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BRIEF TECHNICAL NOTE

Four-Point Bending Test of Determining Stress-Strain Curves Asymmetric between Tension and Compression

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Abstract A method of determining both uniaxial tension and compression stress-strain curves from the result of a single four-point bending test was demonstrated. Stress-strain curves of magnesium showing tension-compression asymmetry due to twinning deformation and those of an S45C steel due to the Bauschinger effect were calculated. The Mayville-Finnie equation was modified slightly for this calculation. The calculation is sensitive to small change in the slope of bending curve, revealing an aspect of inverse problem.

Keywords Inverse problem · Four-point bending · Stress-strain curve · Twinning · Bauschinger effect

An inverse problem in elasto-plastic bending of a material may be to determine uniaxial stress-strain (s-s) curve of the material. This classical subject was discussed by Nadai [1]. Later, Mayville and Finnie studied this problem and derived a formula [2]. Despite the potential of the formula and also the usefulness of bending test, very few application has been found.

Let us consider four-point bending of a prismatic bar (Fig. 1). The bar is simply supported at points A and B, and a load *P*/2 is subjected at points C and D equally. Pure bending with the radius of curvature ρ occurs between C and D, where a constant bending moment of $M = Pd/2$ is subjected. The bending strains of the outermost surfaces ϵ_1 (compression) and ϵ_2 (tension) are measured with two pieces of strain gauge. Accordingly, two curves of bending load vs. bending strain relation (P-ϵ curves) are obtained: $\epsilon_1 = \epsilon_1(P)$ and $\epsilon_2 = \epsilon_2(P)$. The bar has a constant rectangular cross section, where a local y coordinate is taken as in Fig 2. Yield stress in compression is Y_1 and in tension Y_2. A neutral axis NN is located at $y = y_0$. When s-s curves are asymmetric between tension and compression, the neutral axis is away from the centroid axis at $y = h/2$ while bending.

A P-ϵ curve can be derived readily from the exact shape of s-s curve, if it is known. This is a forward problem in bending. For example, a material with an ideal elastic-perfect plastic s-s behavior illustrated in Fig. 3(a) shows P-ϵ curves of Fig. 3(b). The calculation was carried out by assuming that the modulus of elasticity E was 100 GPa, the yield stresses Y_1 in compression, 100 MPa, Y_2 in tension, 200 MPa, and the dimensions, b=5 mm, h=1 mm, d=10 mm. Properties of P-ϵ curves listed below are common to any elasto-plastic material for $Y_1 < Y_2$:

(i) The linear relationship in elastic bending is given by

$$P = \frac{bh^2E}{3d}\epsilon. \tag{1}$$

(ii) When the compression side starts yielding, tension and compression curves branch off from the linear relationship at a point where the load is P_1 and the strain is $\epsilon_1 = Y_1/E$. Hence, $P_1 = bh^2Y_1/(3d)$.

(iii) When the tension side starts yielding, the two curves change the slopes at a point where the load is P_2 and the strain is $\epsilon_2 = Y_2/E$.

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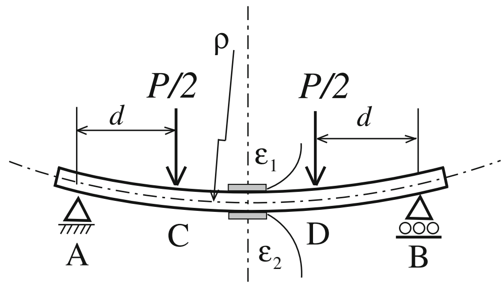

Fig. 1 Four-point bending of a simply supported bar with two pieces of strain gauge glued on the outermost surfaces

The inverse problem is to determine s-s curves from the result of bending. Without solving the problem, we can determine the values of E, Y_1 and Y_2 from P-ϵ curves. Let us start from the P-ϵ curves of Fig. 3(b). The slope of linear relationship determines E as 100GPa from equation (1). The first and second deflection point are found at 0.1 % and 0.2 % strain, so that Y_1 and Y_2 are determined as 100MPa and 200MPa, respectively. However, the deflection under the load P_2 might be difficult to determine by eyes.

