-30)) innovated in remanufacturing processes by using a closed-loop recycling system including a producer and a seller, in which the producer is allowed to resell the recycled products through the seller. They used two Stackelberg games, once with a producer-led model, once with a seller-led model, and also a Nash game with equal powers for both members, to optimize the players' decisions. Finally, they presented a cost-sharing contract to optimize the profts of the entire network. By using laboratory and experimental methods, Marcos et al. [\(2021\)](#page-24-28) analyzed the efect of various uncertainties such as supply, demand, process, and environmental conditions on the recycling of lithium-ion batteries in an electric vehicle closed-loop recycling system.

Research gap

It is clear that the two important currents in the literature, namely municipal waste management and channel optimization, have the most correlation with the subject of our research. However, some newly published review papers such as (Nematollahi and Tajbakhsh [2020](#page-24-29); Nematollahi et al. [2021\)](#page-24-30) by reviewing 247 recent articles, showed that in the quantitative and mathematical literature on food and agricultural channels, not much research has been done on municipal waste recycling system optimization. Therefore, managing and mathematically analyzing the issue of recollecting and recycling food waste materials in nutritional products industries is a signifcant gap in the literature that requires getting more attention. Also, Guo et al. [\(2017](#page-23-31)) by reviewing most of the articles in the feld of channel optimization found that the majority of the researchers are using single contracts in their studies to facilitate the members' relationships. for instance, a buy-back mechanism between a supplier and a manufacturer, between a manufacturer and a retailer, or between a reproducer and a collection agency, in which one of the participants in the contract usually plays as a leader and the other one as a follower. In fact, in the literature and on the recycling of agricultural and urban food waste, the issue of optimization using hybrid and multiple contracts has received less attention. A hybrid contract means when there is more than one contract between the channel members, for example, if there is both a QF and a revenue-sharing contract between manufacturer and retailer, we call it a hybrid contract. Another type of supply contract is multiple contracts that are concluded between several diferent members of the network. For example, if a cost-sharing contract is considered among a manufacturer, supplier, and retailer, it is called a multiple contract.

This study intends to address this literature gap on municipal food waste recycling optimization by converting the collected urban waste into organic fertilizers, using mathematical models and novel, hybrid, and multiple contracts under the demand disruption due to Covid-19 and two sources of uncertainty, namely quality and capacity.

Besides, while QF contracts have been used as efficient approaches to optimize the channels in diferent studies (for example (Heydari et al. [2020\)](#page-23-32)), the majority of these contracts have been developed in the feld of commodity production orders and inventory planning decisions. However, in this study, a QF contract has been used to lease facilities and recycling capacity in peri-urban regions to follow government laws prohibiting production activities in urban areas, which is a new work in the literature.

According to Table [1](#page-5-0) and in comparison with other papers similar to this study, we can refer to (Li et al. [2021\)](#page-24-31) and (Beheshti et al. [2022\)](#page-23-15). Li et al. ([2021\)](#page-24-31) used two capacity reservation and QF contracts to expand the retailer's capacity, while in this study, two option and QF contracts have been compared to rent recycling capacity from suburban areas. They also considered demand and price to be uncertain in a forward structure, while in this study, quality and capacity are assumed to be uncertain in a reverse flow. Beheshti et al. ([2022](#page-23-15)) also used a novel QF contract, with two modes for lead time, to recycle municipal food waste in a closed-loop recycling system under uncertain quality conditions. But, in this study, the combination of QF with SCS and TWCS contracts has been implemented in a reverse recycling

system in which both quality and capacity are considered to be uncertain.

Eventually, our major innovations can be listed below:

- Managing the channel members' relationships in a municipal solid waste recovering system under the uncertainty of the capacity, quality, and demand disruption due to the pandemic quarantine regulations
- Renting recycling facilities and their related technologies from peri-urban areas to comply with governmental rules and prevent the pollution of soil and plants by converting municipal food waste into organic fertilizers
- Reaching channel optimization using and comparing hybrid and multiple contracts such as option contract, QF contract, SCS contract, TWCS contract, and mathematical programming models

Problem description

Apart from economic issues, another important social responsibility, which should be well considered to have a sustainable society and a favorable environment, is the issue of recycling municipal food waste. Today, the expansion of urbanization, the increasing rate of migration to urban areas, and the spread of various diseases such as Covid-19, have caused an increase in the level of food waste in various communities. Therefore, restaurants, cafes, hotels, and chain stores in urban areas these days are faced with large volumes of expired food due to fuctuations and uncertainties related to demand forecasting. Although these food and agricultural wastes can no longer be used by humans, they can still be considered a suitable source of producing natural fertilizers. While most of the agricultural market is monopolized by chemical fertilizers, organic and natural fertilizers are much more nutritious for the soil and plants. Thus, food channel managers should look for suitable solutions to facilitate the collection and conversion of this expired food into organic and natural fertilizers. This conversion also requires up-to-date technologies, appropriate facilities, and sufficient capacity, but in most modern societies, due to the observance of social satisfaction and the prevention of air pollution and noise pollution, manufacturing operations are restricted in residential parts of the city. Since commercial food services are normally built near the city centers, and recycling activities are prohibited there, they must rent their relevant facilities from factories in the suburbs. This research, by using mathematical programming models and hybrid and multiple supply contracts, deals with the issue of relationship management and channel optimization in renting food waste recycling facilities from outside of the city and converting them into natural fertilizers.

In this paper, two uncertainty factors have been studied in two separate scenarios. The frst scenario envisages a three-tier recycling system including a commercial food service near the city center, an out-of-town organic fertilizer factory, and a food waste collection agency. In exchange for a reward, the food waste collection agency receives and inspects municipal food waste from restaurants and chain stores throughout the city. Expired food items that have the necessary quality for recycling are accepted after inspection and sold to the commercial food service. The commercial food service also leases the relevant technologies from the fertilizer factory in suburbs, and fnally sells the recycled organic fertilizers to its special customers in the agricultural sector. As the process of equipping the workshop is a time-consuming activity, the fertilizer factory expects the commercial food service to announce its requested capacity previously, but because of the uncertainty of the collected food waste quality, the exact amount of accepted waste is not clear in advance. Therefore, to properly manage the relationship in renting facilities in the frst scenario, once a hybrid QF and SCS contract and once again a hybrid QF and TWCS contract have been applied to optimize the channel relationships. According to these combined contracts, the commercial food service frst issues an initial order for the fertilizer factory in order for preparing the workshop, and then, when the accurate number of confrmed items is specifed, it can modify its initial order up to b% more and a% less, through a secondary order. Also, the commercial food service and the food waste collection agency can share some parts of the disposing and renting costs to optimize the entire system's profts. Figure [1](#page-7-0) shows the material and money fows in the first scenario.

Moreover, in the second scenario, a two-tier recycling system consisting of two fertilizer factories located in suburban areas is considered in such a way that the frst factory's remanufacturing capacity is assumed to be uncertain. In this scenario, the frst factory collects and recycles expired food from restaurants and chain stores in exchange for proposing a reward and sells the produced organic fertilizers to the agricultural industries. Since the frst factory's remanufacturing capacity is uncertain, in case of capacity shortage, it can rent some parts of the second factory's facilities and carry out its process there. In this section, to better manage the renting relationship, once a QF contract and once an option contract has been used to optimize the channel. In the option contract, the frst factory reserves some of the second factory's facilities in exchange for paying the option price through an initial order in advance, and after determining the exact amount of its additional required facilities, it can pay the exercise price and rent its requested capacity up to the ceiling of its initial order. Figure [2](#page-8-0) shows the material and money transfer fows in the second scenario. In addition,

Table [2](#page-9-0) also defnes the notations, parameters, and variables used in problem modeling.

Subscripts S, T, f, and o represent SCS and TWCS contracts in the frst scenario, QF, and option contracts in the second scenario, respectively. Besides, Superscripts D, C, and H also refer to decentralized, centralized, and coordinated schemes, respectively.

The following assumptions are also considered in modeling the problem mathematically:

- Logically $m \geq d_{\rm S}$, otherwise, collecting the food waste in the frst scenario and under the SCS contract will not be justifed for the food waste collection agency.
- Logically $V \ge m + L$, otherwise, recycling food waste in the frst scenario will not be justifed for commercial food service.
- Logically $L \geq t$, otherwise, the first factory under the QF contract in the second scenario will not do any recycling activity in its workshops.
- Logically $V \ge d_f + L$, otherwise, recycling food waste in the second scenario and under the QF contract will not be justifed for the frst factory.
- Logically $e \geq t$, otherwise, the first factory under the option contract in the second scenario will not do any recycling activity in its workshops.

