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Estimation of Uncertainty for Contour Method Residual Stress Measurements

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Abstract This paper describes a methodology for the estimation of measurement uncertainty for the contour method, where the contour method is an experimental technique for measuring a two-dimensional map of residual stress over a plane. Random error sources including the error arising from noise in displacement measurements and the smoothing of the displacement surfaces are accounted for in the uncertainty analysis. The output is a two-dimensional, spatially varying uncertainty estimate such that every point on the cross-section where residual stress is determined has a corresponding uncertainty value. Both numerical and physical experiments are reported, which are used to support the usefulness of the proposed uncertainty estimator. The uncertainty estimator shows the contour method to have larger uncertainty near the perimeter of the measurement plane. For the experiments, which were performed on a quenched aluminum bar with a cross section of 51×76 mm, the estimated uncertainty was approximately 5 MPa ($\sigma/E=7\cdot10^{-5}$) over the majority of the cross-section, with localized areas of higher uncertainty, up to 10 MPa ($\sigma/E=14\cdot10^{-5}$).

Keywords Residual stress measurement · Contour method · Uncertainty quantification · Repeatability · Aluminum alloy 7050-T74 · Quenching

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Introduction

The contour method is a residual stress measurement technique that is capable of generating a two-dimensional map of residual stress over a plane in a body. The theoretical underpinning of the method is given by Prime [1], and is based on the fact that when a residual stress bearing body is cut in half, stresses are released and redistributed within the body, which causes deformation. The out-of-plane displacements are directly related to the stresses released, and the technique comprises measurement of the out-of-plane displacements at the cut plane and use of the measured displacements to determine residual stress at the cut plane via elastic stress analysis (Fig. 1). The contour method has been used to measure residual stress for a variety of conditions [2–5].

The steps of a contour method measurement are as follows. First, the part is securely clamped and cut in half at the location of the measurement using a wire electric discharge machine (EDM). The goal of the clamping is to minimize movement of the part during cutting. The use of wire EDM is preferred because the cutting action is localized to the cut tip and because the quality of the cut is high. After the cut, the cut surface topography of each half is measured using a precision metrology device, typically a laser profilometer or coordinate measuring machine. The measured surface displacements contain high frequency noise, which is caused by measurement error and surface roughness from EDM cutting. Data processing is performed to extract the form of the surfaces (filtering out the noise) and to average the displacements from the opposite cut faces. The final step in a contour method measurement is to apply the processed displacements as boundary conditions to an elastic finite element model of the part. The resulting stresses in the finite model represent the initial residual stress at the plane of the measurement.

An uncertainty estimate is required to assess the quality of a measurement [6]. Typically the reported uncertainty is the

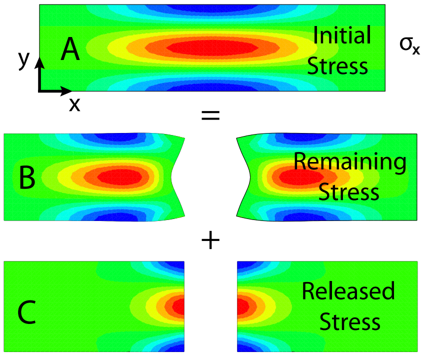


Fig. 1 Contour method steps: Configuration (**A**) is a body containing residual stresses with the color scale corresponding to σ_{xx}; in configuration (**B**), the body has been cut in half, creating a new stress-free surface; configuration (**C**) shows the stresses that were released in going from A to B, which can be found by reversing the cut surface deformation. Assuming elastic behavior, superposition provides A=B+C, and since σ_{xx} on the cut plane in B is zero (*free surface*), then σ_{xx} on the cut plane in A must be equal to σ_{xx} on the cut plane in C

precision of the measurement, which is only affected by random error sources [7]. The two most common approaches for uncertainty estimation are propagating uncertainty through a data analysis equation [8] and making a series of repeat measurements to find the variability in the results (i.e., first order uncertainty [9]). Uncertainty propagation is not appropriate for the contour method because the technique does not have an analytical equation to calculate stress from the measurement data. Instead the contour method relies on a finite element computation. Repeat measurements are possible for certain contour method measurements, but would be undesirable for many situations due to the additional effort required.

Currently, only limited work has been performed for uncertainty estimation of the contour method. Some researchers have used an *overall* uncertainty estimate (e.g., a single uncertainty value for every measured point on the cross-section) based on the noise present in the surface topography data [10, 11] and measuring the surface topography with different instruments [12]. Another study found the uncertainty using differences in residual stress from different levels of smoothing [13], but required that the surface use a similar level of smoothing in both spatial directions. The objectives of this work are to develop a single measurement uncertainty estimator for the contour method, where the estimated uncertainty is a function of in-plane position and is applicable for arbitrary surface smoothing, and to show the uncertainty estimator is a useful predictor of measurement uncertainty.

