RESEARCH ARTICLE

Numerical study on high‑frequency efect of rail corrugation on subway‑induced environmental vibrations

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Abstract

Rail corrugation is a common phenomenon in railway engineering, but its high-frequency efects on environmental vibrations are neglected in most previous research. Therefore, a hybrid numerical method was proposed in this paper to analyze subway-induced ground vibrations, especially in the high-frequency range caused by rail corrugation. The analysis model composed of a three-dimensional (3D) load generation subsystem and a two-dimensional (2D) wave propagation subsystem was established based on the vehicle-track coupling method and fnite element method, and validated by the measured data. Then the high-frequency efects under diferent tunnel depths and rail fasteners were further studied. The results show that high-frequency vibrations propagate radially from the tunnel wall to the surrounding soil and transmit to the ground by the dominant path under diferent tunnel depths. The increase of tunnel depths could result in more serious high-frequency efects in the vibration amplifcation region. When the depth changes from 17 to 29 m, the 250 Hz ground vibration at around 30 m away from the track increases by 5.6 dB. Besides, it was found that in the commonly used range, the reduction of fastener stifness can efectively eliminate high-frequency ground vibrations, while there is a signifcant nonlinear relationship between fastener damping and high-frequency vibration. The fndings of this paper could provide references for parameter design in subway construction and rail corrugation remediation, and help create better living environments.

Keywords Environmental vibrations · Rail corrugation · Numerical analysis · Hybrid method · Subway · Parametric study

Introduction

Nowadays, subway-induced environmental noise and vibration are more and more prominent with the increase of operating years (Connolly et al. [2016;](#page-10-0) Sadeghi et al. [2019](#page-11-0)). As a vital factor causing this problem, the strong wheel-rail impact is closely related to track irregularities, especially the

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rail corrugation which has become a common problem in railways worldwide (Oostermeijer [2008;](#page-11-1) Meehan et al. [2016\)](#page-11-2).

Over the years, a great deal of research work has been made to fnd out how the rail corrugation formed. Wheelrail feedback vibration (Grassie and Kalousek [1993](#page-10-1); Grassie [2009\)](#page-10-2), wheel-rail self-excited vibration (Chen et al. [2010](#page-10-3); Kurzeck and Hecht [2010](#page-11-3)), and wheel-rail contact vibration (Jin et al. [2005](#page-11-4); Wu and Thompson [2005\)](#page-11-5) are thought to be three major generation mechanisms. Besides, through the established models and measurements (Collette et al. [2008](#page-10-4); Eadie et al. [2008;](#page-10-5) Meehan et al. [2016;](#page-11-2) Ng et al. [2019\)](#page-11-6), sleeper distance, speed distribution, and track parameters were proved to have signifcant infuences on the rail corrugation growth. The formed rail corrugation with the dominant wavelength of 20–120 mm will cause a severe high-frequency wheel-rail impact which will lead to more annoying interior noise. Under the subway operation speed of 40–80 km/h, the significant high-frequency noise between 200 and 700 Hz in the vehicle was found to correspond to the excitation frequency caused by rail corrugation (Liu et al. [2011](#page-11-7); Zhao et al. [2017;](#page-11-8) Feng et al. [2018](#page-10-6)).

