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Phase-Stepping Photoelasticity by Use of Retarders with Arbitrary Retardation

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Abstract The present paper describes a new phase-stepping algorithm for the analysis of isochromatics and isoclinics using retarders with arbitrary retardation. A retarder used in the proposed method is not necessarily a quarter-wave plate specified for the wavelength of the light used. Not only the isochromatic and isoclinic parameters but also the retardation of the input and output retarders are determined simultaneously by the proposed phase-stepping method. Thus, accurate analysis can be performed by the proposed method even if accurate quarter-wave plates are not used. In addition, any wavelength of visible light can be used in a single polariscope without requiring matching the wavelength of the quarter-wave plate. Thus, a multi-wavelength technique is easily combined with the proposed method for accuracy improvement, phase-unwrapping, or correction of ambiguity.

Keywords Phase-stepping method · Photoelasticity · Isochromatics · Isoclinics · Retarder · Arbitrary retardation

Introduction

There are several techniques for the analysis of fringe patterns in the fields of optical methods in experimental mechanics and optical metrology [1, 2]. Among them, the phase-stepping method is one of the most important and widely accepted techniques. In photoelasticity or in the field of birefringent measurement, the retardation (isochromatic parameter) and the principal direction (isoclinic parameter) can be evaluated accurately by a phase-stepping method [3–16]. In order to obtain accurate results by the phase-stepping method, however, an accurate quarter-wave plate such as quartz or mica must be used because the retardation error in a quarter-wave plate is one of the important error sources in phase-stepping photoelasticity [17–20]. However, a quarter-wave plate made of quartz is usually expensive, and the field of view is very small; the typical diameter is 10 ~ 30 mm. Usually, the accuracy of a quarter-wave plate with a large field of view is poor, and thus error is introduced during analysis. If accurate results could be obtained with low-priced equipment, digital photoelasticity could become more popular in several fields.

Ajovalasit et al. [17] investigated the influence of the quarter-wave plate error on the results of the analysis of isochromatics and isoclinics. They [18] proposed a phase-stepping method for reducing the influence of the quarter-wave plate errors using right- and left-handed circularly polarized light. Barone et al. [19] improved this method by combining linearly and circularly polarized light. Although accurate phase analysis can be performed by their methods, the influence of the quarter-wave plate error cannot be entirely excluded in the determination of the isochromatic parameter because the use of quarter-wave plates is a premise of their methods. Phase-stepping techniques using linearly polarized light does not require a quarter-wave plate. However, these techniques are not suitable for the determination of the isochromatic parameter because the phase jump is not revealed and

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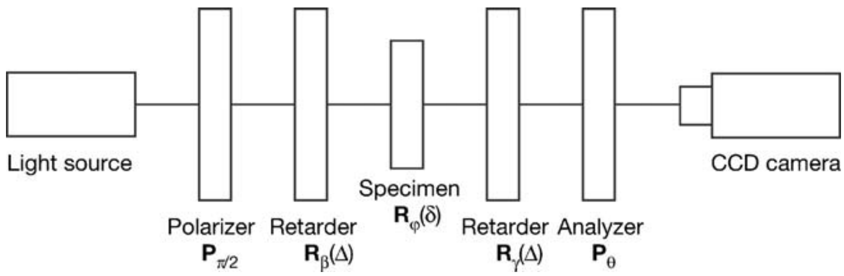
Fig. 1. Arrangement of optical elements for the proposed method



subsequent phase-unwrapping cannot be easily performed [6, 19].

In the present paper, a new phase-stepping algorithm for the analysis of isochromatics and isoclinics using retarders with arbitrary retardation is proposed. The retarder used in the proposed method is not necessarily a quarter-wave plate specified for the wavelength of the light used. Thus, any wavelength can be used as an incident light source. Not only are the isochromatic and isoclinic parameters determined but also the retardation of the input and output retarders simultaneously by the proposed method. In other words, the isochromatic and the isoclinic parameters can be evaluated even if the retardation of the retarder is unknown.

