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RESEARCH ARTICLE

Effective allocation of resources in water pollution treatment alternatives: a multi-stage gray group decision-making method based on hesitant fuzzy linguistic term sets

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Abstract
With the significant economic shift, water pollution treatment has gradually become a key problem which needs to be deeply investigated for the sustainable development of China. In the face of specific water pollution incidents, multiple alternatives are often required to work together in order to achieve better results. However, due to the limitation of resources, alternatives must be ranked to realize the effective allocation of resources, which means the more highly ranked ones should possess more disposable resources. Furthermore, the water pollution treatment process is a multi-stage and multi-objective process. In each stage, decision-makers may have different emphasis and thus have different preferences for the treatment alternatives. How to effectively aggregate decision-makers' preferences in different stages into an overall preference so as to form a ranking of treatment alternatives under global constraints has turned into a problem worthy of discussion. Under such background, this paper proposes a multi-stage gray group decision-making method, where decision-makers use Group-G1 to rank and weight the criteria, and in this way, the weights of decision-makers and criteria in each stage could be determined. Considering the difference and deficiency of the cognitive level of decision-makers, this paper adopts the form of hesitant fuzzy linguistic term sets (HFITS) to express the evaluation information of decision-makers. And then, gray incidence analysis is selected to rank the alternatives. After ranking the alternatives in each stage, the multi-stage rankings will be aggregated into an overall ranking and the resource allocation is made according to the priorities of the alternatives. Finally, an example of water pollution treatment alternatives ranking based on a cyanobacterial bloom in Taihu Lake, China, is given to illustrate the proposed approach.

Keywords Water pollution treatment · Resources allocation · Group decision-making · Group-G1 · Hesitant fuzzy linguistic term sets · Gray incidence analysis

Introduction

With the continuous improvement of industrialization and urbanization, China has achieved remarkable economic growth (Yuan et al. 2019). Nevertheless, at the same time, more and more environmental pollution problems have gradually arisen, such as air pollution and water pollution. These pollution problems seriously endanger people's health and restrict the sound and rapid development of the economy of China (Ebenstein 2012; Wang and Yang 2016). Therefore, the study of various pollution problems has become a hot topic in China. China has a network of rivers, numerous lakes, and considerable territorial waters, yet considering the population, China is essentially a water-scarce country. Insufficient water supply has seriously affected the sustainable development of society, economy, and ecological environment. What is worse, irregular discharge of industrial sewage and domestic sewage is becoming more and more serious, which not only leads to severe water pollution, but also aggravates the problem of water shortage in China (Lu et al. 2015). In recent years, the water pollution level has shown an overall upward trend and almost all the major river systems in China have suffered from varying degrees of pollution. Thus, the Chinese government has taken different measures in water pollution treatment, such

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as optimizing the industrial structure (Guo et al. 2015), establishing a long-term supervision and management mechanism (Xia et al. 2011), and using various technical means for comprehensive treatment (Qu and Fan 2010). Unremitting efforts have achieved some effects and water pollution has been alleviated to a certain extent. Chinese government will take water pollution treatment as a significant issue for a long time to come and pay close attention to the latest progress in this field (Zhang et al. 2010a, b; Liu and Yang 2012).

The effective treatment of water pollution often does not depend on a single alternative, but a number of alternatives work together to achieve the corresponding treatment objectives (Fu and Wang 2011). However, due to the limitation of resources, especially human resources and financial resources, it is not possible to equally allocate resources to all the alternatives. Instead, existing alternatives should be sorted and the resource allocation is made according to the priorities of the alternatives, so as to maximize the utility (Chen et al. 2018). From the perspective of promoting the efficient use of resources, this is extremely important and also in line with China's development status. For water pollution treatment alternatives, the diversity and complexity make them rather difficult to be selected and ranked, under which circumstance multiple criteria are taken into consideration comprehensively. In addition, water pollution is a large social problem, which often requires the joint participation of different groups. Group decision-making method is proposed and aimed at solving this kind of problems. Group decision-making is a research method that selects the best option from a set of alternatives by assembling the viewpoints of multiple decision-makers (Liu et al. 2016).

However, water pollution treatment is a long-term process, which is normally divided into three stages, namely preliminary stage, mid stage, and later stage, and the three stages form a complete treatment system together. It is worth noting that decision-makers' emphasis may vary in different stages. For example, in the preliminary stage of water pollution treatment, in order to control the pollution and eliminate the panic of the public, decision-makers usually give priority to whether the treatment alternatives could achieve results in a relatively short time. As for the mid stage, decision-makers tend to take the maturity and implementation cost of the alternatives as the priority. While in the later stage, they may consider the comprehensive effect after the implementation of treatment alternatives, for instance, whether the secondary pollution is caused, whether the coordinated development of resources and environment could be obtained, etc. In summary, for a group of water pollution treatment alternatives, at different stages of pollution treatment, as the weight of the criteria measured by decision-makers to evaluate the alternatives alters with the evolution of water pollution treatment process, the ranking of alternatives given by decision-makers will change accordingly.

Therefore, the selection and sequencing of water pollution treatment alternatives is a comprehensive application of group decision-making and dynamic decision-making. The existing research on the sequencing of water pollution treatment alternatives is mainly based on the expression of one-stage preferences of decision-makers, i.e., only one-time sequencing of alternatives is given (Shi et al. 2014; Qu et al. 2019), while relatively less research has been done on the situation that decision-makers' preferences may change with the evolution of stages. Hence, the purpose of this paper is to aggregate the preferences of decision-makers at different stages, so as to get the final ranking of the alternatives, that is, to get the global optimal alternative under multi-stage objective constraints, in order to solve the problem of resource allocation in the process of water pollution treatment.

Considering that the sequencing of water pollution treatment alternatives is a multi-stage decision-making process, it is necessary to determine the proportion of different stages in the overall situation. Meanwhile, the importance of the viewpoints given by different decision-makers varies at different stages, so it is necessary to assign weights to each decision-maker in stages, which is an issue to be solved in the decision-making process. In previous studies, a series of methods were proposed. For instance, the bias between the consensus level of decision-makers' preferences and the comprehensive preference of the group was used as the principle of weight assignment (Zhu and Hipel 2012), decision preference distance was used to measure the weights of decision-makers and the evaluation criteria (Yu and Lai 2011), etc. While these methods enrich the research of multi-stage group decision-making issue to a certain extent, the study on this kind of problem is still relatively limited up to now. In view of the background of water pollution treatment alternative ranking, this paper adopts Group-G1 method to express the ranking given by each decision-maker on the importance of evaluation criteria at different stages of pollution treatment. On the one hand, the method could solve the problem of ranking evaluation criteria by each decision-maker; on the other hand, it could determine the weights of decision-makers in group decision-making. This method of weight assignment is now widely used in various decision-making scenarios, such as regional development and pollution treatment (Qu et al. 2016), which has been proved to be a useful tool.

Owing to the differences and deficiencies in cognitive ability and expertise level of decision-makers, it is tough for them to quantify their expressions, which could only be expressed in abstract linguistic terms (Flores-Carrillo et al. 2017; Dong et al. 2018). Based on this fact, decision-making issues with linguistic terms have attracted broad attention (Ren and Lützen 2017). However, the traditional decision-making issues expressed by linguistic terms usually only choose one linguistic term to express one's cognition. When a decision-maker cannot present his preference accurately, it is not enough to use only one linguistic term. Therefore, the concept of hesitant fuzzy linguistic term sets (Rodriguez et al. 2011) was proposed to solve the traditional problem of preference expression. Considering that the decision-makers involved in the evaluation of water pollution treatment alternatives could not quantify their perception of the criteria, and could not give a definite preference expression, this paper describes the evaluation information of each decision-maker in the form of hesitant fuzzy linguistic term sets and then carries on the following work via gray incidence analysis.