Although high-frequency efects of rail corrugation are dominant, many feld measurements (Sanayei et al. [2013](#page-11-9); Qi et al. [2019\)](#page-11-10) and simulation studies (Lopes et al. [2014](#page-11-11), [2016](#page-11-12); Xu et al. [2015;](#page-11-13) Kouroussis et al. [2017\)](#page-11-14) proved that subwayinduced environmental vibrations were mainly concentrated in the low-frequency range below 100 Hz, except that indoor vibrations may peak near 200 Hz due to structural resonance. However, it should be noted that the measured data is only responsible for the specifc case and afected by many factors, including track conditions, tunnel depths, rail fasteners, and so on. It means that there may be no rail corrugation on the rail surface at all, or high-frequency vibrations caused by slight rail corrugation were attenuated greatly through tracks, tunnels, and soil layers. And for numerical simulations, it is well known that calculation results are completely determined by the model parameters. The track irregularities (Kouroussis et al. [2014](#page-11-15); Kuo et al. [2016](#page-11-16); Yang et al. [2017](#page-11-17)) used to calculate excitation loads were usually obtained from the track spectrum only considering mediumlong wavelengths. What is more, even if the rail roughness with a wavelength less than 1.0 m measured using a CAT trolley (Jin et al. [2018\)](#page-10-7) or simulated by the Sato spectrum (Xu et al. [2015\)](#page-11-13) was considered, it does not mean that there must be rail corrugations. Besides, the adopted Rayleigh damping coefficients (Shih et al. 2017 ; Xing et al. 2019) determined by the target frequencies which were set below 100 Hz may reduce high-frequency vibration responses. The results obtained through both feld measurements and numerical simulations are reasonable for certain scenes in environmental vibration assessments and predictions. But it is undeniable that chosen parameters in some simulation studies may underestimate the high-frequency environmental vibrations caused by rail corrugation.

Due to the fact that human perception of environmental changes is increasing with the people's rising living improvements, high-frequency environmental vibrations may also afect the human auditory system and nervous system (Sroczyński et al. [1978;](#page-11-20) Sutinen et al. [2007\)](#page-11-21). In order to improve the living environment quality and reduce the risk of high-frequency vibration infuence, the high-frequency environmental vibration should also be prevented and controlled. In this paper, a hybrid numerical model considering rail corrugations was frstly proposed to reproduce high-frequency ground vibrations presented in the authors' previous fndings (Xin et al. [2020\)](#page-11-22). Then more in-depth theoretical analysis was further carried out, where transmission and variation characteristics of high-frequency vibrations caused by rail corrugation under diferent tunnel depths and rail fasteners were analyzed. The results can provide a theoretical reference for the remediation and prevention of high-frequency environmental vibrations caused by the subway.

Numerical analysis method

The hybrid numerical method adopted here is based on the two-step approach. For reproducing high-frequency environmental vibrations, mesh sizes of the numerical model must meet the propagation requirements of the short vibration wave, which means larger calculation time and computational resources compared with common subwayinduced vibration problems. In order to balance the accuracy and efficiency, the vibration system is divided into two subsystems which are co-simulated through fastening forces in the modeling strategy, as shown in Fig. [1](#page-2-0). The 3D load generation subsystem is coupled by the vehicle, track, and equivalent foundation, which is used to obtain the fastening forces. Then the forces are directly applied in the 2D wave propagation subsystem composed of the track, tunnels, and diferent soil layers to reproduce the high-frequency vibrations and study their characteristics.

Load generation subsystem

When a train runs on the track, the wheel-rail interaction forces will change randomly due to track irregularities and deformations, and further afect fastening forces in both the time domain and frequency domain. In order to consider these infuencing factors, the train and the track are spatially coupled through the wheel-rail contact in the load generation subsystem, as shown in Fig. [2](#page-2-1). What is more, the foundation including base slabs, the tunnel, and soil layers are simplifed as uniformly distributed viscoelastic elements (Xing et al. [2019](#page-11-19); Yang et al. [2019](#page-11-23)), not only exerting a supporting efect but also aiming to save the huge computation time.

The vehicle is composed of seven rigid bodies including one car body, two bogies, and four wheelsets, which are connected by the frst suspension and the second suspension. All the structures except for the suspensions are represented by the lumped mass ignoring their elastic deformation, while the suspensions are simulated by the spring-damper system. In the vehicle model, 31 degree-of-freedoms (DOFs) are considered. The car body and each bogie contain 5 DOFs including transverse, vertical, rolling, pitching, and yawing. And the wheel-set contains 4 DOFs except for pitching. The motion equation of the vehicle running at a constant speed is established by the Hamilton method and fnally expressed as Eq. (1) (1) .

