



Measuring the integrated risk of China's carbon financial market based on the copula model

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Abstract

Measuring the risks of the carbon financial market is of great significance for investment decision-making, risk supervision, and the healthy development of the carbon trading market. Different from previous studies based on traditional VaR (value at risk), this study measures the integrated risk of China's carbon market based on the Copula-EVT (Extreme Value Theory)-VaR model which can explore the unique strength of the copula and EVT-VaR models, of which the copula model is applied to capture the dependence between the different risk factors of carbon price volatility and macroeconomic fluctuation, while the EVT-VaR is used to explore the risk value. The empirical results show that the traditional VaR that only considers a single risk factor from carbon price volatility is likely to overestimate the risk. In addition, compared with other methods that do not consider the interdependence between risk factors, using the copula function to measure the carbon market integration risk is more effective, and backtesting also confirms this conclusion. This paper provides a specific reference for carbon emission companies to participate in the carbon market. It provides a theoretical basis for the supervision of the risk management of the carbon market.

Keywords Carbon emission · Carbon price volatilities · Copula function · Extreme value theory · VaR · Macroeconomic risk

Introduction

Global warming has been a broad concern by countries worldwide in recent years. As an essential part of market mechanisms to mitigate climate change, the carbon market was established and developed rapidly since the Kyoto Protocol. Twenty-one emissions trading systems (ETSs) have been in operation globally. European Union Emissions Trading System (EU ETS), which started trial operation in early 2005 and officially launched in early 2008, has become the largest and most active carbon emissions. Since 2013, China has established eight pilot carbon markets in Shenzhen, Beijing, Shanghai, Guangdong, Tianjin, Hubei, Chongqing, and Fujian. In December 2017, China further launched its

national carbon market and had been in operation since July 16, 2021. As a result, China's emissions trading markets (CETS) show rapid growth. As of December 31, 2021, the cumulative transaction volume of carbon emission allowances in the national carbon market has reached 179 million tons, with a transaction value of 7.684 billion yuan. Accordingly, the CETS has played an essential role in achieving regional emissions reduction targets.

The “Measures for the Administration of Carbon Emissions Trading (Trial)” issued in January 2021 stipulates that the trading products in the national carbon market are carbon emission allowances (CEAs). The transaction method may adopt agreement transfer, one-way bidding, or other methods that meet the regulations. In October 2021, the Ministry of Ecology and Environment issued the “Notice on Doing a Good Job in the Payment of Carbon Emission Allowances in the First Performance Cycle of the National Carbon Emissions Trading Market.” This notice requires the provincial carbon market authorities to quickly complete the quota approval and payment of the first compliance cycle to ensure that enterprises complete compliance. The national carbon market takes the power

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generation industry as the first industry to be covered, and 2225 critical emission units are included. The carbon emissions of these companies exceed 4.5 billion tons of carbon dioxide. After 2022, the national carbon market is expected to gradually incorporate the building materials and steel industries, introduce institutional investors, reduce the total amount of carbon allowances, and increase trading varieties.

However, the carbon market is a complex system and faces more significant uncertainty than the traditional stock market. Given that China's carbon market has developed for a short period, this situation is even more serious. Several risk factors, such as carbon market price volatility, interest rate change, and macro-economy fluctuation, have complicated interdependence. Measuring the carbon market risk without considering their interdependence will lead to biased results. Thus, it is necessary to consider the dependence between different risk factors for accurately measuring China's carbon market risk.

Based on establishing a unified national carbon emission trading system, the macroeconomy greatly influences China's carbon market. Accurately measuring the risks of the carbon market is of great significance for the healthy development of the carbon market. Compared with previous studies, this paper pays more attention to considering the interdependence structure between the carbon price and macroeconomics, which improves the accuracy of carbon market risk measurement to a certain extent.

Compared with existing studies, the contributions of this study are as follows. First, this study considers the risk factors of carbon price itself and the risk factors of the external macroeconomic environment. Because of the interdependence between these two risk factors, we choose the optimal binary copula connection function to measure the overall risk of the carbon market. This measurement method can describe the nonlinear correlation between different sequences and contains all the dependent information between random variables. Combined with the Monte Carlo simulation method, the distribution of the joint distribution can be simulated. Then, the value at risk can be effectively measured. Second, the extreme value theory is introduced to describe the tail characteristics of the carbon price. The distribution characteristics of China's Hubei carbon market return rate and macroeconomic index return rate are fitted to obtain their marginal distributions.

The article is mainly divided into four parts: the following part is the literature review part, which mainly introduces the current research literature related to carbon emissions, and the research method part mainly expounds on the research data and related measurement methods. The empirical research part analyzes the research questions of this paper empirically. The last part is the conclusion and suggestion

part, which expounds on the critical conclusions obtained in this paper and puts forward relevant policy suggestions.

Literature review

As an effective solution for reducing carbon emissions, the carbon market has become a hot topic for scholars at home and abroad in recent years. During the past few years, more and more attention has been paid to the studies of the volatility characteristics of the carbon price, the influencing factors of the carbon price, and the measurement of carbon market risks.

In terms of the research on the volatility characteristics of the carbon price, most scholars used the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model to describe the volatility of carbon prices. They believe that carbon prices are similar to the general financial time series, with prominent peak and thick tail characteristics, price volatility being of a specific long-term nature, and volatility aggregation (Zhang and Xu 2020; Fu and Zheng 2020; Chevallier 2010).

Regarding the influencing factors of the carbon price, Ji et al. (2021) believed that the reason for China's low carbon price was oversupply in the carbon allowance market, insufficient demand, and low auction prices. The expansion and centralized trading of the carbon market are the main reasons for the rise in carbon prices. In addition, many scholars believe that carbon allowance prices are not only affected by internal factors such as allowance supply and demand but also by some external factors, including macroeconomic shocks, energy prices, political events, and other factors. Wen et al. (2020) analyzed the correlation between China's carbon market and the stock market. They believed that China's carbon price is closely related to the stock price of energy-intensive industries. Zhang and Sun (2016) found that coal prices have the most significant impact on carbon market prices in the energy market, while natural gas and oil markets have less impact on carbon prices. Furthermore, it is believed that falling energy prices have a more significant impact on carbon prices than rising energy prices, and there is a certain asymmetry. Yang et al. (2018) studied the impact of policies on carbon prices and set energy, economy, climate, and other factors as control variables. The research found that policies played a vital role in the price discovery and stability of the carbon market. Among these external factors, macroeconomic factors are considered by more and more scholars as to the influencing factors of the carbon market. Shahbaz et al. (2022) explored the impact of financial inclusion on carbon emissions. The study results proved that financial inclusion has a negative impact on pollutant emissions. They believe that the vital role of financial inclusion in promoting carbon emission reduction should

be emphasized. Khan et al. (2021) examined the impact of export diversification and composite risk indices on CO₂ emissions in the Regional Comprehensive Economic Partnership (RCEP) countries from 1987 to 2017. Empirical results suggest that economic growth in RCEP countries will be the initial contributor and ultimate inhibitor of CO₂ emissions. It is believed that export diversification plans should be effectively adjusted and economic growth accelerated in an environmentally friendly manner. Dou et al. (2022) explored the spillover effects of economic policy uncertainty on carbon futures prices. Through wavelet decomposition, it is believed that although economic policy uncertainty (EPU) cannot predict the volatility of carbon futures prices, in the long run, the impact of EPU on carbon futures prices is negative.