• Logically $V \geq d_o + e$, otherwise, recycling of food waste in the second scenario and under the option contract will not be justifed for the frst factory.

Modeling and solution method

The issue has been investigated under two separate scenarios. In the frst scenario, it is assumed that the collected municipal food waste quality is uncertain, and the food waste collection agency, after inspection, will accept θ percent of the waste (θ has a uniform distribution function between 0 and 1). However, in the second scenario, it is assumed that the organic fertilizer factory recycling capacity (i.e., C_p), is uncertain, and if this plant runs out of capacity, it can rent the remanufacturing facilities from other factories $(C_p$ also have a uniform distribution function between 0 and U). Figure [3](#page-10-0) illustrates the modeling and problem-solving process under both the investigated scenarios.

First scenario: uncertainty in the collected municipal food waste quality

Initially, the proposed reward is considered to be predetermined through an agreement between the commercial food

QuantityFlexibility

Fig. 1 Material and money flows in the first scenario

service and store managers previously, and a hybrid QF and SCS contract is used to optimize the channel. In addition, the decision-making on the proposed reward is then placed under the authority and control of the collection agency, and eventually, a hybrid QF and TWCS contract has been implemented to manage the system relationships.

Hybrid QF and SCS contract

According to the QF contract, and the range in which θR_S is located, the proft function of the commercial food service will be a piecewise function as below:

$$
\prod_{M_S} = \begin{cases}\n V \theta R_S - m \theta R_S - L(1 - a) q_S & 0 \le \theta R_S \le (1 - a) q_S \\
 V \theta R_S - m \theta R_S - L \theta R_S & (1 - a) q_S \le \theta R_S \le (1 + b) q_S \\
 V(1 + b) q_S - m(1 + b) q_S - L(1 + b) q_S & (1 + b) q_S \le \theta R_S\n \end{cases}
$$
\n(1)

If $0 \leq \theta R_S$ ≤ $(1 - a)q_S$, the commercial food service purchases θR_S items of accepted waste and makes a net profit equal to $V\theta R_S - m\theta R_S - L(1 - a)q_S$ by converting them into organic fertilizer. If $(1 - a)q_S \leq \theta R_S \leq (1 + b)q_S$,

the commercial food service rents θR_S capacity units and earns $V\theta R_S - m\theta R_S - L\theta R_S$ through recycling activities in fertilizer factory. Eventually, if $(1 + b)q_s \leq \theta R_s$, the commercial food service can only rent $(1 + b)q_s$ capacity units. Therefore, it purchases merely $(1 + b)q_s$ accepted items from the collection agency and receives $V(1 + b)q_S - m(1 + b)q_S - L(1 + b)q_S$.

The mathematical expectation of the commercial food service's proft function is calculated through Eq. ([2\)](#page-8-1).

$$
E\left[\prod_{M_{S}}\right] = \begin{pmatrix} \frac{(1-a)q_{S}}{R_{S}} \\ \int_{0}^{a} ((V-m)\theta R_{S} - L(1-a)q_{S})f(\theta)d\theta \\ \frac{(1+b)q_{S}}{R_{S}} \\ + \int_{\frac{(1-a)q_{S}}{R_{S}}} ((V-m-L)\theta R_{S})f(\theta)d\theta \\ + \int_{\frac{(1+b)q_{S}}{R_{S}}} ((V-m-L)(1+b)q_{S})f(\theta)d\theta \end{pmatrix} \tag{2}
$$

The food waste collection agency's function under the QF contract in the first scenario is also developed according to Eq. [\(3\)](#page-9-1)

QuantityFlexibility

Fig. 2 Material and money transfer fow in the second scenario

Table 2 Notations, parameters, and variables used in the models

$$
\Pi_{CO_S} = \begin{cases} m\theta R_S - d_S \theta R_S & 0 \le \theta R_S \le (1+b)q_S\\ m(1+b)q_S - d_S \theta R_S - S(\theta d_S - (1+b)q_S) & (1+b)q_S \le \theta R_S \end{cases}
$$
(3)

If $0 \le \theta R_S \le (1 + b)q_S$ the commercial food service purchases entire θR_S accepted items and the food waste collection agency's profit will be equal to $m\theta R_S - d_S\theta R_S$. If $(1 + b)q_s \leq \theta R_s$, the commercial food service purchases merely $(1 + b)q_S$ units of collected items and the food waste collection agency must also pay for the disposal fee of confrmed but not recovered materials.

At last, the mathematical expectation of the food waste collection agency's proft function is calculated through Eq. [\(4](#page-9-2)).

$$
E\left[\prod_{COS}\right] = \int_{0}^{\frac{(1+b)q_S}{R_S}} ((m-d_S)\theta R_S)f(\theta)d\theta
$$

+
$$
\int_{\frac{(1+b)q_S}{R_S}}^{1} ((m+S)(1+b)q_S - (d_S+S)\theta R_S)f(\theta)d\theta
$$
 (4)

The calculated proft functions are eventually simplifed as below:

$$
E\left[\prod_{M_S}\right] = -\frac{L(1-a)^2 q_S^2}{2R_S} + (V-m-L)(1+b)q_S
$$

$$
-\frac{(V-m-L)(1+b)^2 q_S^2}{2R_S}
$$
 (5)

$$
E\left[\prod_{CO_S}\right] = -\frac{(d_S + S)R_S}{2} + (m + S)(1 + b)q_S - \frac{(S + m)(1 + b)^2 q_S^2}{2R_S}
$$
\n(6)

Finally, by adding Eq. (5) (5) (5) and (6) (6) , the mathematical expectation of the whole recycling system proft function can be obtained through Eq. [\(7](#page-9-5)).

$$
E\left[\prod_{SC_S}\right] = -\frac{L(1-a)^2 q_S^2}{2R_S} + (V+S-L)(1+b)q_S
$$

$$
-\frac{(V+S-L)(1+b)^2 q_S^2}{2R_S} - \frac{R_S(d_S+S)}{2}
$$
 (7)

Proposition 1: The mathematical expectation of the commercial food service's proft is a concave function in initial order and its optimal value under the decentralized scheme is:

$$
q_S^{D*} = \frac{R_S(V-m-L)(1+b)}{(L(1-a)^2 + (V-m-L)(1+b)^2)}
$$
(8)

For all proofs see Appendix.

Proposition 2: The mathematical expectation of the entire recycling system's proft is a concave function in initial order and its optimal value under the centralized scheme is:

$$
q_S^{C*} = \frac{R_S(V+S-L)(1+b)}{(L(1-a)^2 + (V+S-L)(1+b)^2)}
$$
(9)

Fig. 3 Modeling and problem-solving process under both scenarios

Corollary 1: By comparing Eq. ([8\)](#page-9-6) and Eq. ([9\)](#page-9-7) and also considering the aforementioned assumptions, it is concluded that $q_S^{C*} > q_S^{D*}$. That is, the optimum quantity of the initial order under the centralized scheme is greater.

Corollary 2: The commercial food service is interested in ordering smaller amounts for q_S because if $\theta R_S < (1 - a)q_S$, it has to pay for the additional capacity.

Corollary 3: The food waste collection agency would prefer the commercial food service order larger amounts for q_S because if θR_S becomes greater than $(1 + b)q_S$, it has to pay for the disposal costs.

Corollary 4: By placing the values obtained from Eq. [\(8](#page-9-6)) and Eq. (9) (9) in Eq. (5) (5) , Eq. (6) (6) , and Eq. (7) (7) , and comparing them with each other, it becomes clear that:

$$
\begin{array}{l}E\Big[\prod_{SC_S}\big(q_S^{C*}\big)\Big] > E\Big[\prod_{SC_S}\big(q_S^{D*}\big)\Big], \\ E\Big[\prod_{CO_S}\big(q_S^{C*}\big)\Big] > E\Big[\prod_{CO_S}\big(q_S^{D*}\big)\Big], \\ E\Big[\prod_{M_S}\big(q_S^{C*}\big)\Big] < E\Big[\prod_{M_S}\big(q_S^{D*}\big)\Big] \end{array}
$$

Although q_S^C ^{*} is more profitable than q_S^D ^{*} for the food waste collection agency and the whole recycling system, it causes the commercial food service to lose some parts of its proft. Therefore, since deciding on the amount of initial order is under the authority of commercial food service, it will choose q_S^{D*} in favor of itself.

