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RESEARCH ARTICLE

Estimating CO_2 emissions using a fractional grey Bernoulli model with time power term

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Abstract

Global warming caused by CO_2 emissions will directly harm the health and quality of life of people. Accurate prediction of CO_2 emissions is highly important for policy-makers to formulate scientific and reasonable low-carbon environmental protection policies. To accurately predict the CO_2 emissions of the world's major economies, this paper proposes a new fractional grey Bernoulli model (FGBM(1,1,t^α)). First, this paper introduces the modeling mechanism and characteristics of the FGBM(1,1,t^α) model. The new model can be transformed into other grey prediction models through parameter adjustment, so the new model exhibits high adaptability. Second, this paper employs four carbon emission datasets to establish a grey prediction model, calculates model parameters with three optimization algorithms, adopts two evaluation criteria to evaluate the accuracy of the model results, and selects the optimization algorithm and model results that yield the highest model accuracy, which verifies that the FGBM(1,1,t^α) model is more feasible and effective than the other six grey models. Finally, this paper applies the FGBM(1,1,t^α) model to predict the CO_2 emissions of the USA, India, Asia Pacific, and the world over the next 5 years. The forecast results reveal that from 2020 to 2024, the CO_2 emissions of India, the Asia Pacific region, and the world will gradually rise, but that in USA will slowly decline over the next 5 years.

Keywords CO_2 emissions · Grey Bernoulli model · Grey wolf optimizer · Particle swarm optimizer · Quantum genetic algorithm · Forecasting

Nomenclature

r-AGO	order accumulative generation operator
r-IAGOr	order inverse accumulative generation operator
NGM (1,1)	Nonhomogeneous grey model
SIGM	Self-adaptive intelligence grey model
GMP	Grey polynomial model
GM (1,1,t^α)	Grey model with time power term
FGM (1,1)	Fractional grey model
NGBM (1,1,k,c)	Nonhomogeneous grey Bernoulli model with a grey action quantity of $bt+c$
GWO	Grey wolf optimization
PSO	Particle swarm optimization
QGA	Quantum genetic algorithm
MAPE	Mean absolute percentage error
MAE	Mean absolute error

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Introduction

Global warming caused by the greenhouse effect is one of the factors that seriously threatens human survival and development, and the increase in CO_2 emissions is considered the main cause of the greenhouse effect. According to the Global Climate 2015–2019 report released by the World Meteorological Organization, the growth rate of CO_2 in the atmosphere from 2015 to 2019 was 18% higher than that during the previous 5 years, and the average temperature was 0.2 °C higher than that during the previous 5 years. Hence, this period encompasses the hottest 5 years on record. British Petroleum (BP) World Energy Statistics (2020) indicated that in 2019, the global CO_2 emissions reached 34,169 million tons. Among the various countries, the USA and India rank second and third worldwide, respectively, in terms of their CO_2 emissions, and the total CO_2 emissions of these two countries account for 21.8% of the world emissions. The Asia Pacific region accounts for 50.5% of the global CO_2 emissions. Upon entering the twenty-first century, the growth of global CO_2 emissions is soaring. Thus, from 2009 to 2019, the CO_2 emissions in the USA basically remained at approximately 5000 MT, revealing a trend of continuous fluctuation. The CO_2 emissions in India grew the fastest, by 55.4%. The growth rate of the CO_2 emissions in the Asia Pacific region reached 30.4%, which is also one of the fastest growing economies in the world. From the perspective of global CO_2 emissions, although the growth rate is not high, with an average annual growth rate of 1.1%, CO_2 emissions remain on the rise. To achieve the target of the Paris Agreement, more than 20 countries put forward carbon neutrality goals at the 2020 Climate Ambition Summit, and more than 40 countries established new commitments to independently improve national contributions. Therefore, accurate prediction of future CO_2 emission data of the USA, India, Asia Pacific region, and the world can help policy-makers formulate more scientific and reasonable environmental policies to truly achieve the goal of carbon emission reduction.

The existing models for CO_2 emission forecast can be classified into three categories. The first category includes nonlinear intelligent models, such as the least squares support vector machine (Sun and Liu 2016), extreme learning machine (Sun and Sun 2017), generalized regression neural network (Heydari et al. 2019); and improved chicken swarm optimization (ICSO-SVM) model using the ICSO algorithm to optimize support vector machine parameters (Wen and Cao 2020). The second category involves the statistical analysis model, which studies the quantitative relationship between CO_2 emissions and influencing factors and applies the relationship equation to predict CO_2 emissions. These approaches include trend analysis (Köne and Büke 2010), logistic equations (Meng and Niu 2011), improved Gaussian process regression (Fang et al. 2018), panel quantile regression (Zhu et al. 2018), log-average decomposition index (Xu et al. 2019), and comprehensive methods combining multiple regression analysis, input–output techniques, and structural decomposition analysis (Xia et al. 2019). The last category is the grey prediction model, which was first proposed by Deng (1982). The grey prediction model was established founded on a small amount of incomplete information to describe the development trend of objects more accurately. Compared to machine learning–based and statistical prediction methods founded on large data samples, the grey prediction model can realize the simulation and prediction of small data samples. The classical grey model GM(1,1) and its extended grey prediction model are widely used in the fields of energy, environment, and social management (Tsai 2016; Liu et al. 2020; Liu et al. 2021a; Wang et al. 2020b; Liu et al. 2021b).

There are two main kinds of grey single-variable forecasting models: the first-order grey differential model GM(1,1) and the grey Bernoulli model GBM(1,1). Deng first proposed the GBM(1,1) model in 1985; namely, a power exponent was introduced into the differential Bernoulli equation. When the exponent is equal to 2, the model is also referred to as the grey Verhulst model. Chen et al. (2008) proposed the nonlinear grey Bernoulli model NGBM(1,1) for the first time. Compared to the general GM(1,1) model, the NGBM(1,1) model can better reflect the nonlinear growth trend of data series. Chen et al. (2010) proposed a new grey prediction model NGBM(1,1), by optimizing the background value and power index simultaneously. Subsequently, many scholars improved the NGBM(1,1) model from different perspectives; e.g., Pao et al. (2012) proposed an iterative method to optimize the parameters of the NGBM(1,1) model. Wang (2017) optimized the background coefficient and initial conditions and considered the weighted method in analysis. Guo et al. (2016) proposed a new model by combining the self-memory principle of a dynamic system with the NGBM(1,1) model. Ma et al. (2019) constructed the NGBM(1, n) model by combining the GMC(1, n) model and Bernoulli equation. Wu et al. (2019b) extended the first-order accumulation operation and established the FANGBM(1,1) model based on fractional accumulation. Liu and Xie (2019) proposed the establishment of the WBGM(1,1) model by combining the fitting performance of the NGBM(1,1) model with the Weibull cumulative distribution. Şahin (2020) proposed the OFANGBM(1,1) model based on the integral mean value theorem. Wu et al. (2020a) established the NGBM(1,1, k, c) model by combining the NGM(1,1, k, c) model with the NGBM(1,1) model. Jiang and Wu (2021) constructed a nonlinear grey Bernoulli model based on the fractional order reverse accumulation, namely, the FANGBM(1,1) model. At present, a simpler and more convenient conformable fractional cumulative grey model (CFGM) was proposed (Ma et al., 2020). On this basis, Xie et al. (2020) proposed the conformable fractional grey model in opposite direction (CFGOM). Zheng et al. (2021) proposed the conformable fractional nonhomogeneous Bernoulli model (CFNHGBM (1,1, K)).

The grey prediction model is widely implemented in CO_2 emission prediction. Lin et al. (2011) applied the grey model to predict CO_2 emissions in Taiwan. Pao et al. (2012) used the NGBM model to predict the CO_2 emissions and real GDP growth of China. Lotfall et al. (2013) forecast CO_2 emissions based on a grey model and an autoregressive integrated moving average (ARIMA) model. The prediction accuracy of these two methods was compared according to the root mean square error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE). Gao et al. (2015) established a new discrete fractional order cumulation model, i.e., FAGM(1,1, D), to predict CO_2 emissions. Hamzacebi and Karakurt (2015) used the grey prediction model to predict the energy-related CO_2 emissions in Turkey. Yuan et al. (2017) established a linear programming model reflecting the relationship between the economic development and CO_2 emissions in China and employed the GM(1,1) model to predict the parameters of the planning model. Wang and Ye (2017) developed a nonlinear multivariable grey model to discuss the relationship between economic growth and CO_2 emissions. Xu et al. (2019) combined an adaptive grey model with the buffer rolling method to predict the greenhouse gas emissions in China from 2017 to 2025. Wang and Li (2019) adopted the nonequidistant grey Verhulst model to analyze the relationship between CO_2 emissions and economic growth. Wu et al. (2020b) implemented a conformable fractional nonhomogeneous grey model to predict the CO_2 emissions of BRICS countries. Chiu et al. (2020) proposed a multivariate grey prediction model using neural networks based on feature selection and residual correction to predict China's carbon emissions. Based on the grey Verhulst model, Duan and Luo (2020) introduced an extrapolation method to optimize the background value and predicted the CO_2 emissions for three coal resources in China. Wang et al. (2020a) established the metabolic nonlinear grey model (MNGM)-ARIMA method, established a new MNGM-BPNN combination model based on the MNGM model and back-propagation (BP) neural network and analyzed the CO_2 emissions in China, the USA, and India with these two methods. Zhou et al. (2021) proposed a method to process original sequence data by means of an average weakening buffer operator based on the grey rolling mechanism of the new information priority principle and predicted the trend of CO_2 emissions in China. Xie et al. (2021) established a new continuous conformable fractional nonlinear grey Bernoulli model to forecast CO_2 emissions from fuel combustion in China.

Scholars have greatly promoted the optimization and application of grey models. However, although the existing studies have optimized the grey model considering the structure or parameters, each optimization method only improves the model performance to a certain extent, and the accuracy remains insufficient. In addition, most models only apply one optimization algorithm to determine the optimal parameters, and research on the application of multiple optimization algorithms is rare. Therefore, to better predict carbon emissions, based on the optimization of existing models, namely, NGBM(1,1) and FAGM(1,1,t^α) (Wu et al. 2019a), this paper proposes a new grey prediction model FGBM(1,1,t^α) and considers a variety of optimization algorithms to determine the optimal structural parameters of the model. The main contributions of this paper are as follows:

(1). Based on the advantages of the grey Bernoulli prediction model NGBM(1,1) and FAGM(1,1, t^α), a new model, i.e., FGBM(1,1,t^α), is proposed. The new model can be transformed into other grey prediction models by changing its parameters.
(2). In the FAGM(1,1,t^α) model, application of the trapezoidal integral method to obtain an approximate solution yields errors, but this paper provides an analytical solution of the time response function in the FGBM(1,1,t^α) model, and the result is more accurate.
(3). Grey wolf optimization (GWO), particle swarm optimization (PSO), and quantum genetic algorithm (QGA) are applied to solve the parameters of the FGBM(1,1,t^α) model.
(4). The FGBM(1,1,t^α) model is used to predict the CO_2 emissions of four countries and regions over the next 5 years.