In terms of risk measurement, the current main risk measurement methods are the stress test method, scenario test method, and VaR method. VaR is the most popular and utilized to measure financial market risk. Specific to the risk measurement of the carbon market, Fu and Zheng (2020) used the ARMA-EGARCH-SGED (Autoregressive Moving Average-Exponential Generalized Autoregressive Conditional Heteroscedasticity-Skewed Generalized Error Distribution) model to characterize the carbon market in China and compared the risk of the seven carbon pilots by calculating VaR. The backtest results show that the VaR estimation is valid. Segnon et al. (2016) used EU carbon quotas as the research object to model the fluctuations of carbon prices and compared the VaRs obtained from different models to determine the quality of the model. Zhu et al. (2019) used Empirical Mode Decomposition (EMD) and ARMA-EGARCH to construct a multi-time scale VaR model. Result shows that this method can effectively reduce extreme events and obtain a more accurate European carbon market-overall risk measurement. Since extreme value theory is more effective in describing extreme events in measuring market risk, extreme value theory has been widely used in different risk fields, including financial markets, insurance markets, and energy markets. Extreme value theory provides a solid theoretical basis for the study of extreme risks. Research has found that introducing extreme value theory into VaR measurement can effectively measure extreme risks. Feng et al. (2012) used extreme value theory to measure the spot market and futures market risks in EU carbon emission trading. The empirical results showed that using extreme value theory to estimate VaR is more effective than traditional methods. It is believed that the downside risk of the carbon market was higher. Qiu et al. (2020) established a GARCH-EVT (Generalized Autoregressive Conditional Heteroscedasticity-Extreme Value Theory) model for six pilot markets in China's carbon market. They believed that there are apparent extreme risks in the pilot carbon markets. Therefore, it is suitable to use extreme value theory to capture tail risks.

However, the research above mentioned mainly focused on the single factor risk from carbon price volatilities, without considering the dependence with other risk factors at the same time, such as interest rate, exchange rate, macroeconomic fluctuating, in order to overcome these deficiencies and measure the risk of carbon market more accurately, several scholars tried to consider various risk factors to measure the carbon market integration risk based on the copula connection function. For example, Zhang et al. (2020) consider the three risk factors of interest rate, exchange rate, and carbon price to calculate the integration risk commercial banks faced when participating in carbon trading and believe that vine-copula can effectively measure the integration risk of the carbon market. Marc et al. (2011) used the copula function to analyze the interdependence structure between the EU carbon allowance futures price, commodity index, and financial market. He believed that ignoring the correlation between different risk factors may underestimate the portfolio's risk. Reboredo and Ugando (2015) combined the GARCH (Generalized Autoregressive Conditional Heteroscedasticity) model, extreme value theory, and copula function to analyze the interdependence structure between the EU carbon market and the fossil fuel market. They measured the downside risk of the carbon market and the fossil fuel market combination. Taking carbon allowance trading into the fossil fuel investment portfolio can significantly reduce the downside risk. Zhang and Li (2018) conducted risk measurements on six Chinese commercial banks participating in carbon financing. The copula function combines credit risk and market risk, and the risk is integrated and measured. Finally, the VaR of each of the six banks is calculated and compared using Monte Carlo simulation.

It can be seen from the above analysis that most of the literature mainly focused on the European Union Allowance (EUA) market, the current representative carbon market. In contrast, the studies on China's pilot carbon market are insufficient. In addition, the risk measurement of the carbon market, most of the literature mainly considered a single risk factor, namely carbon price volatility, and failed to consider the dependence with other risk factors, such as macroeconomic fluctuation, which may lead to overestimation risk of China's carbon market. Thus, the aim of this study is to fill this gap by measuring the integrating risk of China's carbon market based on the Copula-EVT-VaR model.

Methodology

In order to better carry out market risk measurement and risk management, when measuring the risk of China's carbon finance market, this article comprehensively considers the correlation between the two risk factors of carbon price risk and macroeconomic risk in the carbon market

transaction process. First, use financial time series modeling to capture the distribution characteristics of the two sequences of the carbon price and stock price, and then use the Euclidean distance to select the best fitting copula function to connect the distribution of the two sequences, and finally use Monte Carlo simulation calculate the VaR of the integrated risk.

Marginal distribution of returns

ARMA-GARCH model

One of the critical characteristics of financial time series is volatility. Under normal circumstances, volatility will show time-varying and agglomeration, and the series often have autocorrelation and conditional heteroscedasticity. The ARMA-GARCH (Auto regressive Moving Average-Generalized Autoregressive Conditional Heteroscedasticity) model can fully describe these characteristics. The ARMA-GARCH model can effectively eliminate the autocorrelation and heteroscedasticity of carbon price volatilities and stock price fluctuations. ARMA is usually used to describe the historical law of price changes, AR(p) is the autoregressive part, and MA(q) is the moving average process. The GARCH model is also called the heteroscedasticity model, which is evolved from the ARCH model and is used to describe the heteroscedasticity of the time series. The GARCH model has been widely used in the financial field (Berkes et al. 2003; Engle 2001; Panorska 1995; Nelson 1990). The GARCH model is described by the mean value equation and the conditional variance equation. Adding the ARMA process to the mean equation of the GARCH model can better mine the residual information of the time series and make the residual series obey more the assumption of independent and identical distribution better to describe the time series' volatility (Liu and Shi 2013). Therefore, this article uses the ARMA-GARCH model to study the marginal distribution of carbon price risk and exchange rate risk. The general form of the ARMA-GARCH model is

$$\begin{cases} r_t = \mu + \sum_{i=1}^p \phi_i r_{t-i} + \sum_{j=1}^q \theta_j \varepsilon_{t-j} \\ \delta_t^2 = \alpha_0 + \sum_{h=1}^m \alpha_h \varepsilon_{t-h}^2 + \sum_{k=1}^n \beta_k \delta_{t-k}^2 \\ \varepsilon_t | I_{t-1} = \delta_t \eta_t, \eta_t \sim i.i.d(0, 1) \end{cases} \quad (1)$$

In the mean value equation, r_t represents the logarithmic return rate of the financial time series, ε_t represents the residual, and in the conditional variance equation, δ_t^2 is the conditional variance of the residual. I_{t-1} represents all the information sets before time t , and η_t is an independent and identically distributed time series with a mean of 0 and a variance of 1.