Methods

The methodology for contour method uncertainty estimation is based on quantifying the uncertainty for random error sources. Two random error sources identified for the contour method are the uncertainty due to noise in the displacement surfaces and the uncertainty arising from smoothing of the displacement surfaces. The uncertainty due to noise in the displacement surfaces is caused by the inherent surface roughness of the EDM cuts as well as the measurement error present in displacement measurement signal. Both of these uncertainty sources show up as high frequency noise in the displacement surface. The uncertainty associated with the noise in the displacement data will be called the "displacement" error. The other error source addressed is the uncertainty arising from the smoothing used in data processing that is required to extract the underlying form of the measured data. The amount of smoothing is selected based upon limited knowledge. Even if the optimal amount of smoothing were selected, there is no guarantee that the extracted form will exactly match the underlying trend in the displacement data. Thus, some amount of error is introduced from the data processing and this will be called the "model error." This approach for uncertainty estimation is similar to the methodology developed by Prime and Hill for uncertainty estimation in the slitting method for residual stress measurement [14].

Model Error

For contour method measurements, the analysis to extract the form of the experimental surface displacement (and simultaneously filter out the noise) is typically accomplished by fitting the measured surface displacements to a bivariate analytical surface. To illustrate the model error concept, consider a general bivariate, tensor product analytical surface

$$f(x,y) = \sum_{i=0}^{m} \sum_{j=0}^{n} C_{ij} P_i(x) P_j(y) \quad (1)$$

where x and y are spatial dimensions in the cut plane, C_{ij} are coefficients, $P_i(x)$ and $P_j(y)$ are known basis functions, and m and n are the highest order terms included in the series corresponding to the x and y spatial dimensions. The amount of smoothing is related to the choice of the fitting model (i.e., Equation (1) and parameters m and n). The choice of the fitting model and parameters affects the contour method result. Earlier work on the slitting method for residual stress measurement [14] treated the smoothing model as a source of uncertainty, referred to as the "model error." Adapting that approach to the contour method, the model error is estimated by taking the standard deviation of the computed residual stresses over a range of smoothing parameters. The model error definition used is

$$U_{Model} = std\left(\sigma_{m,n}, \sigma_{m+1,n}, \sigma_{m,n+1}, \sigma_{m-1,n}, \sigma_{m,n-1}\right) \quad (2)$$

where U_{Model} is the model error, σ is residual stress determined by the contour method for a given smoothing model, and the subscripts refer to the amount of smoothing used in the contour data analysis. This definition of the model error includes a range of smoothing in both spatial directions and both higher and lower in spatial frequency, which provides a measure of the sensitivity of the computed residual stress to the selected fitting model. The model error is assumed to have a Gaussian distribution, which has been found to be appropriate for many experimental error sources [9], and implies that one standard deviation represents a confidence interval of 68 %.

Displacement Error

The contribution of the noise in the measured displacement field to the total uncertainty can be quantified using a Monte Carlo approach [15]. A statistical measure of the random noise in the experimental data is calculated from the residual of the displacement data, after the form of the surface has been extracted by fitting, where the residual is the difference between the measured surface profile and the smoothed surface profile. To estimate the influence of noise on measured residual stress, the contour method data analysis is repeated with normally distributed noise added to the measured displacements (i.e., the displacement data now contain the noise that was originally present plus random noise that was artificially introduced into the analysis). The added normally distributed noise has a standard deviation equal to the standard deviation of the residual of the displacement data. The residual stress is computed after introduction of the additional noise, and the process is repeated several times to develop a set of residual stress results, each computed with different random noise (but in every case the standard deviation of the noise is the same). The displacement error is estimated by computing the standard deviation of the set of residual stress results with added noise, at each spatial location.

Total Uncertainty

The two uncertainty sources are combined together to estimate the total uncertainty. First, the root sum square of the two uncertainty sources is calculated according to

$$U_{RSS}(x,y) = \sqrt{U^2_{Disp}(x,y) + U^2_{Model}(x,y)}. \quad (3)$$

Next, the mean value of the root sum square uncertainty for all points on the cross-section is calculated

$$\overline{U}_{RSS} = \frac{\sum_{i=1}^{N} U_{RSS}(x_i, y_i)}{N} \quad (4)$$

where (x_i, y_i) are a set of N points having roughly uniform spacing over the cross section. The pointwise measurement uncertainty is then defined as the greater of the root sum square uncertainty at each point, or the mean value

$$U_{TOT}(x,y) = \max\left(U_{RSS}(x,y), \overline{U}_{Rss}\right). \quad (5)$$

The inclusion of the mean value places a floor on the estimated uncertainty and ensures that it remains at a reasonable level over the entire cross section.