This paper is organized as follows: the second section first introduces the basic theory of the NGBM(1,1) and FAGM(1,1,t^α) models. Second, a new grey prediction model FGBM(1,1,t^α) is constructed, which mainly includes the modeling mechanism, model characteristics, and solution method. Then, three common optimization algorithms (GWO, PSO, and QGA) are employed to solve the parameters of the FGBM(1,1,t^α) model, and two widely adopted error measures are introduced. The third section considers the CO_2 emission data of four economies to verify the feasibility and effectiveness of the FGBM(1,1,t^α) model. The fourth section applies the FGBM(1,1,t^α) model to predict the CO_2 emissions of the USA, India, Asia Pacific region and the world over the next five years. The fifth section contains the conclusion of this paper.

Prerequisite knowledge

Grey Bernoulli model NGBM(1,1) and FAGM(1,1,t^α)

This section mainly introduces the grey Bernoulli model NGBM(1,1) and the FAGM(1,1,t^α) model as follows:

Definition 1: Given a nonnegative sequence $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\}$, $X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\}$ is referred to as the first-order generating sequence of $X^{(0)}$, where

$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, \cdots n$. Based on Chen et al. (2008), it can be found that the expression of NGBM(1,1) is:

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b(x^{(1)}(t))^{\gamma} \tag{1}$$

The above is a nonlinear equation, and the exponent exponent $\gamma \neq 1$.

The parameters a, b of the NGBM(1,1) model are calculated as follows:

$$(a, b)^T = \left(B^T B\right)^{-1} B^T Y \tag{2}$$

$$B = \begin{pmatrix} -z^{(1)}(2) & \left(z^{(1)}(2)\right)^{\gamma} \\ -z^{(1)}(3) & \left(z^{(1)}(3)\right)^{\gamma} \\ \vdots & \vdots \\ -z^{(1)}(n) & \left(z^{(1)}(n)\right)^{\gamma} \end{pmatrix},\ Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix}$$

where $Z^{(1)} = \left(z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(n)\right)$, $z^{(1)}(k) = 0.5x^{(1)}(k-1) + 0.5x^{(1)}(k), k = 2, 3, \ldots, n$, and n is the sample number of the modeling sequence.

The time response function of the NGBM(1,1) model is as follows:

$$\hat{x}^{(1)}(k) = \{[(x^{(0)}(1))^{1-\gamma} - \frac{b}{a}] \cdot e^{-a(1-\gamma)(k-1)} + \frac{b}{a}\}^{\frac{1}{1-\gamma}}, k = 2, 3, \cdots n \tag{3}$$

The predicted values of the model are as follows:

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1), k = 2, 3, \cdots n \tag{4}$$

Definition 2: Given a nonnegative sequence $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\}$, $r \in R^{+}$, and its r-th order accumulation sequence is $X^{(r)} = \{x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n)\}$ (Wu et al, (2013). Denoted as matrix A^r, the r-AGO matrix that satisfies $X^{(r)} = \mathrm{A}^r X^{(0)}$ is:

$$\mathrm{A}^r = \begin{pmatrix} \begin{bmatrix} r \\ 0 \end{bmatrix} & 0 & 0 & \cdots & 0 \\ \begin{bmatrix} r \\ 1 \end{bmatrix} & \begin{bmatrix} r \\ 0 \end{bmatrix} & 0 & \cdots & 0 \\ \begin{bmatrix} r \\ 2 \end{bmatrix} & \begin{bmatrix} r \\ 1 \end{bmatrix} & \begin{bmatrix} r \\ 0 \end{bmatrix} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \begin{bmatrix} r \\ n-1 \end{bmatrix} & \begin{bmatrix} r \\ n-2 \end{bmatrix} & \begin{bmatrix} r \\ n-3 \end{bmatrix} & \cdots & \begin{bmatrix} r \\ 0 \end{bmatrix} \end{pmatrix}_{n \times n} \tag{5}$$

with $\begin{bmatrix} r \\ i \end{bmatrix} = \frac{r(r+1)\cdots(r+i-1)}{i!} = \begin{pmatrix} r+i-1 \\ i \end{pmatrix} = \frac{(r+i-1)!}{i!(r-1)!}, \begin{bmatrix} 0 \\ i \end{bmatrix} = 0, \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 1.$

Denoted as matrix D^r, the r-IAGO matrix that satisfies $X^{(0)} = D^r X^{(r)}$ is:

$$\mathrm{D}^r = \begin{pmatrix} \begin{bmatrix} -r \\ 0 \end{bmatrix} & 0 & 0 & \cdots & 0 \\ \begin{bmatrix} -r \\ 1 \end{bmatrix} & \begin{bmatrix} -r \\ 0 \end{bmatrix} & 0 & \cdots & 0 \\ \begin{bmatrix} -r \\ 2 \end{bmatrix} & \begin{bmatrix} -r \\ 1 \end{bmatrix} & \begin{bmatrix} -r \\ 0 \end{bmatrix} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \begin{bmatrix} -r \\ n-1 \end{bmatrix} & \begin{bmatrix} -r \\ n-2 \end{bmatrix} & \begin{bmatrix} -r \\ n-3 \end{bmatrix} & \cdots & \begin{bmatrix} -r \\ 0 \end{bmatrix} \end{pmatrix}_{n \times n} \tag{6}$$

with: $\begin{bmatrix} -r \\ i \end{bmatrix} = \frac{-r(-r+1)\cdots(-r+i-1)}{i!} = (-1)^i \frac{r(r-1)\cdots(r-i+1)}{i!}, \begin{bmatrix} -r \\ i \end{bmatrix} = 0, i > r.$

Matrices A^r and D^r satisfy $A^r D^r = I_n$.

The whitening differential equation of the FAGM(1,1,t^{α}) model is:

$$\frac{dx^{(r)}(t)}{dt} + ax^{(r)}(t) = bt^{\alpha} + c, r > 0, \alpha \geq 0 \tag{7}$$

Applying integration to Eq. (7) in the time interval of [k − 1, k], we obtain:

$$\int_{k-1}^{k} \frac{dx^{(r)}(t)}{dt} dt + a \int_{k-1}^{k} x^{(r)}(t) dt = b \int_{k-1}^{k} t^{\alpha} dt + c \int_{k-1}^{k} dt \tag{8}$$

Based on the trapezoid equation and $z^{(r)}(k) = 0.5x^{(r)}(k-1) + 0.5x^{(r)}(k), k = 2, 3, \ldots, n,$

Equation (8) is written as:

$$x^{(r)}(k) - x^{(r)}(k-1) + az^{(r)}(k) = b\frac{k^{1+\alpha} - (k-1)^{1+\alpha}}{1+\alpha} + c \tag{9}$$

Based on Eq. (9), unknown parameters a, b, c of FAGM(1,1,t^{α}) satisfy:

$$(\hat{a}, \hat{b}, \hat{c})^T = \left(B^T B\right)^{-1} B^T Y \tag{10}$$

$$B = \begin{pmatrix} -z^{(r)}(2) & \frac{2^{1+\alpha}-1}{1+\alpha} & 1 \\ -z^{(r)}(3) & \frac{3^{1+\alpha}-2^{1+\alpha}}{1+\alpha} & 1 \\ \vdots & \vdots & \vdots \\ -z^{(r)}(n) & \frac{n^{1+\alpha}-(n-1)^{1+\alpha}}{1+\alpha} & 1 \end{pmatrix},\quad Y = \begin{pmatrix} x^{(r)}(2) - x^{(r)}(1) \\ x^{(r)}(3) - x^{(r)}(2) \\ \vdots \\ x^{(r)}(n) - x^{(r)}(n-1) \end{pmatrix} \tag{11}$$

The time response function of the FAGM(1,1,t^{α}) model is:

$$\begin{aligned} \hat{x}^{(r)}(t) &= e^{-\hat{a}t}\left\{x^{(0)}(1)e^{\hat{a}} + \hat{b}\int_1^t s^{\alpha} e^{\hat{a}s} ds + \tfrac{\hat{c}}{\hat{a}}\left(e^{\hat{a}t} - e^{\hat{a}}\right)\right\} \\ &= \left(x^{(0)}(1) - \tfrac{\hat{c}}{\hat{a}}\right)e^{-\hat{a}(t-1)} + \hat{b}e^{-\hat{a}(t-1)}\int_1^t s^{\alpha} e^{\hat{a}(s-1)} ds + \tfrac{\hat{c}}{\hat{a}} \\ &= \left(x^{(0)}(1) - \tfrac{\hat{c}}{\hat{a}}\right)e^{-\hat{a}(t-1)} + \tfrac{\hat{b}}{2}e^{-\hat{a}(t-1)}\sum_{i=1}^{t-1}\left(i^{\alpha} e^{\hat{a}(i-1)} + (i+1)^{\alpha} e^{\hat{a}i}\right) + \tfrac{\hat{c}}{\hat{a}} \end{aligned} \tag{12}$$

Moreover, the restored value of $\hat{x}^{(0)}(k), k = 2, 3, \ldots, n$ is given as: $\hat{X}^{(0)} = D^r \hat{X}^{(r)}$.

Description of the FGBM(1,1,t^{α})

The expression of the FGBM(1,1,t^{α}) model is as follows:

$$\frac{dx^{(r)}(t)}{dt} + ax^{(r)}(t) = (bt^{\alpha} + c)(x^{(r)}(t))^{\xi} \tag{13}$$

where $0 \le r \le 1, 0 \le \alpha \le 4, 0 \le \xi \le 3, \xi \ne 1$.