POT model of extreme value theory

Since financial time series often have the characteristics of fat-tailed distribution, they usually do not satisfy the assumption of normal distribution. There are two common models in extreme value theory to measure tail risk, one is the extreme value model POT (Peak Over Threshold) that exceeds the threshold, and the other common model is the interval selection extreme model BMM (Block Maxima Group of Models). This model is more suitable for modeling data with a certain periodicity or seasonality. The POT model determines the tail data according to the threshold, which is more accurate in describing the financial time series (Marimoutou et al. 2009; Corcoran 2002; McNeil and Frey 2000). Considering that the fluctuation of the yield of the carbon price and the Shanghai Composite Index do not change with seasons, this article is more suitable to use POT to model the tail of the sequence. The most important step of adopting this method is to determine the threshold of the tail. DuMouchel (1983) proposed a more straightforward quantitative method, which believed that the number of sequences exceeding the critical value should be selected corresponding to 90% of the total sample. The quantile is used as a threshold to fit the data well.

According to the extreme value theory, the cumulative distribution function when the value of x exceeds the critical value μ is

$$\begin{cases} F(y) = 1 - \frac{N_\mu}{N} (1 + \xi y)^{-1/\xi}, \xi \neq 0 \\ F(y) = 1 - \frac{N_\mu}{N} \exp(-y), \xi = 0 \end{cases} \quad (2)$$

This distribution is called the generalized Pareto distribution (Pareto). $y = (x - \mu)/\beta$, $\beta > 0$, β is the proportional parameter, ξ is the shape parameter, N_μ represents the number of exceeding the critical value μ in the sequence, and N is the total number.

Because the tail data is mainly used in the VaR calculation, it is crucial to fit the tail data distribution. This article first uses the ARMA-GARCH model to obtain the standard residual sequence of the return sequence and then uses the data's upper and lower tails, respectively. The Pareto distribution of the extreme value theory is fitted, and the middle part is fitted with the empirical cumulative distribution function. The estimation methods adopt the maximum likelihood estimation method and the non-parametric Gaussian kernel estimation method, respectively. The final distribution function of standardized residuals is

$$F_{\xi, \beta}(\eta_t) = \begin{cases} 1 - \frac{n u_L}{n} \left(1 + \xi \frac{\eta_t - u_L}{\beta} \right)^{-1/\xi}, & -\infty < \eta_t \leq u_L \\ cdf(\eta_t), & u_L < \eta_t \leq u_u \\ 1 - \frac{n u_u}{n} \left(1 + \xi \frac{\eta_t - u_u}{\beta} \right)^{-1/\xi}, & u_u < \eta_t \leq \infty \end{cases} \quad (3)$$

where u_L and u_u are the thresholds of the upper and lower tails respectively. η_1 and η_2 respectively represent the standard residual sequence of the carbon price return rate and the Shanghai Composite Index return rate series after the ARMA-GARCH model is fitted.

Choose the best fit copula function

Copula connection function is widely used in the study of financial sequence to study the structure of dependencies between variables. It can connect edge distributions with different distributions, fit complex nonlinear correlations between variables, and form a joint distribution function. Financial time series usually do not show a simple linear relationship. If we use simple linear correlation to model, we may get wrong results. The copula function can effectively obtain the nonlinear correlation structure between variables. Therefore, we use the copula function to capture the dependency structure between the Hubei carbon price return series and the Shanghai Composite Index return series. According to Sklar (1959), if F is used to represent the joint distribution of random variable x_1, x_2, \dots, x_n , and $F_1(x_1), F_2(x_2), \dots, F_n(x_n)$ is the marginal distribution of each random variable, then there is a copula function C , such that:

$$F(x_1, x_2, \dots, x_n) = C(F_1(x_1), F_2(x_2), \dots, F_n(x_n)) \tag{4}$$

If the distribution functions of all random variables are continuous, then C is unique. If the joint distribution function has an inverse function, the copula function can be obtained according to the above formula:

$$C(u_1, u_2, \dots, u_n) = F(F_1^{-1}(u_1), F_2^{-1}(u_2), \dots, F_n^{-1}(u_n)) \tag{5}$$

u_i represents a random variable that obeys a uniform distribution. If the joint distribution function is differentiable, the joint density function can be written as

$$f(x_1, x_2, \dots, x_n) = \frac{\partial F(x_1, x_2, \dots, x_n)}{\partial x_1 \partial x_2 \dots \partial x_n} = \prod_{i=1}^n f_i(x_i) \frac{\partial C(u_1, u_2, \dots, u_n)}{\partial u_1 \partial u_2 \dots \partial u_n} \tag{6}$$

$f_i(x_i)$ is the density function of the marginal distribution $F_i(x_i)$.

Then, the density function of the copula function is

$$c(u_1, u_2, \dots, u_n) = \frac{\partial C(u_1, u_2, \dots, u_n)}{\partial u_1 \partial u_2 \dots \partial u_n} = \frac{f(F_1^{-1}(u_1), F_2^{-1}(u_2), \dots, F_n^{-1}(u_n))}{\prod_{i=1}^n f_i(F_i^{-1}(u_i))} \tag{7}$$

The estimation of copula parameters can be estimated by the log-likelihood function of the following formula:

$$\log LF(\varphi_1, \varphi_2, \dots, \varphi_n; \theta; x) = \sum_{i=1}^n \log(f_i(\varphi_i; x_i)) + \log\{c(\theta; F_1(x_1), F_2(x_2), \dots, F_n(x_n))\} \tag{8}$$

where φ represents the parameter of the marginal distribution function, and θ is the parameter of the copula function. This paper uses the more commonly used two-stage maximum likelihood estimation to estimate the parameters.

Since the copula function does not restrict marginal distribution, different GARCH models can be selected to fit the two sets of return series according to the fluctuations of the carbon price and the Shanghai Composite Index series. In previous studies, the more common binary copula function forms include the t-copula function, Gumbel copula function, Frank copula function, Gaussian copula function, Clayton copula function. In this paper, the optimal copula function is determined by calculating Euclidean distance. The copula model is constructed using the marginal distribution obtained after fitting the generalized Pareto distribution and the empirical cumulative distribution.