Fig. 1 Diagram of the solution of the hybrid numerical method

Fig. 2 Front diagram of the load generation subsystem model

$$
\begin{bmatrix}\nM_c & 0 & 0 \\
0 & M_b & 0 \\
0 & 0 & M_w\n\end{bmatrix}\n\begin{bmatrix}\n\ddot{\delta}_c \\
\ddot{\delta}_b \\
\ddot{\delta}_w\n\end{bmatrix} +\n\begin{bmatrix}\nC_{ll} & C_{cb} & C_{cw} \\
C_{bc} & C_{bb} & C_{bw} \\
C_{wc} & C_{wb} & C_{ww}\n\end{bmatrix}\n\begin{bmatrix}\n\dot{\delta}_c \\
\dot{\delta}_b \\
\dot{\delta}_w\n\end{bmatrix} +\n\begin{bmatrix}\nK_{cc} & K_{cb} & K_{cw} \\
K_{bc} & K_{bb} & K_{bw} \\
K_{wc} & K_{wb} & K_{ww}\n\end{bmatrix}\n\begin{bmatrix}\n\delta_c \\
\delta_b \\
\delta_w\n\end{bmatrix} =\n\begin{bmatrix}\n0 \\
0 \\
Q_{wr}\n\end{bmatrix}
$$
\n(1)

where *M*, *C*, and *K* refer to the matrix of mass, damping, and stifness, respectively; subscript *c*, *b*, and *m* stand for the car body, the bogie, and the wheelset, respectively; δ , $\dot{\delta}$, and *̈* represent displacement, velocity, and acceleration vector, respectively; and Q_{wr} denotes the wheel-rail contact forces.

The track system is composed of rails, fasteners, track slabs, and foundations. The rails are simulated by the beam elements based on the Timoshenko beam theory, while track slabs are modeled by the solid elements with the consideration of the actual size. The fasteners and foundations are modeled by the linear spring-damper elements, where the equivalent stifness of foundations is determined by static calculation to obtain the same structural displacements compared with the complete system. The motion equation of the track system is expressed as Eq. [\(2](#page-2-3)).

$$
\begin{bmatrix} M_r & 0 \\ 0 & M_s \end{bmatrix} \begin{bmatrix} \ddot{\delta}_r \\ \ddot{\delta}_s \end{bmatrix} + \begin{bmatrix} C_{rr} & C_{rs} \\ C_{sr} & C_{ss} \end{bmatrix} \begin{bmatrix} \dot{\delta}_r \\ \dot{\delta}_s \end{bmatrix} + \begin{bmatrix} K_{rr} & K_{rs} \\ K_{sr} & K_{ss} \end{bmatrix} \begin{bmatrix} \delta_r \\ \delta_s \end{bmatrix} = \begin{Bmatrix} Q_{rw} \\ 0 \end{Bmatrix}
$$
 (2)

where subscript *r* and *s* represent the rail and the slab, respectively.

The wheel-rail contact theory plays an important role in coupling the vehicle and track models. The wheel-rail creep force is realized by Kalker's linear theory and nonlinearly corrected by Shen's theory, while the wheel-rail normal force is based on the nonlinear Hertz elastic contact theory.

Most particularly, when the rail surface is afected by rail corrugation, a high level of the wheel-rail contact force can be experienced. This may lead to that the wheel losing contact with the rail, and the wheel-rail normal force is zero. Considering this possibility, the wheel-rail normal force is calculated by Eq. [\(3](#page-3-0)).

$$
P(t) = \left\{ \frac{1}{G} \left[\delta_{wi}(t) - \delta_{ri}(t) - \delta_{0i}(t) \right] \right\}^{1.5}, \delta_{wi}(t) - \delta_{ri}(t) \ge \delta_{0i}(t)
$$
\n(3)

where G represents the contact coefficient depending on the wheel-rail curvatures (Mazilu [2010](#page-11-24)); $\delta_{wi}(t)$ is the vertical displacement of the *i*th wheel at time *t*; and $\delta_{ri}(t)$ is the vertical displacement of the rail under the *i*th wheel at time *t*. $\delta_{0i}(t)$ denotes the track irregularity on the rail surface under the *i*th wheel at time *t*, which is composed of the short-wavelength and medium-long wavelength random irregularities.

