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Experimental and Numerical Investigation of Free Vibrations of Composite Sandwich Beams with Curvature and Debonds

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Abstract In this paper experimental and numerical results concerning the dynamic response of composite sandwich beams with curvature and debonds are reported. Sandwich beams made of carbon/epoxy face sheets and polyurethane foam core material were manufactured with four different radii of curvature and debonds between the top and bottom interface of face sheet and foam core. Dynamic response was obtained using the impulse frequency response technique under clamped-clamped boundary condition. Experimental results were compared with numerical finite element model results. A combined experimental and numerical FE approach was used to determine the material properties of the skin and foam core materials based on modal vibration and static flexure tests. Results indicate that the fundamental frequency increases with increasing curvature angle, however, for higher frequencies; the natural frequencies are not significantly affected. Also, it is found that face/core debond causes reduction of the natural frequencies due to stiffness degradation.

Keywords Layered structures · Debonding · FEA · Non-destructive testing

Introduction

Sandwich composites are a special class of composite materials used in structural applications (aircraft, spacecraft, submarine, ships and boats, etc.) due to their high strength and stiffness to weight ratios [1]. Such sandwich composites comprise of two thin and stiff face sheets joined together by a relatively thick, lightweight and compliant slab known as the core. The faces possess high elastic modulus and strength to carry axial and bending loads while the core carries the through-the-thickness shear loads and holds skins in place away from neutral axis to maintain high flexural stiffness. Structural members having variable curvature show very complex structural behavior. Therefore, the accurate prediction of the behavior of curved sandwiches is of fundamental importance in the design of these structures. The effects of curvature on the vibration of a curved beam were determined earlier in references [2, 3]. It was shown that even a slight amount of curvature has a significant effect on the fundamental frequency for simply supported and clamped beams. However, earlier studies are limited to beams made of isotropic material [4, 5]. In the case of curved sandwich beams, the finite element (FE) method has been used by Ahmed [6] to determine the effect of various geometric parameters on the resonant frequencies. Theoretical models for analyzing composite sandwich panel structural behavior have been developed by Zenkert [1], while a detailed classification of computational models

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for sandwich panel and shells was listed by Noor et al. [7]. It appears that even though the influence of curvature on the vibration behavior of composite sandwich beams has been shown to be significant by analytical and numerical methods, little or no effort has been made to experimentally investigate the effect of curvature and debonds on the dynamic behavior of composite sandwich beams. Recently, Sakiyama et al. [8] presented a detailed method to analyze the free vibrations of sandwich arches with elastic or viscoelastic cores, and various axis-shape and boundary conditions were considered in the study. Vaswani et al. [9] investigated vibration and damping analysis of curved sandwich beams with a viscoelastic core. The frequency parameter of a curved sandwich beam is seen to decrease with increasing curvature. On the other hand, Kwon [10] observed the general tendency of the natural frequencies and loss factors of cylindrical hybrid composite panels with various damping treatments.

Strength and stiffness of the sandwich structure are significantly affected due to debonds, because of loss of shear and tension transfer between face sheet and core. Debonds at the face/core interface can easily occur during the manufacturing process or under service conditions such as impact loading and wave slamming [11], and they may propagate further leading to total collapse of the sandwich composite [12, 13]. It has been shown that dynamic characteristics such as the natural frequencies and mode shapes are significantly affected because of the presence of debonds [14–16]. Also, modal parameters such as natural frequencies, mode shapes and damping loss factors have been used effectively to detect damage and monitor the structural integrity of the structure [17–20]. Rossettos [21] investigated fundamental frequencies of slightly curved sandwich beams with delamination using non-dimensionalized variational principle based on the principle of minimum potential energy, where the strain energy is based on the model by Hoff [22] developed for straight sandwich beams. The results clearly indicate that even a slight curvature tends to increase the fundamental frequency sharply.Studies found in the literature discuss the mechanical and dynamic properties of flat sandwich panels containing face/core debonds [13, 23–27]. However, vibration response of curved sandwich composite beams containing face/core debonds is not fully understood. Thus,there is a need to study the effect of debonds and curvature on the vibration response of sandwich beams using a combined experimental/numerical approach.

In this paper, a combined experimental and finite element (FE) method is presented to analyze the free vibration response of flat and curved composite sandwich beams containing debonds/delaminations. Composite sandwich beams with four different radii of curvature with debonds (at the top and bottom interface of face sheet and core) are tested and compared with flat beams with and without debonds, respectively. The effects of key variables such as curvature and the location of debonds on the natural frequencies and mode shapes of the sandwich beam are investigated. Material properties of the carbon/epoxy face sheets and polyurethane foam have been determined using the combined experimental/numerical approaches based on static and vibration methods. The material properties are then used in the two-dimensional (2D) and three dimensional (3-D) FE models and predicted modal responses are compared with those of experiments. The results can be useful in providing the foundation for the non-destructive detection and evaluation of debonds in flat and curved sandwich beams.

Experiments

Composite Sandwich Beam Specimen Manufacturing

Composite sandwich beams investigated in this study were manufactured at Design Intent Engineering in Farmington Hills, Michigan. Polyurethane foam with a density of 199 kgm^{-3} having a thickness, t_f, of 12.7 mm was used as the core material for all the composite sandwich beams. Woven carbon fiber fabric with fibers oriented at ±45° and having a nominal thickness of 0.3 mm was selected as the face sheet material. Two layers of such woven fabric with epoxy were used to make up the face sheets on each side of the core, thus the resulting thickness of the face sheets, t_s, on each side was 0.762 mm, and the total thickness of the sandwich, t, was 14.224 mm. The geometric configuration of flat composite sandwich beams is as shown in Fig. 1(a).