VaR and backtesting

In the financial market, VaR is widely used to measure market risk. VaR is defined as the maximum loss in a portfolio under a given confidence level in a certain period. The most significant advantage of this indicator is that it uses a single value to summarize the risks within a specific time interval. For the distribution function $F(X)$ of any sequence of returns, when the confidence level is P , the sequence of VaR can be expressed by the following formula:

$$P(X > VaR_p) = p, VaR_p = F^{-1}(1 - P) \tag{9}$$

This paper uses Monte Carlo simulation to calculate the integrated value of the risk of the two risk factors after fitting with the Copula function to measure the risk of the next day. Finally, the validity and accuracy of VaR need to be tested. The commonly used method is the Kupiec backtest test. Kupiec (1995) constructed the following LR statistics based on the Bernoulli distribution:

$$LR = -2\ln[(1 - p)^{T-N} p^N] + 2\ln\left[\left(1 - \frac{N}{T}\right)^{T-N} \left(\frac{N}{T}\right)^N\right] \tag{10}$$

N is the number of days of failure, T is the number of days actually investigated, N/T is the failure rate, and P is the given confidence level. The null hypothesis of this hypothesis test is: $p = N/T$, by observing whether the actual failure rate is close to the estimated failure rate, the quality of the model is determined. If the null hypothesis holds, it is considered that the likelihood estimator LR approximately obeys the chi-square distribution with 1 degree of freedom. Therefore, when the confidence level

is given, the value of LR can be used to judge whether to reject the null hypothesis.

Empirical analysis

Data processing

Since the Hubei market is the most active trading market with the most significant carbon emissions trading volume in China’s carbon emissions trading pilots, this paper selects the closing price of Hubei’s daily carbon allowance (HBEA) and the closing price of the Shanghai Stock Exchange Index (SSEC) as the research samples. The sample period is from April 2, 2014, to January 27, 2022. The data comes from the Hubei Carbon Emission Rights Exchange and the Choice Financial Terminal database. After removing the data whose transaction volume is 0, there are a total of 1888 data. Figure 1 shows the relative price after the same initial price transformation. Figure 1 shows that from 2014 to the present, the Hubei carbon allowance price and the Shanghai Stock Exchange index price have undergone significant changes, and the trend of change shows a certain degree of correlation. In 2015, affected by the stock market crash in China, the Shanghai Stock Exchange fell sharply, and the trading price of the carbon emissions trading market was once sluggish. However, until 2018, carbon trading prices gradually increased, and the stock market began to pick up gradually. In order to make the data stable, the price data converted into the rate of return data. The calculation formula of the rate of return is

$$r_t = \ln p_t - \ln p_{t-1} \tag{11}$$

where r_t represents the return in period t . p_t and p_{t-1} denote the settlement price in period t and $t - 1$ respectively.

Figures 2 and 3 are the time series of Hubei carbon price volatility and the time series of the Shanghai Stock Exchange returns. It can be seen from the two figures that both have a certain degree of volatility clustering. Table 1 shows the descriptive statistics of all return series of variable prices. Firstly, we found that the standard deviation of the rate of return on carbon prices is significantly greater than the standard deviation of the rate of return on the Shanghai Stock Exchange, indicating that the carbon market has been established for a short time and faces more uncertainties. Therefore, the volatility of carbon prices is greater than that of SSEC. Secondly, the skewness values of the two return rate series are both less than 0, and

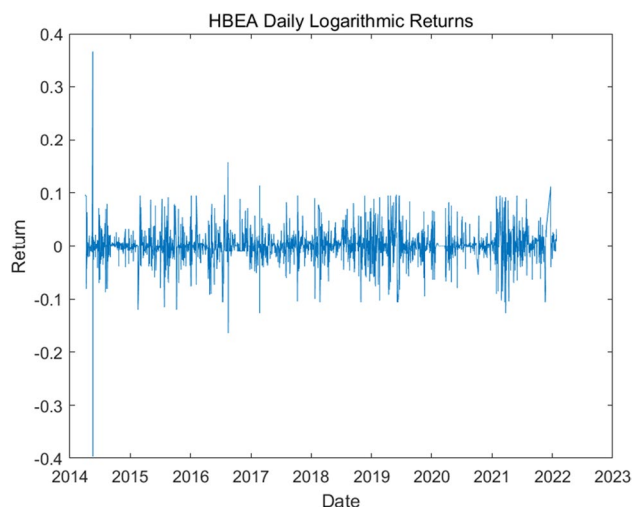


Fig. 2 Timing diagram of the carbon price return sequence

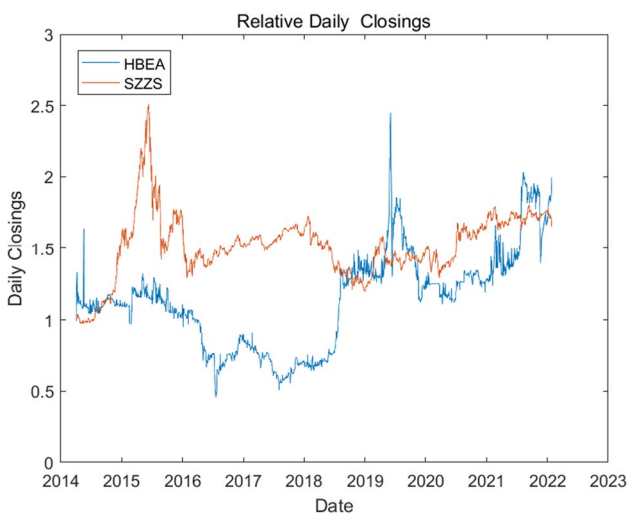


Fig. 1 Timing chart of relative price

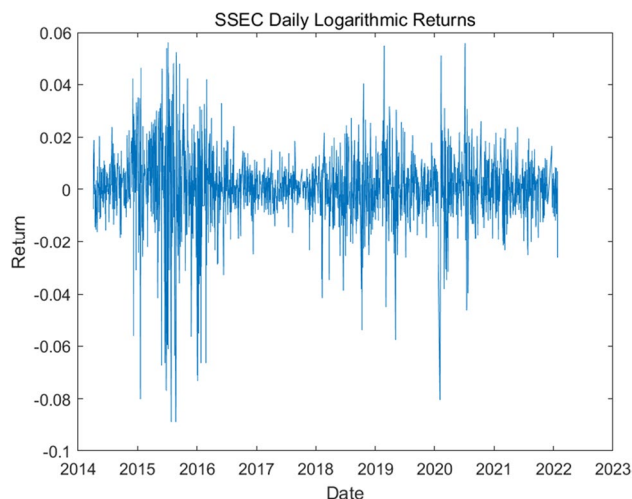


Fig. 3 Time sequence diagram of the Shanghai Composite Index’s return rate series

Table 1 Descriptive statistics of return rate series

	Maximum	Minimum	Mean	Std. dev	Skewness	Kurtosis	Jarque–Bera
HBEA	0.364643	-0.396753	0.000367	0.033124	-0.287641	23.52391	33,145.28 (0.0000)
SSEC	0.056036	-0.088732	0.000265	0.013921	-1.111336	10.47126	4777.258 (0.0000)

Table 2 Descriptive statistics of return rate series

Series	ADF	1% level	5% level	10% level	<i>P</i> value
HBEA	-51.7580	-3.4336	-2.8629	-2.5675	0.0001
SSEC	-41.9312	-3.4336	-2.8629	-2.5675	0.0000

the kurtosis values are far greater than 3, indicating that the two return series have obvious sharp peaks and thick tails. The observed value of the JB statistic in Table 3 is relatively large, and its associated probability *P* value = 0. Therefore, the two return rate series do not obey the normal distribution.