The theory of the proposed method is described in the present paper. Then, the effectiveness of the method is demonstrated by simulation and experiment. It is emphasized that the proposed method is valuable since accurate phase analysis can be performed without expensive optical elements. Additionally, any wavelength of visible light can be utilized in a single polariscope setup without requiring wavelength matched quarter-wave plate. Thus, a multi-wavelength technique is easily combined with the proposed method for accuracy improvement, phase-unwrapping, or correction of ambiguity.

Phase-Stepping Method Using Retarders with Arbitrary Retardation

An arrangement of optical elements in a polariscope shown in Fig. 1 is considered in this paper. This setup consists of a monochromatic light source, a linear polarizer whose optical axis is vertical, a retarder with retardation Δ whose fast axis makes an angle β with the *ox* axis (horizontal axis), a birefringent specimen with retardation δ whose fast axis subtends an angle φ with the *ox* axis, a retarder with retardation Δ whose fast axis makes an angle γ with the *ox* axis, a linear polarizer (analyzer) whose optical axis makes an angle θ with the *ox* axis, and a monochromatic CCD camera. In this setup, a common retardation Δ is assumed for the input and output retarders and the value of the retardation must lie in the range of $0 < \Delta < \pi$ radians. In addition, it is assumed that the retardation Δ of the retarders is uniformly distributed over the field of view although D'Acquisto et al. [20] reported that the retardations of the quarter-wave plates used in their study were not uniform. A general circular polariscope is obtained by setting the retardation $\Delta = \pi/2$ rad. On the other hand, the retardation Δ is treated as unknown in the proposed method.

The angle φ of the principal axis of the specimen is interpreted as the principal stress direction, i.e., the isoclinic parameter. Similarly, the retardation δ of the specimen, that is, the isochromatic parameter relates the principal stress difference as [6]

$$\delta = 2N\pi = 2\pi \frac{C_\sigma d}{\lambda}(\sigma_1 - \sigma_2), \tag{1}$$

where N is the isochromatic fringe order, C_σ is the stress-optic coefficient, d the thickness of the specimen, λ is the wavelength of the monochromatic incident light, and σ_1 and σ_2 are the principal stresses.

Table 1 Optical arrangements and light intensity equations for the proposed method

β, rad	γ, rad	θ, rad	Light intensity, *I*
π/4	0	0	$I_1 = \frac{1}{2}\left\{1 + \sin\delta\sin\Delta\sin 2\varphi - \cos\Delta\left(\cos^2 2\varphi + \cos\delta\sin^2 2\varphi\right)\right\}$
−π/4	0	0	$I_2 = \frac{1}{2}\left\{1 - \sin\delta\sin\Delta\sin 2\varphi - \cos\Delta\left(\cos^2 2\varphi + \cos\delta\sin^2 2\varphi\right)\right\}$
π/4	−π/4	−π/4	$I_3 = \frac{1}{2}\left(1 + \sin\delta\sin\Delta\cos 2\varphi + \cos\Delta\sin^2\frac{\delta}{2}\sin 4\varphi\right)$
−π/4	−π/4	−π/4	$I_4 = \frac{1}{2}\left(1 - \sin\delta\sin\Delta\cos 2\varphi + \cos\Delta\sin^2\frac{\delta}{2}\sin 4\varphi\right)$
−π/4	−π/4	π/2	$I_5 = \frac{1}{2}\left\{1 - \cos\delta\sin^2\Delta + \sin\delta\sin 2\Delta\sin 2\varphi + \cos^2\Delta\left(\cos^2 2\varphi + \cos\delta\sin^2 2\varphi\right)\right\}$
−π/4	π/4	π/2	$I_6 = \frac{1}{2}\left\{1 + \cos\delta\sin^2\Delta + \cos^2\Delta\left(\cos^2 2\varphi + \cos\delta\sin^2 2\varphi\right)\right\}$
π/4	π/4	π/2	$I_7 = \frac{1}{2}\left\{1 - \cos\delta\sin^2\Delta - \sin\delta\sin 2\Delta\sin 2\varphi + \cos^2\Delta\left(\cos^2 2\varphi + \cos\delta\sin^2 2\varphi\right)\right\}$