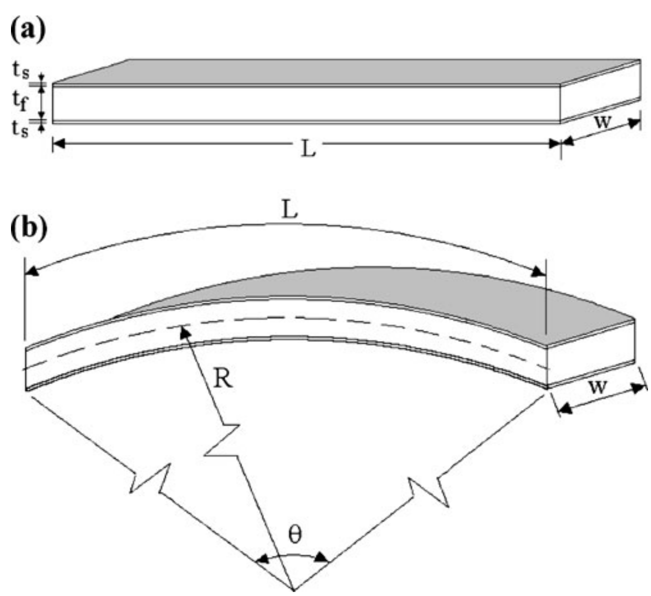


Fig. 1 Geometry of the (**a**) flat sandwich beams and (**b**) curved sandwich beams

The length and width of sandwich beams were, L=254 mm and w=25.4 mm, respectively. Curved composite sandwich beams [Fig. 1(b)] with four different open angles, θ equal to 15 deg, 30 deg, 45 deg and 60 deg were selected, and the corresponding radii of curvature, R, (970.2 mm, 485.08 mm, 323.39 mm, and 242.54 mm) were calculated by keeping the arc length of the sandwich beams, L_{arc}, equal to the length of the flat sandwich beams, L.

The sandwich beams were manufactured using the hand lay-up process. This method is one of the oldest methods which is the best suited for short production series. The process uses a two-sided mold made with wood. The carbon fiber fabric was placed on the mold after spraying the release agent so that the final product does not stick to the mold. An appropriate amount of resin was applied and distributed over the surface of the fabric uniformly with brush. After face coat was impregnated, polyurethane foam material was placed on top of the carbon fabric. Then another coat of resin was uniformly distributed on the surface of the foam core and another layer of fabric was placed. The mold was clamped down to force the sandwich panels such that the total thickness of the sandwich was not greater than 14.224 mm. After curing time which is about 24 h, mold was opened and the specimens from the manufactured panel were cut in a rectangular shape with dimensions of 25.4*254 mm (width*length). To make the sandwich beams with different curvatures, four molds having radii of curvature 970.2 mm, 485.08 mm, 323.39 mm, and 242.54 mm, which corresponds to an open angle of 15°, 30°, 45° and 60°, respectively, were made out of balsa wood and the above procedure was used to cure the sandwich.

Flat and curved sandwich beams with debonds at the interface between face sheet and core were made by inserting a thin Teflon film between the face and core. The masking tape dimensions were 50.8 mm in length and 25.4 mm in width, therefore the size of the debond was about 20% of the length of the beam. Masking tape was placed at the center of the beam at the top and bottom interface to produce debonds on the top and bottom of the beam. The masses of the composite sandwich beams were measured and it was seen that the differences in the masses of sandwich beams (flat and curved) with and without debonds were less than 5%.

Vibration Tests

Modal vibration experiments were performed on flat and curved beam specimens with and without debonds. All the specimens (Table 1) were tested under clamped-clamped (C-C) boundary conditions. The C-C boundary condition was achieved by clamping the free ends of the specimen with angular vises as shown in Fig. 2. After clamping, the span length of the beam was reduced to 230 mm. The basic impulse frequency response vibration test [18] was performed using a HP 3582A Fast Fourier Transform (FFT) spectrum analyzer with a data-acquisition computer. A slight hammer tap excited the specimens and the responses were detected by a miniature piezoelectric accelerometer (PCB 309A) attached to the specimen with bees wax. A conditioning amplifier was used to provide power to the accelerometer and amplify signals from it. The signals were processed by a digital FFT algorithm in the analyzer to convert the analog time domain signals into the frequency domain data. The frequency response functions were plotted on a computer monitor using a LABVIEW program, and the first five modal frequencies, f_n, were recorded. Vibration tests were repeated by removing the specimen and clamping it again in the vise and testing. For each sandwich beam specimen, three readings were taken and since there were three identical specimens of each sandwich beam listed in Table 1, a total of nine readings were taken and a mean value was calculated.

Vibration tests were also performed on polyurethane foam and carbon/epoxy skin materials to determine their material properties under free-free boundary conditions by combined experimental/numerical approaches. The foam material tested had a L*W*t = 254*25.4*12.7 mm, while the composite skin had a L*w*t = 254*25.4*2.54 mm. Vibration tests were performed on these specimens under free-free (F-F) boundary conditions using the same method described above for C-C boundary condition. The F-F boundary condition was obtained by suspending the foam

Table 1 The dimensions and configurations of the sandwich beams

Material	Curvature angle θ (degree)	Debond	Length L (mm)	Width W (mm)	Face sheet thickness t_s (mm)	Core thickness t_f (mm)
Carbon-epoxy face sheet/ polyurethane foam core/ carbon-epoxy face sheet	Flat	No debond	230	25.4	0.762	12.7
	15	Upper face/core debond				
	30					
	45	Lower face core debond				
	60					

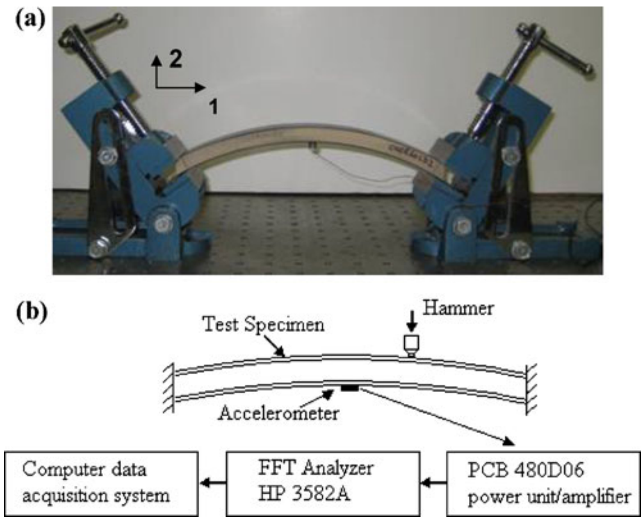


Fig. 2 (**a**) Photograph and (**b**) schematic diagram of experimental setup for vibration test with clamped-clamped boundary condition (1: Length direction, 2: Thickness direction)

and composite skin specimens using thin nylon strings. First five natural frequencies were recorded as mentioned before using the piezoelectric accelerometer and FFT analyzer.