The gray incidence analysis is an important element of gray system theory, which has its own advantage in possessing uncertain, gray, or fuzzy information. The gray incidence analysis has been successfully applied in many fields, such as identification of key indices for industrial development (Liu et al. 2017), inducement to accidents (Zhou and Hu 2012), and energy planning (Yang et al. 2018). Hesitant fuzzy linguistic term sets have a great extent of uncertainty, so it is suitable to adopt the gray incidence analysis to carry on the analysis.

Against this background, this paper aims to propose a gray group decision-making method to evaluate the water pollution treatment alternatives at different stages, and determine the global final ranking of alternatives under multi-stage and multi-objective constraints, so as to provide some reference for the effective allocation of resources among various water pollution treatment alternatives. The organization of this paper is as follows: "The proposed multi-stage gray group decision-making method" constructs a gray group decision-making method to aggregate the preferences of decision-makers at all stages and form the final preferences of the whole situation. "Problem description" introduces the cyanobacterial bloom in the Taihu Lake watershed of China and the corresponding treatment alternatives. "An illustrative example" shows different examples. Based on the evaluation information given by decision-makers and the weights of criteria at each stage, the final ranking of the alternatives is obtained in order to achieve the effective allocation of resources in the process of water pollution treatment. Then, "Conclusions and discussions" are presented, which describe the main contributions and shortcomings of this paper, and discuss the possible new research directions in the future.

The proposed multi-stage gray group decision-making method

Group-G1

The Group-G1 weighting method was proposed in 2002 for weighting evaluation criteria. It is mainly used in the case when decision-makers show inconsistent opinions on the importance of evaluation criteria. The detailed steps are shown as follows:

Firstly, the set of evaluation criteria $C = \{c_1, c_2, \ldots, c_m\}$ and the set of decision-makers$D = \{d_1, d_2, \ldots, d_n\}$ are determined, where we assume the criteria and decision-makers remain unchanged throughout the whole process.

Each decision-maker sorts the criteria in descending order according to the relative importance degree and gets a new sequence $c_1^* > c_2^* > \cdots > c_m^*$. The decision-makers then express the importance ratio of the criterion c_{i-1}^* compared with the criterion c_i^* with the numerical value r_i shown in Table 1. And in this way, the weight of the criterion c_m^* and other criteria could be obtained on the basis of Eq. (1) and Eq. (2):

$$\omega_m = \left(1 + \sum_{i=2}^{m} \prod_{k=i}^{m} r_k\right)^{-1} \quad (1)$$

$$\omega_{i-1} = r_i \omega_i i = m, m-1, \ldots, 2 \quad (2)$$

Above are the weights of criteria determined by each decision-maker.

Next, each decision-maker converts his own ranking of criteria into numerical values. The higher the ranking is, the higher the numerical value would be. The detailed transformation method is shown in Eq. (3), where o_{ij} is the sequence order of the ith criterion in the ranking of decision-maker j, S_{ij} represents the numerical value of the ith criterion determined by decision-maker j:

$$S_{ij} = m + 1 - o_{ij} j = 1, 2, \ldots, n \quad (3)$$

Thus, the mean value of all the criteria is calculated in Eq. (4):

$$\overline{S_i} = \frac{1}{n} \sum_{j=1}^{n} S_{ij} i = 1, 2, \ldots, m \quad (4)$$

The mean value of the criteria $\overline{S_i}$ is a reflection of group opinions; then, all the evaluation criteria could be resorted from high to low according to $\overline{S_i}$.

In group decision-making process, we normally follow such principal that the decision-makers who could reflect group opinions better should be assigned higher weights. As is described above, group opinions could be represented by $\overline{S_i}$. In order to

Table 1 Assignment table of r_i

r_i	Definition
1.0	c_{i-1}^* and c_i^* are equally important
1.2	c_{i-1}^* is slightly more important than c_i^*
1.4	c_{i-1}^* is obviously more important than c_i^*
1.6	c_{i-1}^* is strongly more important than c_i^*
1.8	c_{i-1}^* is absolutely more important than c_i^*

reflect the similarity between group opinions and individual opinion, in other words, the similarity degree between the rankings of the criteria given by the group and the individual, Spearman's rank correlation coefficient could be used, which is a helpful tool to evaluate the correlation between two statistical variables (Bartholomew et al. 2013). If we assume the group ranking sequence of the criteria is $A_o = (\alpha_{1o}, \alpha_{2o}, \ldots, \alpha_{mo})$, the ranking sequence of the criteria determined by decision-maker j is $A_j = (\beta_{1j}, \beta_{2j}, \ldots, \beta_{mj})$; then, the similarity degree between the two sequences could be defined by Eq. (5):

$$\rho_{oj} = 1-\frac{6\sum_{i=1}^{m}\left(\alpha_{io}-\beta_{ij}\right)^2}{m(m^2-1)} j = 1, 2, \ldots, n \tag{5}$$

The weights assigned to decision-makers are based on the distance between individual and group opinions; then, the weight of decision-maker j could be defined in Eq. (6):

$$\omega^j = \frac{\rho_{o,j}}{\sum_{i=1}^{n}\rho_{o,i}} \tag{6}$$

According to the weight assigned to each decision-maker, the comprehensive weight θ_i of the criterion c_i could be obtained in Eq. (7), where ω_i^j is the weight assigned to c_i by decision-maker j.

$$\theta_i = \sum_{j=1}^{n}\omega^j \times \omega_i^j i = 1, 2, \ldots, m \tag{7}$$

According to above steps, the weight of each decision-maker and the comprehensive weight of each criterion at a single stage are obtained. However, the evaluation of water pollution treatment alternatives needs to be carried out at different stages: then, an overall ranking of alternatives is obtained. If the evaluation process of water pollution treatment alternatives is divided into p stages, the above steps need to be repeated p times, which means each decision-maker sorts the criteria and assigns weights to them in p rounds, so as to acquire the weight of each decision-maker and each criterion at different stages.

Hesitant fuzzy linguistic term sets

As the evaluation and selection process of water pollution treatment alternatives is a process with great complexity and uncertainty, it is rather tough for decision-makers to use numerical values to express their assessments; then, in such circumstance, linguistic terms show their great merits. However, decision-makers may hesitate between different linguistic terms in the process of decision-making. For instance, when measuring the performance of a criterion, a decision-maker may express "it is between medium and high." Then, based on this background, the paper introduces the concept of hesitant fuzzy linguistic term sets (Rodriguez et al. 2011) to express the evaluation information of decision-makers. Hesitant fuzzy linguistic term sets (HFLTS) could be used to represent the hesitant preferences when assessing a linguistic variable, which could increase the flexibility of eliciting and representing linguistic information (Liao et al. 2014). The HFLTSs have attracted wide attention recently due to their distinguished power and efficiency in representing uncertainty and vagueness within the process of decision-making.

Definition 1 (Rodriguez *et al.* 2011) Let $S = \{s_\alpha | \alpha = 0| 1| \ldots| 2\tau\}$ be a given linguistic term set, where $2\tau + 1$ is the granularity of the set S, if H_s is a continuous ordered subset about S, then, H_s is a hesitant fuzzy linguistic term set on S.