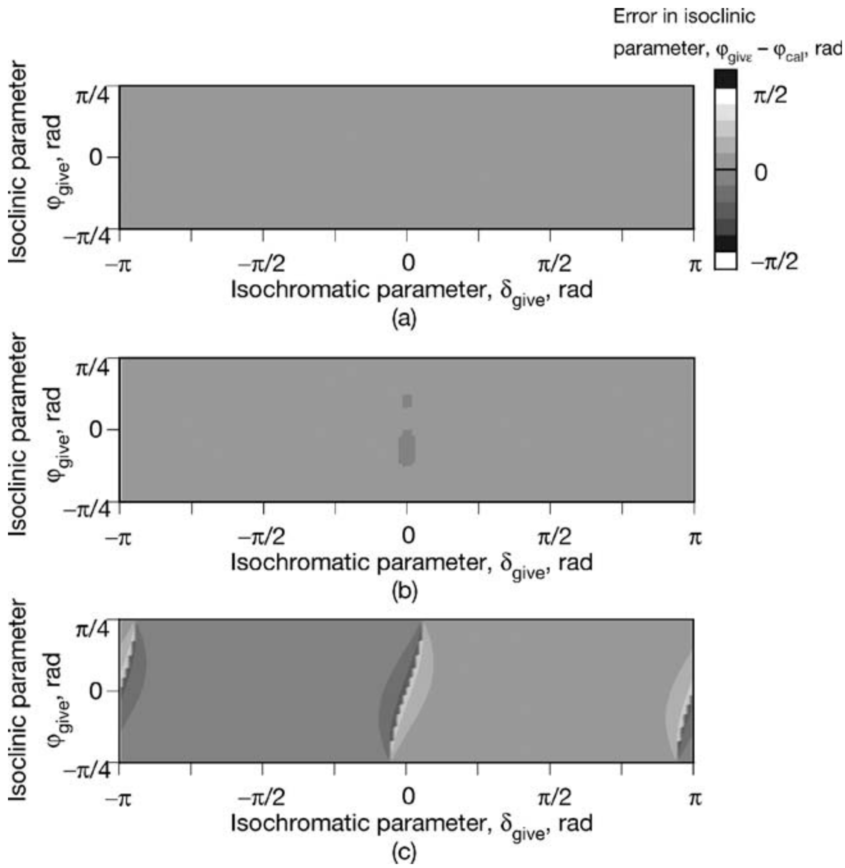


Fig. 2. Simulated error distributions of the isoclinic parameter as the function of the isoclinic and isochromatic parameters for (**a**) the proposed method, (**b**) Barone's method, and (**c**) Patterson's method ($\Delta = 5\pi/8$ rad)