Theorem 1. The time response function of FGBM (1,1,t^{α}) is derived as:

$$x^{(r)}(k) = \{([x^{(0)}(1)]^{1-\xi} - \frac{c'}{a'})e^{-a'(k-1)} + \frac{c'}{a'} + b'e^{-a'(k-1)} \int_1^k \tau^{\alpha} e^{a'(\tau-1)} d\tau\}^{\frac{1}{1-\xi}}, k = 1, 2, 3 \cdots, n \tag{14}$$

The reduction value of $\hat{x}^{(r)}(k)$ is $\hat{x}^{(0)}(k)$,

$$\hat{x}^{(0)}(k) = D^r \hat{x}^{(r)}(k), k = 1, 2, 3, \cdots, n \tag{15}$$

Proof, Multiplying both sides of Eq. (13) by $x^{(r)}(t)^{-\xi}$. Let $y^{(r)} = [x^{(r)}(t)]^{1-\xi}$, one can obtain:

$$\frac{dy^{(r)}(t)}{dt} + a(1-\xi)y^{(r)}(t) = (bt^{\alpha} + c)(1-\xi) \tag{16}$$

Let the left side of Eq. (13) be 0, $a(1-\xi) = a', b(1-\xi) = b', c(1-\xi) = c'$,we can get:

$$\frac{dy^{(r)}(t)}{dt} + a(1-\xi)y^{(r)}(t) = \frac{dy^{(r)}(t)}{dt} + a'y^{(r)}(t) = 0 \tag{17}$$

The general solution formula of the equation is as follows:

$$y^{(r)}(t) = Ce^{-a't} \tag{18}$$

$$\frac{dC(t)}{dt} = e^{a't}(b't^{\alpha} + c') \tag{19}$$

To perform the definite integral on interval $[1, t]$, we know that:

$$C(t) = \int (b't^{\alpha} + c)e^{a't}dt = C(1) + \int_1^t (b'\tau^{\alpha} + c)e^{a'\tau}d\tau = C(1) + b'\int_1^t \tau^{\alpha} e^{a'\tau}d\tau + \frac{c'}{a'}(e^{a't} - e^{a'}) \tag{20}$$

$$C(1) = y^{(r)}(1)e^{a'} = y^{(0)}(1)e^{a'} \tag{21}$$

$$\begin{aligned} y^{(r)}(t) &= e^{-a't}[y^{(0)}(1)e^{a} + b'\int_1^t \tau^{\alpha} e^{a'\tau}d\tau + \frac{c'}{a'}(e^{a't} - e^{a'})] \\ &= (y^{(0)}(1) - \frac{c'}{a'})e^{-a'(t-1)} + \frac{c'}{a'} + b'e^{-a'(t-1)} \textstyle\int_1^t \tau^{\alpha} e^{a'(\tau-1)}d\tau \end{aligned} \tag{22}$$

And because $y^{(r)}(t) = [x^{(r)}(t)]^{1-\xi}$, $y^{(r)}(1) = y^{(0)}(1) = [x^{(r)}(1)]^{1-\xi} = [x^{(0)}(1)]^{1-\xi}$, the time response function can be derived from Eq. (22) by

$$x^{(r)}(k) = [(x^{(0)}(1)^{1-\xi} - \frac{c'}{a'})e^{-a'(k-1)} + \frac{c'}{a'} + b'e^{-a'(k-1)} \int_1^k \tau^{\alpha} e^{a'(\tau-1)}d\tau]^{\frac{1}{1-\xi}} \tag{23}$$

Although α is not an integer, $\int_1^k \tau^{\alpha} e^{a'(\tau-1)}d\tau$ can be integral by numerical integration method to obtain the real number.

The predicted values $\hat{x}^{(0)}(k)$ can be obtained by:

$$\hat{x}^{(0)}(k) = D^r \hat{x}^{(r)}(k), k = 1, 2, 3, \cdots, n \tag{24}$$

Parameters estimation of the FGBM(1,1,t^{α})

By integrating on $[k-1, k]$ both side of Eq. (16) at the same time, the following conclusion can be obtained:

$$y^{(r)}(k) - y^{(r)}(k-1) + a'\int_{k-1}^{k} y^{(r)}(t)dt = \frac{k^{\alpha+1} - (k-1)^{\alpha+1}}{\alpha+1}b' + c' \tag{25}$$

According to the integral median theorem, we can get:

$$\int_{k-1}^{k} y^{(r)}(t)dt = \lambda y^{(r)}(k) + (1-\lambda)y^{(r)}(k-1) \tag{26}$$

where $0 \le \lambda \le 1$.

The results are as follows:

$$y^{(r)}(k) - y^{(r)}(k-1) + a'[\lambda y^{(r)}(k) + (1-\lambda)y^{(r)}(k-1)] = \frac{k^{\alpha+1} - (k-1)^{\alpha+1}}{\alpha+1}b' + c' \tag{27}$$

According to the commonly used method to solve the parameters of grey prediction model, the least square criterion of FGBM (1,1,t^{α}) model can be described as the following unconstrained optimization problems:

$$\min_{a',b',c'} \sum_{k=2}^{n} \left\{ y^{(r)}(k) - y^{(r)}(k-1) + a'[\lambda y^{(r)}(k) + (1-\lambda)y^{(r)}(k-1)] - \frac{k^{\alpha+1}-(k-1)^{\alpha+1}}{\alpha+1} b' - c' \right\}^2 \tag{28}$$

The solution of this optimization problem is:

$$[\hat{a}'\ \hat{b}'\ \hat{c}']^T = \left(B^T B\right)^{-1} B^T Y \tag{29}$$

where

$$B = \begin{pmatrix} -[\lambda y^{(r)}(2) + (1-\lambda)y^{(r)}(1)] & \frac{2^{\alpha+1}-1^{\alpha+1}}{1+\alpha} & 1 \\ -[\lambda y^{(r)}(3) + (1-\lambda)y^{(r)}(2)] & \frac{3^{\alpha+1}-2^{\alpha+1}}{1+\alpha} & 1 \\ \vdots & \vdots & \vdots \\ -[\lambda y^{(r)}(n) + (1-\lambda)y^{(r)}(n-1)] & \frac{n^{\alpha+1}-(n-1)^{\alpha+1}}{1+\alpha} & 1 \end{pmatrix}, Y = \begin{bmatrix} y^{(r)}(2) - y^{(r)}(1) \\ y^{(r)}(3) - y^{(r)}(2) \\ \vdots \\ y^{(r)}(n) - y^{(r)}(n-1) \end{bmatrix} \tag{30}$$

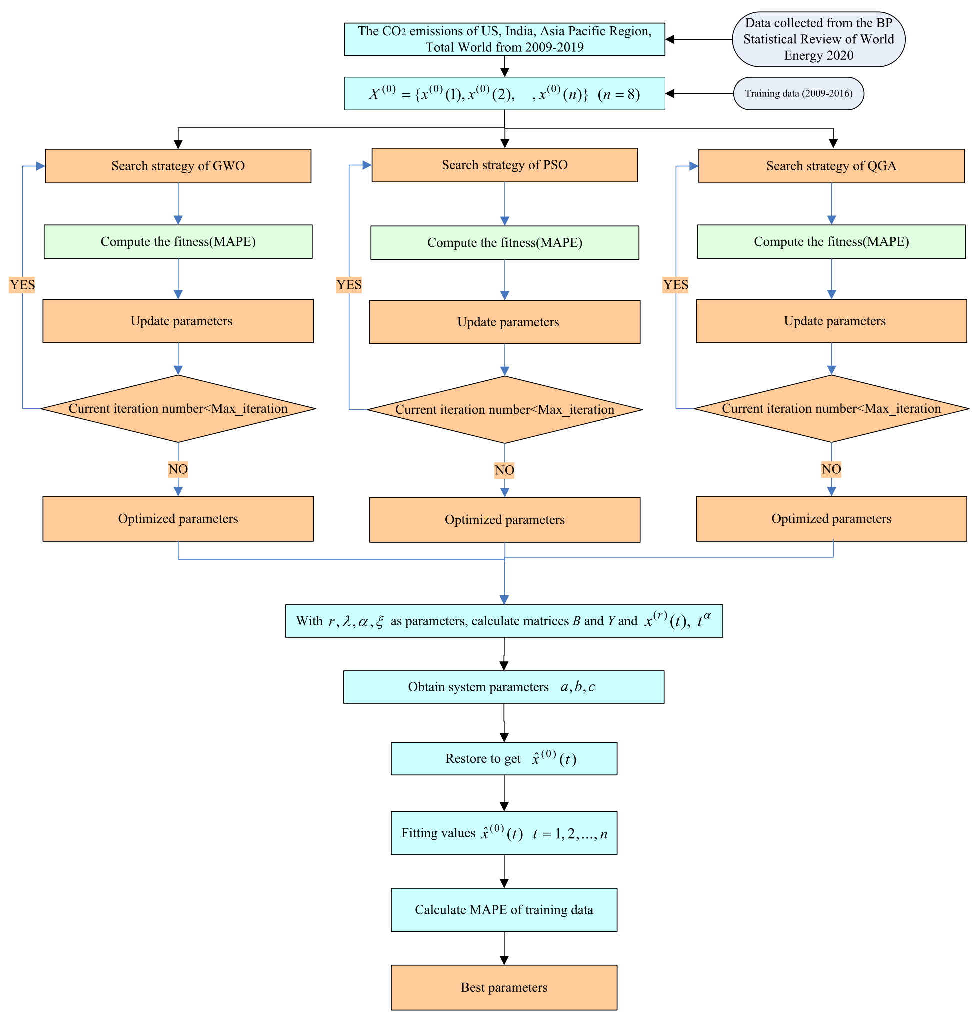
In order to facilitate readers to understand the solution process of the FGBM (1,1,t^α) model, Fig. 1 shows the algorithm implementation process of FGBM (1,1,t^α) model.

Properties of the FGBM(1,1,t^α) model

The expression of the FGBM(1,1,t^α) model indicates that the FGBM(1,1,t^α) model can be transformed into other grey prediction models when different values are assigned to parameters ξ and r, α, such as GM(1,1) (Deng 1982), FGM(1,1) (Wu et al. 2013), FAGM(1,1,t^α) (Wu et al. 2019a), NGM(1,1,k,c) (Cui et al. 2009), GM(1,1,t^2) (Qian et al. 2012), GM(1,1,t^α) (Qian et al. 2012), NGBM(1,1) (Chen et al. 2008), FANGBM(1,1) (Wu et al. 2019a), and NGBM(1,1,k,c) (Wu et al. 2020a). The relationships between FGBM (1,1,t^α) with other grey models are shown in the Fig. 2.

To minimize the model error, we must determine the optimal values of parameters r, λ, α, ξ and adopt the MAPE as the main criterion. The optimization problems when solving the optimal parameters are as follows:

Fig. 1 Algorithm implementation process of FGBM (1,1,)