Therefore, the food waste collection agency, to create coordination in the channel, offers the commercial food service to order q_S^{C*} instead of q_S^{D*} , and if θR_S becomes less than(1 – *a*) q_S , it will pay $\% \beta_s$ of the extra rental costs. In this contract, which is called SCS and is widely used in the literature (Zang et al. [2022\)](#page-24-32), the food waste collection agency and the commercial food service share some parts of the costs and their potential risks between themselves to optimize the entire system's proft. According to this contract, m also changes to m_S , and its value must be calculated in such a way that none of the channel members will lose in comparison with the decentralized scheme.

The commercial food service and the food waste collection agency's expected proft functions under the proposed SCS mechanism are expressed as:

$$
E\left[\prod_{M_{s}^H}\right] = \begin{pmatrix} \frac{(1-\omega)_{\rm{sg}}}{R_{\rm{S}}} & (V-m_{\rm{S}}-\beta_{\rm{S}}L)\theta R_{\rm{S}} - (1-\beta_{\rm{S}})L(1-a)q_{\rm{S}})f(\theta)d\theta\\ \frac{(1+b)_{\rm{sg}}}{R_{\rm{S}}} & (V-m_{\rm{S}}-L)\theta R_{\rm{S}})f(\theta)d\theta\\ + \frac{\int_{\frac{(1-b)_{\rm{sg}}}{R_{\rm{S}}}}}{1} & (V-m_{\rm{S}}-L)(1+b)q_{\rm{S}})f(\theta)d\theta \end{pmatrix} \tag{10}
$$

$$
E\left[\prod_{co_{s}^{H}}\right] = \begin{pmatrix} \frac{(1-a)q_{S}}{R_{S}}\\ \int_{0}^{1}((m_{S}-d_{S}+\beta_{S}L)\theta R_{S}-\beta_{S}L(1-a)q_{S})f(\theta)d\theta\\ +\frac{\int_{0}^{1+bq_{S}}}{R_{S}}\\ +\int_{0}^{1}((m_{S}-d_{S})\theta R_{S})f(\theta)d\theta\\ +\int_{\frac{(1+b)q_{S}}{R_{S}}}^{1}((m_{S}+S)(1+b)q_{S}-(d_{S}+S)\theta R_{S})f(\theta)d\theta\\ \end{pmatrix}
$$
\n(11)

After some simplifcation the proft functions will be as follows:

$$
E\left[\prod_{M_S^H}\right] = -\frac{(1 - \beta_S)L(1 - a)^2 q_S^2}{2R_S} + (V - m_S - L)(1 + b)q_S
$$

$$
-\frac{(V - m_S - L)(1 + b)^2 q_S^2}{2R_S}
$$
 (12)

$$
E\left[\prod_{CO_S^H}\right] = -\frac{\beta_S L(1-a)^2 q_S^2}{2R_S} + (m_S + S)(1+b)q_S
$$

$$
-\frac{(m_S + S)(1+b)^2 q_S^2}{2R_S} - \frac{d_S + S)R_S}{2}
$$
 (13)

The entire recycling system's proft function also remains unchanged.

Proposition 3: The mathematical expectation of the commercial food service's proft under the SCS contract is a concave function in initial order and its optimal value from the commercial food service's point of view is:

$$
q_S^{H*} = \frac{R_S(V - m_S - L)(1 + b)}{((1 - \beta_S)L(1 - a)^2 + (V - m_S - L)(1 + b)^2)}
$$
(14)

Proposition 4: Under the SCS contract, the commercial food service's decision about q_S is the same as the centralized scheme if and only if:

$$
m_S = \beta_S (V - L) - (1 - \beta_S) S \tag{15}
$$

Corollary 5: Based on Proposition [4,](#page-11-0) a closed-form relation between the two contract parameters, namely β_s and m_s is obtained. Now we need to get the value of β_s in such a way that none of the members of the system will lose in comparison with the decentralized scheme. Then the appropriate value of m_S can be obtained by using Eq. [\(15](#page-11-1)).

Proposition 5: The maximum value of β_s under the SCS mechanism for which the commercial food service is convinced to accept the proposed contract is obtained as follows:

$$
\beta_S \le \beta_S^{\max} = 1 - \frac{\left(\frac{L(V - m - L)^2 (1 - a)^2 + (V - m - L)^3 (1 + b)^2}{(L(1 - a)^2 + (V - m - L)(1 + b)^2)^2}\right)}{\left(\frac{L(V + S - L)^2 (1 - a)^2 + (V + S - L)^3 (1 + b)^2}{(L(1 - a)^2 + (V + S - L)(1 + b)^2)^2}\right)}
$$
(16)

Proposition 6: The minimum value of β_s under the SCS mechanism for which the food waste collection agency is convinced to accept the proposed contract is obtained as follows:

$$
\beta_S \ge \beta_S^{\min} = \frac{\left(\frac{(m+S)(V-m-L)}{(L(1-a)^2 + (V-m-L)(1+b)^2)} - \frac{(m+S)(V-m-L)^2(1+b)^2}{2(L(1-a)^2 + (V-m-L)(1+b)^2)^2}\right)}{\left(\frac{(V+S-L)^2}{(L(1-a)^2 + (V+S-L)(1+b)^2)} - \frac{L(V+S-L)^2(1-a)^2 + (V+S-L)^3(1+b)^2}{2(L(1-a)^2 + (V+S-L)(1+b)^2)^2}\right)}
$$
(17)

Corollary 6: If β_S is set in the range of $\left[\beta_S^{\text{min}}, \beta_S^{\text{max}}\right]$, the system will be properly optimized by using the proposed contract. The exact value of β_s can be also determined according to the bargaining power between the commercial food service and the food waste collection agency. Besides, after determining the exact value of β_s , the amount of m_s can be obtained through Eq. ([15](#page-11-1)).

Hybrid QF and TWCS contract

In this section, it is assumed that the proposed reward (d_T) is under the authority and control of the food waste collection agency and is not known from the beginning. In addition, the collected food waste (R_T) needs to be computed again according to the new value for (d_T) .

Like (Govindan and Popiuc [2014](#page-23-33); Heydari and Ghasemi [2018\)](#page-23-28), spoiled food that is collected corresponding to the specifed amount of reward is:

$$
R_T = \begin{cases} \frac{d_T}{d_{\text{max}}} Q & d_T \le d_{\text{max}}\\ Q & d_T \ge d_{\text{max}} \end{cases} \tag{18}
$$

By substituting R_S of Eqs. ([5\)](#page-9-3), ([6\)](#page-9-4), and ([7\)](#page-9-5) with R_T from Eq. [\(18](#page-12-0)), the mathematical expectation of the commercial food service, the food waste collection agency, and the entire system's proft functions will be as follows:

$$
E\left[\prod_{M_T}\right] = \frac{-L(1-a)^2 q_T^2}{2\left(\frac{d_T}{d_{\text{max}}}Q\right)} + (V-m-L)(1+b)q_T
$$

$$
-\frac{(V-m-L)(1+b)^2 q_T^2}{2\left(\frac{d_T}{d_{\text{max}}}Q\right)}
$$
(19)

$$
E[\prod_{CO_T}] = -\frac{(d_T + S)\left(\frac{d_T}{d_{\text{max}}}Q\right)}{2} + (m + S)(1 + b)q_T
$$

$$
-\frac{(S + m)(1 + b)^2 q_T^2}{2\left(\frac{d_T}{d_{\text{max}}}Q\right)}
$$
(20)

$$
E[\prod_{SC_T}] = -\frac{L(1-a)^2 q_T^2}{2\left(\frac{d_T}{d_{\text{max}}}Q\right)} + (V+S-L)(1+b)q_T
$$

$$
-\frac{(V+S-L)(1+b)^2 q_T^2}{2\left(\frac{d_T}{d_{\text{max}}}Q\right)} - \frac{\left(\frac{d_T}{d_{\text{max}}}Q\right)(d_T+S)}{2}
$$
(21)

In this section, the commercial food service and the food waste collection agency have equal power. Therefore, they decide on their optimal strategies through a Nash game approach at the same time.