In order to better extract the characteristics of the return rate series, it is necessary to test the stability of the return rate series. Table 2 shows the ADF test results of the carbon price rate of return and the Shanghai Composite Index rate of return. The results show that the two ADF statistics of the concomitant probability *P* value of the quantity is 0, indicating that the two sets of return rate series are stable.

Determination of marginal distribution

Fitting of ARMA-GARCH model

Figures 4 and 5 are the autocorrelation and partial correlation diagrams of the carbon price return rate and the Shanghai

Composite Index return rate, respectively. It can be seen from Figs. 4 and 5 that there are apparent autocorrelation in the carbon price return rate and the Shanghai Composite Index return rate series. It is more obviously affected by historical price fluctuations. Therefore, selecting the ARMA model to model its return data to eliminate serial autocorrelation is suitable. The ARCH effect test is performed on the return residuals after the ARMA model fitted, and the results show that the series has heteroscedasticity. Therefore, it is necessary to continue to establish the GARCH model for the volatility of the return series to eliminate the heteroscedasticity. The above analysis makes it reasonable to establish the ARMA-GARCH model to determine the marginal distribution of the sequence for the two sets of return rate series. Many studies on risk measurement have shown that the first-order or second-order ARMA-GARCH model has better fit the characteristics of the return rate sequence. Therefore, this article uses the ARMA(1,1)-GARCH(1,1) model, respectively. The carbon price return sequence fitted with the Shanghai Composite Index return sequence. Table 3 shows the corresponding parameter estimation results. Figures 6 and 7 show the autocorrelation and partial correlation diagrams of the residual series of the carbon price return and the residual series of the Shanghai Composite Index after the model fitted. The model eliminates the autocorrelation of the series. The ARCH effect test on the residuals after the model fitted (see Table 4). The associated probability *P* values of

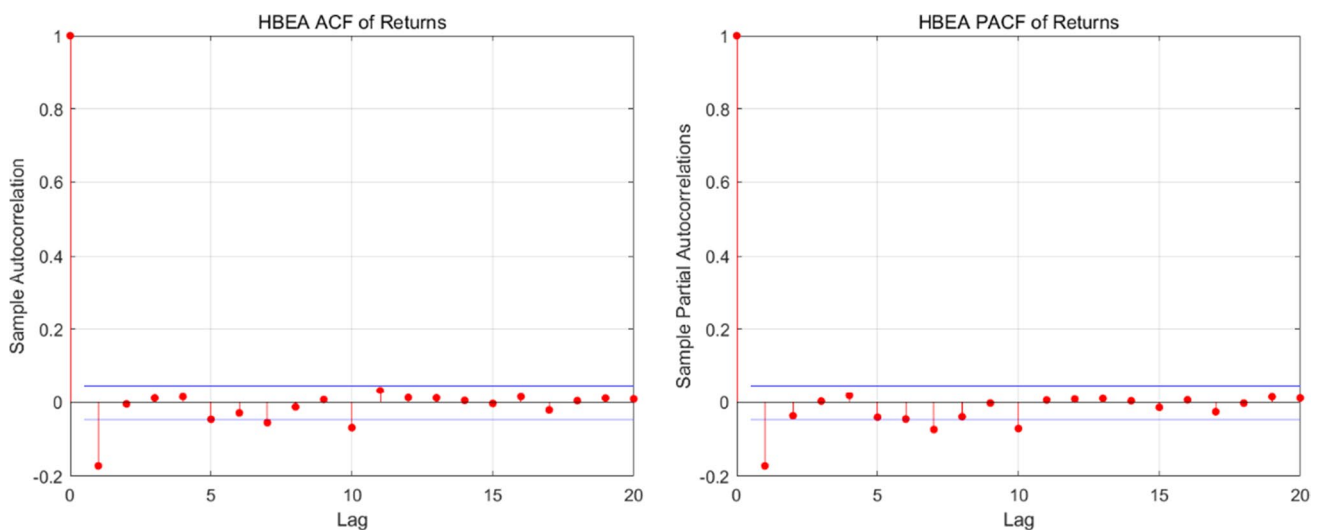
**Fig. 4** Autocorrelation graph and partial correlation graph of Hubei carbon price yield

Fig. 5 Autocorrelation graph and partial correlation graph of Shanghai Composite Index yield

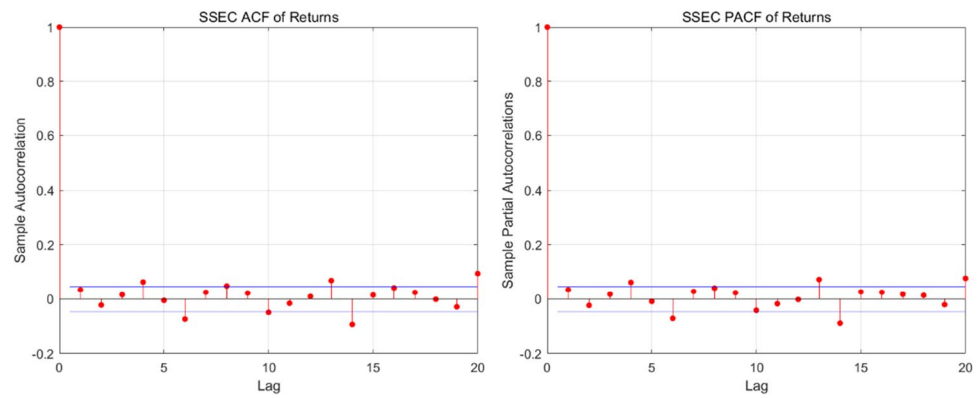


Table 3 Parameter estimation results of ARMA(1,1)-GARCH(1,1)

		AR(1)	MA(1)	C	ARCH(1)	GARCH(1)
HBEA	Coefficient	0.1395	-0.4188	0.0002	0.6373	0.2966
	Z-statistic	2.0289**	-7.1320***	23.3502***	34.1084***	16.9404***
		(0.0425)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
SSEC	Coefficient	-0.1159	0.1366	1.34E-06	0.0929	0.9058
	Z-statistic	-0.1298	0.1534	4.9639***	15.8978***	186.6356***
		(0.8968)	(0.8781)	(0.0000)	(0.0000)	(0.0000)

the *F* statistic are 0.9848 and 0.7153, respectively. Therefore, we considered that ARMA(1,1)-GARCH(1,1) eliminates the sequence’s heteroscedasticity.