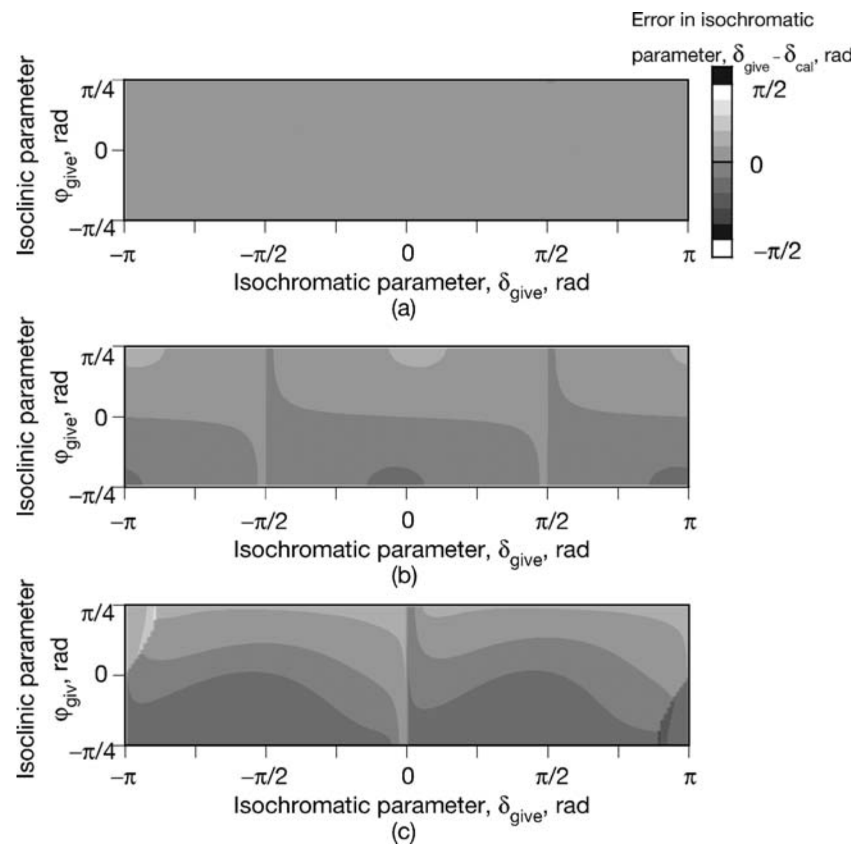


Fig. 3. Simulated error distributions of the isochromatic parameter as the function of the isoclinic and isochromatic parameters for (**a**) the proposed method, (**b**) Barone's method, and (**c**) Patterson's method ($\Delta = 5\pi/8$ rad)

The equation of the light intensity emerging from the analyzer can be obtained by use of Mueller or Jones calculus [5, 6, 21]. The proposed phase-stepping method is based on seven acquisitions, that is, seven combinations of the angular positions β, γ and θ of the retarders and the analyzer are used. The arrangements of β, γ and θ, and the corresponding light intensities $I_1 \sim I_7$ used in the proposed method are shown in Table 1. Here, the amplitude and the background bias are omitted in this table. Using the seven light intensity values $I_1 \sim I_7$ shown in Table 1, the isoclinic parameter (principal direction) φ, the retardation Δ of the retarder, and the isochromatic parameter (retardation) δ can be obtained as

$$\tan 2\varphi = \frac{I_1 - I_2}{I_3 - I_4} = \frac{\sin\delta \sin\Delta \sin 2\varphi}{\sin\delta \sin\Delta \cos 2\varphi} \quad (2)$$

for $\sin\delta \neq 0$,

$$\cos\Delta = \frac{I_5 - I_7}{2(I_1 - I_2)} = \frac{\frac{1}{2}\sin\delta \sin\Delta \sin 2\varphi}{\sin\delta \sin\Delta \sin 2\varphi} \quad (3)$$

for $\sin\delta \neq 0$ and $\sin 2\varphi \neq 0$,

$$\tan\delta = \frac{2(I_3 - I_4)\sin\Delta}{(-I_5 + 2I_6 - I_7)\cos 2\varphi} = \frac{\sin\delta \sin^2\Delta \cos 2\varphi}{\cos\delta \sin^2\Delta \cos 2\varphi} \quad (4)$$

for $\cos 2\varphi \neq 0$.

Since the retardation Δ must be the positive value, the sign of the function sinΔ in equation (4) can be determined as positive. Using equations (2) to (4), the phase values δ and φ of the isochromatic and isoclinic parameters are determined even if the retardation Δ of the retarder is unknown. At the points where the retardation Δ of the retarder is not obtainable, the isochromatic parameter δ cannot be determined. However, the values of the retardation at these points can be obtained by adopting the obtained values at other points since the retardation of the retarder is assumed as constant and uniform. Otherwise, if the retardation of the retarder is known in advance by a preliminary calculation or a calibration, this difficulty is easily overcome by inputting the value of the retardation into equation (4). The proposed method can be used for the case when the wavelength of the light does not match the quarter-wave plates in a circular polariscope, or the retardation of the quarter-wave plates deviates from the designed value even if the matching wavelength for the wave plate is used. In other words, any wavelength of visible spectrum can be used without taking the matching wavelength of the quarter-wave plate into account. Thus, accurate analysis can be performed by the proposed method without accurate and expensive quarter-wave plates such as a retarder made of quartz.