The fiber stresses σ_1 and σ_2 in Fig. 2 are yet to be determined. The strains on the outermost surfaces are

$$\epsilon_1 = \frac{y_0}{\rho} \text{ and } \epsilon_2 = \frac{h - y_0}{\rho}, \tag{2}$$

and then the curvature is given by

$$\frac{1}{\rho} = \frac{\epsilon_1 + \epsilon_2}{h}. \tag{3}$$

The equilibrium of the bending stress and that of the bending moment acting on cross sectional area A (see Fig. 2) are

$$\int_A \sigma dA = 0 \text{ and } \int_A \sigma \bar{y} dA = M, \tag{4}$$

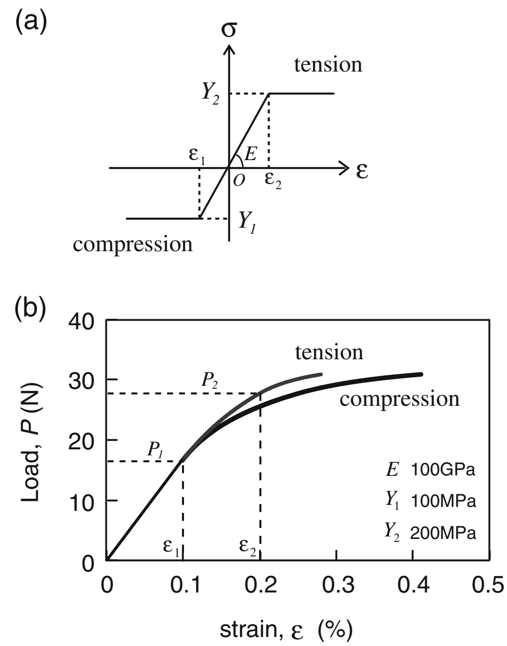

Fig. 3 An elastic-perfect plastic stress-strain curve, (**a**), and bending load-strain curves, (**b**)

where $\bar{y}$ is the distance from the neutral axis, such that $\bar{y} = y - y_0$. Reminding $\epsilon = \bar{y}/\rho$ and using equations (2) and (3),

$$b\int_{-y_0}^{h-y_0} \sigma d\bar{y} = b\rho \int_{-\epsilon_1}^{\epsilon_2} \sigma d\epsilon = 0, \text{ and } b\int_{-y_0}^{h-y_0} \sigma \bar{y} d\bar{y} = b\rho^2 \int_{-\epsilon_1}^{\epsilon_2} \sigma \epsilon d\epsilon = M. \tag{5}$$

Calculus of the first variation gives

$$\sigma_1 \delta\epsilon_1 = \frac{1}{bh^2}(2M(\delta\epsilon_1 + \delta\epsilon_2) + (\epsilon_1 + \epsilon_2)\delta M), \tag{6}$$

$$\sigma_2 \delta\epsilon_2 = \frac{1}{bh^2}(2M(\delta\epsilon_1 + \delta\epsilon_2) + (\epsilon_1 + \epsilon_2)\delta M), \tag{7}$$

which are equal to the expressions derived by Mayville and Finnie in [2]. In these incremental forms the causal

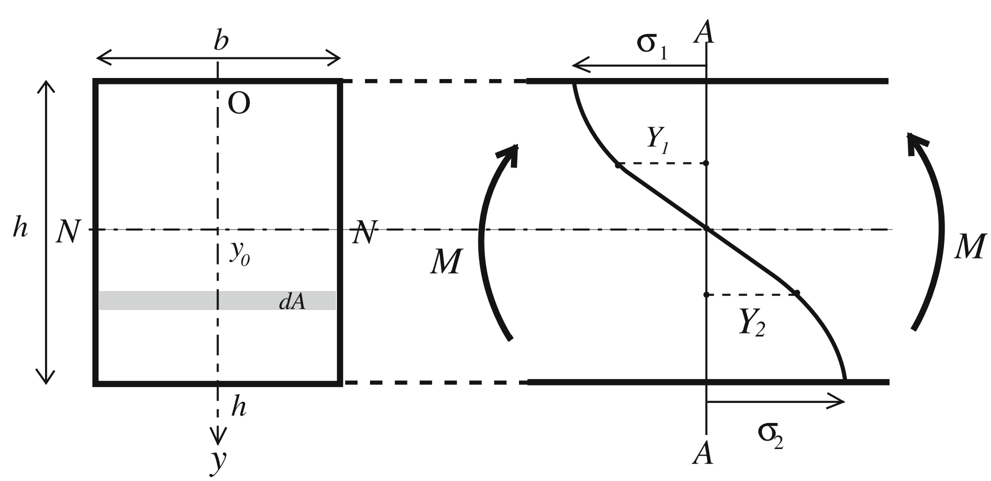

Fig. 2 The cross sectional area A of a bar (*left*) with distribution of bending stress as depending on the distance from neutral axis NN (*right*)