Numerical Experiment

To assess the validity of the uncertainty estimate a numerical experiment was conducted using linear elastic finite element analysis. The geometry of the finite element model was a block with a cross-section of 50 mm (x-direction) by 25 mm (y-direction) and a length of 300 mm (z-direction). The finite element mesh used 93,750 eight-node linear brick elements with generic aluminum material properties (E=70 GPa, ν= 0.3). The model was initialized with an equilibrium residual stress field that is a tensor product of a Gaussian in x and biquadratic in y

$$\sigma_{zz}(x,y) = 72.115 \cdot f(x)g(y) - 105.6 \quad (6)$$

$$f(x) = 10\left(e^{-x^2/184.32} - e^{-x^2/128}\right) - 0.261 \quad (7)$$

$$g(y) = \frac{12.5^4 - y^4}{5000} - 1 \quad (8)$$

for x between −25 and 25 mm, y between −12.5 and 12.5 mm, and stress in MPa. The resulting stress after an equilibrium step is referred to as the "known" residual stress. The steps used in the numerical experiment are as follows: determine the known stress, simulate cutting the part in half, record the resulting displacements on the plane of interest (these become the simulated experimental data), and estimate the residual

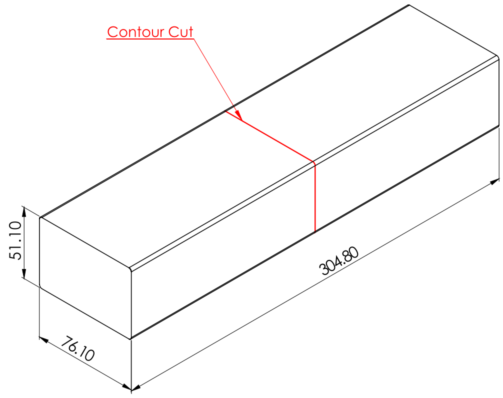


Fig. 2 Diagram of aluminum block and contour measurement location. Dimensions are in mm

stress using the contour method, which includes fitting the displacement field. After the residual stress is calculated, the uncertainty is computed using equation (5).

To evaluate the usefulness of the uncertainty estimate, the estimated residual stress (i.e., the "measured" value from the numerical experiment) is compared to the known residual stress. The uncertainty estimator is assumed to be performing appropriately when the estimated stress±the uncertainty estimator provides a bound that overlaps the known stress for a percentage of points equal to a stated confidence interval. The total uncertainty in this case is only the model error, because there is no noise in the simulated displacement data.

Physical Experiment

To show whether the uncertainty estimate is reasonable, it is useful to apply the methodology to a physical experiment. The physical experiment took the form of a series of repeated measurements on five 7050-T74511 extruded aluminum bars, in a set of experiments similar to those performed in a recent repeatability study for the contour method [16]. The bar cross section is 51.1 mm thick and 76.1 mm wide, with a length of 304.8 mm. The bars had residual stresses induced by heat treating to a T74 temper [17], which consisted of solution heat treatment at 477 °C for 3 h, immersion quenching in room temperature water with 16 % polyalkylene glycol (Aqua-Quench 260), and a dual artificial age at 121 °C for 8 h then 177 °C for 8 h. Based on earlier work, this treatment is expected to produce compressive residual stress on the bar exterior and tensile residual stress on the bar interior, both having magnitudes greater than 100 MPa [16, 18].

Contour method measurements were performed at the mid-length of each bar, as can be seen in Fig. 2. Since the bars have identical geometry, and were processed identically, they are expected to contain identical distributions of residual stress.