Proposition 7: The commercial food service and the food waste collection agency's proft functions in the decentralized scheme are concave in q_T and d_T respectively.

Because of the concavity of both functions, optimal amounts of q_T^D and d_T^D will be calculated through solving Eq. ([22](#page-12-1)) and Eq. ([23](#page-12-2)) concurrently which are separately gained from the frst-order optimality condition on Eq. ([19\)](#page-12-3) with respect to q_T and Eq. [\(20](#page-12-4)) with respect to d_T .

$$
\frac{\partial E[\prod_{M_T}]}{\partial q_T} = \frac{-L(1-a)^2 q_T}{\left(\frac{d_T}{d_{\text{max}}} Q\right)} + (V - m - L)(1+b) \\
-\frac{(V - m - L)(1+b)^2 q_T}{\left(\frac{d_T}{d_{\text{max}}} Q\right)} = 0
$$
\n(22)

$$
\frac{\partial E[\prod_{c} c_{r}]}{\partial d_{T}} = -\frac{d_{T}Q}{d_{\text{max}}} - \frac{SQ}{2d_{\text{max}}} + \frac{2d_{\text{max}}(S+m)(1+b)^{2}q_{T}^{2}}{2d_{T}^{2}Q} = 0
$$
\n(23)

Therefore, both members of the system in the decentralized scheme announce their optimal strategies as follows and at the same time.

$$
q_T^{D*} = \frac{Q(S+m)(V-m-L)^3(1+b)^5}{2d_{\text{max}}(L(1-a)^2 + (V-m-L)(1+b)^2)^3} - \frac{SQ(V-m-L)(1+b)}{2d_{\text{max}}(L(1-a)^2 + (V-m-L)(1+b)^2)}
$$
(24)

$$
d_T^{D*} = \frac{(S+m)(V-m-L)^2(1+b)^4}{2(L(1-a)^2+(V-m-L)(1+b)^2)^2} - \frac{S}{2}
$$
 (25)

Moreover, we then obtain the optimal values of the decision variables according to the centralized scheme in which the whole system is assumed to be under the control of one decision-maker.

Proposition 8: The entire recycling system's profit is a concave function with respect to q_T and d_T jointly. Therefore, the optimal amount of the initial order and the proposed reward from the entire channel point of view will be as follows:

$$
q_T^{C*} = \begin{pmatrix} \frac{LQ(V+S-L)^3(1-a)^2(1+b)^3}{2d_{\text{max}}(L(1-a)^2+(V+S-L)(1+b)^2)^3} \\ + \frac{Q(V+S-L)^4(1+b)^5}{2d_{\text{max}}(L(1-a)^2+(V+S-L)(1+b)^2)} \\ - \frac{Q(V+S-L)(1+b)}{2d_{\text{max}}(L(1-a)^2+(V+S-L)(1+b)^2)} \end{pmatrix}
$$
(26)

$$
d_T^{C*} = \frac{L(V+S-L)^2(1-a)^2(1+b)^2}{2(L(1-a)^2 + (V+S-L)(1+b)^2)^2} + \frac{(V+S-L)^3(1+b)^4}{2(L(1-a)^2 + (V+S-L)(1+b)^2)^2} - \frac{S}{2}
$$
\n(27)

Corollary 7: According to the aforementioned assumptions, it is clear that q_T^{C*} is greater than q_T^{D*} and also d_T^{C*} is greater than d_T^{D*} . Besides, the whole recycling system's interest in the centralized scheme will increase compared to the decentralized one. Therefore, by providing a suitable contract, the commercial food service can encourage the food waste collection agency to consider a larger amount of reward, and the food waste collection agency can also use an incentive plan to encourage the commercial food service to place a larger initial order.

Therefore, similar to (Wang et al. [2020\)](#page-24-33), a TWCS contract has been applied in this section. It means that the commercial food service pays $\lambda\%$ of the disposal costs, and the food waste collection agency pays β_T % of the additional capacity hiring costs. Thus, the food waste collection agency and the commercial food service are encouraged to consider larger reward and initial order, respectively. According to this contract, *m* also changes to m_T again, and its value must be calculated in such a way that none of the recycling system

members will lose in comparison with the decentralized scheme.

The commercial food service and The food waste collection agency's expected proft functions under the proposed TWCS mechanism are expressed as:

$$
E\left[\prod_{M_T^H}\right] = \begin{pmatrix} \frac{(1-\omega_{RT})}{\left(\frac{d_T}{\omega_{\text{max}}}\right)} \left((V-m_T-\beta_T L)\theta\left(\frac{d_T}{d_{\text{max}}}\mathcal{Q}\right) - (1-\beta_T)L(1-a)q_T\right)f(\theta)d\theta\\ \frac{(1+b)q_T}{\left(\frac{d_T}{\omega_{\text{max}}}\mathcal{Q}\right)} \left((V-m_T-L)\theta\left(\frac{d_T}{d_{\text{max}}}\mathcal{Q}\right)\right)f(\theta)d\theta\\ + \frac{(1+b)q_T}{\left(\frac{d_T}{d_{\text{max}}}\mathcal{Q}\right)} \left((V-m_T-L+\lambda S)(1+b)q_T - \lambda S\theta\left(\frac{d_T}{d_{\text{max}}}\mathcal{Q}\right)\right)f(\theta)d\theta\\ + \frac{(1+b)q_T}{\left(\frac{d_T}{d_{\text{max}}}\mathcal{Q}\right)} \end{pmatrix} \tag{28}
$$

$$
E\left[\prod_{M_T^H}\right] = \left(\frac{-\frac{(1-\beta_T)L(1-a)^2q_T^2}{2\left(\frac{d_T}{d_{\max}}Q\right)} + (V-m_T-L+\lambda S)(1+b)q_T}{-\frac{\lambda S\left(\frac{d_T}{d_{\max}}Q\right)}{2} - \frac{(V-m_T-L+\lambda S)(1+b)^2q_T^2}{2\left(\frac{d_T}{d_{\max}}Q\right)}}{2\left(\frac{d_T}{d_{\max}}Q\right)}\right)
$$
(30)

$$
E\left[\prod_{CO_{T}^{H}}\right] = \left(\frac{-\frac{\beta_{T}L(1-a)^{2}q_{T}^{2}}{2\left(\frac{d_{T}}{d_{\max}}Q\right)} + (m_{T} + (1-\lambda)S)(1+b)q_{T}}{-\frac{(d_{T} + (1-\lambda)S)\left(\frac{d_{T}}{d_{\max}}Q\right)}{2} - \frac{(m_{T} + (1-\lambda)S)(1+b)^{2}q_{T}^{2}}{2\left(\frac{d_{T}}{d_{\max}}Q\right)}}{2(31)}
$$
\n(31)

$$
E\left[\prod_{\substack{C O_{\eta}^H\\(d_{\max}^H\\(d_{\max}^H\\d_{
$$

The entire recycling system's proft function also remains unchanged.

Proposition 9: The commercial food service and the food waste collection agency's proft functions under the TWCS contract are concave in q_T and d_T respectively.