Definition 2 (Rodriguez *et al.* 2011) Let S be the linguistic term set expressed in grammar G_H and define the function of transforming grammar G_H into hesitant fuzzy linguistic term set H_s as E_{G_H}; then, the rules of transformation are as follows:

$$E_{G_H}(s_t) = \{s_t | s_t \in S\}$$

$$E_{G_H}(\boldsymbol{lessthan} s_n) = \{s_t | s_t \in \boldsymbol{S and} s_t < s_n\}$$

$$E_{G_H}(\text{no}\boldsymbol{morethan} s_n) = \{s_t | s_t \in \boldsymbol{S and} s_t \le s_n\}$$

$$E_{G_H}(\boldsymbol{more\ than} s_n) = \{s_t | s_t \in \boldsymbol{S and} s_t > s_n\}$$

$$E_{G_H}(\text{no}\boldsymbol{less\ than} s_n) = \{s_t | s_t \in \boldsymbol{S and} s_t \ge s_n\}$$

$$E_{G_H}\left(\boldsymbol{between} s_n \boldsymbol{and} s_q\right) = \left\{s_t | s_t \in \boldsymbol{S and} s_n \le s_t \le s_q\right\}$$

According to the above definitions, the hesitant fuzzy linguistic term set could be applied to the actual scenario of water pollution treatment alternative evaluation.

Set of evaluation criteria is defined as $C = \{c_1, c_2, \ldots, c_m\}$.

Set of decision-makers is defined as $D = \{d_1, d_2, \ldots, d_n\}$.

Set of treatment alternatives is defined as $A = \{a_1, a_2, \ldots, a_l\}$.

Then, for decision-maker j, the evaluation value of alternative a_u under criterion c_i is H_s^{ui}; a hesitant fuzzy linguistic term evaluation matrix of j could be constructed as follows:

$$H^j = \begin{pmatrix} H_s^{11} & H_s^{12} & \cdots & H_s^{1m} \\ H_s^{21} & H_s^{22} & \cdots & H_s^{2m} \\ \cdots & \cdots & \cdots & \cdots \\ H_s^{l1} & H_s^{l2} & \cdots & H_s^{lm} \end{pmatrix} \tag{8}$$

Each decision-maker gives his own evaluation value in the form of hesitant fuzzy linguistic term sets, and then, the rank of alternatives should be determined according to the weight of criteria, the weight of decision-makers, and the evaluation matrix given by each decision-maker.

Definition 3(Xu 2005) *Let $S = \{s_\alpha | \alpha = 0| 1| \ldots | 2\tau\}$ be a given linguistic term set, $s_\alpha, s_\beta \in S$ are two linguistic terms of S, then, the distance between s_α and s_β could be defined as follows:*

$$d(s_\alpha, s_\beta) = \frac{|\alpha - \beta|}{2\tau + 1} \tag{9}$$

where $2\tau + 1$ is the number of linguistic terms in the set S.

Definition 4 (Zhu and Xu 2013) *Different hesitant fuzzy linguistic term sets may have different numbers of linguistic terms in most circumstances, which leads to a dilemma when making comparisons between them. In order to make comparisons more reasonably, linguistic terms should be added to the shorter one.*

Let $b = \{b_l | l = 1| \ldots | \#b\}$ be a hesitant fuzzy linguistic term set, where $\#b$ is the number of linguistic terms in set b; b^+ and b^- are the maximum and minimum of linguistic terms in set b, respectively; then, linguistic term $\overline{b}$ could be added to set b.

$$\overline{b} = \xi b^+ \oplus (1-\xi) b^-, 0 \le \xi \le 1 \tag{10}$$

Without loss of generality, $\xi = \frac{1}{2}$ in this paper.

Definition 5 (Liao *et al.* 2014) *Let $S = \{s_\alpha | \alpha = 0| 1| \ldots | 2\tau\}$ be a given linguistic term set, $H_s^p = \left\{ s_{\delta_l^p} | l = 1| \ldots | \#H_s^p \right\}$ and $H_s^q = \left\{ s_{\delta_l^q} | l = 1| \ldots | \#H_s^q \right\}$ are two hesitant fuzzy linguistic term sets on S, where $\#H_s^p$ and $\#H_s^q$ represent the numbers of linguistic terms in H_s^p and H_s^q, respectively. Let $\#H_s^p = \#H_s^q = L$, that is, H_s^p and H_s^q have the same number of linguistic terms (otherwise, additional linguistic terms could be added into the shorter one in the form of Eq. (10).) Then, the Euclidean distance between H_s^p and H_s^q could be defined as in Eq. (11):*

$$d\left(H_s^p, H_s^q\right) = \left[\frac{1}{L} \sum_{l=1}^{L} \left(\frac{|\delta_l^p - \delta_l^q|}{2\tau + 1} \right)^2 \right]^{\frac{1}{2}} \tag{11}$$

According to the analysis above, each decision-maker would give his evaluation matrix of all the alternatives based on the criteria, which is composed of l evaluation sequences and each sequence represents the assessment of one alternative. Now the issue remains to be solved is that how to sort the alternatives in line with these sequences. One common method to deal with the issue is to set a reference sequence and then compare the sequences with the reference sequence. The rank of the alternatives is based on the distance between the evaluation sequences and the reference sequence. The gray incidence analysis (Deng 1989) is introduced in this paper to measure the similarity between the sequences.

Gray incidence analysis

The basic idea of the gray incidence analysis is to measure whether the correlation between different sequences is close, according to the similarity degree of geometric shape of the sequence curves. Based on the gray incidence analysis model proposed by professor Deng (1989), the classical definition of gray incidence degree is shown as follows.

Assume system behavior sequences to be:

$$\begin{aligned} X_0 &= (x_0(1), x_0(2), \cdots, x_0(n)) \\ X_1 &= (x_1(1), x_1(2), \cdots, x_1(n)) \\ &\vdots \\ X_i &= (x_i(1), x_i(2), \cdots, x_i(n)) \\ &\vdots \\ X_m &= (x_m(1), x_m(2), \cdots, x_m(n)) \end{aligned}$$

For $\zeta \in (0, 1)$, let

$$\gamma(x_0(k), x_i(k)) = \frac{\min_i \min_k |x_0(k) - x_i(k)| + \zeta \max_i \max_k |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \zeta \max_i \max_k |x_0(k) - x_i(k)|} \tag{12}$$

$$\gamma(X_0, X_i) = \frac{1}{n} \sum_{k=1}^{n} \gamma(x_0(k), x_i(k)) \tag{13}$$

Then, $\gamma(X_0, X_i)$ is defined as the gray incidence degree between X_0 and X_i (Deng 1989), ζ is the distinguishing coefficient, in general, $\zeta = 0.5$.

In Eq. (12), $|x_0(k) - x_i(k)|$ is an estimate of the distance between two points, as is discussed in Eq. (11); the Euclidean distance between two hesitant fuzzy linguistic term sets is determined. Thus, when the decision-makers' evaluation information is given in the form of hesitant fuzzy linguistic sets, Eq. (11) could be used to replace $|x_0(k) - x_i(k)|$ in Eq. (12); then, the gray incidence degree based on hesitant fuzzy linguistic term sets could be obtained.