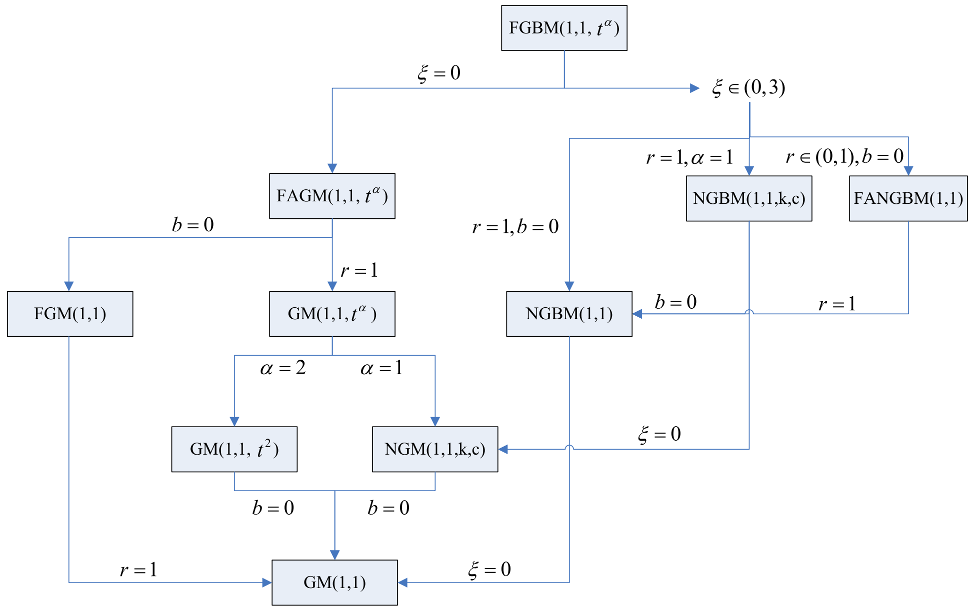


Fig. 2 Relationships between FGBM (1,1,) with other grey models

$$\min_{r,\lambda,\alpha,\xi} f(r,\lambda,\alpha,\xi) = \frac{1}{n-1}\sum_{t=2}^{n}\left|\frac{x^{(0)}(t)-\hat{x}^{(0)}(t)}{x^{(0)}(t)}\right| \times 100\% \quad (31)$$

$$s.t.\begin{cases} 0 \le r \le 1, 0 \le \lambda \le 1, 0 \le \alpha \le 4, 0 \le \xi \le 3, \xi \ne 1 \\ [\hat{a}'\ \hat{b}'\ \hat{c}']^T = \left(B^T B\right)^{-1} B^T Y \\ B = \begin{pmatrix} -[\lambda y^{(r)}(2) + (1-\lambda) y^{(r)}(1)] & \frac{2^{\alpha+1}-1^{\alpha+1}}{1+\alpha} & 1 \\ -[\lambda y^{(r)}(3) + (1-\lambda) y^{(r)}(2)] & \frac{3^{\alpha+1}-2^{\alpha+1}}{1+\alpha} & 1 \\ \vdots & \vdots & \vdots \\ -[\lambda y^{(r)}(n) + (1-\lambda) y^{(r)}(n-1)] & \frac{n^{\alpha+1}-(n-1)^{\alpha+1}}{1+\alpha} & 1 \end{pmatrix}, Y = \begin{bmatrix} y^{(r)}(2) - y^{(r)}(1) \\ y^{(r)}(3) - y^{(r)}(2) \\ \vdots \\ y^{(r)}(n) - y^{(r)}(n-1) \end{bmatrix} \\ \hat{x}^{(r)}(t) = [(x^{(0)}(1)^{1-\xi} - \frac{\hat{c}'}{\hat{a}'})e^{-\hat{a}'(t-1)} + \frac{\hat{c}'}{\hat{a}'} + \hat{b}' e^{-\hat{a}'(t-1)} \int_1^t \tau^{\alpha} e^{\hat{a}'(\tau-1)} d\tau]^{\frac{1}{1-\xi}} \\ \hat{x}^{(0)}(t) = \left(\hat{x}^{(r)}(t)\right)^{(-r)}, t = 1, 2, 3, \cdots, n \end{cases} \quad (32)$$

The above optimization problems are essentially nonlinear programming problems with equality constraints, which can be solved with intelligent optimization algorithms or heuristic algorithms. In this paper, GWO, PSO, and QGA are adopted to solve the parameters. The reason for choosing these three algorithms is: GWO algorithm has the characteristics of simple structure, has few parameters to be adjusted, and is easy to implement. Among them, there are adaptive convergence factors and information feedback mechanism, which can achieve a balance between local optimization and global search. Therefore, GWO algorithm has good performance in solving the problem accuracy and convergence speed. The advantage of PSO algorithm is that it has memory ability. In the implementation of intelligent search, it can combine the individual and global optimal location to realize location. It has a very fast speed of approaching the optimal solution and can effectively optimize the parameters of the system. The population coding method used by QGA greatly enriches the diversity of the population, at the same time, the quantum revolving gate is used to update the population and evolve based on the information of the current optimal individual. It has the characteristics of strong adaptability, fast convergence and suitable for global search. The parameters of the algorithm are shown in Table 1.

Error metric

This section considers two error metrics widely used for prediction models to test the effectiveness and applicability of the grey prediction model, as indicated in Table 2.

Validation of FGBM(1,1,t^{α})

In this section, four cases are presented to check the accuracy of the FGBM(1,1,t^{α}), and the forecasting results are compared to those obtained with other grey prediction models, such as NGM(1,1), SIGM, GMP, FGM, NGBM(1,1,k,c), and FAGM(1,1,t^{α}). In Sections 3.1 to 3.4, the raw data in Table 1 and the above seven grey models are used to simulate and

Table 1 The parameters setting for the three algorithms

	Initial population/particle number	Max iterations	Other parameters
GWO	30	300	Default
PSO	30	300	Vmax = 0.6; wMax = 0.9; wMin = 0.2; c1 = 2; c2 = 2;
QGA	30	300	Default

Table 2 Error metrics of prediction model

Name	Abbreviation	Formulation
Mean absolute percentage error	MAPE	$\frac{1}{n-1}\sum_{k=2}^{n}\left\vert\frac{x^{(0)}(k)-\hat{x}^{(0)}(k)}{x^{(0)}(k)}\right\vert\times 100$
Mean absolute error	MAE	$\frac{1}{n-1}\sum_{k=2}^{n}\vert\hat{x}^{(0)}(k)-x^{(0)}(k)\vert$

predict the CO_2 emissions of the USA, India, Asia Pacific region, and the world. The original time series data from 2009 to 2016 are used to build FGBM(1,1,t^{α}), NGM(1,1), SIGM, GMP, FGM, NGBM(1,1,k,c), and FAGM(1,1,t^{α}), and the original time series data from 2017 to 2019 are used to check the accuracy of the above grey prediction models.

CO_2 emissions of the USA

According to Statistics Review of World Energy 2020, the USA produces the highest CO_2 emissions worldwide. In 2019, the CO_2 emissions of the USA reached 4964.69 million tons, accounting for 14.5% of the total global CO_2 emissions, ranking second in the world. The task of emission reduction in the USA is far from realization. In this regard, the USA has put forward the Zero Carbon Action Plan (ZCAP), which deeply considers stabilization of domestic employment, economic innovation, and coordinated development of the environment in the postepidemic era. Therefore, this section adopts the USA as an example to test the accuracy of the FGBM(1,1,t^{α}) model. The parameters and MAPE values of FGBM(1,1,t^{α}) calculated with the three optimization algorithms are listed in Table 3. Figure 3 shows the number of iterations of the three optimization algorithms and the relationship between MAPE and the parameters. The MAPE and test_MAPE values for QGA are 1.1320% and 1.5805%, respectively, which are the smallest among those for the three optimization algorithms, and the performance is the best. Therefore, the QGA results are selected here. The fitting and prediction results for each model are shown in Fig. 4 and Table 4, and the error measurement results for each model are shown in Fig. 5 and Table 5. The prediction value of the FGBM(1,1,t^{α}) model is the closest to the actual value. Based on the fitting values, the two error measurement indexes of the FGBM(1,1,t^{α}) model are the best. This also indicates that the FGBM(1,1,t^{α}) model is more accurate than the other grey models in predicting the CO_2 emissions in the USA.

CO_2 emissions of India

According to Statistics Review of World Energy 2020, in 2019, the CO_2 emissions in India reached 2480.35 million tons, accounting for 7.3% of the global CO_2 emissions, ranking third in the world. From 2010 to 2019, the CO_2 emissions in India increased, at an annual growth rate of 5.48%. With the rapid growth of the Indian economy and population, its CO_2 emissions also increased notably, which is estimated to account for 11% of the global CO_2 emissions by 2030. In the face of the call of the international community to promote carbon emission reduction, the Indian government has promised to reduce greenhouse gas emissions by 33–35% from 2015 to 2030. Therefore, this section chooses India as an example to test the accuracy of the FGBM(1,1,t^{α}) model. The parameters and MAPE values of the FGBM(1,1,t^{α}) model calculated with the three optimization algorithms are listed in Table 6. The MAPE and test_MAPE values for GWO are 0.5090% and 0.7130%, respectively, which are the smallest among the three optimization algorithms, and the performance is the best. Therefore, GWO is selected here. Figure 6 shows the number of iterations and the relationship between MAPE and the parameters. The fitting and prediction results for each model are shown in Fig. 7 and Table 7, and the error measurement results for each model are shown in Fig. 8 and Table 8. The prediction value of the FGBM(1,1,t^{α}) model is the closest to the actual value. Based on the prediction and fitting values, the two error measurement indexes of the FGBM(1,1,t^{α}) model are the best. This further demonstrates that the FGBM(1,1,t^{α}) model is more accurate than the other grey models in predicting the CO_2 emissions of India.

CO_2 emissions of the Asia Pacific region

According to Statistics Review of World Energy 2020, in 2019, the CO_2 emissions of the Asia Pacific region amounted to 17,269.46 million tons, accounting for 50.54% of the world's CO_2 emissions. From 2010 to 2019, the CO_2 emissions in the Asia Pacific region indicated a rising trend, at an average annual growth rate of 2.60%. This section selects the Asia Pacific region as an example to test the accuracy of the FGBM(1,1,t^{α}) model. The parameters and MAPE values of the FGBM(1,1,t^{α}) model calculated with the three optimization algorithms are shown in Table 9. The MAPE and test-MAPE values obtained with PSO are 0.1076% and 0.6854%,

Table 3 Parameters and MAPEs of the FGBM (1,1,t^{α}) model based on different optimization algorithms(Case 1)

Algorithm	r(Parameter 1)	λ(Parameter 2)	α(Parameter 3)	ξ(Parameter 4)	MAPE(%)	test_MAPE(%)
GWO	0.0016	0.1922	1.3618	0.0125	1.2046	1.9173
PSO	0.0000	0.1796	1.3533	1.3194	1.2100	1.8242
QGA	0.6068	0.5624	0.1232	0.8031	1.1320	1.5805