For each measurement, the single-measurement uncertainty estimator was applied using the previously described approach. In addition, the mean and standard deviation of the population was determined. The uncertainty estimator is assumed to be performing appropriately when the measured

Fig. 3 Contour plots of residual stress from the numerical experiment: (**a**) known residual stress, (**b**) true error, and (**c**) model error

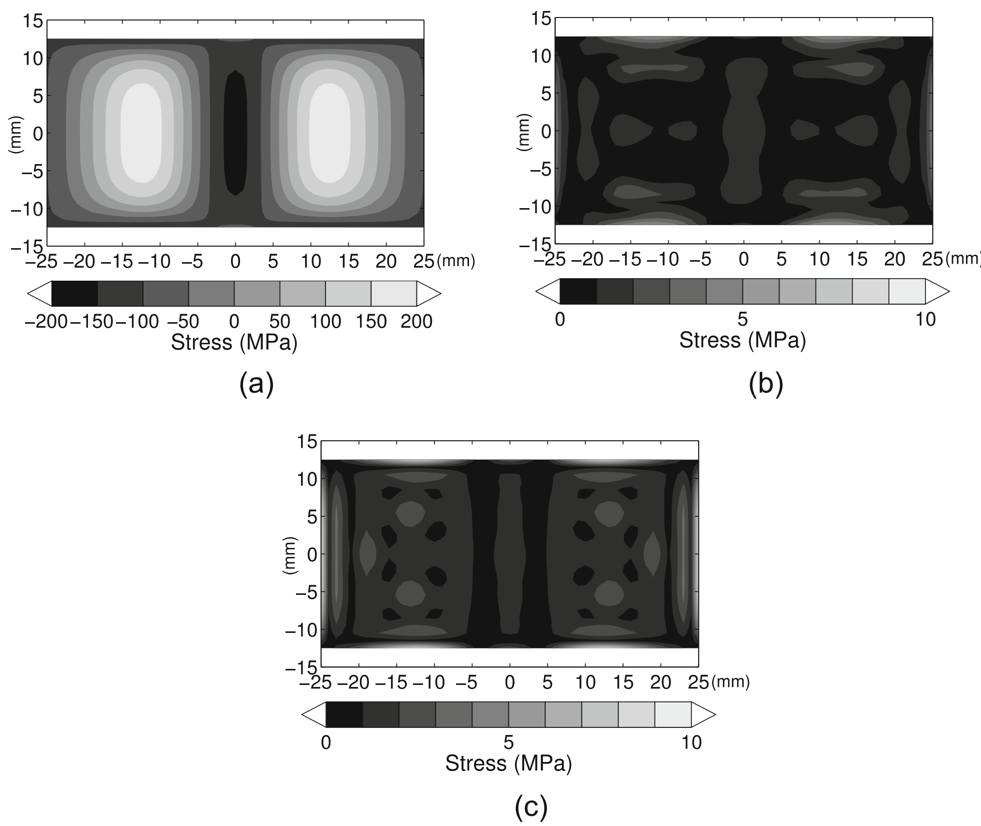
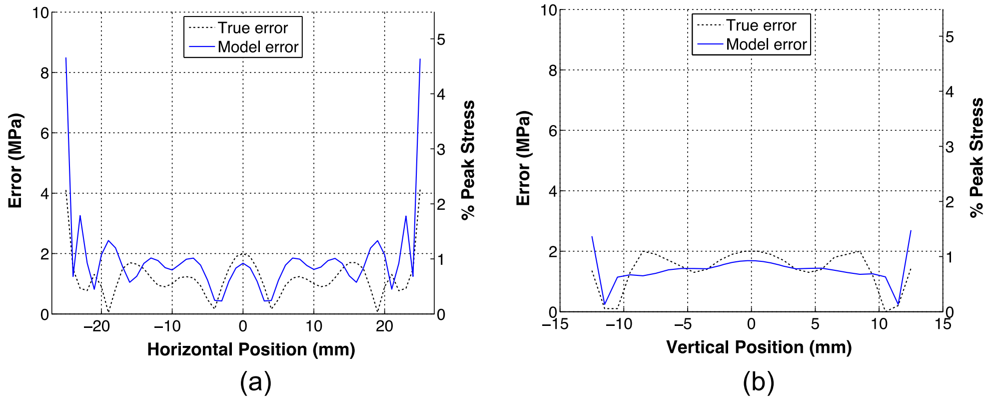


Fig. 4 Line plots of the model error and true error in the (**a**) horizontal direction at the mid-thickness (y=0) and (**b**) vertical direction at the mid-width (x=0)



residual stress±the estimated uncertainty provides a bound that includes the mean stress, for a fraction of in-plane locations equal to a stated confidence interval. The mean value was chosen because, with a series of repeated measurements, the mean is the best estimate of the underlying residual stress field, an estimate that improves as the number of measurements increases.