Simulations and Results

Error Distributions

The effectiveness of the proposed method is evaluated by simulation. In the first step, the errors in the

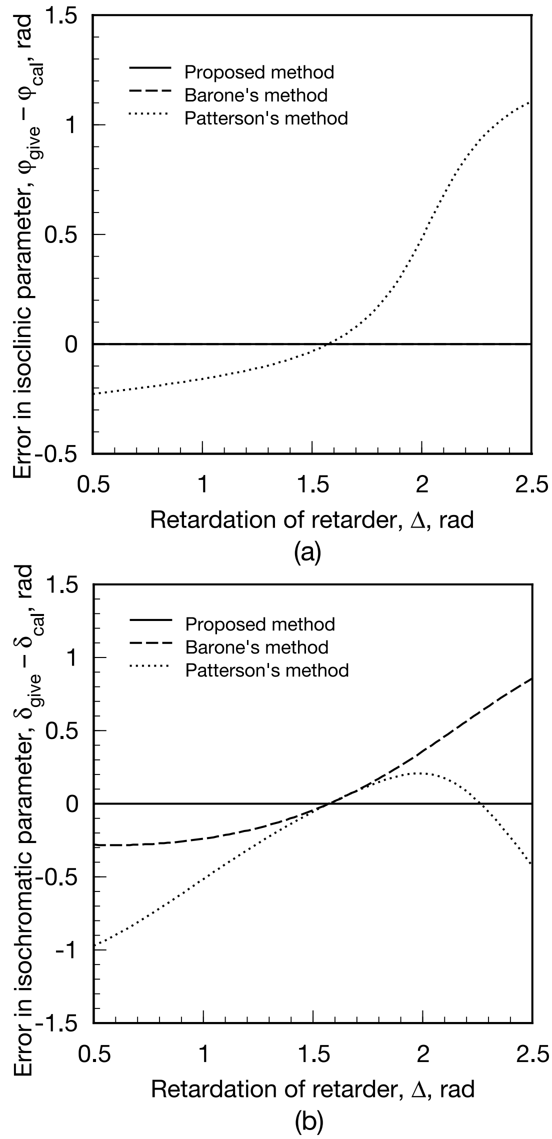


Fig. 4. Simulated error distributions of (**a**) the isoclinic and (**b**) isochromatic parameters as the function of the retardation of the retarder (φ_{give} = π/6 rad and δ_{give} = π/6 rad)

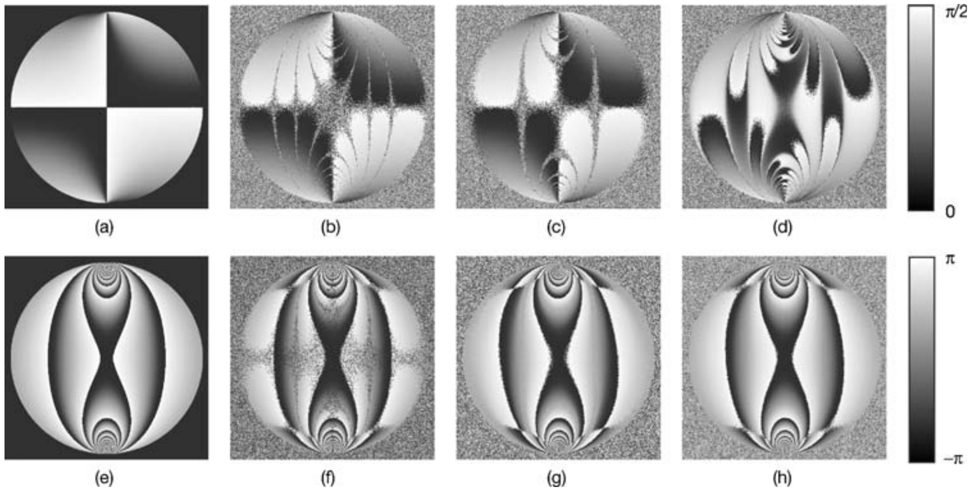


Fig. 5. Simulated phase distributions of the disk under compression: (**a**) theoretical φ; (**b**) φ obtained by the proposed method; (**c**) φ obtained by Barone's method; (**d**) φ obtained by Patterson's method; (**e**) theoretical δ; (**f**) δ obtained by the proposed method; (**g**) δ obtained by Barone's method; and (**h**) δ obtained by Patterson's method