After the complete system matrices are assembled, the static analysis is frst performed to obtain the initial condition for the subsequent dynamic analysis. And then the dynamic responses of tracks are calculated using Newmark-β integration method with the integration step of 5×10^{-5} s to ensure convergence of results. Further, the vertical fastening forces at any position can be obtained by Eq. [4](#page-3-1).

$$
f_i = k_0 (z_{up,i} - z_{down,i}) + c_0 (z_{up,i} - z_{down,i})
$$
 (4)

where *z* and *ż* denote vertical displacement and acceleration; k_0 and c_0 are stiffness and damping of the *i*th rail fastener; and the subscript *up* and *down* represents the up node and down node of the *i*th rail fastener.

Wave propagation subsystem

The problem of subway-induced environmental vibrations is simplifed as a 2D plane strain problem, under the condition that the soil deformation can be considered as linear elastic (Shen [2007](#page-11-25)) and satisfy the requirements of displacement coordination between diferent layer interfaces.

Therefore, the wave propagation subsystem is modeled by the 4-node plane strain elements, according to the typical section of track-tunnel-soil system, as shown in Fig. [3.](#page-3-2) The model size $(h \times w)$ is large enough to satisfy the literature criteria that the horizontal truncation length is more than 10 times the outer diameter of tunnel on both sides (Huang et al. [2017](#page-10-8)) and the depth should be more than 6–7 times the tunnel buried depth (Zhang [2002](#page-11-26); Shen [2007](#page-11-25)). Furthermore, the effects of model size on the target vibrations need to be investigated to determine the fnally optimal size. The mesh size meets the requirement of at least 6 elements per wavelength (Shih et al. [2016](#page-11-27)), whose specific information is listed in the "[Validation of established model"](#page-4-0) section. To avoid the refection of vibration waves on the intercepted boundary surface, the artifcial viscoelastic boundary is adopted and discretized by spring-damper elements with the stifness and damping determined through the method proposed by Liu et al. ([2006\)](#page-11-28).

In this paper, the Caughey damping (Hall [2018](#page-10-9); Han et al. [2021](#page-10-10)) is adopted to both cover the dominant frequency range of subway-induced vibrations below 100 Hz and high-frequency components induced by rail corrugations above 200 Hz. The damping matrix with four frequency points can be expressed as Eq. (5) (5) .

$$
[C] = a_0[M] + a_1[K] + a_2[K][M]^{-1}[K] + a_3[K][M]^{-1}[K][M]^{-1}[K]
$$
\n(5)

Fig. 3 Diagram of the wave propagation subsystem model

where a_0 , a_1 , a_2 , and a_3 are the coefficient related to damping ratio and characteristic frequency.

The damping ratio at *n*th mode can be expressed as Eq. ([6](#page-4-1)), which is assumed as constant in this study. So if the damping ratio and the characteristic frequencies are obtained, the above coefficient can be calculated through simultaneous equations and the system damping is further determined.

$$
\zeta_n = \frac{a_0}{2\omega_n} + \frac{1}{2}a_1\omega_n + \frac{1}{2}a_2\omega_n^3 + \frac{1}{2}a_3\omega_n^5
$$
\n(6)

where ζ_n and ω_n are the damping ratio and natural frequency at *n*th modal, respectively.

Besides, how to transfer the fastening forces generated by the 3D load generation subsystem into equivalent excitations used in the 2D wave propagation subsystem is a crucial concern for the reasonable simulation results. The vertical fastening forces corresponding to the typical subway section are applied in the model, and this method is proved to efficiently analyze the qualitative behavior of the vertical subway-induced environmental vibrations, as specifed in the ["Validation of established model](#page-4-0)" section.

Once the motion equation of the wave propagation system is established, environmental vibrations are solved by Newmark-β method with the integration step of 5×10^{-4} to cover the high-frequency vibration responses.

