Int Adv Econ Res (2016) 22:199–209
DOI 10.1007/s11294-016-9577-9

Algebra of Integrated Time Series: Evidence from Unit Root Analysis

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Published online: 28 April 2016

Abstract It is argued if $x_t \sim I(1)$ and $y_t \sim I(1)$, then running a regression x_t on y_t would produce spurious results because e_t would generally be $I(1)$. However, there may exist a 'b' such that $e_t = x_t - by_t$ is $I(0)$, then running a regression x_t on y_t would not produce spurious results. This special case of two integrated time series is known in the literature as cointegration. In this particular case, x_t and y_t are said to be cointegrated. In our review of the development of the concept of cointegration, we identified that the underlying reason for this special case to arise is the proposition that if $x_t \sim I(d_x)$, $y_t \sim I(d_y)$, then $z_t = bx_t + cy_t \sim I(\max(d_x,d_y))$. In this research, we offer evidence against this proposition.

Keywords Cointegration analysis · Unit root · Time series · Econometric modeling · Economic policy · Policy analysis

JEL Classification C22 · C50 · E60

Introduction

In economics, when a historical perspective is overlooked in a descriptive research design, misleading conclusions may often follow.[1] By historical perspective, we refer to the understanding of a subject matter in light of its previous stages of intellectual

[1]The knowledge that in response to the financial and economic crisis of 2007–2009, economists are open for re-evaluating alternative approaches to neoclassical paradigm gave us an additional strength to carry out this research (Neck 2014).

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development and successive advancement. We therefore put our arguments against unit roots and cointegration analysis in a historical perspective.[2]

The recognition of a spurious regression problem in the late 1970s contributed decisively to the development of unit roots and cointegration (Granger and Newbold 1974; Hendry 1980, 1986; Granger 1981, 1986). A spurious regression problem arises when a regression analysis indicates a relationship between two or more unrelated time series variables because each variable has either a trend, or is nonstationary, or both. While working with economic time series data, researchers, attempting to account for a spurious regression problem, began pretesting for nonstationarity before estimating regressions. If data were found to be nonstationary on the basis of an appropriate unit root test, researchers would routinely purge the nonstationarity by differencing and then estimating regression equations using only differenced data as a solution for the spurious regression problem. The practice of purging the nonstationarity by differencing would also result in the loss of valuable information from economic theory about the long-run equilibrium properties of the data (Kennedy 2003).

It was in this context that Granger proposed that if two nonstationary variables were $I(1)$ process, the bivariate dynamic relationship between the two nonstationary variables would be misspecified when both of the nonstationary variables were differenced. This class of models has since become a dominant paradigm in empirical economic research and is known in the literature as a cointegrated process (Hamilton 1994).

In our review of the development of the concept of cointegration, we identified that the most important proposition of two integrated series was that if $x_t \sim I(d_x)$, $y_t \sim I(d_y)$ then $z_t = bx_t + cy_t \sim I(\max(d_x, d_y))$ (Granger 1981). Put simply, this proposition states that the sum of the two time series variables of a different order of integration will always yield another time series variable that will retain the "order of integration property" of the two series that has the higher order of integration. Granger's proposition was subsequently clarified by Hendry (1986), Engle and Granger (1987), Cuthbertson et al. (1992), and the Royal Swedish Academy of Science (2003). The clarified proposition included both the sum and the difference of the two time series. In this research, we offer evidence against this proposition.

Test of Order of Integration and Data

A time series is said to be strictly stationary if both its marginal and all joint distributions are independent of time. For practical purposes, however, it is the weak stationarity or covariance-stationarity that is more useful. A time series is said to be weakly stationary or covariance-stationary if the first two moments, mean and autocovariances, of a series do not depend on time. A stationary time series that does not need differencing is said to be integrated of order zero and is denoted $I(0)$. A nonstationary time series that becomes stationary after first differencing is said to be integrated of order one and is denoted $I(1)$. In general, a time series that requires differencing d times to become $I(0)$ is said to be integrated of order d and is denoted $I(d)$ (Granger 1986). Since the number

[2] See Temin (2013) for an eloquent description of how or why economic history vanished both from the faculty and the graduate program at Massachusetts Institute of Technology (MIT), and subsequently its cost consequences to current economic education and overall societal scholarship.

d equals the number of unit roots in the characteristic equation for the time series (Said and Dickey 1984, page 599) unit root tests are often used to determine the order of integration of a series. Thus, we describe below the unit root tests that we will use in our analysis.

Consider the following difference equation:

$$Y_t = \rho Y_{t-1} + e_t \tag{1}$$

where e_t is a white noise error term. When $\rho = 1$, equation (1) is known as a pure random walk model, a nonstationary stochastic process. This model can be alternatively expressed as:

$$\Delta Y_t = \delta Y_{t-1} + e_t \tag{2}$$

where $\delta = (\rho - 1)$ and Δ is the first-difference operator (e.g. $Y_t - Y_{t-1}$).