Finite Element Analysis

In this section, first, two-dimensional (2-D) FE models that were developed to determine the properties of foam core and composite skin material will be discussed, followed by the 2-D FE model developed to determine the modal vibration response of flat and curved composite sandwich beams with and without debonds. Finally, a brief discussion on the three-dimensional (3-D) FE models developed for flat and curved sandwich beam (15 deg only) without debonds and the comparison of the results will also be included. Hypermesh® 7.0 was used to develop all the FE models and post-processing of the results. ABAQUS® V 6.5 standard was used to predict the modal frequencies and mode shapes of the cantilever and free-free (F-F) specimens discussed above.All the 2-D FE models were developed using 4-noded CPS4 plane stress solid elements and the 3-D models were developed using C3D8 hexagonal elements. For 2-D models, C-C boundary conditions for composite sandwich beams were simulated by constraining the displacements in the x, and y directions along both the edges, and for 3-D models C-C boundary conditions was achieved by constraining the displacements in x, y and z along both the edges. For F-F boundary condition (for foam and composite skin specimens only) no constraints were imposed. The solid elements do not have rotational degrees of freedom. A FE mesh sensitivity study was performed for all the models to ensure that the finite element meshes were fine enough to give satisfactory results.

Two-dimensional FE Study

Material characterization models

Two-dimensional (2-D) FE model developed for polyurethane foam material (L*W*t = 254*25.4*12.7 mm) had 250 elements along the length and 15 elements along the through-the-thickness (TTT) direction. This mesh density was found to give satisfactory results. To make the analysis simple, the foam material was assumed to be isotropic. Natural frequencies were extracted using the eigenvalue extraction method in ABAQUS using the F-F boundary condition, and Young's modulus of the foam material, E_{foam} was initially assumed to be 104 MPa. To determine the actual modulus of the foam a combined experimental/numerical approach was used. In this approach, E_{foam} is used as the curve fitting parameter in the FE model, which minimizes the function F, by the total square error method:

$$F = \sum_{n} (f_P - f_E)_n^2$$

F Total error function to be minimized
f_P Predicted frequency (Hz)
f_E Measured frequency (Hz)
n mode number

The 2-D FE model developed for the carbon/epoxy composite skin material (L*W*t = 254*25.4*2.54 mm) had 250 elements along the length and 10 elements along the through-the-thickness (TTT) direction. This mesh density was found to give satisfactory results. During modeling the skin material was modeled as orthotropic and therefore orthotropic material properties were assigned. As mentioned before the carbon/epoxy skin material selected was

Table 2 Material properties of carbon/epoxy skin and polyurethane foam determined using combined experimental/numerical approach and used in the FE model

Part	Material	E_1 (MPa)	E_2 (MPa)	E_2 (MPa)	υ_{12}	υ_{13}	υ_{23}	G_{12} (MPa)	G_{13} (MPa)	G_{23} (MPa)	ρ (kgm^{-3})
Face sheet	Carbon/epoxy	10658	10658	10658	0.26	0.26	0.26	4000	4000	4000	1446
Core	Polyurethane foam	115	–	–	0.3	–	–	–	–	–	199

Table 3 Natural frequencies of polyurethane foam and carbon/epoxy skin samples

Mode no	Polyurethane foam			Carbon/epoxy skin		
	Natural frequency (Hz)		%Diff[a]	Natural frequency (Hz)		%Diff[a]
	EXP f_n (Hz)	FEA E_{foam}=115MPa f_n (Hz)		EXP f_n (Hz)	FEA (see Table2 for material properties) f_n (Hz)	
1	152	153.1	0.74	116	117	1.28
2	420	415.3	−1.12	327	324	−0.93
3	797	796.1	−0.11	659	635	−3.80
4	1276	1279.6	0.28	1113	1049	−6.10
5	1876	1850.7	−1.37	1715	1565	−8.70

[a] FEA as reference

made of woven fabric, thus the, longitudinal modulus E_1, transverse moduli E_2 and E_3, Poisson's ratios υ_{12}, υ_{13} and υ_{23} and the shear modulus G_{12}, and transverse shear moduli G_{13} and G_{23}, were assumed to be equal in all the three directions. (i.e. $E_1 = E_2 = E_3$, $\nu_{12} = \nu_{13} = \nu_{23}$ *and* $G_{12} = G_{13} = G_{23}$) (See Fig. 2 for orientation). Further, only the Young's moduli (E_1, E_2 and E_3) were used as the curve fitting parameter, and the combined/experimental approach as explained before was used to minimize the difference between predicted and experimental frequencies using F-F boundary conditions.

Composite sandwich models

2-D FE models of flat and curved sandwich beams were developed using Hypermesh and ABAQUS as mentioned before. The core and face sheets were modeled as separate components in the FE model with coincident nodes along their boundaries. Material properties used for foam and skin material given in Table 2 were determined using the combined experimental/numerical approach described earlier. As mentioned before, the face sheets are considered as linear, elastic, and orthotropic, while the core is assumed linear, elastic, and isotropic. Due to the limitations of time and resources, only in-plane modes such as flexure in the plane of the model was predicted and compared with the corresponding results from experiments. 3-D FE model was developed to confirm that the modes that are compared are in-plane and flexural only, which will be discussed later in section Three-dimensional FE Study.