calculated isoclinic and isochromatic parameters due to the retardation values of the retarder are calculated. The light intensity values expressed by the equations in Table 1 are generated as the function of the given isoclinic and isochromatic parameters φ_{give} and δ_{give} for several retardation values of the retarders. Here, the light intensities are not digitized such that the effect of the quantizing error is minimized. Then, the isoclinic and isochromatic parameters are calculated from those light intensities. In order to compare the proposed method with other methods, the phase-stepping algorithm for reducing the influence of the quarter-wave plate errors proposed by Barone et al. [19] and conventional algorithm proposed by Patterson and Wang [8] are also examined. Figure 2 shows an example set of the distributions of the error $\varphi_{give} - \varphi_{cal}$ in isoclinic parameter, i.e., the difference between the provided isoclinic parameter φ_{give} for the calculation of the light intensities and the calculated isoclinic parameter φ_{cal} by the proposed method, Barone's method, and Patterson's method. Similarly, the error distributions $\delta_{give} - \delta_{cal}$ of the isochromatic parameter are shown in Fig. 3. In these figures, the retardation of the retarders is set to 5π/8 rad, that is, the quarter-wave plate error is π/8 rad. No errors are seen for both of the isoclinic and isochromatic parameters obtained by the proposed method. In the Barone's algorithm, the quarter-wave plate mismatch dose not influence the isoclinic parameter [19]. However, it is observed that the isoclinic parameter obtained by the Barone's method shown in Fig. 2(b) contains small errors near the region where the isochromatic parameter equals zero because the square function of the isochromatics, i.e., $\sin^2(\delta/2)$, is present in the denominator in the calculation of the isoclinic parameter [19]. The isochromatic parameter by Barone's algorithm in Fig. 3(b) contains much error although their method does reduce the influence of quarter-wave plate errors. Because their method supposes the use of quarter- wave plates with only small error, much error appears in the isochromatic parameter for the retardation error π/8 rad of the retarder. It is observed from Figs. 2(c) and 3(c) that the results obtained by Patterson's algorithm are greatly influenced of using the retarder other than quarter-wave plates, as reported by Barone et al [19].

Figure 4 illustrates the errors in the isoclinic and isochromatic parameters $\varphi_{give} - \varphi_{cal}$ and $\delta_{give} - \delta_{cal}$ calculated as the function of the retardation Δ of the retarder. Here, the given isoclinic and isochromatic parameters φ_{give} and δ_{give} are set to π/6 rad and π/6 rad, respectively. No errors are seen in the isoclinic parameter obtained by the proposed and Barone's methods. On the other hand, the isoclinic parameter obtained by Pattersons's method contains much error

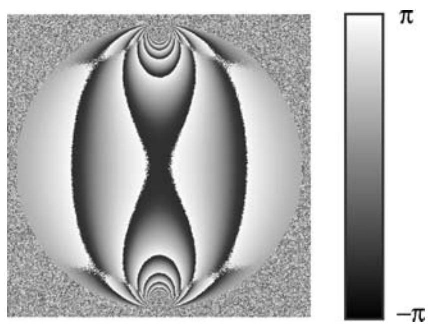


Fig. 6. Simulated phase distribution of the isochromatic parameter obtained by the proposed method