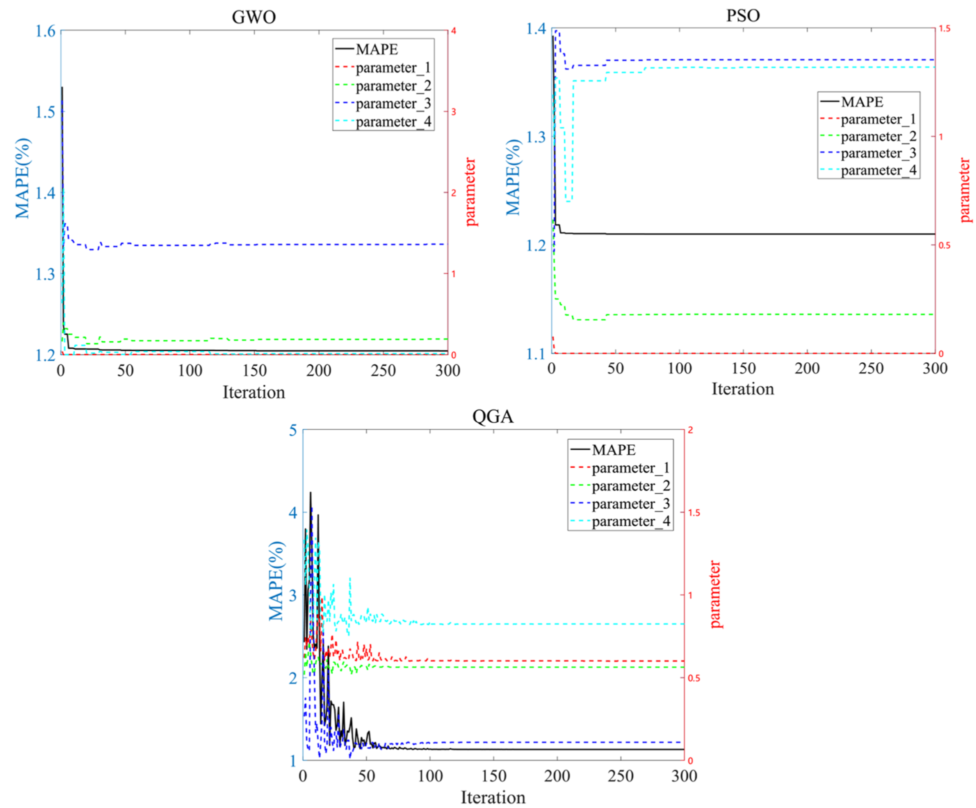


Fig. 3 Iterations, MAPE, and parameters of the three optimization algorithms

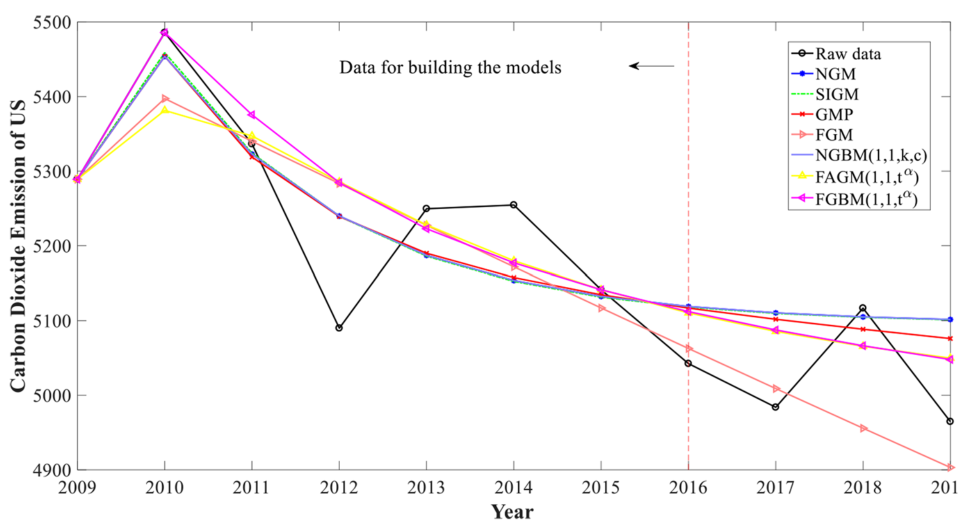


Fig. 4 Results of CO_2 emissions of USA

respectively, which are the smallest and the best among the three optimization algorithms, so the PSO algorithm is selected here. Figure 9 shows the relationship between the number of iterations, MAPE, and parameters. The fitting and prediction results for each model are shown in Fig. 10 and Table 10, and the error measurement results for each model are shown in Fig. 11 and Table 11. The predicted value of the FGBM(1,1,t^{α}) model is the closest to the actual value. According to the prediction and fitting values, the two error metrics of the FGBM(1,1,t^{α}) model are the best. This also indicates that the FGBM(1,1,t^{α}) model is more accurate than the other grey prediction models in predicting the CO_2 emissions in the Asia Pacific region.

Total CO_2 emissions of the world

According to Statistics Review of World Energy 2020, in 2019, the global CO_2 emissions reached 34,169.00 million tons, which has increased greatly over the past few decades. From 2010 to 2019, the global CO_2 emissions increased by

Table 4 Fitting and prediction results of CO_2 emissions of USA

Year	data	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^{α})	FGBM
2009	5289.14	5289.14	5289.14	5289.14	5289.14	5289.14	5289.14	5289.14
2010	5485.72	5453.79	5458.8	5453.91	5397.34	5453.79	5381.41	5485.72
2011	5336.44	5322.85	5324.18	5318.94	5340.07	5322.85	5347.20	5375.56
2012	5089.97	5239.79	5239.46	5239.19	5283.4	5239.79	5285.63	5284.68
2013	5249.6	5187.09	5186.15	5189.98	5227.34	5187.09	5227.87	5222.82
2014	5254.57	5153.67	5152.6	5157.66	5171.88	5153.67	5179.98	5177.19
2015	5141.41	5132.46	5131.49	5134.68	5117	5132.46	5141.41	5141.41
2016	5042.43	5119.01	5118.21	5116.87	5062.7	5119.01	5110.48	5112.17
Year	data	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^{α})	FGBM
2017	4983.87	5110.47	5109.85	5101.92	5008.98	5110.47	5085.63	5087.57
2018	5116.79	5105.06	5104.59	5088.55	4955.83	5105.06	5065.69	5066.42
2019	4964.69	5101.62	5101.28	5076.05	4903.25	5101.62	5049.79	5047.92

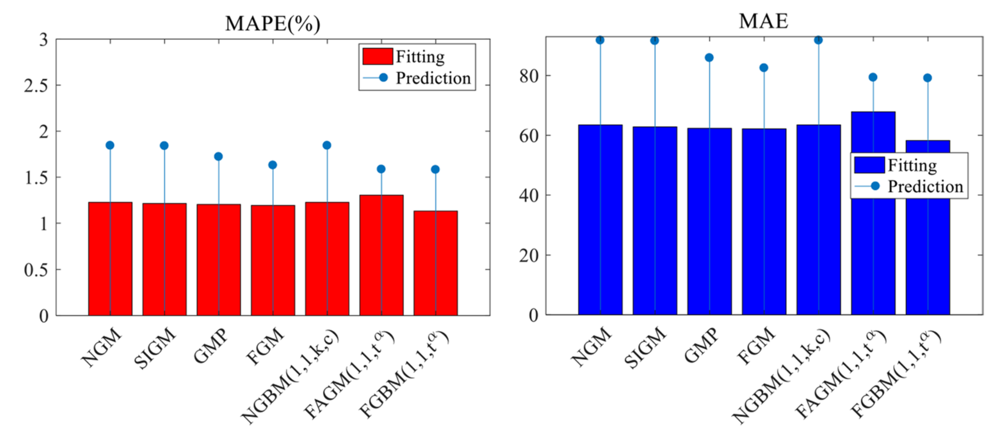


Fig. 5 Error metrics of CO_2 emissions of USA

Table 5 Error metrics of CO_2 emissions of USA

Fitting	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^{α})	FGBM
MAPE(%)	1.2262	1.2146	1.2037	1.1934	1.2262	1.3043	**1.1320**
MAE	63.4644	62.8228	62.315	62.1516	63.4644	67.8710	**58.2463**
Prediction	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^{α})	FGBM
MAPE(%)	1.8425	1.8391	1.7212	1.629	1.8425	1.5848	1.5805
MAE	91.7527	91.5873	85.8817	82.5036	91.7527	79.3186	79.0989

Table 6 Parameters and MAPEs of the FGBM (1,1,t^{α}) model based on different optimization algorithms(Case 2)

Algorithm	r(Parameter 1)	λ(Parameter 2)	α(Parameter 3)	ξ(Parameter 4)	MAPE(%)	test_MAPE(%)
GWO	0.0005	0.4788	0.4393	0.0067	0.5090	0.7130
PSO	1.0000	0.5173	3.3792	0.0000	0.5893	0.9856
QGA	0.0045	0.5024	2.2159	0.0001	0.5126	7.7080

1.102% annually, exhibiting a slow rising trend. However, economic recovery after the COVID-19 pandemic will facilitate a global rebound in CO_2 emissions. Therefore, this section adopts the world as an example to test the accuracy of the FGBM(1,1,t^{α}) model. The parameters and MAPE values of the FGBM(1,1,t^{α}) model calculated with the three optimization algorithms are summarized in Table 12. The MAPE and test_MAPE values obtained with QGA are 0.168% and 2.352%, respectively, which are the smallest and the best among the three optimization algorithms. Therefore, QGA is selected here. Figure 12 shows the relationship between the number of iterations, MAPE, and parameters.