Because of the concavity of both functions, optimal amounts of q_T^H and d_T^H will be calculated through solving Eq. (32) (32) (32) and Eq. (33) (33) (33) concurrently which are separately gained from the frst derivative of Eq. [\(30](#page-13-2)) with respect to q_T and the first derivative of Eq. ([31\)](#page-13-3) with respect to d_T .

$$
\frac{\partial E\left[\prod_{M_T^H}\right]}{dd_T} = \begin{pmatrix} -\frac{d_{\text{max}}(1-\beta_T)L(1-a)^2q_T}{d_TQ} + (V-m_T-L+\lambda S)(1+b) \\ -\frac{d_{\text{max}}(V-m_T-L+\lambda S)(1+b)^2q_T}{d_TQ} \end{pmatrix} = 0
$$
\n(32)

$$
\frac{\partial E\left[\prod_{CO_{T}^{H}}\right]}{\partial d_{T}} = \frac{d_{\max}\beta_{T}L(1-a)^{2}q_{T}^{2}}{2d_{T}^{2}Q} - \frac{d_{T}Q}{d_{\max}} - \frac{(1-\lambda)SQ}{2d_{\max}} + \frac{d_{\max}(w_{T} + (1-\lambda)S)(1+b)^{2}q_{T}^{2}}{2d_{T}^{2}Q} = 0
$$
\n(33)

Therefore, both members of the system under the TWCS mechanism announce their optimal strategies as follows and at the same time.

$$
q_T^{H*} = \begin{pmatrix} \frac{\beta_T L Q (V - m_T - L + \lambda S)^3 (1 - a)^2 (1 + b)^3}{2d_{\text{max}} ((1 - \beta_T) L (1 - a)^2 + (V - m_T - L + \lambda S)(1 + b)^2)^3} \\ + \frac{Q (m_T + (1 - \lambda) S) (V - m_T - L + \lambda S)^3 (1 + b)^5}{2d_{\text{max}} ((1 - \beta_T) L (1 - a)^2 + (V - m_T - L + \lambda S)(1 + b)^2)^3} \\ - \frac{(1 - \lambda) S Q (V - m_T - L + \lambda S)(1 + b)}{2d_{\text{max}} ((1 - \beta_T) L (1 - a)^2 + (V - m_T - L + \lambda S)(1 + b)^2)} \end{pmatrix}
$$
(34)

$$
d_T^{H*} = \begin{pmatrix} \frac{\beta_T L (V - m_T - L + \lambda S)^2 (1 - a)^2 (1 + b)^2}{2((1 - \beta_T)L(1 - a)^2 + (V - m_T - L + \lambda S)(1 + b)^2)^2} \\ + \frac{(m_T + (1 - \lambda)S)(V - m_T - L + \lambda S)^2 (1 + b)^4}{2((1 - \beta_T)L(1 - a)^2 + (V - m_T - L + \lambda S)(1 + b)^2)^2} - \frac{(1 - \lambda)S}{2} \end{pmatrix}
$$
(35)

Next, the values of the proposed contract terms, i.e. β_T , λ and m_T , must be calculated in such a way that the profit of none of the system participants becomes less than their decentralized scheme. Finally, similar to (Heydari and Mosanna [2018\)](#page-23-34), a mathematical nonlinear model is used to optimize the whole system's interest through calculating the optimal values of contract parameters according to the existing constraints.

$$
\max Z = E\Big[\prod_{SC_T} \left(q_T^{H*}, d_T^{H*}\right)\Big]
$$
\n(36)

$$
E\left[\prod_{M_T^H} (q_T^{H*}, d_T^{H*})\right] \ge E\left[\prod_{M_T} (q_T^{D*}, d_T^{D*})\right] + \alpha(E\left[\prod_{sc_i} (q_T^{H*}, d_T^{H*})\right] - E\left[\prod_{SC_T} (q_T^{D*}, d_T^{D*})\right])
$$
(37)

$$
E\left[\prod_{CO_T^{H}}(q_T^{H*}, d_T^{H*})\right] \ge E\left[\prod_{co_i} (q_T^{D*}, d_T^{D*})\right] + \alpha(E\left[\prod_{sc_i} (q_T^{H*}, D_T^{H*})\right] - E\left[\prod_{SC_T} (q_T^{D*}, d_T^{D*})\right])
$$
\n(38)

$$
0 \le m_T, 0 \le \beta_T, \lambda \le 1 \tag{39}
$$

where Eq. (36) (36) (36) intends to optimize the whole system's interest as an objective function. Equations (37) and (38) (38) respectively ensure that the commercial food service and the food waste collection agency's profts under the proposed contract are at least equal to their decentralized schemes plus α % of the added profit to the whole system. In other words, these constraints guarantee enough motivation for the commercial food service and the food waste collection agency to accept the proposed contract. Constraint (39) also certifes that the optimal solution values are gained reasonably.

Second scenario: uncertainty in the recycling capacity

In this scenario, where the frst factory's recycling capacity is uncertain, according to an agreement, it can rent some parts of the second factory's facilities in necessary situations. In this section, frst the option contract and then the QF contract are implemented and fnally compared for renting the recycling capacity.

Option contract

In this section, an option contract has been applied to optimize and manage the facility renting relationships between the two factories. Under this contract, the proposed reward and the initial order are considered as the frst factory's decision variables and the option price as the second factory's decision variable. In addition, a Stackelberg two-player game approach has also been used to solve the problem.

Under the option contract, the frst factory's function is calculated according to Eq. ([40\)](#page-14-3)

$$
\prod_{F1_o} = \begin{cases}\nVR_o - d_oR_o - tR_o - oq_o & R_o \le C_P \\
VR_o - d_oR_o - tC_P - oq_o - e(R_o - C_P) & C_P \le R_o \le C_P + q_o \\
V(C_P + q_o) - d_oR_o - tC_P - oq_o - eq_o - S(R_o - (C_P + q_o)) & C_P + q_o \le R_o\n\end{cases}
$$
\n(40)

The expected value of the frst factory's proft under the option contract, by considering the continuous uniform probability distribution function between $(0, U)$ for C_p , is simplifed as follows:

$$
E\left[\prod_{F1_o}\right] = ((V - d_o - t)R_o - oq_o)
$$

$$
-\frac{1}{U}((V - e + S)\frac{(R_o - q_o)^2}{2} - (t - e)\frac{R_o^2}{2})
$$
(41)

Also, the proft function of the second factory, under the option contract is:

$$
\prod_{F2_o} = \begin{cases}\n oq_o - kq_o & R_o \le C_P \\
 oq_o + e(R - C_P) - kq_o & C_P \le R_o \le C_P + q_o \\
 oq_o + eq_o - kq_o & C_P + q_o \le R_o\n\end{cases}
$$
\n(42)

The expected value of the second factory's proft by considering the continuous uniform probability distribution function between $(0, U)$ for C_p , is simplified as follows:

$$
E\left[\prod_{F2_o}\right] = (o - k)q_o - \frac{1}{U}(e\frac{(R_o - q_o)^2}{2} - e\frac{R_o^2}{2})
$$
(43)

By substituting the relation $R_o = \frac{d_o}{d_{\text{max}}} Q$ in Eq. ([41](#page-14-4)) and Eq. ([43\)](#page-14-5), the expected value of the frst and second proft functions are rewritten as:

$$
E\left[\prod_{F1_o}\right] = \left((V - d - t)\left(\frac{d_o}{d_{\text{max}}}\mathcal{Q}\right) - oq_o\right)
$$

$$
-\frac{1}{U}\left((V - e + S)\frac{\left(\left(\frac{d_o}{d_{\text{max}}}\mathcal{Q}\right) - q_o\right)^2}{2} - (t - e)\frac{\left(\frac{d_o}{d_{\text{max}}}\mathcal{Q}\right)^2}{2}\right) \tag{44}
$$

$$
E\left[\prod_{F2_o}\right] = (o - k)q_o - \frac{1}{U} \left(e^{\frac{\left(\left(\frac{d_o}{d_{\text{max}}}Q\right) - q_o\right)^2}{2}} - e^{\frac{\left(\frac{d_o}{d_{\text{max}}}Q\right)^2}{2}}\right)
$$
\n(45)

Finally, the mathematical expectation of the entire system's proft function is obtained as the summation of Eq. ([44\)](#page-14-6) and Eq. [\(45](#page-14-7)):

$$
E\left[\prod_{s c_0}\right] = \left((V - d - t)\left(\frac{d_o}{d_{\text{max}}}\mathcal{Q}\right) - kq_o\right)
$$

$$
-\frac{1}{U}\left((p + S)\frac{\left(\left(\frac{d_o}{d_{\text{max}}}\mathcal{Q}\right) - q_o\right)^2}{2} - t\frac{\left(\frac{d_o}{d_{\text{max}}}\mathcal{Q}\right)^2}{2}\right) \tag{46}
$$

The frst factory decides on the reward and the initial rental capacity, and the second factory decides on the option price, and the decisions of both members of the system also afect each other's proft functions. So here we face a twoplayer game in which the second factory will play the role of leader and the frst factory will play the role of follower. To solve this model, a Stackelberg method has been applied, which means that the leader, i.e., the second factory, decides on the option price and then the frst factory, by knowing the second factory's policy, decides on the rental capacity and the reward. Of course, this process is done in a backward procedure.