In the scenario of water pollution treatment alternative evaluation, let reference sequence to be

$$H_s^0 = \left(H_s^{01}, H_s^{02}, \cdots H_s^{0m}\right),$$

and let the evaluation sequences of alternatives determined by decision-maker j to be:

$$\begin{aligned} H_s^1 &= \left(H_s^{11}, H_s^{12}, \cdots H_s^{1m}\right) \\ H_s^2 &= \left(H_s^{21}, H_s^{22}, \cdots H_s^{2m}\right) \\ &\vdots \\ H_s^l &= \left(H_s^{l1}, H_s^{l2}, \cdots H_s^{lm}\right) \end{aligned}$$

Then, under the measurement of decision-maker j, the gray incidence degree of alternative a_u and the reference sequence under the criterion c_i could be defined as follows:

$$\gamma^{ui} = \gamma\left(H_s^{0i}, H_s^{ui}\right) = \frac{\min_i \min_u d\left(H_s^{0i}, H_s^{ui}\right) + \zeta \max_i \max_u d\left(H_s^{0i}, H_s^{ui}\right)}{d\left(H_s^{0i}, H_s^{ui}\right) + \zeta \max_i \max_u d\left(H_s^{0i}, H_s^{ui}\right)} \tag{14}$$

That is,

$$\gamma^{ui} = \gamma\left(H_s^{0i}, H_s^{ui}\right) = \frac{\min_i \min_u \left[\frac{1}{L}\sum_{l=1}^{L}\left(\frac{|\delta_l^{0i}-\delta_l^{ui}|}{2\tau+1}\right)^2\right]^{\frac{1}{2}} + \zeta \max_i \max_u \left[\frac{1}{L}\sum_{l=1}^{L}\left(\frac{|\delta_l^{0i}-\delta_l^{ui}|}{2\tau+1}\right)^2\right]^{\frac{1}{2}}}{d\left(H_s^{0i}, H_s^{ui}\right) + \zeta \max_i \max_u \left[\frac{1}{L}\sum_{l=1}^{L}\left(\frac{|\delta_l^{0i}-\delta_l^{ui}|}{2\tau+1}\right)^2\right]^{\frac{1}{2}}} \tag{15}$$

where the evaluation of alternative a_u under the criterion c_i depends on the value of $\gamma\left(H_s^{0i}, H_s^{ui}\right)$.

In "Group-G1," the weight of each criterion has been determined, which is $\theta^i (i = 1, 2, \dots, m)$; then, for decision-maker j, the gray incidence degree between alternative a_u and the reference sequence is defined in Eq. (16):

$$G_u^j = \sum_{i=1}^{m} \theta^i \gamma^{ui} \tag{16}$$

Then, the preference towards alternative a_u of decision-maker j could be defined in Eq. (17):

$$\varphi_u^j = G_u^j / \sum_{u=1}^{l} G_u^j \tag{17}$$

Considering the weights of multiple decision-makers, the group preference towards the alternative a_u at a given stage is shown as follows:

$$\varphi_u = \sum_{j=1}^{n} \omega^j \varphi_u^j \tag{18}$$

The ranking of the alternative a_u at the stage is determined by the value of φ_u; then, the next step of this paper is to identify the weight of each stage in order to evaluate the alternatives from a global perspective.

Identification of the weight of each stage

Considering that in the evaluation process of water pollution treatment alternatives, the preferences of decision-makers will alter with the change of objectives at different stages, which results in different alternative rankings. Thus, it is necessary to determine the weight of each stage and aggregate the preferences of decision-makers at different stages so as to get an overall ranking of the water pollution treatment alternatives.

Suppose that the evaluation process of water pollution treatment alternatives is divided into p stages, according to the analysis from "Group-G1" to "Gray incidence analysis," each decision-maker sorts the criteria and assigns weights to them in p rounds; then, the weights of all the criteria in p stages could be obtained. Subsequently, the decision-makers would evaluate the criteria of each alternative. As the criteria weights may change among stages, the ranking of alternatives would also change corresponding to each stage. What remains to be solved is how to aggregate these rankings into a final ranking; thus, the weight of each stage should be identified.

As is discussed in "Gray incidence analysis", the group preference towards the alternatives at a given stage is determined; we could assume the group preference towards alternative a_u at the qth stage to be φ_{qu}, where $q = 1, 2, \dots, p$.

Since subjective weighting may cause deviations in some circumstances, we assume that the weight of each stage is equal, and then, we could make adjustments to them.

In the case of equal stage weight assignment, the group's average preference to alternative a_u is shown in Eq. (19):

$$\overline{\varphi_u} = \sum_{q=1}^{p} \varphi_{qu} / p \tag{19}$$

Then, we could adjust the stage weights according to the following principal: The weight of each stage should make the sum of the deviations between the group preference of each alternative in p stages and their average preferences be the smallest. Meanwhile, in water pollution treatment process, the objectives of the previous stage are often more important than those of the next stage, so another principal to meet is that the weight of the previous stage is no less than the weight of the next stage. Based on these requirements, the following goal planning model could be established.

$$Y = \min \sum_{q=1}^{p} \sum_{u=1}^{l} \left[\mu_q\left(\varphi_{qu} - \overline{\varphi_u}\right)\right]^2$$
$$s.t. \begin{cases} \mu_q \geq 0 \\ \sum_{q=1}^{p} \mu_q = 1, q = 1, 2, \dots p \\ \mu_q \geq \mu_{q+1} \end{cases} \tag{20}$$

After the weight of each stage $\mu_q (q = 1, 2, \dots p)$ is identified, the global comprehensive group preference for alternative a_u is defined as follows:

$$\varphi_u' = \sum_{q=1}^{p} \mu_q \varphi_{qu} \tag{21}$$

Finally, we only need to sort φ_u' by descending order, and in this way, could the final ranking of alternatives be obtained. Resource allocation in water pollution treatment alternatives should be based on the final results so as to realize the reasonable allocation of resources and maximization of utility.

Problem description

The cyanobacterial bloom in Taihu Lake

China is experiencing a golden period of rapid industrialization and modernization (Yang 2014). While enjoying the benefits brought by economic development, the pollution problems are becoming more and more serious. Among various pollution problems, water pollution is closely related to human production and life as all human activities are inseparable from water. Nevertheless, in economically developed areas, especially in the middle and lower reaches of the Yangtze River, due to the influx of heavy sewage, the activities of reclaiming farmlands from lakes and the decrease of aquatic vegetation coverage, water eutrophication is becoming more and more critical, which results in a large outbreak of algal blooms (Zhang et al. 2010a, b). Algal blooms refer to the phenomenon that a large amount of sewage containing nitrogen and phosphorus enters the water body, and cyanobacteria, green algae, and diatoms increase to large numbers so that the water body appears blue or green. Several important freshwater lake basins in China have been plagued by water blooms in recent years; conditions are even worse in the Taihu Lake, the Chaohu Lake, the Dianchi Lake, etc. Even in flowing waters, algal blooms have also appeared in recent years, such as the Hanjiang River, the largest tributary of the Yangtze River. With the gradual deepening of water eutrophication, the impact area of algal blooms will be rapidly expanded, bringing much more harm to human beings.

Cyanobacterial blooms are the most common form in algal blooms (Qin et al. 2015). The greatest harm caused by cyanobacterial blooms in freshwater is that they affect the safety of drinking water sources and aquatic products by producing odorous substances and cyanotoxins (Guo 2007). In particular, the secondary metabolite of cyanobacteria, microcystin, may cause nonalcoholic fatty liver by interfering with lipid metabolism (Carmichael 2001). Chronic exposure to cyanobacterial blooms for a long time could also lead to liver damage. In addition, microcystins may cause gallbladder hardening and atrophy; thus, the threat to human health caused by serious cyanobacterial blooms is increasing. Moreover, if the filter device of waterworks is stuffed by algae blooms, which could not carry out effective water purification work, water supply difficulties may also occur. The accumulation and anaerobic decomposition of cyanobacteria in water sources will make the water resources stinky and seriously affect the lives of residents. At the same time, the death of cyanobacteria will increase the deposition of lake sediment, resulting in the decline of the capacity of flood control. Cyanobacterial blooms may seriously destroy aquatic ecosystems as well (Micheli 1999; Vonlanthen et al. 2012), because cyanobacteria can survive smoothly in extremely harsh ecological environment, which inhibits the survival of other algae. Besides, a large number of cyanobacteria may float on the water surface when cyanobacteria erupts, which shades the sunlight, thus affecting the reproduction of submerged plants, fish, benthic animals, and even causing a large number of deaths of creatures. Therefore, each eruption of cyanobacterial blooms will bring serious disasters for human beings and nature (Paerl and Huisman 2008; Brookes and Carey 2011; Carey et al. 2012).