Validation of established model

In the case reported in China (Xin et al. [2020](#page-11-22)), serious rail corrugations with a wavelength of 40 mm were found on the right rail surface through feld measurements, which far exceed the limits specifed in ISO 3095 (ISO 3095–2005

2005), as shown in Fig. [4.](#page-4-2) More seriously, the high-frequency vibrations at 250 Hz caused by rail corrugation were further found in the living environments, which afected the nearby residents. On the background of this case, the hybrid numerical model of the complete environmental vibration system was established by the proposed method and verifed by measured data of ground vibrations.

Model parameters

The vehicle on the operation is a B-type metro train with a running speed of about 35 km/h, with detailed parameters shown in Table[.1.](#page-4-3) For the track irregularity, the short-wavelength random irregularity below 0.5 m is the measured rail corrugation shown in Fig. [4](#page-4-2), while the medium-long wavelength irregularity is based on the ffth-grade power spectral density formula proposed by the US Federal Railroad Administration (Xing et al. [2019;](#page-11-19) Zhu et al. [2020](#page-11-29)), whose wavelength is in the range of 0.5–200 m.

The track structure is composed of 60 kg/m rail, compressed fasteners, and the common monolithic track.

Table 1 Key parameters of China's B-type subway vehicle

Parameters	Unit	Value	
Mass of car body	kg	18,250	
Mass of bogie frame	kg	3393	
Mass of wheel-set	kg	1527	
Vertical stiffness of primary suspension	MN/m	10	
Vertical stiffness of secondary suspension	MN/m	0.257	
Vertical damping of primary suspension	kN·s/m	10	
Vertical damping of secondary suspension	kN·s/m	30	

Fig. 4 Rail corrugation obtained by feld measurement. (a) Distribution of the rail corrugation on the left rail. (b) Distribution of the rail corrugation on the right rail. (c) Roughness level on the both rails

The vertical stifness of the fastener is 27 kN/mm according to test results, with the damping of 40 kN·s/m. The transverse and longitude stifness are set to 10 kN/mm, with the damping of 2 kN·s/m. The track slabs with the width of 2400 mm and height of 300 mm adopt a continuous layout ignoring the infuence of slab joint. The materials of the track slab and basement slab are C60 concrete and C40 concrete. The vertical stifness of the structure under track slabs in load generation subsystem model is set to 80 kN/ mm according to the static calculation, with the damping of 50 kN·s/m.

The tunnel whose inner and outer diameters are 5.5 m and 6.2 m adopts C50 concrete, and the buried depth is 17.5 m, as shown in Fig. [5.](#page-5-0) The soil is mainly composed of medium-softsoil and divided into three layers. The range of 2.5 m below the surface is artifcial fll, and that of 2.5–11 m is silty sand 1, while the silty sand 2 is below 11 m. The specific parameters of diferent soil are listed in Table [2.](#page-5-1)

The wave propagation subsystem model is in the size of $200 \text{ m} \times 160 \text{ m}$ whose width is 32 times the diameter of the tunnel and height is 9 times the tunnel depth. The maximum element size of the slab, tunnel, and near-feld surrounding soil is about 0.06 m which satisfes the wave propagation requirements of 250 Hz vibration under the minimum shear wave velocity of 125.1 m/s, while the mesh sizes of the far-feld soil are between 0.06 and 1 m. Besides, the damping ratio of the system at diferent modal is set to 3% determined by the main soil type (Liu et al. [2002](#page-11-30); Chen and Liu [2004\)](#page-10-11). And ω_n is set to 1 Hz, 80 Hz, 200 Hz, and 300 Hz considering that the ground vibration in this case is mainly in the range of 1–80 Hz and 200–300 Hz, respectively.

Comparison results

In order to have a comprehensive comparison, the simulation results and measured data (averaged by 20 groups) are compared in the one-third octave band center frequencies of 31.5 Hz, 40 Hz, 50 Hz, and 250 Hz, which can be calculated by Eq. (7) .

$$
VAL_{i} = 20 \log_{10} \left(\frac{a_i}{a_0} \right) \tag{7}
$$

where a_i is the acceleration root mean square value with center frequency f_i in $1/3$ octave frequency band and the reference acceleration a_0 is equal to 10⁻⁶ m/s².