Use the POT model to describe marginal distribution

For most financial time series, since the return rate series often have the characteristics of peaks and thick tails, it cannot be directly considered that the return rate series obey a normal distribution. Therefore, this paper uses the super-threshold model (POT) in extreme value theory to fit the standard residual sequence obtained after the ARMA(1,1)-GARCH(1,1) model further fitted, and a 10% score is selected. The number of digits is the upper and

lower tail thresholds. First, the tail of the sequence fitted with the generalized Pareto distribution using the maximum likelihood method. Then, the empirical cumulative distribution function is estimated using the Gaussian kernel function for the middle part of the sequence. Figure 8 shows the cumulative distribution of the overall experience of the Hubei carbon price residual sequence and the Shanghai Composite Index residual sequence after fitting. Figure 9 compares the results of the generalized Pareto fitting of the upper tail of the residuals and the accumulated experience. The residual sequence fitted by the GPD closely follows the empirical cumulative distribution function. Therefore, we considered that the GPD distribution fits the tail of the residual sequence well.

Fig. 6 Residual autocorrelation and partial correlation graphs of Hubei carbon price return rate

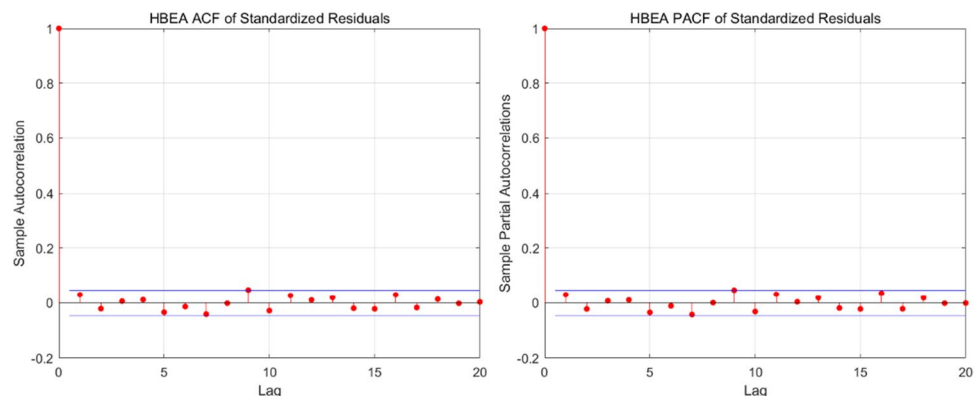


Fig. 7 Autocorrelation and partial correlation diagrams of the Shanghai Composite Index’s yield residuals

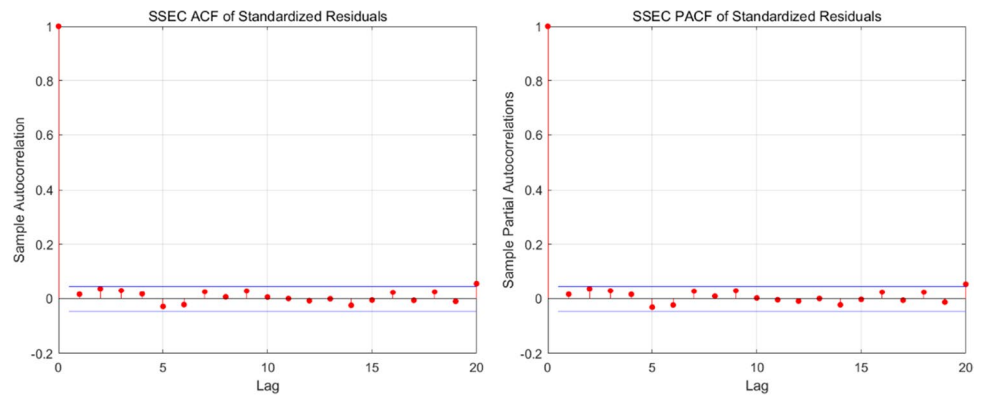


Table 4 ARCH effect test of residual series

	HBEA	SSEC
<i>F</i> -statistic	0.326496 (0.9848)	0.737653 (0.7153)
Obs* <i>R</i> -squared	3.937029 (0.9845)	8.871472 (0.7139)

Selection of copula function and parameter estimation

We have obtained the marginal distribution of the Hubei carbon price return rate and the Shanghai Composite Index return rate through the above analysis. In order to meet the distribution condition of the copula function, we transformed the fitted residual sequence into a uniform distribution in the interval 0 to 1. This paper uses the pseudo-maximum likelihood estimation method (CML) to estimate the parameters of several commonly used copula functions, respectively calculate the Euclidean distance between the estimated copula and the empirical copula, and select the copula function with the smallest distance as the optimal copula, to describe the related structure of the carbon price sequence and the Shanghai Composite Index sequence. Table 5 shows the parameter

estimation results of different copula functions and the Euclidean distance. The smallest Euclidean distance is the Frank copula function and the t-copula function. Therefore, this paper selects these two copula functions to describe the interdependence between the Hubei carbon price series and the Shanghai Composite Index time series.

Calculation of VaR based on Monte Carlo simulation

Calculation of VaR

After determining the marginal distribution of the sequence and the optimal copula function, the Monte Carlo simulation method is used to simulate the random number of the future return rate. The number of random numbers set in this article is 2000. After simulation we obtained the VaRs at different levels of confidence. Figure 10 is the cumulative distribution diagram of the coming day’s return obtained after the Monte Carlo simulation. Table 6 shows the VaR of Hubei carbon price and Shanghai Stock Exchange Index respectively and the integrated VaR calculated based on the copula function.

It can be seen from the results that at the 99% confidence level, the VaR of the Hubei carbon price return rate is – 12.1859%, and the VaR of the Shanghai Composite Index return rate is – 6.2380%. The integrated VaRs obtained