Fine mesh density was used to model the foam and skin material and a convergence study for the first natural frequency was carried out. A mesh density of 15 elements for the foam and 4 elements for the skin material along the TTT, and 250 element divisions along the length were found to give satisfactory results. The same model was used for both the beams with and without debonds. Debond regions were meshed with a set of coincident nodes at the face-core interface. At the interface between core and face sheets, two sets of nodes, one belonging to the face sheet, the other to the core, were placed. These nodes occupy the same geometric position, but they could vibrate independently. Contact definitions cannot be defined between the skin and foam elements, because, ABAQUS does not allow nonlinear contacts for a simple modal analysis. The sets of coincident nodes between the layers representing the interface between face sheet and core were coupled together for the beams without debond. The length of debond was always 50.8 mm length for each case. Natural frequencies for the straight and curved sandwich beams were extracted using the eigenvalue extraction method using C-C boundary conditions.

Table 4 Measured natural frequencies of the sandwich beams from vibration tests

	Curvature angle θ (degree)	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Sandwich beams with no debond	Flat	541	1225	2038	2848	3688
	15	627	1183	2089	2949	3805
	30	741	1139	1935	2682	3496
	45	1093	1213	2075	2813	3660
	60	1108	1171	2006	2713	3536
Sandwich beams with upper debond	Flat	554	1178	2031	2817	3661
	15	600	1080	1970	2665	3516
	30	794	1156	2043	2779	3631
	45	977	1119	1955	2585	3418
	60	1100	1109	1946	2637	3473
Sandwich beams with lower debond	Flat	554	1178	2031	2817	3661
	15	608	1162	2010	2800	3650
	30	800	1156	2060	2756	3603
	45	980	1200	2013	2740	3584
	60	1081	1119	1911	2621	3425

Three-dimensional FE Study

As mentioned before, 3-D FE model was developed for flat and curved sandwich beam (15 deg only) without debond. The main reason for developing the 3-D model was to make sure that the predicted natural frequencies from 2-D models that are being compared with experiments are associated with only in-plane and flexural. Fine mesh density was used to model the foam and skin material and a convergence study for the first natural frequency was carried out. The model developed had a fine mesh density of 350*20*10 elements for the foam along the length, width and TTT, respectively. For the skin material three elements along the TTT, and same number of elements along the width and length were found to give satisfactory results.

Results and Discussion

Vibration Test Results

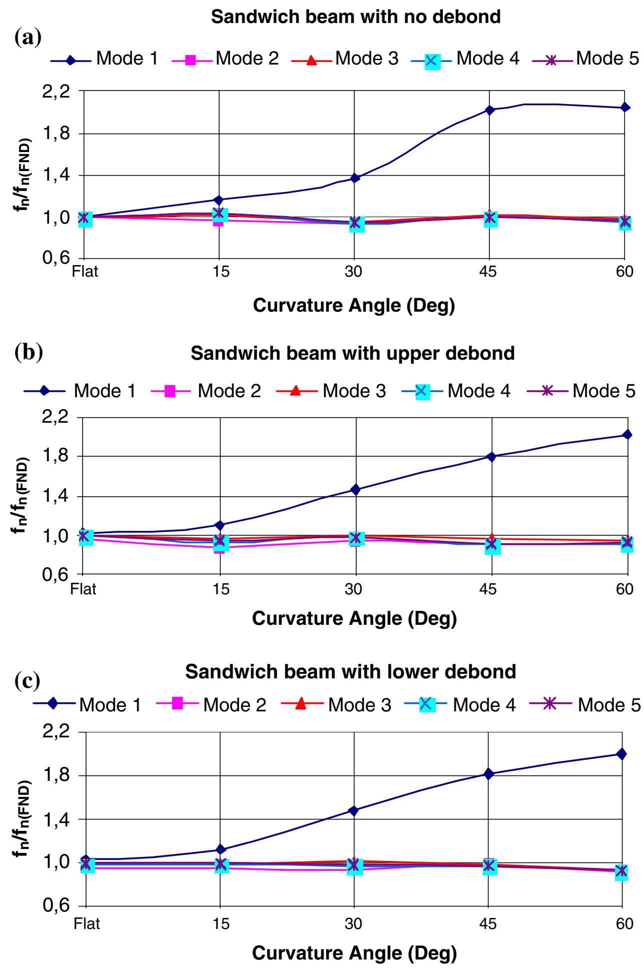
Composite skin and foam specimens

Natural frequencies of the composite skin and foam material specimens described earlier in section "Vibration Tests" are reported here. These results are later compared with numerical FE results and the material properties are determined using the combined experimental/numerical approach as discussed before. The natural frequencies under F-F boundary condition for polyurethane foam and carbon/epoxy skin specimens are tabulated in Table 3.

Composite sandwich beams

In this section, first, the vibration test results for the flat and curved sandwich beams without debonds are presented to investigate the dependence of natural frequencies on the curvature of the beam only. Next, results for beams with debond between upper and lower face sheets and core are presented. Curved sandwich beams with four different curvature angles in the range from 15° to 60° were tested as mentioned before (See Table 1). The first five natural frequencies of the flat and curved sandwich beams with and without debonds are tabulated in Table 4 and Fig. 3 shows the variation of natural frequencies under C-C boundary conditions with increasing curvature, where the frequencies are normalized to the corresponding frequencies of the flat sandwich beam without debonds ($f_{n(FND)}$).

Effect of curvature It is evident from the data presented in Fig. 3(a) that the first natural frequency increase with increasing curvature for the sandwich beams with no debond. For example, the fundamental frequency, f_1, of the flat sandwich beam is 541 Hz, whereas $(f_1)_{60}$=1,108 Hz when the curvature angle is 60° (See Table 4). However, for higher modes we can see that the frequencies are approximately the same. Therefore, we can conclude that only the fundamental frequency is significantly affected by increasing curvature. In this study it should be noted that the mass of all the beams are same. One of the factors which influence the modes is boundary condition. The mass dominated mode is sensitive to curvature because of the boundary condition. Boundary condition affects stiffness also. It is observed that at higher modes, the influence of boundary condition on stiffness is more when compared with the increase in the stiffness due to increase in curvature. All of higher mode frequencies are affected at same ratios. It is difficult to see the effect of the curvature at higher modes. Finally, nearly same frequencies are found at higher modes, whereas, at the initial modes, the mass is dominant. So changing of frequencies are related with only curvature.