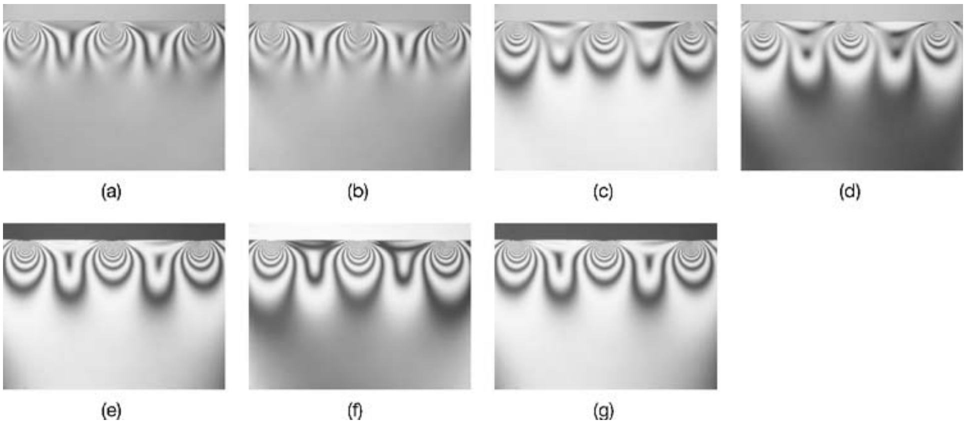


Fig. 7. (**a**) ~ (**g**) Phase-stepped photoelastic fringe patterns of the stress-frozen specimen for the proposed method

except in the region where the retardation of the retarder is π/2 rad. The isochromatic parameter obtained by the proposed method is also determined to be independent of the retardation of the retarder. However, the isochromatic parameters obtained by the other methods are much affected by the retardation values of the retarder.

Disk Under Compression

Next, a disk under compression is simulated. This example is chosen for comparison to other methods in the literature. In this simulation, the light intensity values are calculated from the theoretical stress distributions obtained by the theory of elasticity. These light intensity values, i.e., fringe patterns are digitized into 200 gray levels. Then, random noises of maximum 20 and minimum –20 gray levels are added to the digitized fringe patterns. The phase maps of the isoclinic and isochromatic parameters are then calculated from the noisy fringe patterns. Figure 5 shows the results of the simulation obtained by the proposed method, Barone's method, and Patterson's method for the case where the retardation of the retarder is 37π/90 rad. This is the case where the wavelength of 670 nm is used whereas the matching wavelength of a quarter-wave plate is 550 nm. In Fig. 5, the theoretically obtained phase maps are also shown for comparison. Accurate phase maps are obtained by the proposed and Barone's methods as shown in Fig. 5(b) and (c) except in the region of the isochromatic-isoclinic interaction [6]. The phase map of the isoclinics in Fig. 5(d) is not correct as predicted in previous simulation. The phase maps of the isochromatics in Fig. 5(g) and (h) show a small amount of deviation and unsymmetrical distributions. On the other hand, the phase of the isochromatic parameter obtained by the proposed method in Fig. 5(f) contains errors near the region where the isoclinic parameter is 0 or π/2 rad and the isochromatic parameter is 0 rad. In these regions, the isochromatic parameter is not obtainable because the retardation of the retarder cannot be obtained from equation (3). As mentioned in a previous section, however, it is not necessary to calculate the retardation of the retarder at all points. The value of the retardation can be obtained by a calibration or a preliminarily calculation in the proposed method. Otherwise, the calculated value of the retardation at the other points can be used. Using the average value obtained in the region where the retardation of the retarder can be calculated, the phase

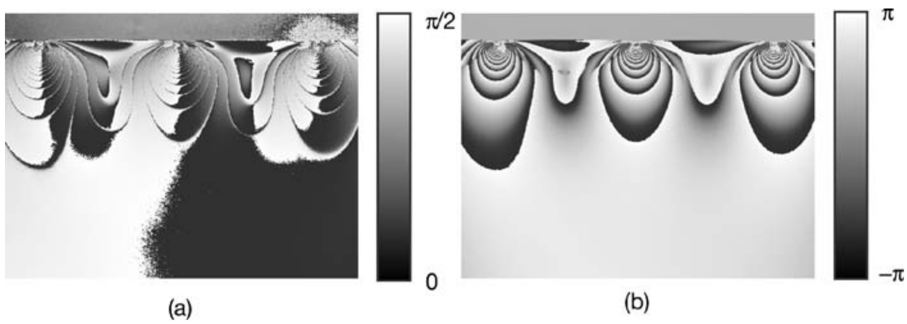


Fig. 8. Phase distributions of (**a**) the isoclinics and (**b**) the isochromatics obtained by the proposed method (λ = 550 nm)