Proposition 10: The frst factory's proft under the option contract is a concave function with respect to initial order and proposed reward jointly, and its best responses will be as:

$$
q_o^{BR} = \frac{(V - o - t)QU}{2Ud_{\text{max}} + (e - t)Q} - \frac{oU}{(V - e + S)}
$$
(47)

$$
d_o^{BR} = \frac{(V - o - t)U}{\left(2U + (e - t)\left(\frac{Q}{d_{\text{max}}}\right)\right)}
$$
(48)

Proposition 11: After placing Eq. [\(47\)](#page-15-0) and Eq. ([48\)](#page-15-1) in Eq. ([45](#page-14-7)), the second factory's expected profit function becomes concave in o and the optimal value of the option price is obtained as follows:

$$
o^* = \frac{\frac{eVUQ^2 - eUQ^2t}{(2Ud_{\text{max}} + (e-t)Q)^2} + \frac{UQt - VUQ - kUQ}{2Ud_{\text{max}} + (e-t)Q} - \frac{kU}{(V-e+S)}}{(\frac{eUQ^2}{(2Ud_{\text{max}} + (e-t)Q)^2} - \frac{2UQ}{2Ud_{\text{max}} + (e-t)Q} - \frac{eU}{(V-e+S)^2} - \frac{2U}{(V-e+S)})}
$$
(49)

$$
E\left[\prod_{F|_f}\right] = \begin{pmatrix} (V - d - t)R_f - L(1 - A)q_f + \frac{tR_f^2}{2U} \\ + \frac{1}{V}((V - L + S)(1 + B)q_f(R_f - (1 + B)q_f) + L(1 - A)q_f(R_f - (1 - A)q_f)) \\ - \frac{1}{U}((V - L + S)R_f(R_f - (1 + B)q_f) + LR_f(R_f - (1 - A)q_f)) \\ + \frac{1}{U}((V - L + S)\frac{(R_f - (1 + B)q_f)^2}{2} + L\frac{(R_f - (1 - A)q_f)^2}{2}) \end{pmatrix}
$$
(53)

Also, the second factory's proft function in this section is formulated as follows:

$$
\prod_{F2_f} = \begin{cases} L(1-A)q_f - k(1+B)q_f 0.17 \text{em} & R_f \le C_P \\ L(1-A)q_f - k(1+B)q_f & C_P \le R_f, R_f - C_P \le (1-A)q_f \\ L(R_f - C_P) - k(1+B)q_f & C_P \le R_f, (1-A)q_f \le R_f - C_P \le (1+B)q_f \\ L(1+B)q_f - k(1+B)q_f & C_P \le R_f, (1+B)q_f \le R_f - C_P \end{cases} \tag{54}
$$

Proposition 12: After revealing the second factory's fnal and optimal decision about the option price, the frst factory's best responses will also be determined accurately and the optimal values of d_o and q_o will be calculated as follows:

$$
d_{o}^{*} = \frac{(V - \frac{e^{VUQ^{2} - eUQ^{2}t}}{(2Ud_{\max} + (e - i)Q)^{2}} + \frac{UQt - VUQ - kUQ}{2Ud_{\max} + (e - i)Q} - \frac{eU}{(V - e + S)} - t)U}{(2Ud_{\max} + (e - i)Q)^{2} - \frac{2UQ}{2Ud_{\max} + (e - i)Q} - \frac{eU}{(V - e + S)^{2}} - \frac{2U}{(V - e + S)} - t)U}
$$
\n
$$
(2U + (e - t)\left(\frac{Q}{d_{\max}}\right))
$$
\n
$$
(V - t)(V - e + S)QU - \frac{\left\{\begin{array}{c}e^{VUQ^{2} - eUQ^{2}t} \\(2U^{2}d_{\max} + (p - t + S)QU\end{array}\right\}}{2Ud_{\max} + (e - i)Q} + \frac{UQt - VUQ - kUQ}{2Ud_{\max} + (e - i)Q} - \frac{kU}{(V - e + S)}\end{array}\right\}}
$$
\n
$$
q_{o}^{*} = \frac{\left\{\begin{array}{c}e^{VUQ^{2} - eUQ^{2}t} \\(2U^{2}d_{\max} + (e - i)Q) & - \frac{2UQ}{2Ud_{\max} + (e - i)Q}\end{array}\right\}}{2Ud_{\max} + (e - i)Q)(V - e + S)}
$$
\n(51)

QF contract

⎞ ⎟ ⎟ ⎟ ⎟ \mathbf{I} ⎠

In this part, the frst factory decides on the initial order and then the second factory adjusts A and B to create coordination in the channel so that the frst factory's decision becomes similar to a situation in which the whole system's interest is maximized.

The frst factory's proft function in the second scenario and under the QF contract is formulated as follows:

$$
\prod_{f|_{F}} = \begin{cases} \n\forall R_f - d_f R_f - tR_f - L(1-A)q_f & R_f \leq C_P \\ \n\forall R_f - d_f R_f - tC_P - L(1-A)q_f & C_P \leq R_f, R_f - C_P \leq (1-A)q_f \\ \n\forall R_f - d_f R_f - tC_P - L(R_f - C_P) & C_P \leq R_f, (1-A)q_f \leq R_f - C_P \leq (1+B)q_f \\ \n\left(\begin{array}{cc} V(C_P + (1+B)q_f) - d_f R_f - tC_P & C_P \leq R_f, (1+B)q_f \leq R_f - C_P \\ -L(1+B)q_f - S(R_f - (C_P + (1+B)q_f)) \end{array} \right) & C_P \leq R_f, (1+B)q_f \leq R_f - C_P \end{cases} \tag{52}
$$

The expected value of the frst factory's proft, by considering the continuous uniform probability distribution function between $(0, U)$ for C_p , is simplified as follows:

The expected value of the second factory's proft function, by considering the continuous uniform probability distribution function between $(0, U)$ for C_p , is simplified as follows:

$$
E\left[\prod_{F2_f}\right] = \left(\begin{array}{c} -k(1+B)q_f + L(1-A)q_f + \frac{1}{U}\left(\begin{array}{c}L(1+B)q_f(R_f - (1+B)q_f) \ -L(1-A)q_f(R_f - (1-A)q_f)\end{array}\right) \\ + \frac{1}{U}\left(\frac{L(R_f - (1+B)q_f)^2}{2} - L\frac{(R_f - (1-A)q_f)^2}{2}\right) \\ - \frac{1}{U}\left(LR_f(R_f - (1+B)q_f) - LR_f(R_f - (1-A)q_f)\right) \end{array}\right) \tag{55}
$$

By placing the relation $R_f = \frac{d_f}{d_{\text{max}}} Q$ in Eq. ([53](#page-15-2)) and Eq. [\(55](#page-16-0)), the first and second factories' expected profit functions are rewritten as follows:

$$
E\left[\prod_{F|_f}\right] = \begin{pmatrix} (V-d_f-t)(\frac{d_f}{d_{\max}}Q) - L(1-A)q_f + \frac{t(\frac{d_f}{d_{\max}}Q)^2}{2U} \\ + \frac{1}{U}((V-L+S)(1+B)q_f((\frac{d_f}{d_{\max}}Q) - (1+B)q_f) + L(1-A)q_f((\frac{d_f}{d_{\max}}Q) - (1-A)q_f)) \\ - \frac{1}{U}((V-L+S)(\frac{d_f}{d_{\max}}Q)((\frac{d_f}{d_{\max}}Q) - (1+B)q_f) + L(\frac{d_f}{d_{\max}}Q)((\frac{d_f}{d_{\max}}Q) - (1-A)q_f)) \\ + \frac{1}{U}((V-L+S)(\frac{d_f}{d_{\max}}Q) - (1+B)q_f) + L(\frac{d_f}{d_{\max}}Q - (1-A)q_f) \\ + \frac{1}{U}(V-L+S)(\frac{d_f}{d_{\max}}Q) - (1+B)q_f) + L(\frac{d_f}{d_{\max}}Q - (1-A)q_f) \end{pmatrix} \tag{56}
$$

$$
E\left[\prod_{F2_f}\right] = \begin{pmatrix} -k(1+B)q_f + L(1-A)q_f \\ +\frac{1}{U}(L(1+B)q_f((\frac{d_f}{d_{\max}}Q) - (1+B)q_f) - L(1-A)q_f((\frac{d_f}{d_{\max}}Q) - (1-A)q_f)) \\ +\frac{1}{U}(L(\frac{(\frac{d_f}{d_{\max}}Q) - (1+B)q_f)}{2} - L(\frac{(\frac{d_f}{d_{\max}}Q) - (1-A)q_f)^2}{2}) \\ -\frac{1}{U}(LR_f((\frac{d_f}{d_{\max}}Q) - (1+B)q_f) - L(\frac{d_f}{d_{\max}}Q)((\frac{d_f}{d_{\max}}Q) - (1-A)q_f)) \end{pmatrix}
$$
(57)

Finally, the mathematical expectation of the entire system's proft function is calculated by summing Eq. ([56](#page-16-1)) and Eq. ([57\)](#page-16-2) as Eq. ([58](#page-16-3)).