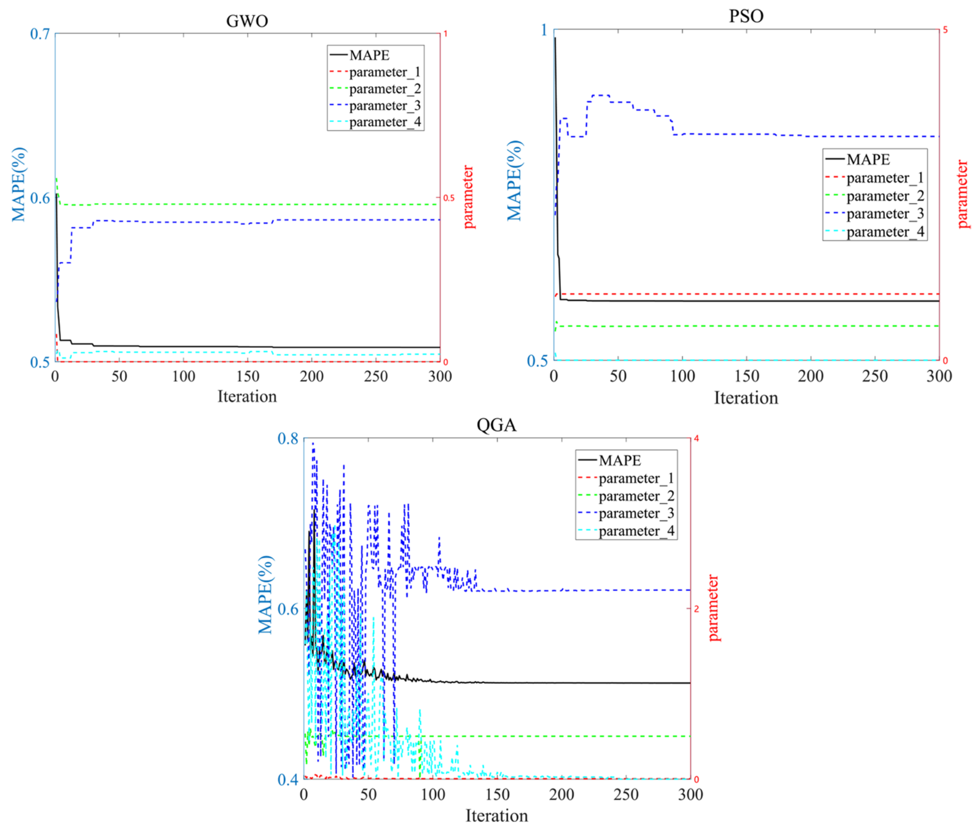


Fig. 6 Iterations, MAPE, and parameters of the three optimization algorithms

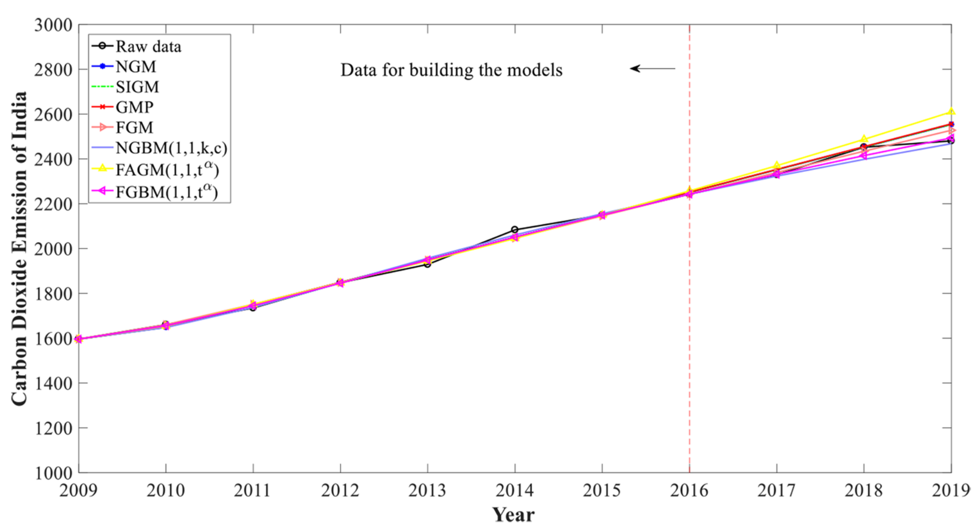


Fig. 7 Results of CO_2 emissions of India

The fitting and prediction results for each model are shown in Fig. 13 and Table 13, and the error measurement results for each model are shown in Fig. 14 and Table 14. The predicted value of the FGBM(1,1,t^{α}) model is the closest to the actual value. Based on the prediction and fitting values, the two error metrics of the FGBM(1,1,t^{α}) model are the best. This further verifies that the FGBM(1,1,t^{α}) model is more accurate than the other grey prediction models in predicting the global CO_2 emissions.

The above results reveal that although FGBM(1,1,t^{α}) achieves the best effect in predicting the global CO_2 emissions among the many models, due to the sudden increase in the original data of CO_2 emissions in 2016, the fitting effect of the model is good, but the prediction error is large. The predicted global CO_2 emissions over the next 5 years may be far from the real value. To resolve this problem, global carbon emission data can be divided into two parts, namely, Organisation for Economic Co-operation and Development

Table 7 Fitting results and prediction results of CO_2 emissions of India

Year	data	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^α)	FGBM
2009	1596.24	1596.24	1596.24	1596.24	1596.24	1596.24	1596.24	1596.24
2010	1660.65	1649.02	1649.03	1649.8	1660.65	1648.09	1654.19	1656.16
2011	1735.15	1749.11	1749.12	1749.45	1751.62	1737.49	1751.26	1745.24
2012	1848.13	1849.31	1849.32	1849.41	1848.95	1846.63	1847.28	1845.79
2013	1929.35	1949.63	1949.64	1949.65	1948.07	1956.97	1944.88	1949.24
2014	2083.54	2050.06	2050.07	2050.17	2047.3	2060.50	2045.31	2051.17
2015	2149.38	2150.62	2150.63	2150.95	2145.88	2155.62	2149.30	2149.38
2016	2242.89	2251.29	2251.3	2251.99	2243.4	2242.89	2257.37	2242.89
Year	data	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^α)	FGBM
2017	2352.08	2352.09	2353.26	2391.72	2323.4	2352.08	2369.91	2331.375
2018	2453	2453.01	2454.75	2498.25	2398.17	2453.00	2487.30	2414.902
2019	2554.03	2554.04	2556.47	2605.12	2468.07	2554.03	2609.87	2493.722

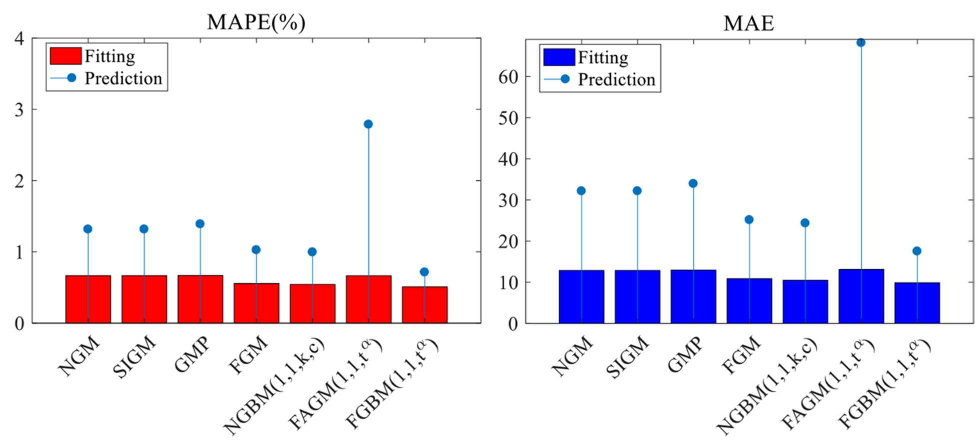


Fig. 8 Error metrics of CO_2 emissions of India

Table 8 Error metrics of CO_2 emissions of India

Fitting	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^α)	FGBM
MAPE(%)	0.6654	0.6656	0.6685	0.5555	0.5429	0.6646	**0.5090**
MAE	12.8783	12.8825	12.9674	10.8937	10.4717	13.1051	**9.8836**
Prediction	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^α)	FGBM
MAPE(%)	1.3154	1.3158	1.3889	1.0248	0.9953	2.7872	**0.7130**
MAE	32.1452	32.1552	33.9366	25.1379	24.3432	68.1367	**17.5085**

Table 9 Parameters and MAPEs of the FGBM (1,1,t^α) model based on different optimization algorithms(Case 3)

Algorithm	r(Parameter 1)	λ(Parameter 2)	α(Parameter 3)	ξ(Parameter 4)	MAPE(%)	test_MAPE(%)
GWO	0.2262	0.5336	3.8539	0.1932	0.1095	0.8011
PSO	0.2731	0.5320	3.6715	0.0000	0.1076	0.6854
QGA	0.2187	0.5337	3.9843	0.2037	0.1100	1.2388

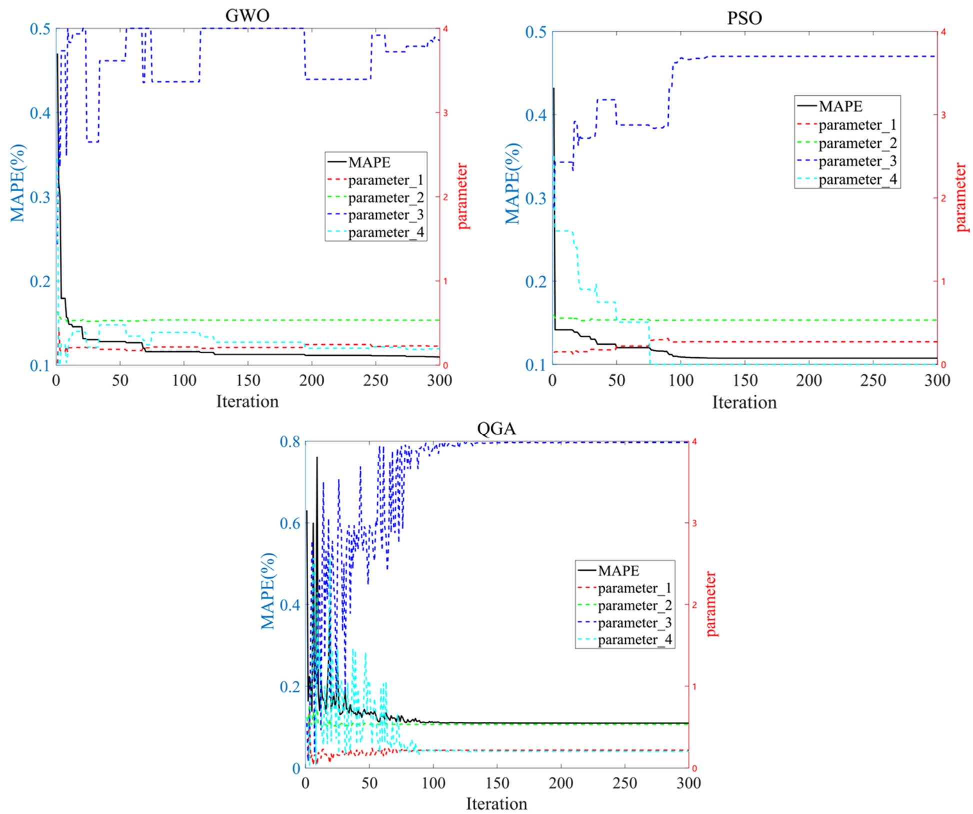


Fig. 9 Iterations, MAPE, and parameters of the three optimization algorithms

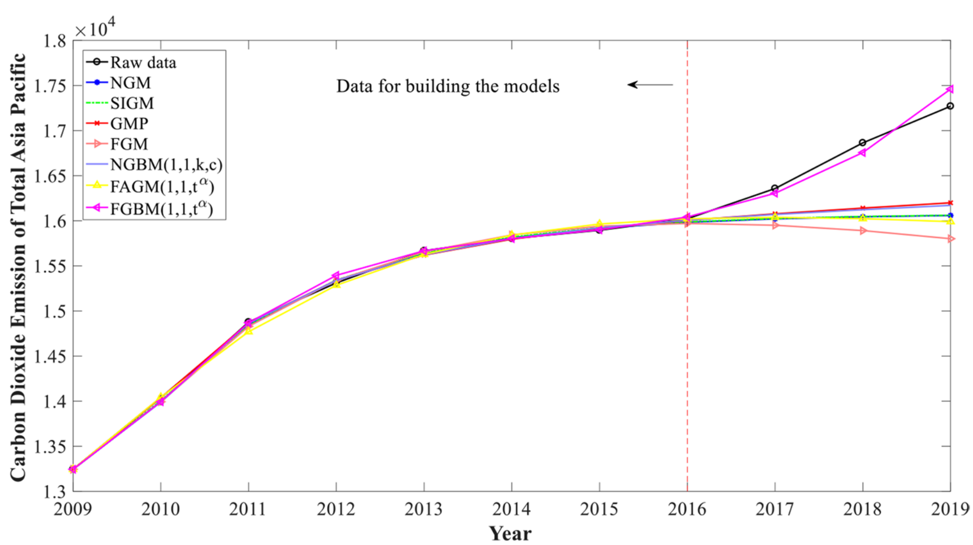


Fig. 10 Results of CO_2 emissions of Asia Pacific

Table 10 Fitting results and prediction results of CO_2 emissions of Asia Pacific