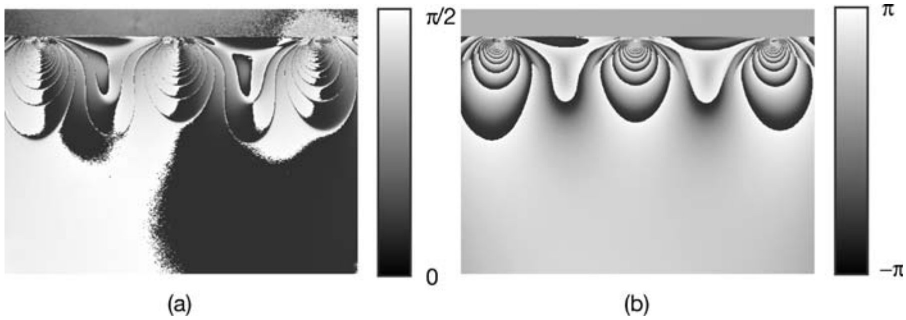


Fig. 9. Phase distributions of (**a**) the isoclinics and (**b**) the isochromatics obtained by the proposed method (λ = 600 nm)

map of the isochromatic parameter is calculated as shown in Fig. 6. Unlike the phase maps in Fig. 5(f), (g), and (h), a symmetrical and accurate phase distribution is obtained.

The above simulations show that the isoclinic and isochromatic parameters can be determined accurately by the proposed method using retarders with known retardation.

Experiments and Results

Two polymeric retarders, 100 mm in diameter and 1.2 mm in thickness, are used as input and output retarders. These retarders are cut from a single sheet. The cost of this sheet type retarder is about 1/3000 ~ 1/1000 of the quarter-wave plate made of quartz. The value of the retardation of this retarder is still unknown. The polarizer and the analyzer are also cut from a sheet type polarizer in order to construct a low-priced polariscope with a large field of view.

A simple static problem is analyzed to evaluate the effectiveness of the proposed method. A stress-frozen specimen made of epoxy resin, 100 mm in width, 80 mm in height, and 6 mm in thickness, is subjected to three concentrated loads on the upper edge and the stresses are frozen in an oven. The photoelastic fringe patterns are collected by a monochromatic CCD camera with a resolution of 640 $\times$ 480 pixels and 256 gray levels. Two monochromatic lights of wavelengths of 550 nm and 600 nm emitted from a halogen lamp with interference filters are used as the light source in order to validate the independence of the proposed method to wavelength.

Figure 7 shows the seven photoelastic fringe patterns for the proposed phase-stepping method at the wavelength of 550 nm. Using the proposed algorithm, the phase distributions of the isoclinic and isochromatic parameters are determined as shown in Fig. 8 for the wavelength λ = 550 nm and in Fig. 9 for λ = 600 nm. Note that no interpolation or filtering is adopted in these results. It is observed from Figs. 8(a) and 9(a) that good phase distributions of the isoclinics are obtained by the proposed method except in regions of isochromatic-isoclinic interaction. The average retardation value Δ of the retarder is obtained from equation (3) as 1.42 rad for λ = 550 nm and 1.34 rad for λ = 600 nm. The phase values of the isochromatic parameter in Figs. 8(b) and 9(b) are calculated using these retardation values. Very smooth phase distributions are obtained as shown in Figs. 8(b) and 9(b). The phase values of the isochromatic parameter for the two wavelengths are different from each other because the phase value depends on the wavelength. However, similar phase distributions are obtained for two different wavelengths as shown in Figs. 8(b) and 9(b). In addition, no deviation of the phase distribution and no noise are observed.

From the results of simulation and experiment, the effectiveness of the proposed method is validated. That is, the phase analysis of the isochromatics and the isoclinics can be performed without accurate quarter-wave plates. In addition, any wavelength of the visible light can be used in a single polariscope. Thus, a multi-wavelength technique is easily combined with the proposed method for accuracy improvement, phase-unwrapping, or correction of ambiguity.

Conclusions

This paper describes a new phase-stepping algorithm for the analysis of isochromatics and isoclinics using retarders with arbitrary retardation. The phase distributions of the isoclinics and the isochromatics can be obtained by the proposed method without accurate quarter-wave plates. Any wavelength of the visible light can be used in a single polariscope without matching the wavelength of the quarter-wave plate. It is expected that a low-priced polariscope with accurate analysis such as the propose method would make a digital photoelasticity more popular in several fields.

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