Results

Numerical Experiment

The results of the numerical experiment are presented in Fig. 3. The known residual stress (Fig. 3(a)) has a magnitude of nominally ± 200 MPa. The true error is defined as the absolute difference between the known residual stress and the estimated residual stress, which requires a choice of smoothing parameters. In a typical experiment, the number of fitting coefficients must be balanced with some measure of fit quality. For this work, the displacements were initially fit with a low number of fitting coefficients, and the number of fitting coefficients systemically increased until the RMS of the difference between the surface fit and displacement data plateaued. The true error results are presented in Fig. 3(b). The true error is less than 2 MPa at most points on the cross-section (which is less than 1 % of the peak residual stress value). There are locations near the perimeter of the cross-section where the true error is higher (5 MPa). The model error is plotted in Fig. 3(c). The model error is less than 3 MPa over most of the cross-section with localized regions up to 8 MPa near the perimeter.

Line plots of the model error and the true error can be seen in Fig. 4. The results show that the model error has a similar magnitude as the true error and the distributions are nominally similar, with both having high spots toward the perimeter and in the center.

Figure 5 shows a plot of the locations on the cross-section where the estimated residual stress±the model error does (light gray) or does not (dark gray) overlap the known residual stress. The uncertainty estimator meets this criterion for approximately 68.4 % of points on the cross section, which indicates the uncertainty estimator is performing well (assuming a 68 % confidence interval).

Physical Experiment

The results of the physical experiment can be seen in Fig. 6. The mean of the five contour measurements is shown in Fig. 6(a). The contour method results are typical of a quenched aluminum component with compressive residual stress around the perimeter (−175 MPa) and tensile residual stress in the center (160 MPa). The standard deviation of the five contour method measurements can be seen in Fig. 6(b), being under 10 MPa over most of the interior, with a high of 20 MPa at one edge. The calculated displacement error (for a single measurement from the population) can be seen in Fig. 6(c). The displacement error is very low at all points, but is slightly higher at the perimeter of the cross-section (around 2 MPa) than it is in the interior (around 1 MPa). The model error results (for a single measurement from the population) can be seen in Fig. 6(d). The results show that the model error is significantly higher at the perimeter of the cross-section (around 40 MPa) than in the interior (under 10 MPa at most locations). The total uncertainty estimate is shown in Fig. 6(e).

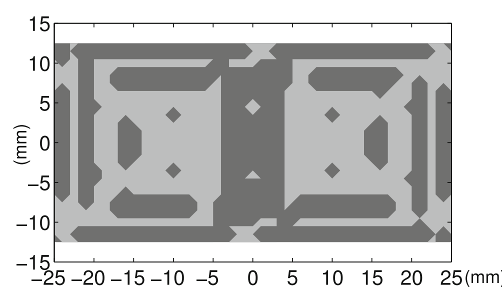


Fig. 5 Diagram of points were the numerical experiment met (*light gray*) and did not meet (*dark gray*) the acceptance criteria of the initialized residual stress falling within the bound of the measured results±total uncertainty (68 % confidence interval); 68 % of the points meet the acceptance criterion