Year	data	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^α)	FGBM
2009	13,244.47	13,244.47	13,244.47	13,244.47	13,244.47	13,244.47	13,244.47	13,244.47
2010	13,993.5	14,039.21	14,004.53	14,037.98	13,993.5	13,989.08	14,044.40	13,993.5
2011	14,876.64	14,850.3	14,843.42	14,866.61	14,828.75	14,862.94	14,769.40	14,869.13
2012	15,310.55	15,338.96	15,343.17	15,339.22	15,339.94	15,338.78	15,284.81	15,394.69
2013	15,666.93	15,633.38	15,640.89	15,619.27	15,655.5	15,623.05	15,626.99	15,666.93
2014	15,802.62	15,810.76	15,818.25	15,795.15	15,842.54	15,802.63	15,840.96	15,801.7
2015	15,894.14	15,917.63	15,923.9	15,914.68	15,939.34	15,921.87	15,963.44	15,898.41
2016	16,022.09	15,982.01	15,986.85	16,003.74	15,969.78	16,005.85	16,022.09	16,041.41
Year	data	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^α)	FGBM
2017	16,357.09	16,020.8	16,024.34	16,076.31	15,949.83	16,069.61	16,037.07	16,304.71
2018	16,863.32	16,044.18	16,046.68	16,139.97	15,890.71	16,122.45	16,022.88	16,755.99
2019	17,269.46	16,058.26	16,059.99	16,198.8	15,800.65	16,170.25	15,989.84	17,459.35

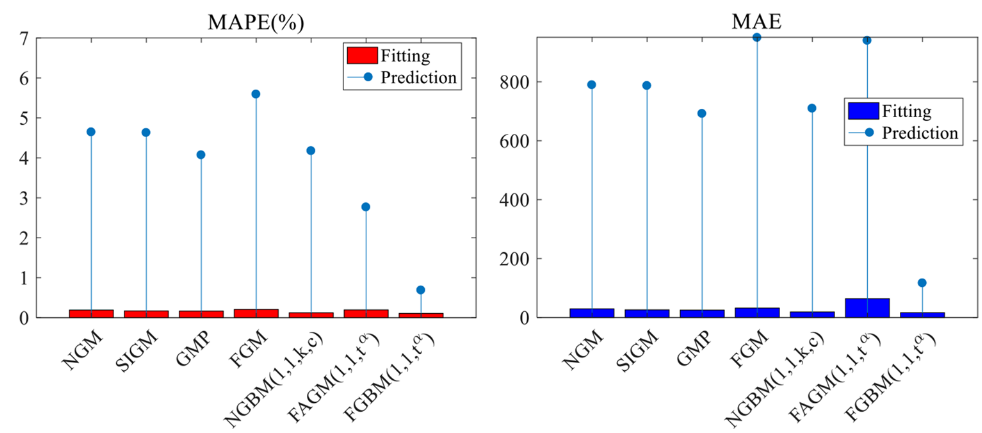


Fig. 11 Error metrics of CO_2 emissions of Asia Pacific

Table 11 Error metrics of CO_2 emissions of Asia Pacific

Fitting	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^α)	FGBM
MAPE(%)	0.1933	0.1696	0.1668	0.2072	0.1234	0.3123	**0.1076**
MAE	29.3878	26.22	25.3164	32.305	19.1706	47.3506	**16.594**
Prediction	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^α)	FGBM
MAPE(%)	4.6423	4.6268	4.0686	5.5876	4.172	4.7834	**0.6854**
MAE	788.8784	786.2858	691.5954	949.5596	709.1898	813.3621	**116.5323**

Table 12 Parameters and MAPEs of the FGBM (1,1,t^α) model based on different optimization algorithms(Case 4)

Algorithm	r(Parameter 1)	λ(Parameter 2)	α(Parameter 3)	ξ(Parameter 4)	MAPE(%)	test_MAPE(%)
GWO	0.0000	0.5671	4.0000	3.0000	0.1891	2.4410
PSO	0.0000	0.5671	4.0000	3.0000	0.1891	2.4410
QGA	0.0624	0.5545	0.1410	0.9082	0.1680	2.3515

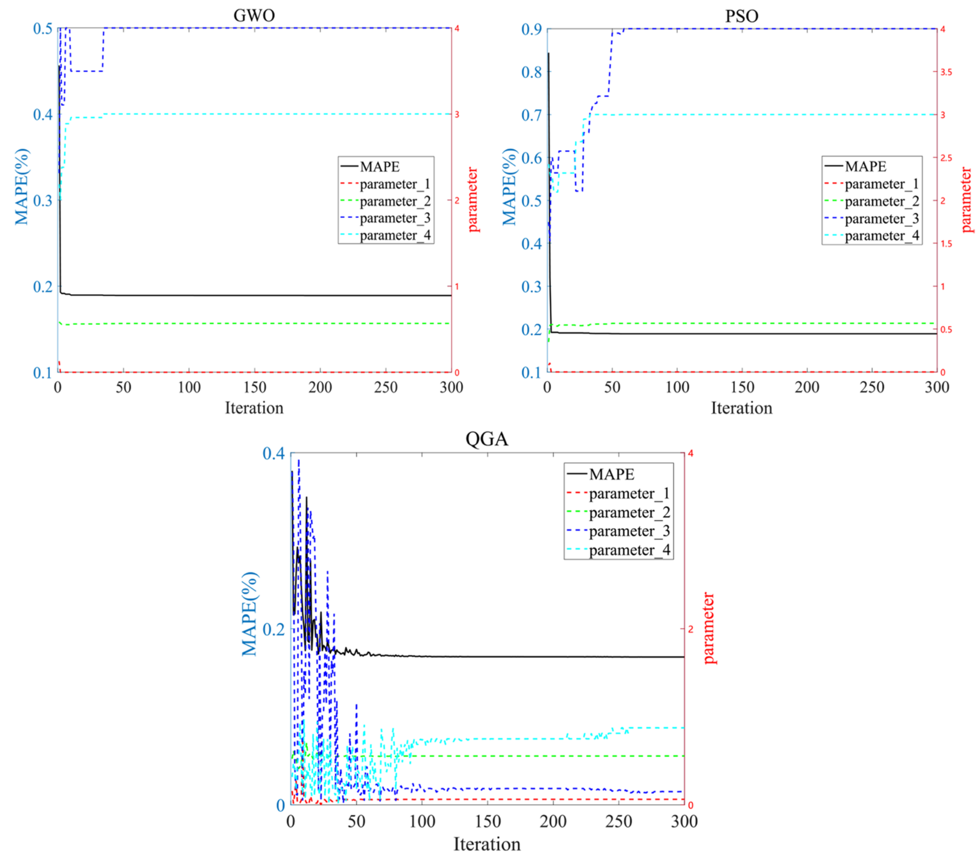


Fig. 12 Iterations, MAPE, and parameters of the three optimization algorithms

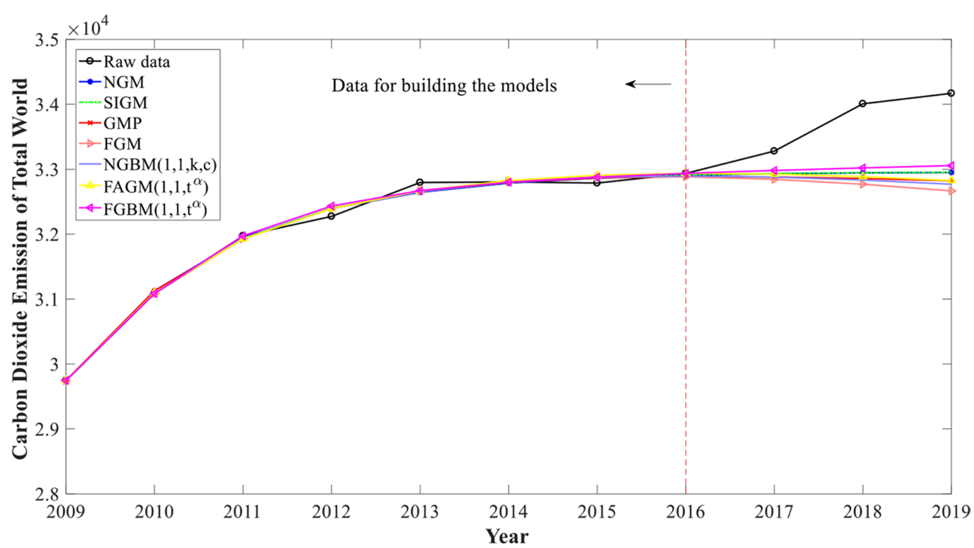


Fig. 13 Results of CO_2 emissions of total world

(OECD) and non-OECD data. The trend of CO_2 emissions based on OECD and non-OECD data is more stable than that based on global data. Therefore, this paper applies the FGBM(1,1,t^{α}) model to fit and predict the CO_2 emissions based on OECD and non-OECD data, as listed in Tables 15, 16, and 17. Tables 15 and 16 reveal that between the OECD and non-OECD data, the prediction error obtained with QGA is the smallest, so QGA is selected to calculate the model parameters. The global CO_2 emissions are determined by adding the OECD and non-OECD-based CO_2 emissions,

Table 13 Fitting results and prediction results of CO_2 emissions of total world

Year	data	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^{α})	FGBM
2009	29,745.21	29,745.21	29,745.21	29,745.21	29,745.21	29,745.21	29,745.21	29,745.21
2010	31,085.53	31,127.57	31,087.02	31,127.78	31,085.54	31,085.54	31,097.99	31,085.61
2011	31,973.37	31,942.26	31,938.15	31,929.02	31,927.62	31,939.21	31,927.01	31,973.1
2012	32,273.53	32,394.85	32,402.62	32,395.41	32,396.32	32,394.97	32,392.79	32,434.11
2013	32,795.55	32,646.28	32,656.09	32,658.89	32,666.91	32,656.47	32,667.86	32,670.83
2014	32,804.72	32,785.96	32,794.40	32,799.41	32,814.93	32,803.29	32,826.66	32,800.37
2015	32,787.20	32,863.56	32,869.89	32,865.42	32,879.79	32,875.37	32,908.30	32,880.00
2016	32,936.07	32,906.67	32,911.08	32,886.28	32,884.68	32,895.7	32,936.07	32,936.28
Year	data	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^{α})	FGBM
2017	33,279.49	32,930.62	32,933.55	32,879.78	32,844.47	32,878.79	32,925.14	32,981.3
2018	34,007.89	32,943.93	32,945.82	32,856.7	32,769.27	32,834.34	32,886.06	33,020.58
2019	34,169.00	32,951.32	32,952.52	32,823.57	32,666.30	32,769.20	32,826.52	33,056.66