Note: $f_{n(FND)}$ = frequency of flat beam with no debond

Fig. 3 Normalized natural frequencies for sandwich beam (**a**) with no debond (**b**) with upper debond (**c**) with lower debond

Similar results were obtained for the beams with debonds. To observe the results of the beams with debonds (presented in Table 4) graphically, natural frequencies of the curved sandwich beam specimens (with upper and lower debonds) normalized with respect to the natural frequency of flat sandwich beams without debonds ($f_{n(FND)}$) are plotted against curvature angle and shown in Fig. 3(b) and (c). From these figures, it can be seen that the first frequency increases with the increasing curvature angle and frequencies for higher modes are approximately the same. These results are consistent with the previous observations, in which no debonds were present.

Effect of debond As mentioned before, one of the objectives of the research is to investigate the effect of debond on vibration response of curved sandwich beams. Therefore, sandwich beams were made with debonds at the lower and upper interface between the face sheet and the core. The results of natural frequency variations for the sandwich beam with and without debonds as a function of curvature angle are presented in Fig. 4 up to 5th mode, in which the frequencies are normalized with respect to the corresponding frequencies of beams with no debonds. From this figure, it can be seen that the effect of debonds is more obvious for the curved beams, i.e. the natural frequencies of the curved beams are significantly affected by the presence of the debond. For flat beams, the differences in the natural frequencies of beams with and without debonds are less when compared with those of curved beams. One obvious reason for this is that the natural frequencies of each mode and each curvature angle decrease due to loss in stiffness caused by the presence of the debond except for the beams with angle of 30°. This is also true for the case of flat beam with UD, where the Mode

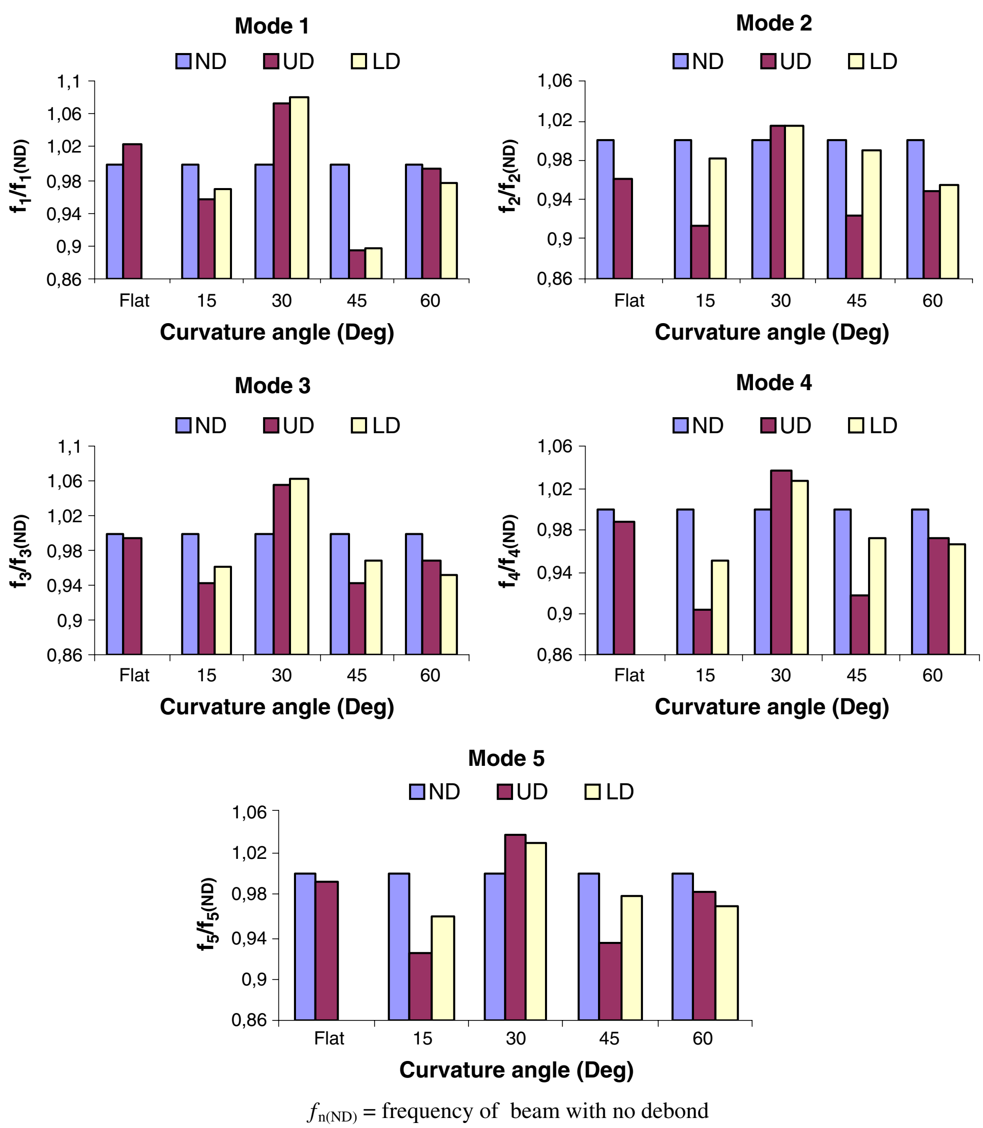


Fig. 4 Measured natural frequencies normalized with respect to corresponding frequencies of the beams with no debonds (ND = no debond, UD = upper debond, ND = lower debond)

1 frequency increases compared with the corresponding frequency for the beam with ND. For 30 deg curvature angle, it is observed that the natural frequencies increase when debonds are present. One possible reason could be due to manufacturing of the sample itself. Further investigation is being carried out to determine this behavior.

Effect of debond location To investigate the effect of debonds on the natural frequencies for each beam configuration, frequencies from Table 4 for flat and θ=15, 30, 45 and 60 degrees versus mode number are plotted as shown in Fig. 5. For the beams with curvature angle of 15° and 45°, natural frequencies are more sensitive to the upper debond. The presence of an upper debond reduces the natural frequency by about 4.5–10% for each mode. The greatest decrease of natural frequency with debond is seen in the beam with curvature angle of 45° for the first mode. For flat beams, the maximum decrease in the natural frequency with debond is about 4% for the second mode.