The area most affected by cyanobacterial blooms in China is the Taihu Lake watershed, which is located in the Yangtze River Delta of China. The whole Taihu Lake watershed is one of the areas with the best economic development and social prosperity in China, and it is the main source of drinking water in the surrounding cities. The Taihu Lake watershed also plays an irreplaceable role in agriculture, flood control and drought relief, shipping tourism, climate regulation, and ecological balance, and even in the development of the whole economy and society (Yang and Liu 2010).

However, the Taihu Lake watershed has been plagued by the outbreak of cyanobacteria since the 1990s (Zhang et al. 2011). At the end of May 2007, a large-scale outbreak of cyanobacteria in Taihu Lake caused serious “lake flooding,” which is a natural phenomenon of anaerobic reactions in the sediment of polluted lakes to produce odor. At the same time, it caused water supply difficulties in Wuxi and seriously affected the daily life of residents (Qin et al. 2010). After the outbreak of cyanobacteria in 2007, the government adopted a number of treatment alternatives to coordinate pollution control and had achieved some results. After 2007, cyanobacteria in Taihu Lake also broke out on a small scale, but the degree was not very serious, basically in a controllable range. With the development of human beings, it is quite difficult to eradicate the outbreak of cyanobacteria thoroughly. However, we can relieve this problem effectively by scientific and reasonable treatment alternatives. By means of long-term and multi-stage treatment work, we could gradually control the growth of cyanobacteria until the problem is eliminated.

During the treatment process, the government found that many alternatives were used for controlling cyanobacteria in Taihu Lake and these alternatives all had certain effects. Considering the multi-stage nature of water pollution treatment, there would be different emphases and objectives in each stage; thus in different stages, the evaluation of these alternatives is not the same. As resources are limited, it is necessary to allocate resources reasonably among various alternatives in order to maximize utility, which could not only offer assistance to the current pollution treatment but also provide references for the follow-up work.

A summary of cyanobacteria pollution treatment alternatives in Taihu Lake

The pollution treatment alternatives for the outbreak of cyanobacteria in Taihu Lake are introduced in this section. They have been applied in the real cases of cyanobacteria control in Taihu Lake and have achieved certain results (Le et al. 2010; Sun et al. 2016). A brief introduction of the alternatives is shown in Table 2.

In summary, a series of treatment alternatives after the outbreak of cyanobacterial blooms in the Taihu Lake watershed have been introduced. They have achieved certain results and alleviated the environmental pressure. However, in order to allocate resources reasonably, we could not distribute resources equally to all the alternatives. Instead, what we should do is to allocate resources according to the priorities of the alternatives. On the one hand, it could provide suggestions for the current work of improving the water quality of the Taihu Lake; on the other hand, it could also provide references for the follow-up treatment work. For the sake of evaluating the alternatives, we should determine the evaluation criteria; then, "Problem description" of the paper is the construction of the evaluation criteria system.

Criteria system for water pollution treatment alternatives

The issue of allocating resources to water pollution treatment alternatives according to the priorities is always a complex problem, as the evaluations and ranking of water pollution alternatives are based on multiple criteria. Thus, it is of great significance to construct a suitable evaluation criteria system.

On the basis of previous researches (Qu et al. 2019; Chen et al. 2018; Du et al. 2019), the evaluation criteria for pollution treatment alternatives are commonly comprised of four categories: economic, technical, environmental, and social, and

Table 2 A brief introduction of the alternatives

Treatment alternatives	A brief introduction
Mechanized salvage of cyanobacteria	Salvaging cyanobacteria could effectively and directly remove nitrogen, phosphorus, and organic matter in the water, which may significantly improve water quality and reduce the outbreak of cyanobacteria. To improve efficiency, large-scale salvage of cyanobacteria could be based on mechanized salvage.
Inter-basin water transfer	At present, the inter-basin water transfer projects in Taihu Lake watershed mainly include the project of "Water diversion from the Yangtze River to the Taihu Lake" and the water diversion project of the Meiliang Lake Pumping Station. The main aim is to divert Yangtze River water to replace the water of Taihu Lake, thus taking away a large number of nitrogen and phosphorus-containing substances and cyanobacteria dry substances in Taihu Lake water, so as to enhance its self-purification capacity..
Controlling pollution sources	Controlling pollution sources refers to the control and reduction of exogenous pollutants into the lake, mainly including a significant reduction in living, industrial, and agricultural pollution, as well as the reduction of surface runoff of nitrogen- and phosphorus-containing substances into the lake. Reducing the inflow of exogenous pollutants into the lake is mainly to reduce the inflow of nitrogen and phosphorus into the lake and to reduce the direct inflow of surface runoff into the lake.
Ecological remediation	Ecological remediation, also known as ecological reconstruction, mainly refers to the restoration of damaged ecosystem to the original level or exceeding the original level through a series of measures, and maintains its long-term stability and enters a virtuous circle. A good ecosystem could inhibit algae growth, purify water, and reduce nitrogen and phosphorus.
Ecological dredging	Due to the long-term impact of cyanobacterial blooms, the sediment of Taihu Lake has a high content of nitrogen, phosphorus and other elements, which also contains a large number of active cyanobacterial provenances. Therefore, regular cleaning of the sediment of Taihu Lake is an effective treatment scheme as well. In the implementation of silt removal work, attention should also be paid to the safety of construction to reduce the disturbance to the water body; otherwise, the work of silt removal will be worthless.
Flocculation sedimentation	The harmful algae blooms could be absorbed by clay flocculation. In recent years, this method has greatly improved the sedimentation efficiency and alleviated the environmental pressure caused by cyanobacterial blooms to a certain extent. However, there is an obvious disadvantage of flocculation sedimentation, that is, the clay will easily cause secondary pollution after cyanobacteria sedimentation, so the use of flocculation sedimentation near drinking water sources should be particularly cautious.

under each category, there are different secondary criteria. Considering the actual situation of water pollution treatment, an evaluation criteria system could be constructed as follows (Table 3).

Multi-stage gray group decision-making process based on hesitant fuzzy linguistic term sets

The process of water pollution treatment is a multi-stage and multi-objective process. At different stages of water pollution treatment, different decision-makers may have different goals and assign different weights to evaluation criteria, thus generating different preferences for alternatives, which leads to different ranking of alternatives at different stages. In order to obtain the global scheme ranking, it is necessary to aggregate the scheme ranking of each stage to get the final ranking.

Generally, water pollution treatment could be divided into three stages, that is, the initial stage, the middle stage, and the later stage. Decision-makers should rank and assign their own criteria weights according to the different objectives of different stages and give their own evaluation to the various criteria of the alternatives. The preference aggregation is carried out by using the gray group decision-making method given in "The proposed multi-stage gray group decision-making method" of this paper, and the concrete steps are as follows:

Step 1. Determination of water pollution treatment alternatives and evaluation criteria

Step 2. The decision-makers rank and assign weights to the evaluation criteria at each stage, and the weight of each criterion and each decision-maker is obtained.

Step 3. The decision-makers evaluate all the criteria of each alternative in the form of hesitant fuzzy linguistic set and determine the reference sequence.

Step 4. The gray incidence degrees of the alternatives at different stages are calculated, and the preferences towards the alternatives are obtained.

Step 5. Identification of the weight of each stage

Step 6. Calculate the comprehensive group preference towards all the alternatives and rank them in descending order according to the preference.

Step 7. Resources are allocated according to the ranking results of alternatives, and the alternative with high comprehensive ranking should be distributed more resources.

An illustrative example

As is discussed above, cyanobacterial blooms in the Taihu Lake watershed have always been a major issue affecting regional environmental safety. Based on the current situation of the continuous outbreak of cyanobacterial blooms in the Taihu Lake watershed, in order to effectively control water pollution and provide references for the follow-up treatment work, it is necessary to determine the ranking of pollution treatment alternatives under three-stage constraints, so as to rationally allocate resources to achieve the optimal treatment effect.