relationship between bending curves and calculated s-s curves is not easy to grasp. In fact, P-ϵ curves were not considered in the previous study.

Present study transforms the incremental equations into a set of ordinary differential equations. Since the strains ϵ_1 and ϵ_2 are functions of the single variable of load P, the stresses are also the case. If we divide both sides of the formulas by an infinitesimally small δP and take $\delta P \rightarrow 0$, we can replace the incremental terms by compliances of P-ϵ curves defined as

$$\frac{d\epsilon_1}{dP} = C_1 \text{ and } \frac{d\epsilon_2}{dP} = C_2. \tag{8}$$

Finally, equations (6) and (7) become

$$\sigma_1 = \frac{d}{bh^2C_1}\left(P(C_1+C_2)+\frac{\epsilon_1+\epsilon_2}{2}\right), \tag{9}$$

$$\sigma_2 = \frac{d}{bh^2C_2}\left(P(C_1+C_2)+\frac{\epsilon_1+\epsilon_2}{2}\right), \tag{10}$$

which can be solved as the simultaneous equations with the curves $\epsilon_1 = \epsilon_1(P)$ and $\epsilon_2 = \epsilon_2(P)$. In elastic range the formulas become well-known one in elastic bending $\sigma = 3Pd/(bh^2)$, which is equivalent to equation (1).

In experiments, narrow bars were prepared from a sheet of 99.8 % magnesium and that of S45C. The width and thickness was 5.0 mm and 2.2 mm, respectively. The specimens were annealed in order to remove their processing histories. In Fig. 1, the span AB and CD was 45 mm and 20 mm, respectively, and thus $d = 12.5$ mm. Uniaxial push-pull tests were also carried out for specimens of 6 mm in gauge with a hydrofluoric Shimadzu Servopulser EHF machine. Owing to large flexure of thin plate samples, present bending test was limited below 1.5 % strain.

P-ϵ curves of a magnesium bar are shown in Fig. 4(a). The curve began with elastic bending, and then showed a marked deflection at a point under the load $P_1 = 27$ N, which gives E=43 GPa from equation (1). The strain at this point ϵ_1 was 0.08 %. Then the yield stress Y_1 is estimated as 34 MPa. These values agree with those in literatures [3]. As the load was increased above P_1, the strain in compression became larger than that in tension for a given load P.

The s-s curves calculated from equation (9) and (10) are shown as solid lines in Fig. 4(b), while the results of uniaxial tension and compression testing as dotted lines. The flow stress of the calculated curves agrees well with that of measured one. It is known that magnesium is plastically deformable at room temperature due to the mixture of slip and twinning, and that the asymmetry is a typical nature of twining in hexagonal metals and alloys [3].

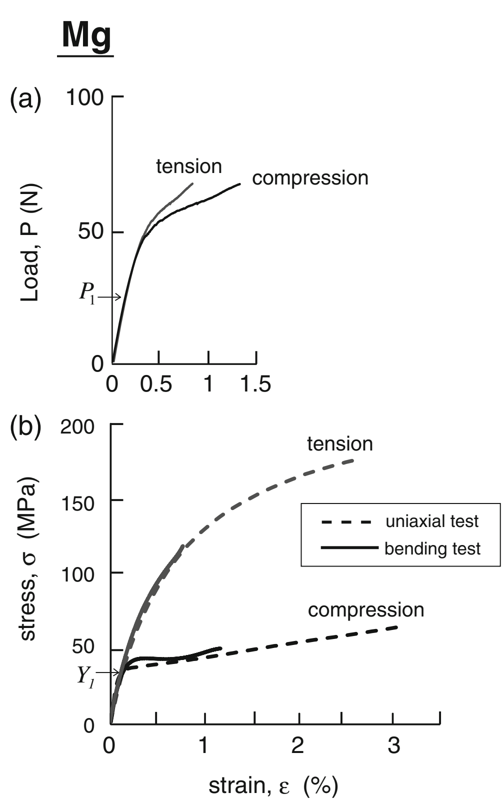

Fig. 4 Bending load-strain curves measured in a magnesium bar, (**a**), and calculated stress-strain curves, (**b**)