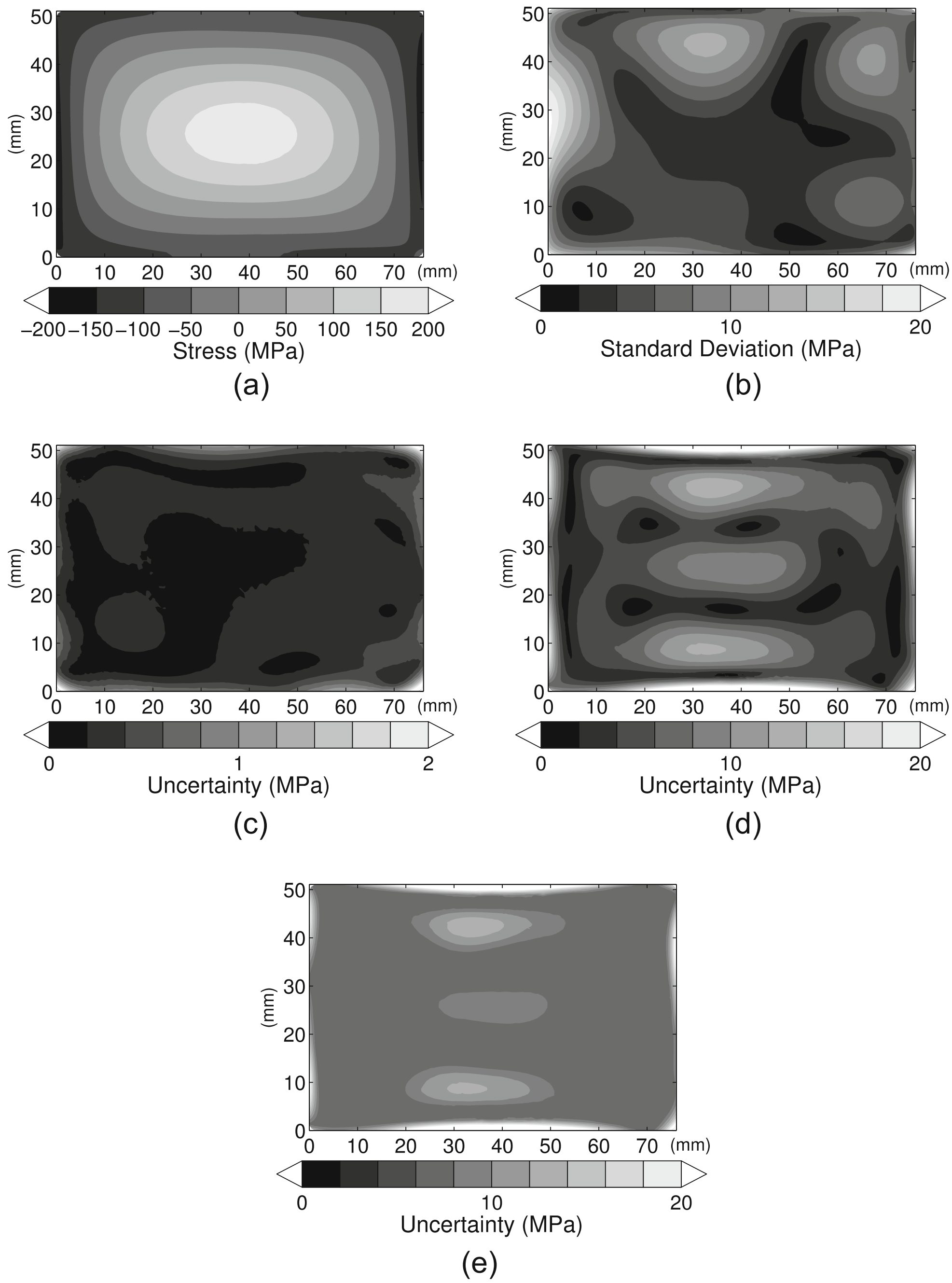


Fig. 6 Results from the physical experiment: (**a**) mean measured residual stress, (**b**) standard deviation of the measured stress, (**c**) displacement error, (**d**) model error, and (**e**) total uncertainty

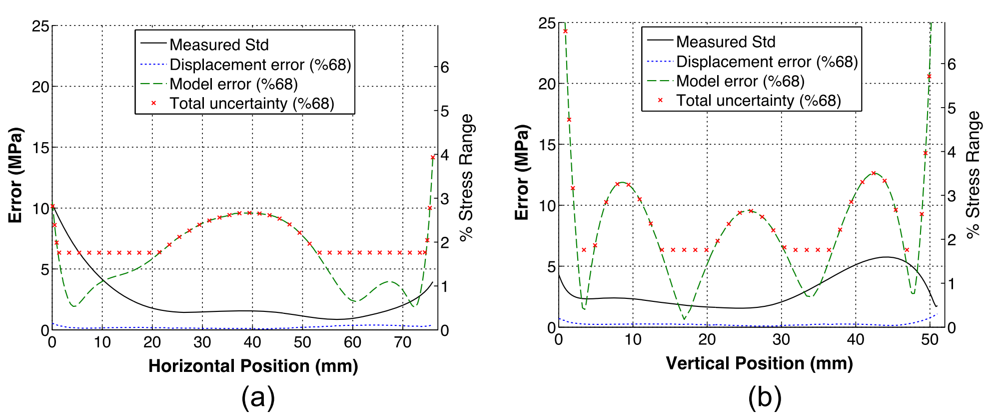


Fig. 7 Line plots of the measured standard deviation, displacement error, model error, and total uncertainty in the (**a**) horizontal direction at the mid-thickness (y= 25.4 mm) and (**b**) vertical direction at the mid-width (x= 38.1 mm)

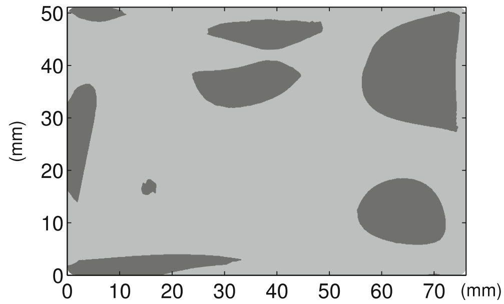


Fig. 8 Diagram of points were the physical experiment met (*light gray*) and did not meet (*dark gray*) the acceptance criteria of the mean residual stress falling within the bound of the measured stress±total uncertainty (68 % confidence interval); 72 % of points meet the acceptance criterion

Line plots of the standard deviation of the measurement population, model error, displacement error, and total uncertainty estimate are shown in Fig. 7 (the latter three being for a single measurement from the population only). The results show that displacement error is very small compared to the model error, and in this case negligibly contributes to the total uncertainty estimate. The model error is approximately the same magnitude as the standard deviation of the population, but is significantly higher at the perimeter of the cross section.