The vertical vibration levels at four positions, including 22.5 m, 30 m, 37.5 m, and 45 m, are shown and compared in Fig. [6.](#page-6-0) It can be seen that the vibration attenuation trends at diferent central frequencies obtained from simulation and measurement are basically consistent, showing frst decreases and then increases with the increase of distance away from the track. Moreover, the simulation results are basically consistent with the measured data in the intersection of 50 Hz and 250 Hz vibration curves. However, the numerical simulation results are lower than the measured data because the load in the model is only taken from a set of fastener forces without considering the infuence of other fasteners (Xu et al. [2015\)](#page-11-13). The other diference may

Fig. 5 Cross-section of the tunnel structure. (a) Tunnel in the scene. (b) Tunnel in design (mm)

(a) Tunnel in the scene \qquad (b) Tunnel in design (mm)

Table 2 Parameters of soil in different layers

Soil type	Elastic modulus (MPa)	Poisson ratio	Density (kg/m^3)	Shear wave velocity (m/s)	Compression wave velocity (m/s)
Artificial fill soil	86.5	0.49	1850	125.1	790.8
Silty sand 1	197.5	0.49	1900	187.7	1357.2
Silty sand 2	285.7	0.49	1910	224.6	1505.8

Fig. 6 The vibration levels at diferent frequency band. (a) The diagram of the comparing position. (b) The measurement results. (c) The simulation results

be due to the fact that the simulation model fails to fully consider the change of actual soil parameters. In general, it can be concluded that the model established in this paper can accurately analyze vibration propagation qualitatively, including both low frequency and high frequency.

High‑frequency ground vibration

In order to have a more comprehensive understanding of high-frequency environmental vibrations caused by the presented rail corrugation, the impacts on ground vibration under diferent tunnel depths and rail fasteners are further analyzed through the validated numerical model. The vibration transmission and variation characteristics under diferent parameters are obtained, in which the rail corrugation adopts the data shown in Fig. [4](#page-4-2). The results are helpful to determine sensitive range on the ground and select mitigation measures, so as to get a better prevention and control effect.

The efect of tunnel depth

Tunnel depth is one of the key design parameters in diferent subway lines. Besides, for the specifc line, its buried depth changes greatly with the line direction. Therefore, it is necessary to discuss high-frequency environmental vibrations when rail corrugation occurs in diferent tunnel depths. Four

Fig. 7 Ground vibration at 250 Hz under diferent buried depths

tunnel depths, including 17 m, 21 m, 25 m, and 29 m, are selected for analysis in this section. The distributions of the ground vibration at 250 Hz are shown in Fig. [7.](#page-6-1)

The results show that the attenuation law of high-frequency vibration at 250 Hz is similar under diferent tunnel depths. Specifcally, the vibration frst increases, then decreases, then increases and decreases again at last along with the transverse distance. To better describe the changes, two special positions are defned. The frst one is called the

maximum point, which is at the position of the peak value, while the second one is at the position of vibration mutation, called the infection point, which is actually caused by vibration amplifcation (Jin et al. [2005](#page-11-4); Ma et al. [2014](#page-11-31)). Under the buried depth of 17 m, the maximum point does not appear above the tunnel, instead, it is located near 8.5 m, and the vibration infection point appears near 36 m. The comparisons show that with the increase of tunnel depths, the positions of the maximum point and the infection point will shift to the right, and the vibration response at its position will also be signifcantly reduced. With the tunnel depth increasing from 17 to 29 m, the maximum point moves from 11.5 to 19.8 m, and the maximum value decreases from 72 to 67 dB with a reduction rate of 7.0%. With the buried depth increasing from 17 to 25 m, the infection point moves from 37.8 to 60.5 m, and the vibration response decreases from 45.6 to 34.7 dB, which is reduced by 23.9%. The deviation may aggravate the high-frequency environmental vibration within the range from 24 to 65 m on the ground surface.