Combined Experimental/Numerical Analysis Results

Composite skin and foam specimens

In this section, the material properties that are indirectly determined using the combined experimental/numerical approach will be discussed. First, to determine the properties of the foam material, all the first five modes were used and the value of E_{foam} was indirectly determined to be 115 MPa. The predicted natural frequencies agree well with the measured values as shown in Table 3. As mentioned before, the density of the foam material was equal to 199 kgm^{-3}. Then, to determine the material properties of skin material, the shear modulus, G_{12}= 4,000 MPa and Poisson's ratio ν_{12}=0.26 were kept constant. Semiempirical micromechanical model approach was used to estimate shear modulus based on Tsai-Hahn [28]. Further, only the Young's moduli (E_1, E_2 and E_3) were used as the curve fitting parameter, and the combined/

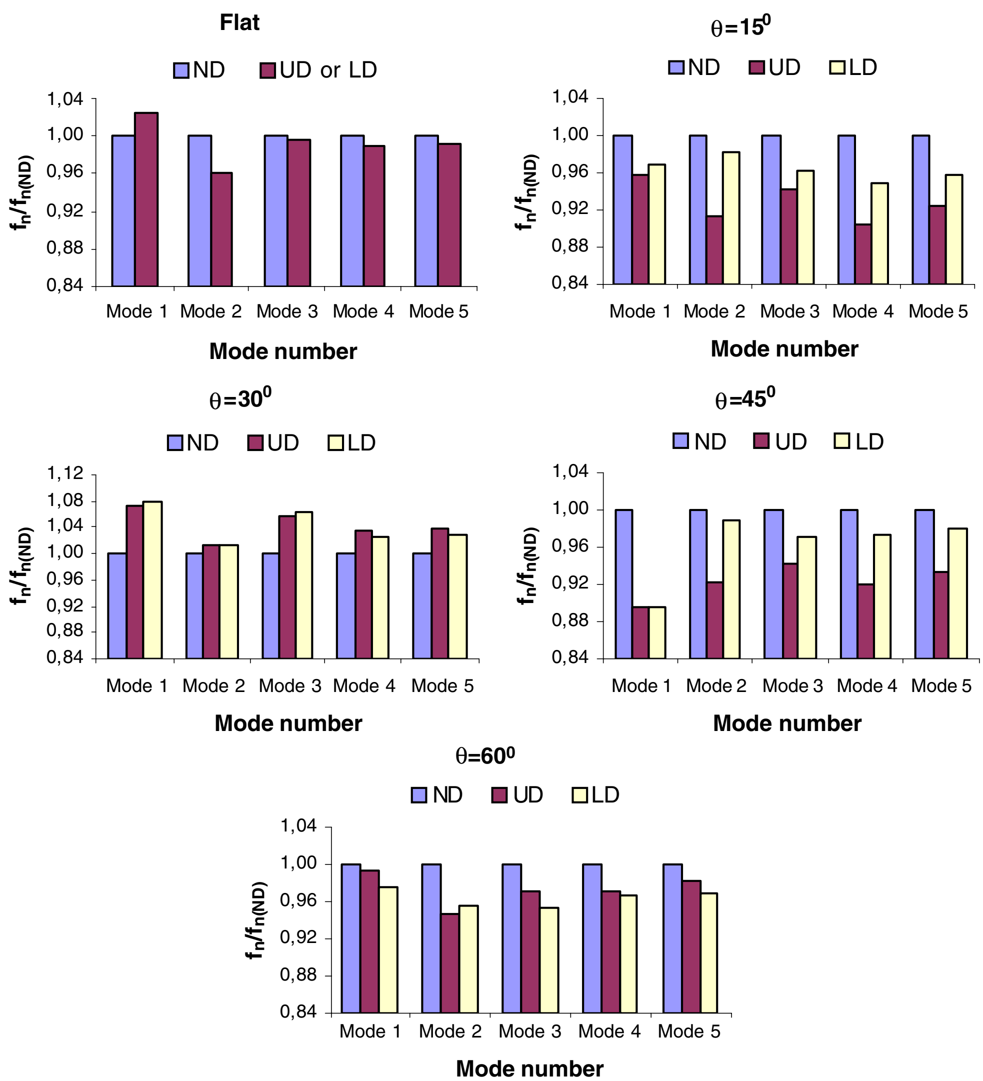


Fig. 5 Data from Fig. 4 replotted to show variation of measured frequencies with mode number for flat and curved beams

Table 5 Comparison of measured natural frequencies with predicted from 2-D and 3D models for flat and curved sandwich beams without debond

Natural frequency (Hz)						
Mode no	Flat sandwich beam			Curved sandwich beam with 15deg curvature		
	EXP	2-D	3-D	EXP	2-D	3-D
1	541	572	574	627	644	646
2	1225	1246	1248	1183	1262	1264
3	2038	2029	2030	2089	2059	2062
4	2848	2846	2847	2949	2878	2882
5	3688	3677	3678	3688	3717	3722

experimental approach as explained before was used to minimize the difference between predicted and experimental frequencies using F-F boundary conditions. The material properties obtained for foam and composite skin using the combined experimental/numerical method are tabulated in Table 2. It is observed again in Table 3 that the predicted natural frequencies (for skin material) agree very well with the measured values.

Composite sandwich beams

Comparison of 2-D and 3-D FE model results Table 5 shows the comparison of first five predicted natural frequencies of flat and curved sandwich beams (15 deg only) with no debonds from 2-D and 3-D models with experiments. The results agree very well with the measured values. The first five mode shapes from 2-D and 3-D models for flat and curved sandwich beam are given in Fig. 6(a) and (b), respectively. It is clear from these figures that the modes being compared are in-plane and flexural modes. Since there was good correlation between the 2-D and 3-D model results, it was decided to use only 2-D model results for further analysis. In the following section more detailed 2-D analysis of the sandwich beams with debonds are modeled and compared with experimental results.