The points where the measured residual stress±the total uncertainty estimate overlaps the mean of the measurement population are shown in Fig. 8. In this case, for 72.2 % of the points the measured residual stress±the uncertainty estimate overlaps the mean of the population (for a 68 % confidence interval on the uncertainty estimate) and there does not appear to be a systematic trend in the locations of the points that do and do not overlap the mean. A line plot of an individual measurement and uncertainty estimate from one of the measurements is shown in Fig. 9 as well as the mean of the measurements.

Discussion

The nominally small model error indicates that the smoothing tends to be robust in the part interior, whereas the smoothing is less stable at the part boundary, which is indicated by the larger uncertainties at the boundary. The numerical experiment shows that the estimated residual stress±the uncertainty estimate appropriately overlaps the known stress for a given percentage of points corresponding to the confidence interval (68 %). This indicates that the model error has reasonable accuracy relative to the true error associated with smoothing. However, these results were for a single, selected amount of smoothing. To determine the performance of the uncertainty estimate for a range of smoothing conditions, the root mean square (RMS) of the true error and the model error were determined and are plotted in Fig. 10. The results show that for a large number of fitting coefficients the model error becomes negligible small, as would be expected since the noise-free surface is better fit as the number of fitting terms increases. The results show that although the model error is somewhat larger than the true error for small numbers of fitting coefficients, it tracks relatively well with the true error for any choice of smoothing parameter, which further gives confidence in the model error definition.

The numerical experiment was repeated with normally distributed noise, with a standard deviation of 4 μm, added to the displacement field to assess several questions regarding the displacement error. The RMS displacement error is strongly dependent on the amount of smoothing. Figure 11 shows that the displacement error increases as the amount of smoothing decreases (more terms added to the model), which is expected because a highly smoothed surface is less influenced by noise. The effect of displacement measurement spacing on the displacement error was found with the numerical experiments by using only a subset of the known surface displacements, corresponding to a measurement spacing that ranged from 0.1 mm×0.1 mm to 5 mm×5 mm. The noisy displacement data for each measurement spacing was used in standard contour method data analysis, and all models evaluated the smoothed displacements at a common set of finite element node locations. The results show that the displacement error varies significantly depending of the measurement spacing (Fig. 12). The maximum displacement error ranges from around 1 MPa for small spacing

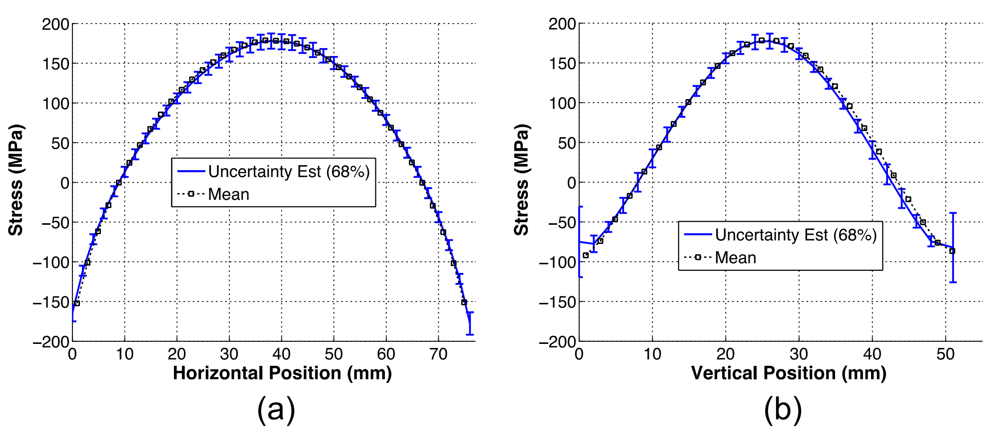


Fig. 9 Line plots comparing the individual measured stress (for one of the measurements from the population) and the mean for the measurement set