To have a more comprehensive understanding of the infuence of tunnel depth on the subway-induced ground vibrations, the comparison results with consideration of the 31.5 Hz, 40 Hz, 50 Hz, and 250 Hz of the two typical positions at 22.5 m and 30.0 m are shown in Fig. [8.](#page-7-0) For the positon at 22.5 m, it can be found that the ground vibrations at diferent frequencies decrease obviously, but high-frequency vibrations at 250 Hz are always the dominant components. Although the main component is low-frequency vibration at the position of 30 m when the tunnel depth is 17 m, the 250 Hz vibration plays a more important role with the increases of tunnel depth due to the amplifcation phenomenon of high-frequency vibrations presented in Fig. [7.](#page-6-1) For example, the high-frequency vibration at around 30 m away from the track increases by 5.6 dB when the depth changes from 17 to 29 m.

Through the above analysis, it can be concluded that the increase of tunnel depth does not mean that it can reduce the infuence of high-frequency vibration at any position, on the

contrary, it may amplify vibrations. Therefore, for the new subway lines, the tunnel depth should be reasonably selected according to the actual placement of the environmentally vibration-sensitive buildings to prevent the possible impact of high-frequency vibration. On the other hand, the subway section with deeper tunnel depth in the operation line may also be afected by high-frequency vibration, to which attention should also be paid to.

To further explore the characteristics of high-frequency vibration propagation at diferent tunnel depths, the harmonic response analysis method is used for the analysis of the ground vibrations under 250 Hz excitation. As shown in Fig. [9,](#page-8-0) the up sub-graph refers to the vertical displacement of the ground, which is normalized due to the small value. The down sub-graph represents the spatial distribution of high-frequency vibration, which is processed by logarithmic due to the large diference among diferent regions.

It can be seen that at diferent buried depths, high-frequency vibration at 250 Hz propagates radially from the tunnel wall to the surrounding soil and forms multiple vibration dominant transmission paths. The ground vibration is mainly afected by the black line path, as shown in the fgure. The angle between the boundary of the transmission path and the horizontal direction basically remains unchanged at diferent buried depths, and the left and right are about 75° and 35°, respectively. With the increase of the tunnel depth, the position of the vibration transmitted to the ground gradually moves to the right, resulting in the deviation of the maximum point and the infection point. Although the increase of tunnel depths can efectively reduce the maximum value of the high-frequency vibration on the ground, the main infuence area of high-frequency vibration increases. In the case that the tunnel depth increases from 17 to 29 m, the main infuence range increases from 17 to 30 m, resulting in the increase of vibration in this area. Therefore, the infuence of tunnel depth can be attributed to the combined efect of the increase of propagation distance and the offset of propagation position.

(a) Vibration distribution under 17 m (b) Vibration distribution under 21m

Fig. 9 Vibration distribution under diferent tunnel depths. (a) Vibration distribution under 17 m. (b) Vibration distribution under 21 m. (c) Vibration distribution under 25 m. (d) Vibration distribution under 29 m

The efect of rail fastener

With diferent kinds of rail fasteners, serious rail corrugation with the dominant wavelength of 30–80 mm may occur on the rail surface. In this section, four types of fasteners widely used in subway lines are selected, including DTVI2 fastener, Cologne-egg fastener, Lord fastener, and Vanguard fastener, whose specific vertical parameters are shown in Table [3.](#page-8-1)

The distribution characteristics of high-frequency vibrations on the ground are compared and analyzed, as shown in Fig. [10.](#page-9-0) It can be seen that under the same rail corrugation, the attenuation laws of high-frequency environmental vibration with diferent fasteners are basically the same, but there are obvious diferences in specifc response values. The vibration curves of DTVI2 fasteners and Cologne-egg fasteners almost coincide. Compared with the other two fasteners, the vibration responses at any positions are the largest except for those near the infection point, that is, up to 74.3 dB. The high-frequency vibration of the Lord

Fig. 10 Ground vibration at 250 Hz under diferent rail fasteners

fastener is between that of DTVI2 fasteners and Vanguard fasteners, which may exceed that of DTVI2 fasteners near the infection point, while Vanguard fasteners show an excellent damping performance that the high-frequency vibration at diferent positions is at the lowest level. According to the statistics on high-frequency vibration, about 30% of vibration responses exceed 68 dB when using DTVI2 fasteners and Cologne-egg fasteners, which are concentrated within the range of 20 m from the tunnel center line. In the case that Lord fasteners are adopted, this index is reduced to 25%, while the vibration levels at any position are lower than 68 dB using Vanguard fasteners.