Fig. 8 Cumulative distribution of experience

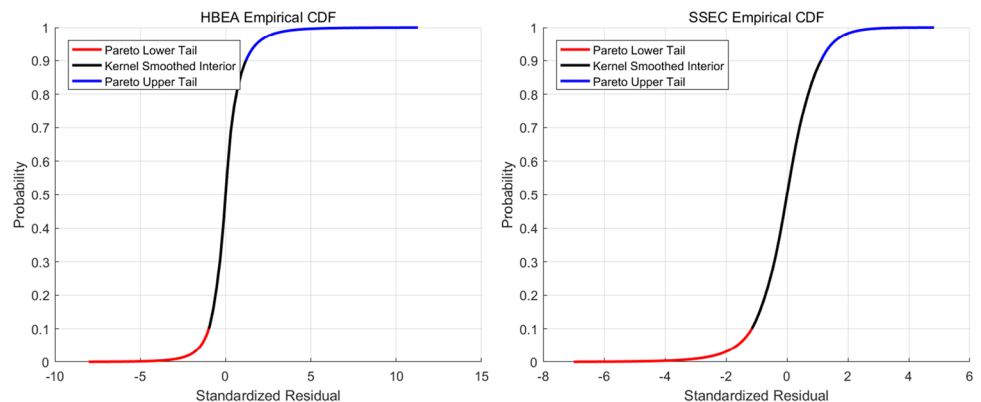
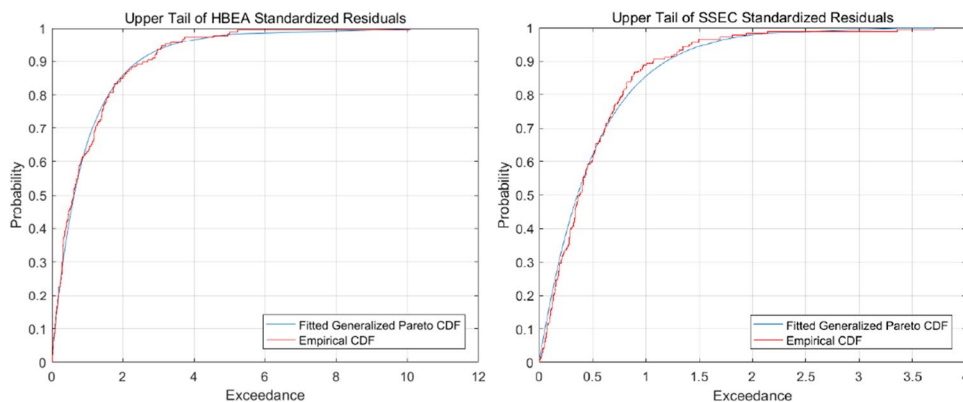


Fig. 9 GPD distribution function and empirical distribution fit test



by Frank copula and t-copula are -5.0731% and -5.0732% , respectively. Therefore, considering the interdependence, the copula-VaR are all smaller than their respective single risk value. It shows that ignoring the correlation between different risk factors will overestimate the risk to a certain extent, affecting investment decision-making and risk management. We also got the same result at other confidence levels.

Comparison of different methods and Kupiec backtesting

This paper calculates the combined rate of return of the carbon price and the Shanghai Composite Index according to the equal weight method and compares the VaR calculated by the historical simulation method, the normal simulation method, the extreme value distribution simulation method, and the Copula-VaR calculation. Figure 11 is a comparison diagram between the actual distribution and the normal distribution. It shows that the actual distribution has

prominent sharp peaks and thick tail compared with the normal distribution. The VaR obtained by normal simulation may underestimate the risk. Table 7 shows the value-at-risk calculation results of different methods. It shows that the VaR calculated by the extreme value theory is the largest, overestimating the risk compared with the other three methods. At a confidence level of 99%, the value-at-risk obtained by normal simulation is the smallest and may underestimate the risk. The value-at-risk result calculated by Copula-VaR is between of them, and the result is more likely to be reasonably estimated.

Table 8 shows the backtest results of different methods at the 99% confidence level. The results show that the statistics calculated by Frank copula-VaR and t-copula-VaR based on the number of failures are less than the critical value at the 99% confidence level and both pass backtesting. However, the backtested LR statistics obtained by the normal simulation method and the extreme value theory simulation method

Table 5 The copula parameter estimation results

	Gaussian copula	t-copula	Frank copula	Clayton copula	Gumbel copula
Parameter estimates	-0.0097	-0.0132	-0.1819	0.0124	1.0000
Euclidean distance	0.0306	0.0279	0.017	0.0534	0.0409

Fig. 10 Cumulative probability distribution of portfolio returns in the future

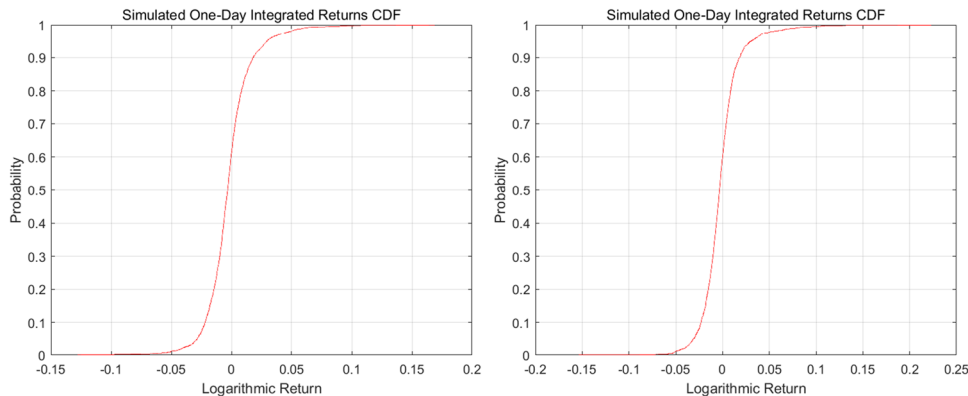


Table 6 VaR of carbon price risk and macroeconomic risk

Confidence level	Frank copula-VaR	t-copula-VaR	HBEA-VaR	SSEC-VaR	Sum-VaR
99%	-5.0731%	-5.0732%	-12.1859%	-6.2380%	-18.4240%
95%	-2.8597%	-3.1356%	-7.2929%	-3.3073%	-10.6002%
90%	-2.1862%	-2.3013%	-5.2210%	-2.0780%	-7.2990%

failed the significance test. The results show that the VaR calculated using the copula method can effectively measure the integration risk of Hubei carbon price and the Shanghai Stock Exchange Index. The VaR calculated after considering interdependence can effectively measure the carbon market integration risk. For market participants, it is conducive to make a reasonable estimate of the risk and avoid reducing investors' enthusiasm due to overestimation of the risk.

Conclusions

This article takes Hubei's carbon price as the research object and takes a macroeconomic risk as a risk factor into the risk measurement of the carbon market. We combine the conditional variance model and extreme value theory to determine the marginal distribution of the return sequence. Finally, using the optimal copula function to describe the interdependence between the two risk factors and calculating the integrated VAR of the carbon market through Monte

Carlo simulation, we can draw the following conclusions from the results.

- (1) We first discovered that the Hubei market and the Shanghai Composite Index have prominent peaks and thick tails in their respective return sequences, and their return rates do not follow a normal distribution. Moreover, the Hubei carbon price return sequence shows greater volatility, indicating that compared with the macroeconomy, the carbon market faces more significant uncertainty due to insufficient market development and insufficient policies, making the carbon market more uncertain. The carbon market may bring more significant risks to investors.
- (2) The generalized Pareto distribution in the extreme value theory is used to fit the tail of the marginal distribution of returns, and the results show that the tail fits better. Compared with the previous assumption of normal distribution, this improves the model's accuracy to a certain extent, indicating that both the Hubei carbon price return series and the Shanghai Composite Index return series have thick tails, and both tails obey the generalized Pareto support distribution. The significance of this method is that the description of the distribution of the tail of the sequence is more accurate, and the accuracy of the VaR measurement is improved. The use of traditional risk assessment methods may lead to misjudgments of China's carbon market risks.
- (3) By comparing different methods of calculating VaR, it is found that the VaR obtained by using extreme value theory and normal distribution simulation is likely to overestimate or underestimate the risk, and the copula method that considers the structure of dependence between the two risk factors Compared with other methods, the result is more reasonable. The results of the backtesting test also confirmed this conclusion.