Comparison of 2-D FE model results with experiments The comparison of measured and predicted natural frequencies of flat and curved beams with and without debonds are presented in Table 6. The results from the experiments and finite element analysis are found to be in good agreement. The difference between experimental and predicted results range from 0.2% to 11%. The predicted results agree very well for flat sandwich beams without debonds, but for beams with debonds the differences in the results are significant. From the predicted results, it is observed that for beams without debonds, the fundamental natural frequency increases with the increasing curvature. However, for higher modes, the frequencies are approximately the same, This is similar to the observations made from the experiments.

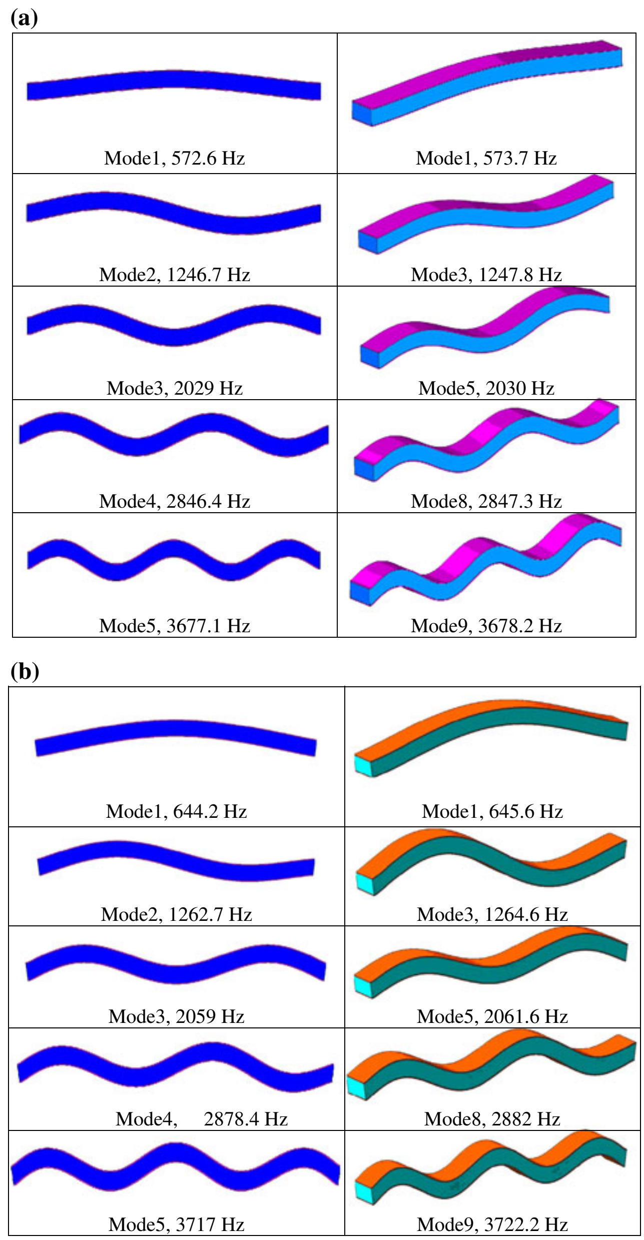


Fig. 6 Comparison of first five mode shapes from 2D and the corresponsing flexural modes from the 3D FE models (**a**) for flat sandwich beams without debond (**b**) for 15 Deg curved sandwich beam without debond

Table 6 Natural frequency comparison of Experimental (EXP) and Numerical (FEA) results for sandwich beams with and without debonds

Natural frequencies of sandwich beams (Hz)

Curvature angle (degree)	Specimen confg.	Mode 1			Mode 2			Mode 3			Mode 4			Mode 5		
		EXP	FEA	%Dif	EXP	FEA	%Dif	EXP	FEA	%Dif	EXP	FEA	%Dif	EXP	FEA	%Dif
Flat	ND	541	572	5,42	1225	1246	1,66	2038	2029	−0,43	2848	2846	−0,07	3688	3677	−0,29
	UD or LD	554	559,3	0,95	1178	1077	−9,30	2031	2059	1,37	2817	2835	0,63	3661	3526	−3,84
15	ND	627	644	2,69	1183	1262	6,26	2089	2059	−1,46	2949	2878	−2,47	3805	3717	−2,36
	UD	600	638	5,97	1080	1088	0,75	1970	1962	−0,38	2665	2566	−3,84	3516	3542	0,74
	LD	608	595	−2,13	1162	1088	−6,77	2010	2060	2,43	2800	2599	−7,73	3650	3577	−2,04
30	ND	741	815	9,13	1139	1269	10,21	1935	2088	7,33	2682	2896	7,39	3496	3745	6,64
	UD	794	782	−1,53	1156	1113	−3,79	2043	2084	1,96	2779	2594	−7,11	3631	3608	−0,65
	LD	800	721	−10,96	1156	1089	−6,15	2060	1931	−6,68	2756	2585	−6,62	3603	3585	−0,50
45	ND	1093	987	−10,74	1213	1242	2,33	2075	2094	0,91	2813	2874	2,12	3660	3743	2,22
	UD	977	982	0,51	1119	1175	4,77	1955	2037	4,03	2585	2696	4,12	3418	3598	4,99
	LD	980	982	0,20	1200	1157	−3,72	2013	1954	−2,99	2740	2774	1,23	3584	3669	2,33
60	ND	1108	1182	6,26	1171	1226	4,49	2006	2134	5,99	2713	2874	5,59	3536	3766	6,10
	UD	1100	1072	-2,58	1109	1251	11,35	1946	2113	7,90	2637	2549	-3,44	3473	3574	2,82
	LD	1081	1052	-2,79	1119	1259	11,12	1911	2097	8,88	2621	2609	-0,46	3425	3627	5,58