Thus, four experts from the field of environment and water conservancy are invited to assess the existing alternatives so as to obtain the overall ranking of alternatives. Four experts are represented by D_1 to D_4; treatment alternatives are represented by A_1 to A_6, which refers to mechanized salvage of cyanobacteria, inter-basin water transfer, controlling pollution sources, ecological remediation, ecological dredging, and flocculation sedimentation separately. The criteria are represented by C_1 to C_8 as mentioned above, and the three stages of water pollution treatment are expressed in terms of T_1 to T_3. The

Table 3 Criteria system for water pollution treatment alternatives

Primary criteria	Secondary criteria	Criteria description
Economic	Implementation cost (C_1)	Refer to the cost of putting the alternative to practice and maintaining operation, including material cost and labor cost
	Economic return (C_2)	Expected economic benefit from the implementation of the alternative
Technical	Scheme maturity (C_3)	Whether the technology of the alternative is consummate
	Scheme timeliness (C_4)	Whether the alternative could control pollution to a certain extent in a relatively short time
	Scheme effectiveness (C_5)	Refer to the degree of pollution treatment the alternative could achieve
Environmental	Restoration of ecological environment (C_6)	Refer to the degree of ecological environment restoration through the implementation of the alternative
	Secondary pollution (C_7)	Whether the implementation of the alternative would result in secondary pollution discharge
Social	Public recognition (C_8)	Refer to the social acceptance of implementing the alternative

experts use a set of linguistic terms with seven language scales for evaluation, where in the set S, s_0 = ***very poor***, s_1 = ***poor***, s_2 = ***mediumpoor***,s_3 = ***medium***, s_4 = ***mediumgood***, s_5 = ***good***, and s_6 = ***verygood***. The ranking and the assignment of the criteria weight at three stages given by the experts are shown in Table 4, and the evaluation matrices are shown in Table 5.

According to the group decision-making method proposed in "The proposed multi-stage gray group decision-making method," the computational process could be shown in the following steps.

Step 1. Determination of the criteria weight given by each expert at three stages

At stage T_1 to T_3, the weight matrices calculated are as follows:

$$W^1 = \begin{bmatrix} 0.0834 & 0.0521 & 0.1441 & 0.2074 & 0.1201 & 0.1201 & 0.1000 & 0.1729 \\ 0.0952 & 0.0567 & 0.1919 & 0.2303 & 0.1371 & 0.0952 & 0.0793 & 0.1142 \\ 0.0952 & 0.0496 & 0.1143 & 0.2303 & 0.1600 & 0.0793 & 0.0793 & 0.1920 \\ 0.0796 & 0.0415 & 0.1114 & 0.2311 & 0.1925 & 0.1337 & 0.0497 & 0.1605 \end{bmatrix}$$

$$W^2 = \begin{bmatrix} 0.1572 & 0.1310 & 0.1572 & 0.0568 & 0.1886 & 0.1091 & 0.1091 & 0.0910 \\ 0.1323 & 0.0787 & 0.1852 & 0.0364 & 0.2222 & 0.1852 & 0.0945 & 0.0656 \\ 0.1224 & 0.0729 & 0.2115 & 0.0455 & 0.1763 & 0.1469 & 0.1224 & 0.1020 \\ 0.1560 & 0.1083 & 0.1560 & 0.0553 & 0.1872 & 0.1300 & 0.1300 & 0.0774 \end{bmatrix}$$

$$W^3 = \begin{bmatrix} 0.1104 & 0.1325 & 0.0767 & 0.0479 & 0.1590 & 0.1908 & 0.1908 & 0.0920 \\ 0.0976 & 0.1639 & 0.0581 & 0.0415 & 0.1366 & 0.2360 & 0.1967 & 0.0697 \\ 0.1218 & 0.1705 & 0.0870 & 0.0518 & 0.0870 & 0.2047 & 0.2047 & 0.0725 \\ 0.1325 & 0.1590 & 0.0767 & 0.0479 & 0.1104 & 0.1908 & 0.1908 & 0.0920 \end{bmatrix}$$

Step 2. Convert the ranking of criteria into numerical values based on Eq. (3) and obtain the mean value of all the criteria at different stages based on Eq. (4).

At stage T_1, the mean value vector of the criteria is":

$$V^1 = (3.25, 1, 5.5, 8, 6, 4.25, 2.5, 6.25)$$

At stage T_2, the mean value vector of the criteria is::

$$V^2 = (6, 3.25, 7.25, 1, 7.75, 5.5, 4.5, 2.25)$$

At stage T_3, the mean value vector of the criteria is::

$$V^3 = (4.5, 5.75, 2.5, 1, 4.75, 8, 7.75, 2.75)$$

Step 3. Determine the group opinion of the criteria ranking based on Step 2 and measure the similarity degree between group and individual opinions.

According to the results in Step 2, the evaluation criteria could be resorted based on the mean value to form a group ranking sequence and the group ranking sequence of the criteria is a reflection of the group opinion.

As for stage T_1, the group ranking sequence of the criteria is A_o = (6, 8, 4, 1, 3, 5, 7, 2) and the individual ranking sequence could be obtained from Table 3. Using Spearman's rank correlation coefficient given in Eq. (5), the similarity degree between group and individual opinions is obtained:

$$\rho_{o1} = 0.9405, \rho_{o2} = 0.8929, \rho_{o3} = 0.9643, \rho_{o4} = 0.9524$$

Similarly, we could obtain the similarity degree between group and individual opinions at stage T_2 and T_3.

At stage T_2, ρ_{o1} = 0.9405, ρ_{o2} = 0.9405, ρ_{o3} = 0.9167, ρ_{o4} = 0.9762, while at stage T_3, ρ_{o1} = 0.9643, ρ_{o2} = 1, ρ_{o3} = 0.9167, ρ_{o4} = 0.9643.

Step 4. Determine the weight of each expert and each criterion at different stages based on Eq. (6) and Eq. (7).

At stage T_1, the weight of the experts is ω^1 = (0.2508, 0.2381, 0.2571, 0.2540) and the weight of criteria is θ^1 = (0.0883, 0.0499, 0.1395, 0.2248, 0.1528, 0.1071, 0.0765, 0.1607).

At stage T_2, the weight of the experts is ω^2 = (0.2492, 0.2492, 0.2429, 0.2587) and the weight of criteria is θ^2 = (0.1422, 0.0980, 0.1771, 0.0486, 0.1936, 0.1427, 0.1141, 0.0838).

At stage T_3, the weight of the experts is ω^3 = (0.2508, 0.2601, 0.2384, 0.2508) and the weight of criteria is θ^3 = (0.1153, 0.1564, 0.0743, 0.0472, 0.1238, 0.2059, 0.1957, 0.0816).

Step 5. Calculation of the group preferences to the alternatives at different stages

Assume the reference sequence to be $[\{s_6\}\{s_6\}\{s_6\}\{$-$s6\}\{s_6\}\{s_6\}\{s_6\}\{s_6\}]$, then, based on the method proposed in "The proposed multi-stage gray group decision-making method,", the gray incidence degree could be obtained.