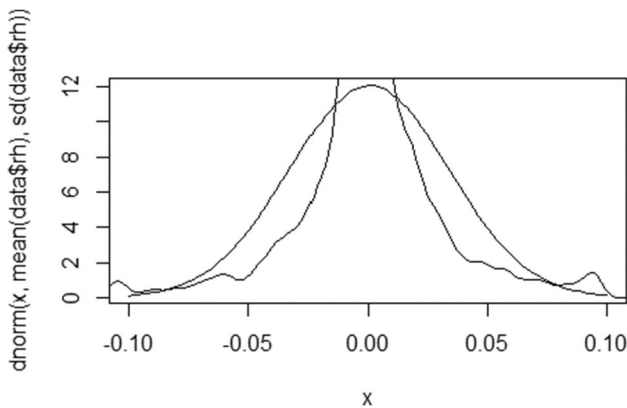


Fig. 11 Comparison of normal distribution and actual distribution

Table 7 VaR calculated by different methods

Confidence level	Frank copula-VaR	t-copula-VaR	HS	Normal	EVT
99.5%	-6.1543%	-5.3864%	-6.2838%	-4.5948%	-8.0700%
99%	-5.0731%	-5.0732%	-5.3031%	-4.1437%	-6.7001%
95%	-2.8597%	-3.1356%	-2.8892%	-2.9112%	-3.9215%
90%	-2.1862%	-2.3013%	-1.7270%	-2.2543%	-2.8632%

Table 8 Kupiec backtest test results

	Frank copula-VaR	t-copula-VaR	HS	Normal	EVT
Number of failures	26	25	18	42	6
LR statistics	2.4345	1.8253	0.0411	21.2357	12.0787
Number of failures	95	78	93	89	47
LR statistics	0.0047	3.1615	0.0204	0.3252	30.4346

Compared with the existing research results, the carbon market integration risk obtained in this paper has a lower failure rate at 99% and 95% confidence levels, indicating that its risk measurement is more effective. Therefore, copula provides an effective method for measuring the integrated risk of the carbon market. For companies participating in carbon emissions trading, measuring the integrated risk of the carbon market and the macroeconomic market is of great significance. From a regulatory perspective, it is necessary to carry out necessary risk control over China's carbon market.

This article provides a specific reference significance for the risk research of the carbon market, and effectively reduce global carbon emissions through risk management, thereby promoting the development of the global carbon market. China's carbon market has continued to develop since its establishment, and changes in macroeconomic and policy factors often bring more extreme risks to the carbon market. Therefore, the government, financial institutions, and emission control enterprises should improve their awareness of risk management from the following aspects and make adequate preparations for the occurrence of extreme risks:

Firstly, carbon allowances are the cornerstone of the entire carbon market. China's carbon price has a distinct "compliance cycle" feature: When the compliance period is approaching, the emission-controlling enterprises concentrate their transactions, which makes the carbon market temporarily active, and the carbon price increases accordingly; while during the non-compliance period, the carbon price drops sharply. Therefore, the government should adequately adjust the supply and demand of carbon allowances: in terms of carbon allowance supply, the total amount of allowances should be calculated scientifically, and the coverage of carbon emission reduction should be expanded. First, a carbon emission accounting system should be established in energy-intensive industries and gradually extended to other industries. In terms of demand, companies should be encouraged to participate in carbon quota trading actively, and a sound reward and punishment mechanism should be established to make carbon emission companies aware of the potential profitability of the

carbon market. In addition, we should learn from the EU carbon market to establish a corresponding market stability reserve mechanism: on the one hand, when the market price falls excessively, repurchase allowances and sell them when the price is too high; on the other hand, set up price fluctuations and other mechanisms to prevent the risk of abnormal fluctuations in carbon prices. These measures can prompt investors to form expectations of long-term increases in carbon prices. Stable market expectations are conducive to investors' active participation in the carbon market and are also more conducive to promoting emission reductions.

Secondly, there are few carbon financial products and services in China's carbon market, and it is not easy to support the considerable transaction scale only by the spot market. It is urgent to provide more risk control tools for the trading entities involved in carbon emissions. At present, the participation of China's financial institutions in the carbon market is mainly concentrated in the field of carbon financing services, and the financial market has formed a relatively complete system in terms of risk management. Therefore, on the one hand, financial institutions can strengthen innovation in carbon financial derivatives and carbon asset management types, design more diversified carbon financial products and services, give full play to the financial attributes of carbon market transactions, and enhance the trading vitality of carbon markets; in terms of carbon asset management, it is necessary to strengthen information disclosure in carbon asset management, make full use of emerging technologies such as big data, accelerate the construction of carbon financial information and data platforms, improve carbon financial risk prevention and control mechanisms, and incorporate carbon financial risks as an essential part of the entire risk management framework, make full use of financial institutions to balance the relationship between innovation incentives and risk management in carbon market investment.

Finally, enterprises participating in the carbon market mainly concentrate on the power generation industry. More high-carbon industries will be included in the national carbon market transaction in the future with the carbon market development. Therefore, cultivating the risk management awareness of emission control enterprises is of great significance to the stable devel-

opment of the carbon market. Firstly, standardized carbon asset management training should be provided for enterprises participating in carbon market trading, and basic information such as carbon market policies and trading rules should be publicized. On the one hand, it enhances the enthusiasm of enterprises to participate in the carbon market. On the other hand, it enhances the risk awareness of carbon-emitting enterprises in participating in the carbon market. Secondly, we must strengthen the training of carbon accounting information processing professionals. By learning the budget management and accounting management of carbon allowances, we can reduce the management cost of corporate carbon assets, help companies identify carbon market risks in advance, and do an excellent job adequate risk preparation; Thirdly, build a reasonable carbon asset management system to provide more support for other industries that will soon be included in the national carbon market, and solve the actual problems faced by enterprises, to help enterprises control and manage the risks they face when participating in the carbon market risk.

Author contribution All authors contributed to the study conception and design. Material preparation, data collection, and analysis were performed by XW and LY. The first draft of the manuscript was written by LY, and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

Declarations

Ethics approval This article does not contain any studies with human participants or animals performed by any of the authors.

Consent to participate Informed consent was obtained from all individual participants included in the study.

Consent for publication All individual participants consent to publish this article.

Competing interests The authors declare no competing interests.

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