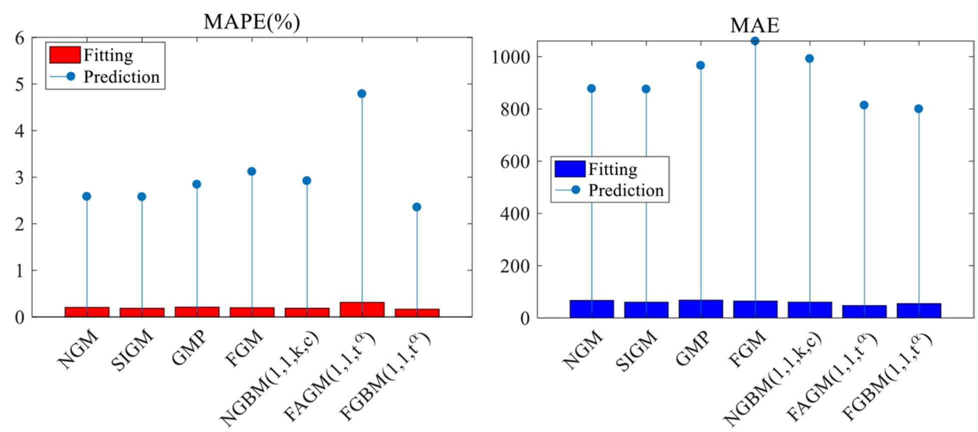


Fig. 14 Error metrics of CO_2 emissions of total world

Table 14 Error metrics of CO_2 emissions of total world

Fitting	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^{α})	FGBM
MAPE(%)	0.2061	0.1857	0.2107	0.1979	0.1861	0.1972	**0.168**
MAE	66.8934	60.4641	68.3522	64.4834	60.6642	64.1159	**54.7167**
Prediction	NGM	SIGM	GMP	FGM	NGBM(1,1,k,c)	FAGM(1,1,t^{α})	FGBM
MAPE(%)	2.5802	2.5742	2.8412	3.1157	2.9172	2.7641	**2.3515**
MAE	876.8394	874.8306	965.4436	1058.7825	991.3511	939.5525	**799.2845**

Table 15 Parameters and MAPEs of the FGBM (1,1,t^{α}) model based on different optimization algorithms(OECD)

Algorithm	r(Parameter 1)	λ(Parameter 2)	α(Parameter 3)	ξ(Parameter 4)	MAPE(%)	test_MAPE(%)
GWO	0.0337	0.7190	1.8236	2.9951	0.2853	1.0751
PSO	0.0327	0.7232	1.3789	3.0000	0.2848	1.0823
QGA	0.0336	0.7198	1.7555	3.0000	0.2851	1.0763

Table 16 Parameters and MAPEs of the FGBM (1,1,t^{α}) model based on different optimization algorithms (non-OECD)

Algorithm	r(Parameter 1)	λ(Parameter 2)	α(Parameter 3)	ξ(Parameter 4)	MAPE(%)	test_MAPE(%)
GWO	0.2690	0.5229	0.2772	0.1436	0.1489	3.1289
PSO	0.2147	0.5267	4.0000	0.0000	0.1557	1.1457
QGA	0.8747	0.5157	3.9997	0.0691	0.1392	1.0637

Table 17 Fitting and prediction results of CO_2 emissions in OECD, non-OECD and total world

Year	OECD	FGBM	Non-OECD	FGBM	Total World	Add_values
2009	12,507.58	12,507.58	17,237.63	17,237.63	29,745.21	29,745.21
2010	12,957.49	12,957.46	18,128.05	18,128.06	31,085.53	31,085.52
2011	12,783.1	12,822.77	19,190.27	19,179.78	31,973.37	32,002.55
2012	12,580.34	12,671.53	19,693.19	19,770.25	32,273.53	32,441.78
2013	12,661.94	12,547.74	20,133.61	20,112.97	32,795.55	32,660.71
2014	12,441.45	12,443.81	20,363.27	20,325.81	32,804.72	32,769.62
2015	12,347.76	12,352.68	20,439.44	20,488.73	32,787.20	32,841.41
2016	12,270.06	12,270.02	20,666.00	20,666.10	32,936.07	32,936.12
Year	OECD	FGBM	Non-OECD	FGBM	Total World	Add_values
2017	12,300.25	12,193.21	20,979.24	20,916.26	33,279.49	33,109.47
2018	12,372.33	12,120.54	21,635.56	21,296.18	34,007.89	33,416.72
2019	12,011.96	12,050.83	22,157.05	21,864.04	34,169.00	33,914.88

Table 18 Error metrics of CO_2 emissions in total world(Add_values)

Fitting	MAPE	MAE	Prediction	MAPE	MAE
FGBM	0.1852	60.2361	FGBM	0.9976	338.4367

Table 19 Predictions for the CO_2 emissions over the next 5 years in OECD and Non-OECD

Year	2020	2021	2022	2023	2024
OECD	11,983.30	11,917.34	11,852.53	11,788.55	11,725.16
Non-OECD	22,680.69	23,810.63	25,322.78	27,291.05	29,794.83

Table 20 Predictions for the CO_2 emissions over the next 5 years

Year	US	India	Total Asia Pacific	Total World
2020	5031.49	2568.18	18,477.36	34,663.99
2021	5016.79	2638.66	19,872.37	35,727.97
2022	5003.49	2705.52	21,707.38	37,175.31
2023	4991.37	2769.12	24,046.64	39,079.60
2024	4980.25	2829.79	26,956.05	41,519.99

as summarized in Table 17. The fitting and prediction errors of the global CO_2 emissions are listed in Table 18 and are lower than those listed in Table 13. Table 19 provides the OECD and non-OECD based CO_2 emissions over the next 5 years. The global CO_2 emissions over the next 5 years can be obtained by adding the two types of emissions. The fitting and prediction results are more accurate than the direct prediction results of the global CO_2 emissions.

Forecasting CO_2 emissions over the next 5 years

In this section, we employ the FGBM(1,1,t^α) model to predict the CO_2 emissions of the USA, India, Asia Pacific region, and the world over the next 5 years (2020–2024). The prediction results are summarized in Table 20 and Fig. 15. The CO_2 emissions in India, the Asia Pacific region, and the world will gradually increase over the 5 years. In addition, the CO_2 emissions in the USA will slowly decline.

Conclusions and policy implications

To better describe the future CO_2 emissions of the USA, India, Asia Pacific region, and the world, a new grey prediction model, i.e., the FGBM(1,1,t^α) model, is proposed based on the NGBM(1,1) and FAGM(1,1,t^α) models, and a precise solution of the new model is obtained via the numerical integration method. Moreover, this paper applies three common optimization algorithms to calculate the model parameters. By changing the model parameters, the FGBM(1,1,t^α) model can be transformed into other models, so the model achieves a strong adaptability. The CO_2 emission fitting and forecasting results in the above four economies indicate that the FGBM(1,1,t^α) model is more effective and accurate than the existing NGM(1,1), SIGM, GMP, FGM, NGBM(1,1,k,c), and FAGM(1,1,t^α). Moreover, we employ the FGBM(1,1,t^α) model to predict the CO_2 emissions of the USA, India, Asia Pacific region, and the world over the next 5 years. The forecast results reveal that from 2020 to 2024, the CO_2 emissions of India, the Asia Pacific region, and the world will gradually rise. The CO_2 emissions of the USA will slowly decline over the next 5 years. Notably, the grey prediction FGBM(1,1,t^α) model can be applied not only in

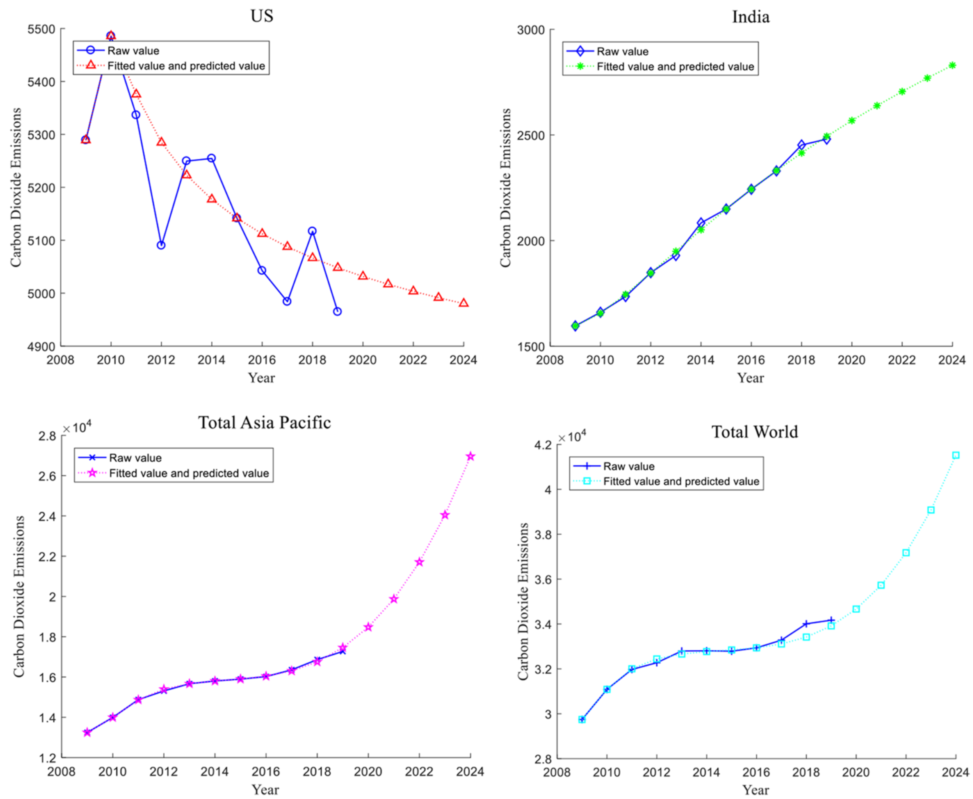


Fig. 15 Predictions for the CO2 emissions over the next 5 years

the prediction of CO_2 emissions but also in the prediction of other data, with a high adaptability.

In order to meet the requirements for carbon emission reduction in the sustainable development goals, several suggestions are put forward based on the above findings. First, promoting the green and low-carbon transformation of the energy system is the key. We should focus on energy conservation and improve the dual control system of total energy consumption and intensity. We should increase the utilization ratio of renewable energy such as wind power and photovoltaic power generation, and develop hydropower, geothermal energy, marine energy, hydrogen energy, biomass energy, and photothermal power generation according to local conditions. Second, the fiscal and tax support should be strengthened to encourage the research and development of green and low-carbon technologies. Financial funds should be used to support the development of green environmental protection industry and energy efficient utilization, and a number of scientific and technological projects should be arranged in the fields of energy conservation and environmental protection, cleaner production, and clean energy. Third, the recycling of renewable resources should be strengthened to realize a green and low-carbon lifestyle. We should promote the classification, reduction, and resource utilization of domestic waste according to local conditions, accelerate the construction of waste material recovery system, and strengthen the recycling of waste paper, waste plastics, waste tires and other resources.

This study still has some limitations. First, the model has many hyper-parameters, which may lead to over-fitting problems. Second, the model is a univariate model, which may ignore some important influencing factors (e.g., economic development, industrial policy, etc.), so the multivariate grey model can also be applied to predict carbon emissions, which is the optimization direction in the future.

Author contribution All authors contributed to the study conception and design. Material preparation, data collection, and analysis were performed by Huiping Wang and Yi Wang. The first draft of the manuscript was written by Yi Wang. All authors read and approved the final manuscript.

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Data availability The datasets used or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent to publish Not applicable.

Competing interests The authors declare no competing interests.

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