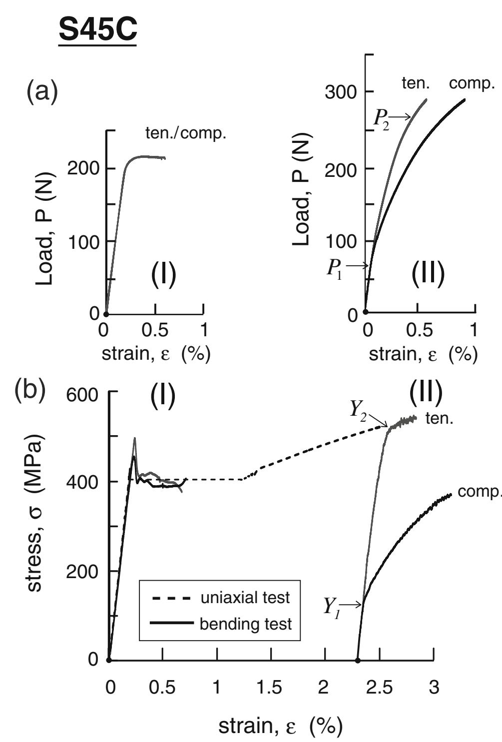

Fig. 5 Bending load-strain curves measured in two S45C bars, (**a**), and calculated stress-strain curves, (**b**). Curves (I) were obtained in a bar with no prestrain and Curves (II) with 2.3 % prestrain

P-ϵ curves measured in S45C steel bars are shown in Fig. 5(a). An annealed S45C is known to show yield point elongation (YPE) followed by strain-hardening, which can exhibit pronounced Bauschinger effect. Curves (I) were measured in an as-annealed specimen to observe YPE, while curves (II) were in a specimen with 2.3 % pre-strain to observe the Bauschinger effect. As expected, the bending curves (I) were equal between tension and compression, but the curves (II) were well-separated between them.

The s-s curves calculated from the P-ϵ curves (I) and (II) are shown in Fig. 5(b). The dotted line was obtained by uniaxial testing when giving the pre-strain. The calculated curves (I) agreed well with the YPE in measured curve. A yield drop was revealed in the calculated curves. A Bauschinger effect was clearly observed in the s-s curve (II) in compression, where the yielding at Y_1 corresponds to the load P_1 in P-ϵ curve (II). It is reasonable that the yield stress in tension remains the same as that of the pre-straining. These results show that this method is very useful for observing Bauschinger effect.

It is seen that later parts of the calculated curves are somewhat wavy. Coarse oscillation are seen in Fig. 4(b) and Fig. 5(b)(I), while fine ones in Fig. 5(b)(II). The oscillation is not the stress-strain behavior of the material because the curves are smooth when measured by uniaxial testing. Experimental data has shown that the oscillation occurred in the values of C_1 and C_2. That is, the calculation is sensitive to the derivative of P-ϵ curves, which means that a high-order precision is needed in the bending experiment. This oscillation was not found in the previous study, since the calculation of s-s curves was carried out in a few points [2]. Some data processing to smooth the P-ϵ curves may be effective to remove oscillation originating from instrumental errors; however, no data processing is given in the results of Figs. 4 and 5.

Conclusively, it was demonstrated first that tension-compression asymmetric s-s curves of pure magnesium and those of S45C steel are determined from the result of four-point bending tests. A set of simultaneous equations was derived from the Mayville-Finnie equations. The calculated s-s curves showed good agreement with the curves measured in uniaxial deformation. The calculation was found to be able to follow closely both yield drop and yield point elongation in steels. It was pointed out that the bending test needs a high-order precision to avoid oscillation in calculated s-s curves.

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