ND No debond, *UD* Upper debond, *LD* Lower debond

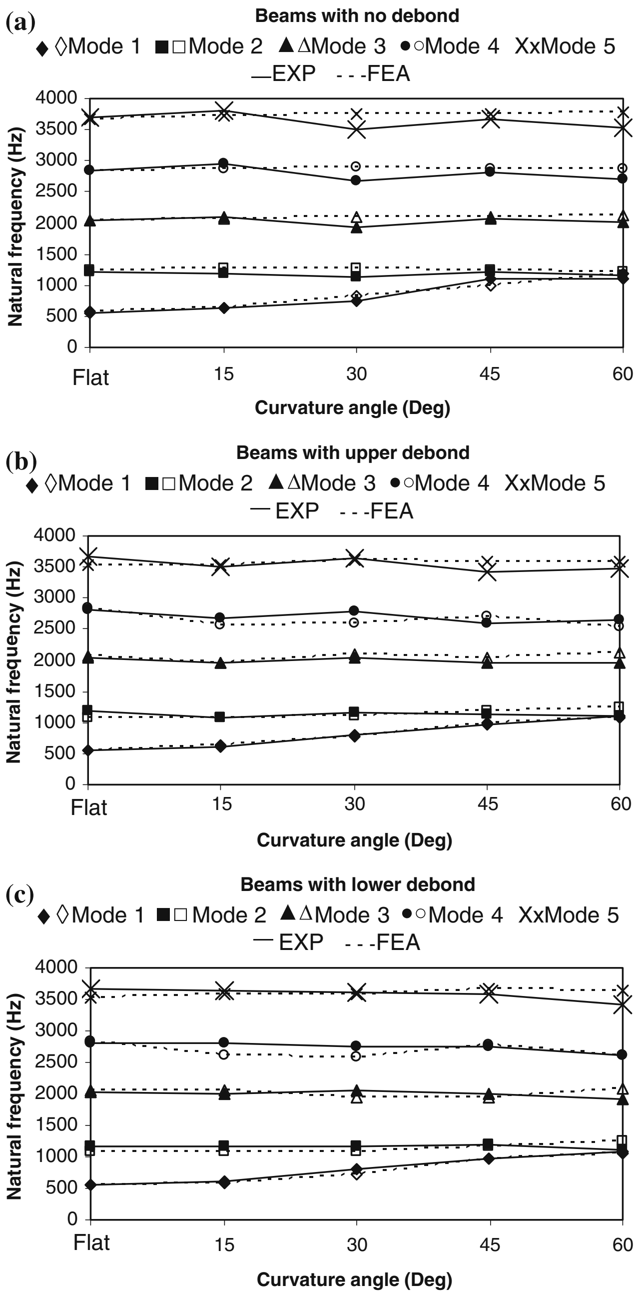


Fig. 7 Natural frequency comparisons of experimental and numerical results for sandwich beams (**a**) with no debond (**b**) with upper debond (**c**) with lower debond

When compared with experimental results, the predicted results generally tend to yield higher values.

The predicted and experimental natural frequencies versus curvature angle for all beams are given in Fig. 7. Similar to the experimental results, predicted results show that increasing the curvature angle of the sandwich beam does not have a significant effect on the higher natural frequencies of the intact sandwich beam. However, increasing the curvature angle from 15° to 60° increases the fundamental natural frequency of the intact sandwich beam 80%. Similarly, the variation of frequency of curved beams is sensitive to the curvature angle for mode 1, but the frequency variation for higher modes shows insensitive results when comparing the mode 1. For all curved beams, frequencies for the first mode increase with curvature angle.

Mode shapes (Mode 1 and 2) of flat and curved beams with no debonds are given in Fig. 8. It is observed that the mode shapes are not affected with increasing curvature (open angle) of the beams. However, if the mode shapes of sandwich beam with curvatures are compared with those of the flat beam [Fig. 6(a)], it is seen that the locations where the maximum displacement points occur change for each mode.

The measured natural frequencies of sandwich beams presented in Fig. 5 show the effect of debonds on the natural frequency. For flat beams with and without debonds, it is observed that reduction in Mode 2 frequency is greater than other modes. It seems that sandwich beams with curvature also behave in a similar manner with presence of debond for the second mode.

Conclusions

In this paper, predictions and measurements of the natural frequencies of flat and curved sandwich beams with and without debonds have been presented. The configurations considered were sandwich beams containing two debonds of the same length between the upper and lower face sheets and core. Debond length was 20% of the beam length. Only clamped-clamped boundary conditions were studied. The experimental vibration responses were studied using a frequency spectrum analyzer. 2D and 3-D finite element models were developed using the ABAQUS computer code for the numerical investigation. The results of the finite element analysis and experimental data showed good agreement except for the differences discussed earlier that deserve further investigation. The conclusions based on

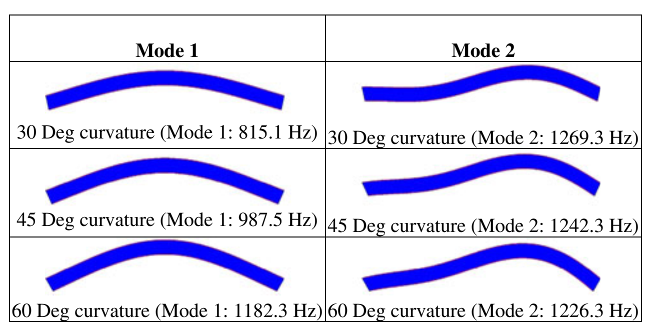


Fig. 8 Mode shapes (modes 1 and 2) of curved sandwich beams from without debonds from 2D FE model under clamped-clamped (C-C) boundary condition

vibration tests and finite element analyses can be summarized as follows:

- The values of the frequencies produced by the FE method agree with the experimental results. Maximum difference between experimental and predicted results is about 11%.
- The results show that the frequencies of sandwich beams considered in this study are quite sensitive to the presence of a face/core debond. Generally, reductions of the natural frequencies with debonds are seen to be greater for curved beams than for flat beams. Maximum reduction in natural frequency is 3.8% for flat beams, whereas for curved beams the maximum reduction in natural frequency is 10.6% due to the debond.
- It was found that decreasing the curvature angle diminishes the first natural frequency for each beam configuration. For higher frequencies, the curvature of the beam does not influence tremendously the natural frequency.

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