According to Table 4, the evaluation matrices could be transformed into the matrices which measure the distance between the alternatives and the reference sequence.

$$H^1 = \begin{bmatrix} 0.1429 & 0 & 0.1844 & 0 & 0.2259 & 0.4286 & 0.2857 & 0.2259 \\ 0.5714 & 0.5051 & 0.2259 & 0.1010 & 0.1844 & 0.2857 & 0.2857 & 0.1429 \\ 0.3642 & 0.5714 & 0 & 0.5051 & 0.2259 & 0.4286 & 0.2259 & 0.3642 \\ 0.5714 & 0.2259 & 0.2259 & 0.5714 & 0.1429 & 0 & 0.1010 & 0.1010 \\ 0.2857 & 0.5051 & 0.3642 & 0.5714 & 0.3642 & 0.2259 & 0.4286 & 0.4286 \\ 0.2259 & 0.5714 & 0.4442 & 0.2857 & 0.4286 & 0.5051 & 0.5714 & 0.5051 \end{bmatrix}$$

$$H^2 = \begin{bmatrix} 0.2857 & 0.1010 & 0.1429 & 0 & 0.3086 & 0.2857 & 0.1429 & 0.1429 \\ 0.5051 & 0.4442 & 0.1010 & 0.1429 & 0.2259 & 0.1429 & 0.2259 & 0.2857 \\ 0.2857 & 0.5051 & 0 & 0.5051 & 0.2259 & 0.4286 & 0.1429 & 0.3642 \\ 0.5051 & 0.2259 & 0.1429 & 0.5051 & 0.1429 & 0 & 0 & 0.1429 \\ 0.2857 & 0.5051 & 0.4286 & 0.5051 & 0.3642 & 0.2259 & 0.3642 & 0.4286 \\ 0.1429 & 0.5051 & 0.5051 & 0.2857 & 0.4286 & 0.5051 & 0.7143 & 0.5714 \end{bmatrix}$$

$$H^3 = \begin{bmatrix} 0.2259 & 0 & 0.1429 & 0 & 0.2259 & 0.2857 & 0.1429 & 0.1429 \\ 0.4286 & 0.5051 & 0.1010 & 0.2259 & 0.2259 & 0.3086 & 0.2259 & 0.2259 \\ 0.2259 & 0.5051 & 0 & 0.4442 & 0.1010 & 0.3642 & 0.1429 & 0.3642 \\ 0.5051 & 0.1429 & 0.1429 & 0.5051 & 0.1429 & 0.1010 & 0 & 0.1429 \\ 0.3642 & 0.5051 & 0.5714 & 0.5051 & 0.3642 & 0.1429 & 0.3642 & 0.4286 \\ 0.1429 & 0.5051 & 0.2857 & 0.2259 & 0.4286 & 0.5051 & 0.6468 & 0.5714 \end{bmatrix}$$

$$H^4 = \begin{bmatrix} 0.1429 & 0 & 0.1010 & 0 & 0.2259 & 0.4286 & 0.2857 & 0.2259 \\ 0.5714 & 0.5051 & 0.3642 & 0.1010 & 0.1010 & 0.2857 & 0.4286 & 0.1429 \\ 0.3642 & 0.5714 & 0.1010 & 0.4286 & 0.2259 & 0.4286 & 0.2857 & 0.3642 \\ 0.5714 & 0.2259 & 0.2857 & 0.5051 & 0.1429 & 0 & 0.1429 & 0.1844 \\ 0.3642 & 0.5051 & 0.4442 & 0.5714 & 0.3642 & 0.1429 & 0.4286 & 0.5051 \\ 0.2259 & 0.5714 & 0.5051 & 0.2857 & 0.4286 & 0.5051 & 0.5051 & 0.5051 \end{bmatrix}$$

Then, based on "Gray incidence analysis," the group preferences to each alternative at different stages could be obtained, which are shown as follows.

At stage T_1, the group preference vector is 0.2115, 0.1838, 0.1629, 0.1834, 0.1316, 0.1327.

At stage T_2, the group preference vector is 0.1946, 0.1654, 0.1742, 0.1977, 0.1386, 0.1294.

At stage T_3, the group preference vector is 0.1981, 0.1611, 0.1596, 0.2128, 0.1430, 0.1256.

Step 6. Identify the weight of each stage.

Based on "Identification of the weight of each stage,", the weight of each stage could be obtained. According to the results calculated above, in the case of equal stage weight assignment, the vector of group's average preference to alternatives could be defined as 0.2014, 0.1701, 0.1656, 0.1980, 0.1377, 0.1292.

Then, we could adjust the weight of each stage based on Eq. (20), and the weights of the three stages are 0.3441, 0.3441, and 0.3117, respectively.

Step 7. Determine the comprehensive group preferences to the alternatives and rank them in descending order.

In line with Eq. (21), the comprehensive group preference vector is shown as follows.

$$\varphi' = (0.2049, 0.1704, 0.1657, 0.1975, 0.1375, 0.1293)$$

Through the calculation process of the above steps, the final priorities of the six alternatives for treating cyanobacterial outbreak in the Taihu Lake watershed are determined under global constraints. We could find that alternative A_1 (mechanized salvage of cyanobacteria) is the best alternative under the three-stage objective constraint of water pollution treatment. And according to the group preference value, 20.49% of the total resources could be allocated to it. Alternative A_4 (ecological remediation) is the global sub-optimal scheme, so the resources allocated to the alternative account is 19.75% of the total amount. For alternatives A_2 (inter-basin water transfer) and A_3 (controlling pollution sources), the proportions of the

Table 4 Ranking and assignment of the criteria weight at three stages

	Criteria ranking at three stages								r_i at three stages						
T_1	C_4	C_8	C_3	C_5	C_6	C_7	C_1	C_2	1.2	1.2	1.2	1.0	1.2	1.2	1.6
	C_4	C_3	C_5	C_8	C_1	C_6	C_7	C_2	1.2	1.4	1.2	1.2	1.0	1.2	1.4
	C_4	C_8	C_5	C_3	C_1	C_6	C_7	C_2	1.2	1.2	1.4	1.2	1.2	1.0	1.6
	C_4	C_5	C_8	C_6	C_3	C_1	C_7	C_2	1.2	1.2	1.2	1.2	1.4	1.6	1.2
T_2	C_5	C_3	C_1	C_2	C_6	C_7	C_8	C_4	1.2	1.0	1.2	1.2	1.0	1.2	1.6
	C_5	C_3	C_6	C_1	C_7	C_2	C_8	C_4	1.2	1.0	1.4	1.4	1.2	1.2	1.8
	C_3	C_5	C_6	C_1	C_7	C_8	C_2	C_4	1.2	1.2	1.2	1.0	1.2	1.4	1.6
	C_5	C_1	C_3	C_6	C_7	C_2	C_8	C_4	1.2	1.0	1.2	1.0	1.2	1.4	1.4
T_3	C_6	C_7	C_5	C_2	C_1	C_8	C_3	C_4	1.0	1.2	1.2	1.2	1.2	1.2	1.6
	C_6	C_7	C_2	C_5	C_1	C_8	C_3	C_4	1.2	1.2	1.2	1.4	1.4	1.2	1.4
	C_6	C_7	C_2	C_1	C_3	C_5	C_8	C_4	1.0	1.2	1.4	1.4	1.0	1.2	1.4
	C_6	C_7	C_2	C_1	C_5	C_8	C_3	C_4	1.0	1.2	1.2	1.2	1.2	1.2	1.6

resources allocated could be 17.04% and 16.57%, respectively. With regard to alternative A_5 (ecological dredging) and alternative A_6 (flocculation sedimentation), the comprehensive group preferences are relatively poor, and 13.75% and 12.93% of the resources could be distributed to them for scheme construction. According to the above calculation results, it could be concluded that the final priorities of the alternatives are as follows, which is $A_1 \succ A_4 \succ A_2 \succ A_3 \succ A_5 \succ A_6$. Thus, the allocation method of the resources for water pollution treatment could be obtained.