According to the above analysis, it can be concluded that Vanguard fasteners are able to signifcantly reduce the highfrequency environmental vibration caused by rail corrugation. Therefore, in the rail corrugation treatment, Vanguard fasteners can be considered to replace the fastener system besides rail grinding, so that the high-frequency vibration can be efectively suppressed when rail corrugation occurs again.

To explore the relationship between high-frequency environmental vibration and fastener parameters, the stifness and damping are set to 5–100 kN/mm and 1–1000 kN·s/m, respectively. The ground vibration at the maximum point is taken as the analysis object, and high-frequency vibration changes at 250 Hz are compared with low-frequency ones at 31.5 Hz, 40 Hz, and 50 Hz. The results shown in Figs. [11](#page-9-1) and [12](#page-9-2) suggest that, in the entire analysis range, the variation of high-frequency vibration with stifness and damping is similar to that of low-frequency vibration. However, there are obvious diferences within the commonly used range.

Diferent from the phenomenon that the low-frequency vibrations frst increase and then decrease with rising fastener stifness, the high-frequency vibration at 250 Hz keeps

Fig. 11 Ground vibration under diferent stifness

increasing in the commonly used range. However, there is a strong nonlinear correlation between the high-frequency vibration and fastener damping. The vibration response frst decreases and then increases with the increase of damping, and the infection point is about 10–20 kN·s/m, which is obviously diferent from the negative correlation between low-frequency vibrations and damping. The above results can be used to explain why the high-frequency vibration on the ground under the Vanguard fasteners is the lowest. Besides, an important conclusion can be drawn that the treatment of high-frequency environmental vibration caused by rail corrugation and low-frequency vibration has to be carried out following diferent principles. The fastener system

Fig. 12 Ground vibration under diferent damping

with temperate damping and lower stiffness should be selected for the elimination of high-frequency vibrations.

Conclusion

In this paper, a hybrid numerical analysis model consisting of a 3D load generation subsystem and a 2D wave propagation subsystem was proposed and validated by measured data. The effects of high-frequency vibration at 250 Hz induced by rail corrugation with diferent tunnel depths and rail fasteners were further analyzed. The results can provide an important theoretical reference for subway design and the remediation of rail corrugation. The main conclusions are drawn as follows:

- (1) Through the comparison with the measured data, it is proved that the hybrid numerical analysis model proposed in this study is able to accurately predict the attenuation trend of subway-induced environmental vibrations, especially in the high-frequency range caused by rail corrugation.
- (2) Under different tunnel depths, the high-frequency vibrations at 250 Hz induced by rail corrugation are transmitted to the ground by the dominant transmission path with the same angle of about 75° and 35°. In the case that the buried depth increases from 17 to 29 m, the 250 Hz vibration above the tunnel can be reduced by 5 dB. However, more serious high-frequency efects will appear in the vibration amplification region, especially at around 30 m away from the track where the 250 Hz vibration increases by 5.6 dB.
- (3) The variation of high-frequency vibration with fastener stifness and damping in the commonly used range is quite diferent from that of low-frequency vibration. Specially, there is a signifcant nonlinear relationship between fastener damping and high-frequency vibration, and the optimal damping is about 10–20 kN·s/m.

Author contribution Sen Wang: formal analysis, data curation, writing—original draft, writing—review and editing, investigation. Tao Xin: conceptualization, methodology, formal analysis, writing—review and editing, funding acquisition. Pengsong Wang: data curation, writing—original draft, writing—review and editing. Yi Yang: writing review and editing, investigation. Peng Chen: project administration, data curation. Lei Zhao: project administration, investigation, funding acquisition. Sihe Zhao: investigation, data curation. Yuming Luo: supervision, investigation.

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Data availability The datasets used and analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Ethics approval and consent to participate Not applicable.

Consent for publication Not applicable.

Competing interests The authors declare no competing interests.

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