Proposition 13: The expected profit function of the first factory is concave respect to the q_f under the QF contract in the

$$
E\left[\prod_{SC_f}\right] = \left(\begin{array}{l} -k(1+B)q_f + (V - d_f - t)(\frac{d_f}{d_{\text{max}}}Q) + \frac{t(\frac{d_f}{d_{\text{max}}}Q)^2}{2U} \\ -\frac{1}{U}((V + S)(\frac{d_f}{d_{\text{max}}}Q)((\frac{d_f}{d_{\text{max}}}Q) - (1+B)q_f)) \\ +\frac{1}{U}((V + S)(\frac{t(\frac{d_f}{d_{\text{max}}}Q) - (1+B)q_f)^2}{2}) + \frac{1}{U}((V + S)(1+B)q_f((\frac{d_f}{d_{\text{max}}}Q) - (1+B)q_f)) \end{array}\right) \tag{58}
$$

second scenario, and the optimal value of q_f from the first factory's perspective is obtained as follows:

$$
q_f^{D*} = \frac{-L(1-A)U + (V - L + S)\left(\frac{d_f}{d_{\text{max}}}Q\right)(1+B) + L\left(\frac{d_f}{d_{\text{max}}}Q\right)(1-A)}{((V - L + S)(1+B)^2 + L(1-A)^2)}
$$
(59)

Proposition 14: The proft of the whole system has also a concave function with respect to the initial order under the QF contract in the second scenario, and its optimal amount from the whole system point of view is obtained as follows:

$$
q_f^{C*} = \frac{-k(1+B)U + (V+S)(1+B)(\frac{Q}{d_{\text{max}}})d_f}{(V+S)(1+B)^2}
$$
(60)

Corollary 8: To optimize the whole system's proft, the second factory must appropriately adjust the relationship between A and B so that the decision of the frst factory becomes similar to q_f^{C*} .

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Proposition 15: *If the second factory regulates the relationship between A and B as the following equation*, *the entire recycling system is well optimized*.

$$
\frac{(1-A)}{(1+B)} = \frac{\begin{pmatrix} -(-L(V+S)U + L(V+S)(\frac{\partial}{d_{\text{max}}})d_f) & & & \\ -(L(V+S)U + L(V+S)(\frac{\partial}{d_{\text{max}}})d_f)^2 & & \\ -(4(LkU - L(V+S)(\frac{\partial}{d_{\text{max}}})d_f)(k(V-L+S)U) & \\ 2(LkU - L(V+S)(\frac{\partial}{d_{\text{max}}})d_f) & & \end{pmatrix} \tag{61}
$$

Discussion and computational results

In this subsection, we will make mathematical calculations and experiments to analyze the efficacy and usefulness of our developed contracts. Table [3](#page-17-0) shows all notations employed in our mathematical structures along with their estimated values. The values of these parameters have been

Table 3 Applied dataset

predicted through conducting interviews with managers and senior experts of Kaleh-Solico companies.

Table [4](#page-17-1) shows the optimal values of the decision variable and SCS contract parameter. Also, if we consider β_S equal to the mean of its upper and lower limits $(\beta_S = \frac{\beta_S^{\text{final}} + \beta_S^{\text{final}}}{2})$, the values of the expected proft functions can be reached in Table [5](#page-17-2).

As you can see in the above tables, in contrast with the decentralized scheme the initial order optimal value rises in the centralized structure. Besides, although the food waste collection agency and the entire system's profts increase in the centralized scheme, the commercial food service's interest falls off. This causes the commercial food service not to accept the decision-making similar to a centralized scheme; But with the proposed SCS contract, the commercial food service's initial order and the entire system's expected proft become the same as their centralized scheme. Now, if we consider β_S between the $\left[\beta_S^{\text{min}}, \beta_S^{\text{max}}\right]$, the commercial food service and the food waste collection agency will both have higher expected profts than their decentralized structure. Therefore, during this contract, the whole channel will be well optimized and the Pareto Improvement conditions will be achieved for all members.

Table 4 Optimal values of the decision variable and SCS contract parameters

First scenario	Decentralized	Centralized	Coordinated
q_S	412.5	484	484
m_S	N/A	N/A	165.6036932
β_S^{min}	N/A	N/A	0.642432851
β_S^{\max}	N/A	N/A	0.664256198

Table 5 Optimal values of the expected proft functions under the SCS contract

First scenario	Decentralized	Centralized	Coordinated
$E\big \prod_{M_S}\big $	29493.75	28607.62667	30452.29688
$E\big[\Pi_{CO_S}\big]$	23435.15625	26238.37333	24393.70313
$E\big[\prod_{SC_S}\big]$	52928.90625	54846	54846

According to Table [6,](#page-17-3) with increasing β_s , the expected proft of the commercial food service decreases, and for $\beta_S = \beta_S^{\text{max}}$, it reaches its minimum value (i.e., 29,493.75, the expected proft of its decentralized scenario). Therefore, by further increasing it, the commercial food service will not accept the proposed contract. Besides, by decreasing the value of β_s , the food waste collection agency's expected profit decreases, and for $\beta_S = \beta_S^{\text{min}}$ it reaches its lowest value (i.e. 23,435.15625, the expected proft of its decentralized scheme). This time, if β_s decreases further, the food waste collection agency will no longer accept the proposed contract. Ultimately, the choice of the exact value for β_s will

Table 6 Sensitivity of the commercial service and collection agency's profit functions to changes in β_s

$\beta_{\rm S}$	$E\left \prod_{M_S}\right $	$E \prod_{CO_S}$
0.642432851	31410.84375	23435.15625
0.643432851	31322.99775	23523.00225
0.644432851	31235.15175	23610.84825
0.645432851	31147.30575	23698.69425
0.646432851	31059.45975	23786.54025
0.647432851	30971.61375	23874.38625
0.648432851	30883.76775	23962.23225
0.649432851	30795.92175	24050.07825
0.650432851	30708.07575	24137.92425
0.651432851	30620.22975	24225.77025
0.652432851	30532.38375	24313.61625
0.653432851	30444.53775	24401.46225
0.654432851	30356.69175	24489.30825
0.655432851	30268.84575	24577.15425
0.656432851	30180.99975	24665.00025
0.657432851	30093.15375	24752.84625
0.658432851	30005.30775	24840.69225
0.659432851	29917.46175	24928.53825
0.660432851	29829.61575	25016.38425
0.661432851	29741.76975	25104.23025
0.662432851	29653.92375	25192.07625
0.663432851	29566.07775	25279.92225
0.664256198	29493.75	25352.25

Table 7 Decision variables contract parameters and proft functions under the TWCS contract

First scenario	Decentralized	Centralized	Coordinated
d_S	32.19140625	121.41	117.0860611
q_T	553.2897949	2448.44	2461.081804
m_T	N/A	N/A	240.9260752
β_T	N/A	N/A	0.86740536
λ_T	N/A	N/A	0.5860944664668
$E\left[\prod_{M_T}\right]$	39560.22034	144718.8314	49667.31295
$E[\prod_{CO_T}]$	42623.74820	39536.01985	133587.5293
$\Pi_{\scriptscriptstyle SC_r}$ \boldsymbol{E}	82183.96854	184254.8513	183254.8423

depend on the bargaining power between the commercial food service and the food waste collection agency.

Moreover, Table [7](#page-18-0) shows decision variables, proposed TWCS contract parameters, and the expected proft functions' values, respectively.