It is obviously shown above that at different stages of water pollution treatment process, the objectives may vary, which results in the change of alternative priorities. At the initial stage of pollution treatment, alternative A_1 (mechanized salvage of cyanobacteria) performs well because of its advantage in controlling pollution to a certain extent in a relatively short time; at the middle stage, alternatives A_1 (mechanized salvage of cyanobacteria) and A_4 (ecological remediation) rank ahead as they possess mature technology and valid treatment effects, while at the final stage, alternative A_4 (ecological remediation) is the best one for the reason that it could ensure the coordinated development of economy and environment. In a word, each alternative has its own merits and demerits at different stages and determining the sequence of alternatives under multi-stage constraints could help us better solve the pollution treatment issue.

Conclusions and discussions

In this paper, a multi-stage gray group decision-making method based on hesitant fuzzy linguistic term sets is proposed to deal with the issue of resource allocation in water pollution treatment so as to offer some references for current and future pollution treatment work. As resources are limited, it is impossible to distribute resources

Table 5 Evaluation matrices

$$H^1 = \begin{bmatrix} \{s_5\} & \{s_6\} & \{s_4,s_5,s_6\} & \{s_6\} & \{s_4,s_5\} & \{s_3\} & \{s_4\} & \{s_4,s_5\} \\ \{s_2\} & \{s_2,s_3\} & \{s_4,s_5\} & \{s_5,s_6\} & \{s_4,s_5,s_6\} & \{s_4\} & \{s_4\} & \{s_5\} \\ \{s_3,s_4\} & \{s_2\} & \{s_6\} & \{s_2,s_3\} & \{s_4,s_5\} & \{s_3\} & \{s_4,s_5\} & \{s_3,s_4\} \\ \{s_2\} & \{s_4,s_5\} & \{s_4,s_5\} & \{s_2\} & \{s_5\} & \{s_6\} & \{s_5,s_6\} & \{s_5,s_6\} \\ \{s_4\} & \{s_2,s_3\} & \{s_3,s_4\} & \{s_2\} & \{s_3,s_4\} & \{s_4,s_5\} & \{s_3\} & \{s_3\} \\ \{s_4,s_5\} & \{s_2\} & \{s_2,s_3,s_4\} & \{s_4\} & \{s_3\} & \{s_2,s_3\} & \{s_2\} & \{s_2,s_3\} \end{bmatrix}$$

$$H^2 = \begin{bmatrix} \{s_4\} & \{s_5,s_6\} & \{s_5\} & \{s_6\} & \{s_3,s_4,s_5\} & \{s_4\} & \{s_5\} & \{s_5\} \\ \{s_2,s_3\} & \{s_2,s_3,s_4\} & \{s_5,s_6\} & \{s_5\} & \{s_4,s_5\} & \{s_5\} & \{s_4,s_5\} & \{s_4\} \\ \{s_4\} & \{s_2,s_3\} & \{s_6\} & \{s_2,s_3\} & \{s_4,s_5\} & \{s_3\} & \{s_5\} & \{s_3,s_4\} \\ \{s_2,s_3\} & \{s_4,s_5\} & \{s_5\} & \{s_2,s_3\} & \{s_5\} & \{s_6\} & \{s_6\} & \{s_5\} \\ \{s_4\} & \{s_2,s_3\} & \{s_3\} & \{s_2,s_3\} & \{s_3,s_4\} & \{s_4,s_5\} & \{s_3,s_4\} & \{s_3\} \\ \{s_5\} & \{s_2,s_3\} & \{s_2,s_3\} & \{s_4\} & \{s_3\} & \{s_2,s_3\} & \{s_1\} & \{s_2\} \end{bmatrix}$$

$$H^3 = \begin{bmatrix} \{s_4,s_5\} & \{s_6\} & \{s_5\} & \{s_6\} & \{s_4,s_5\} & \{s_4\} & \{s_5\} & \{s_5\} \\ \{s_3\} & \{s_2,s_3\} & \{s_5,s_6\} & \{s_4,s_5\} & \{s_4,s_5\} & \{s_3,s_4,s_5\} & \{s_4,s_5\} & \{s_4,s_5\} \\ \{s_4,s_5\} & \{s_2,s_3\} & \{s_6\} & \{s_2,s_3,s_4\} & \{s_5,s_6\} & \{s_3,s_4\} & \{s_5\} & \{s_3,s_4\} \\ \{s_2,s_3\} & \{s_5\} & \{s_5\} & \{s_2,s_3\} & \{s_5\} & \{s_5,s_6\} & \{s_6\} & \{s_5\} \\ \{s_3,s_4\} & \{s_2,s_3\} & \{s_2\} & \{s_2,s_3\} & \{s_3,s_4\} & \{s_5\} & \{s_3,s_4\} & \{s_3\} \\ \{s_5\} & \{s_2,s_3\} & \{s_4\} & \{s_4,s_5\} & \{s_3\} & \{s_2,s_3\} & \{s_1,s_2\} & \{s_2\} \end{bmatrix}$$

$$H^4 = \begin{bmatrix} \{s_5\} & \{s_6\} & \{s_5,s_6\} & \{s_6\} & \{s_4,s_5\} & \{s_3\} & \{s_4\} & \{s_4,s_5\} \\ \{s_2\} & \{s_2,s_3\} & \{s_3,s_4\} & \{s_5,s_6\} & \{s_5,s_6\} & \{s_4\} & \{s_3\} & \{s_5\} \\ \{s_3,s_4\} & \{s_2\} & \{s_5,s_6\} & \{s_3\} & \{s_4,s_5\} & \{s_3\} & \{s_4\} & \{s_3,s_4\} \\ \{s_2\} & \{s_4,s_5\} & \{s_4\} & \{s_2,s_3\} & \{s_5\} & \{s_6\} & \{s_5\} & \{s_4,s_5,s_6\} \\ \{s_3,s_4\} & \{s_2,s_3\} & \{s_2,s_3,s_4\} & \{s_2\} & \{s_3,s_4\} & \{s_5\} & \{s_3\} & \{s_2,s_3\} \\ \{s_4,s_5\} & \{s_2\} & \{s_2,s_3\} & \{s_4\} & \{s_3\} & \{s_2,s_3\} & \{s_2,s_3\} & \{s_2,s_3\} \end{bmatrix}$$

equally; thus, it is of great significance to find a suitable approach to rank the treatment alternatives under multi-stage constraints, in which the weights of decision-makers, evaluation criteria, and the stages should also be taken into consideration. The contributions of this paper could be listed as follows.

First, as the issue of alternative evaluation is a rather complex and significant one, a group decision-making framework is established to reflect different opinions during the decision-making process and then convert the opinions into an overall opinion under the constraints. In addition, traditional evaluation process of water pollution treatment alternatives is merely based on one stage, which neglects the changes of objectives during the water pollution treatment process, so the paper proposes a multi-stage gray group decision-making method to settle this issue.

Second, considering the differences and deficiencies in cognitive ability and expertise level of decision-makers, the paper describes the evaluation information of each decision-maker in the form of hesitant fuzzy linguistic term sets and then uses the gray incidence analysis for follow-up work. And in this way, the decision-makers participating in the evaluation process would not find it difficult to give their evaluation information.

Third, the method proposed in this paper could not only be used for alternative evaluation, such as the illustrative example in "An illustrative example," but also for many other environmental decision-making processes, no matter if the process is based on one stage or more.

Last but not least, the paper puts forward the selection process of water pollution treatment alternatives from the perspective of efficient utilization of resources, which provides a certain reference value for the government to formulate pollution treatment measures and take specific actions in the modern era of increasingly scarce resources.

However, this paper also has its demerits. It assumes the evaluation criteria and alternatives to remain the same during the process of evaluation, where sometimes it is not the case. So in terms of future research, the criteria and alternatives may be dynamically altered in the process of water pollution treatment. In another word, some criteria and alternatives may disappear or join in during the treating process; then, proposing a new method which takes the circumstance into consideration would be an interesting research field. Besides, the decision-makers may also change at different stages of the treating process; thus, considering this situation would be another research direction.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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