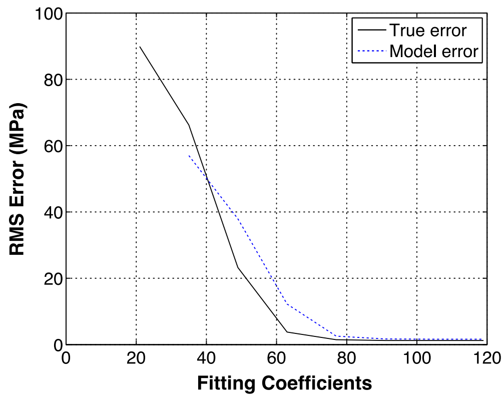


Fig. 10 Comparison for numerical experiment between RMS of true and model errors for various numbers of fitting coefficients

(0.1 mm×0.1 mm) to greater than 50 MPa for large spacing (5 mm×5 mm). These results make practical sense because the high frequency noise will have a less significant effect on the underlying long-range form for lower measurement spacing, with more data points.

Bevington suggests that, in principle, any internal method of determining the uncertainty should agree with that obtained by an external method of determining the uncertainty [7]. Applied to the contour method, this suggests that the contour uncertainty estimate should agree with the precision found from a set of repeatability experiments. The results of the physical experiment demonstrate that the single measurement uncertainty estimate (an internal method, Fig. 6(e)) does qualitatively agree well with the repeatability standard deviation (an external method, Fig. 6(b)). However, a more quantitative assessment criterion was used to determine if a measurement result±uncertainty estimate includes what is best thought to be the true value of stress. This criterion determined the fraction of points where the measurement±total uncertainty overlapped the mean of the repeatability data, which showed the fraction of points to be 72 %, very close to the 68 % confidence interval inherent in the single-standard deviation uncertainty bound.

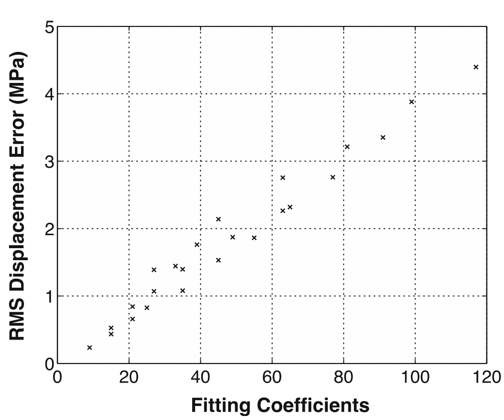


Fig. 11 Displacement error for a range of number of fitting coefficients

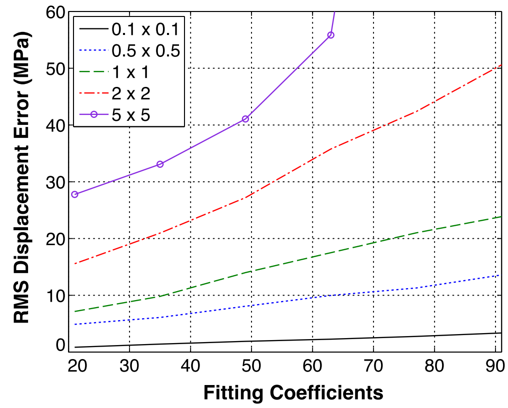


Fig. 12 Displacement error for ranges of measurement spacing and number of fitting coefficients

The uncertainty estimator in this paper covers sources of random error. Bias errors, such as plasticity [19–21] or the bulge error [4, 21, 22] can be significant. The preferred approach, if one can estimate those errors, is to correct for them.

The experimental uncertainties reported here are specific to the samples tested. Because the main quantity measured is a displacement, uncertainties could be expected to increase with the elastic modulus and stress gradients, and to decrease with specimen size, at least until other uncertainty mechanisms dominate.

This experimental work used samples that have a simple geometry and simple stress field. Follow-on work should apply this uncertainty approach to more complicated samples, to show whether the uncertainty estimator provides reasonable results under a broader range of circumstances.

Summary/Conclusions

This paper describes a single measurement uncertainty estimator for the contour method, which accounts for noise in the measured displacement field and error associated with smoothing. The error arising from noise in the measured displacement field is estimated using a Monte Carlo approach by finding the standard deviation of the differences in stress resulting from applying normally distributed noise to the measured surface. The error associated with the smoothing model for the displacement field was found by taking the standard deviation of stresses computed using different levels of smoothing.

The uncertainty estimate was evaluated in the context of both numerical and physical experiments. The bounds of measured residual stress±the total uncertainty estimate included the known stress for the numerical experiment and the mean of a series of repeated measurements for the physical experiment. The results show that the proposed single measurement uncertainty estimator for the contour method performs well under the conditions examined.

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