As you can see in the above table, both the initial order and proposed reward are higher in the centralized scheme. Also, while the commercial food service and the entire system's interests increase in the centralized structure, the expected proft of the food waste collection agency decreases. This causes the food waste collection agency to reject the decision-making similar to a centralized system; However, due to the proposed TWCS contract, the initial order, as well as the proposed reward, become as close as possible to the optimal values of the centralized scheme and the total recycling system's proft increases almost to near the expected proft of the centralized structure; Finally, it can be seen that the commercial food service and the food waste collection agency will both gain higher profts than their decentralized scheme (Pareto Improvement).

According to Table [8](#page-18-1), the optimal decisions i.e. the initial order and also the proposed reward, by increasing the amount of α , change increasingly and decreasingly respectively. Although these changes will lead to more expected profit for the commercial food service, they ultimately reduce the whole system and the food waste collection agency's interests.

Moreover, the decision variables, contract terms, and proft functions' optimal amounts under the option contract for diferent values of e and under the QF contract for diferent values of L in the second scenario are given in Tables [9](#page-19-0) and [10](#page-20-0), respectively. Figure [4](#page-21-0) also graphically shows these changes in proft functions under both contracts simultaneously.

According to the above tables and figures, both the option and QF contracts' expected profts for the frst factory are declining by increasing in e and L, and have a close profitability level so that both can be used with almost equal efficiency. But in any case, it can be said that for $(L \le 145, e \le 95)$ and also for $(L \ge 285, e \ge 235)$, the option contract is a little more proftable for the frst factory but for $(145 < L < 285, 95 < e < 235)$ the first factory prefers the QF contract instead of the option contract. Moreover, the second factory's proft for the option contract is a descending function with respect to e and for the QF contract is an ascending function relative to L. Therefore, it can be said that for $(L \le 240, e \le 190)$, the second factory prefers the option contract and for $(L \ge 240, e \ge 190)$, it prefers QF contract. In addition, the total system's proft for the option contract is also a downward function with respect to e, and for QF contract is a fxed-function in terms of L. So, for $(L \le 230, e \le 180)$ option contract and for $(L \ge 230, e \ge 180)$ QF contract will have more productivity for the entire recycling system.

Comparison of results with previous research

Table 9 Sensitivity of second scenario's components under option contract by changing in e

Heydari and bakhshi ([2022](#page-23-35)) examined the effect of using two types of coordination mechanisms, i.e., conventional contract and option contract on fulflling market demands and providing appropriate delivery service in companion with a 3PL retailer. They realized that when demand uncertainty is low, the option contract can be more benefcial and when the demand uncertainty rises, the conventional contract would work more efficiently. However, in the current study, we have compared the efectiveness of using an option contract with a QF contract to create coordination between two factories which were producing organic fertilizers. Our results showed that on one hand, the option contract could do better when the unit cost of leasing recycling capacity is greater than the exercise price and on the other hand, the QF

contract may make higher profts for both members when the exercise price is greater than the unit cost of leasing recycling capacity.

Yang et al. ([2017\)](#page-24-34) studied two supply chains facing a competitive environment to gain more market share under a cap and trade system. They proved that a single revenuesharing contract not only provides a win–win situation for both participants but also makes more proft for the entire network. However, we showed that a two-way cost/revenue sharing contract may even bring more profts for all of the involved participants than a single cost/revenue sharing contract, as proved by (Wang et al. [2020\)](#page-24-33). Besides, in our work, it has been illustrated that a two-way cost/revenue sharing contract can ensure Pareto-Improvement conditions for all

Table 10 Sensitivity of second scenario's components under QF contract by changing in L

of the channel members, which is also compatible with the results gained by Beheshti et al. ([2022\)](#page-23-15).

Practical implementation and managerial insights

Based on the obtained results, commercial food service managers are suggested to get together with waste collection companies and select the initial order amount by mutual negotiations with them. It is also suitable for commercial food service managers to collaborate with organic fertilizer factories, specifcally when faced with large amounts of food waste (for example, in times of crises such as Covid-19). On the other hand, greater participation in environmentally friendly exercises may also be a decisive and infuential element in enhancing the company's prestige and increasing the demand for its products. Whole system decision-makers are also advised to allow the food waste collection companies to decide on the proposed reward themselves because it will further increase the expected proft of the channel as well as the expected proft of the other members. Food recovering system managers are also offered not to insist on achieving extra added revenue, because insisting on a larger share of the total surplus proft may create worse conditions for all participants.

Fig. 4 Sensitivity of second scenario's proft functions by changing in e and L

In summary, the proposed models developed in this study, as a managerial insight, may support recycling system administrators and decision-makers to:

- Come up with the best way of working with food collection and inspection companies to save time, money, and energy and enter into a win–win relationship with them.
- Achieve a suitable approach for collecting and recycling waste, to prevent diseases caused by the consumption of expired and wasted foodstufs.
- Enter into optimal negotiations and conclude contracts with factories that convert food waste into organic fertilizers, to improve the environmental conditions and eliminate the problems related to the chemical fertilizers
- Achieve the best approach for renting recycling facilities from suburban areas to avoid governmental fnes for banning productive and reproductive activities in urban areas and gaining social satisfaction.
- Enter into negotiations with parallel factories and use the best contract for renting their facilities, in case of lack of reproductive capacity.
- Efectively decide on the proposed reward value to be paid to customers such as hotels and cafes to create appropriate competition with parallel rivals.

Future research

The models analyzed in this study can still be developed in several ways, which can help to bring the models closer to real-world conditions. for example:

- A capacity ceiling can be set for the production factories so that these plants can only use their remaining capacity (not leased) to carry out their recycling operations.
- The two-channel system i.e. traditional one (through the food waste collection agency) or the modern one (via the Internet) can be considered for food waste collection, and the competition between the two channels can also be examined.
- The government can be included in the model as a decision-maker to determine the percentage of reproductive activities or the amount of capacity that each factory can access for its recycling activities inside and outside the city.
- The diferent phases of the problem can be interrelated so that the output of each phase will be the input of the next phase and fnally the results of all phases can be analyzed collectively.

Conclusion

This study deals with the issue of optimization in a recycling urban waste system, under the uncertainty of collected waste and reproductive capacity in which there are restrictions on manufacturing and remanufacturing activities (for example, limitations such as the prohibition of reproductive activities to recycle urban food waste and convert it into organic fertilizers). In the frst scenario of this study, the optimization of a system consisting of a commercial food service, a food waste collection agency, and a fertilizer factory is investigated, in which a hybrid contract (QF and SCS) is used to manage the relationships between members. In this section, to optimize the channel, the initial order is calculated as a commercial food service's decision variable, but the proposed reward is supposed to have a predetermined amount. The system is then re-evaluated, by another hybrid contract (QF and TWCS) and a mathematical nonlinear model to optimize and manage the participant's connection. Besides, in the second scenario, the two factories' relationship is managed under the uncertainty of the frst factory's reproductive capacity. In this section, the initial order and the proposed reward that afects the amount of the collected waste and the overall proft of the system are considered model decision variables. This case has also been evaluated and optimized using two contracts (QF and option). Diferent models of the problem have been studied under three schemes: decentralized, centralized, and under the proposed contracts. Numerical examples show that under the SCS contract, the proftability of the channel reaches its highest possible value (equal to the centralized scheme). Moreover, under the TWCS contract, the profts of all members, as well as the whole system's interest, outweigh their decentralized scheme that shows Pareto Improvement results for all participants. Moreover, it can be seen that the QF contract in the second scenario works better in some periods and the option contract also works better in some other periods for diferent amounts of the rental fee, i.e., L and e.

Supplementary Information The online version contains supplementary material available at<https://doi.org/10.1007/s11356-022-23851-7>.

Author contribution Saeed Beheshti and Jafar Heydari contributed to the study conception and design. Material preparation, data collection, and analysis were performed by Saeed Beheshti and Jafar Heydari. The frst draft of the manuscript was written by Saeed Beheshti, then Saeed Beheshti and Jafar Heydari commented on previous versions of the manuscript. Saeed Beheshti and Jafar Heydari read and approved the fnal manuscript.

Data availability All data generated or analyzed during this study are included in this published article and its supplementary information fles.

Declarations

Ethics approval and consent to participate Not applicable.

Consent for publication Not applicable.

Competing interests The authors declare no competing interests.

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