Testing for the presence of unit roots involves simultaneously determining whether an intercept and/or a time trend belong to the regression model, not including enough of them biases the test in favor of the unit root null hypothesis, whereas including too many of these parameters results in the loss of power (Elder and Kennedy 2001). With an economic time series, the main competing alternative to the presence of a unit root is a deterministic linear time trend. We therefore modify Eq. 2 to include (i) a drift term (Eq. 3), and (ii) a drift term and a linear trend term (Eq. 4):

$$\Delta Y_t = \alpha + \delta Y_{t-1} + e_t \tag{3}$$

$$\Delta Y_t = \alpha + \beta t + \delta Y_{t-1} + e_t \tag{4}$$

Due to its simplicity, we will use the test procedure proposed by Fuller (1976) and Dickey and Fuller (1979, 1981), known as the Dickey-Fuller (DF) test. In the DF test, it is assumed that e_t are uncorrelated and independently and identically distributed (*iid*). If e_t are correlated, Said and Dickey (1984) showed that the DF test may still be used provided that the lag length in the autoregression increases with the sample size. The test modified by Said and Dickey (1984) is known as the augmented Dickey-Fuller (ADF) test, and is given by:

$$\Delta Y_t = \alpha + \beta t + \delta Y_{t-1} + \sum\nolimits_{i=1}^{p} \gamma_i \Delta Y_{t-i} + e_t \tag{5}$$

The problem with the ADF test is that the choice of the lag length is arbitrary. Although the Akaike or Schwarz information criteria are generally used to decide the lag length, they do not always yield identical results. According to Harris (1992), the size and power properties of the ADF test are improved if a fairly generous lag is used. He proposes a formula, $l_i = int\{i(n/100)^{1/4}\}$, to determine the lag length that allows for the order of autogression to grow with sample size. Previously, Schwert (1989) used lag lengths based on the formulas $l_4 = int\{4(n/100)^{1/4}\}$ and $l_{12} = int\{12(n/100)^{1/4}\}$. In contrast, Taylor (2000) recommended selecting a lag length using a data-based algorithm or using a much higher significance level (e.g. 0.2 level rather than the traditional 0.05 level) in the general-to-specific rule framework. Note that the critical values for the ADF test differ very little from the DF critical values, so in practice researchers often used the DF critical values. Above all, if the e_t are not *iid* and are correlated, the DF tests do not have the correct asymptotic size (Phillips and Perron 1988).

In our analysis, we do not use the ADF test for reasons discussed in Luitel and Mahar (2016). Furthermore, using nonparametric methods, Phillips (1987) and Phillips and Perron (1988) show that the Philips-Perron (PP) unit root test takes into account serial correlation in the error terms without adding lagged difference terms and thus has an advantage over DF and ADF test procedures. We will use PP test procedure because it will arguably allow for a wide class of time series models in which a unit root may be present.

Even today much controversy in the literature still surrounds the most powerful test for unit roots. To overcome such controversy, Elliott et al. (1996) proposed a modified version of the Dickey-Fuller *t* test, known as the DF-GLS test, in which the time series was transformed using a generalized least square (GLS) method, rather than OLS, before performing the unit root test. Elliott et al. (1996) argued that the DF-GLS test substantially improved power when an unknown mean or trend was present, and as such, this test dominates all other unit root tests currently in common use such as DF, ADF and PP unit root tests. We will use DF-GLS test in our data analysis.

In order to show the violation of Granger's proposition, we use two sets of data: (i) Gross Domestic Product (GDP) and some of its components and (ii) unemployment rate and its components. Many macroeconomic variables are either a sum or a difference or some algebraic manipulation of other macroeconomic variables. For example, consider the openness index, an economic variable, defined as the sum of exports and imports divided by GDP $\left(\text{Openness Index} = \frac{\text{Exports}+\text{Impo}rts}{GDP}\right)$. We test whether the openness index and all its components are stationary or nonstationary. For a second example of an algebraic manipulation of other macroeconomic variables, we use the unemployment rate, defined as the number of people unemployed and looking for work divided by the labor force (the sum of unemployed plus employed) $\left(\text{Unemployment Rate} = \frac{\text{Unemployed}}{\text{Labor Force}}\right)$. We test whether unemployment rate and all its components are stationary or nonstationary. We obtained annual data on GDP, exports and imports for the United States from 1929 to 2012. Similarly, we obtained annual data on number of people unemployed and number of people employed for the United States from 1947 to 2012. We created other desired variables using the appropriate algebraic manipulation as described above. The literature suggests that data in logarithmic form achieves stationarity better than unlogged data. Nonetheless, we report results both for original series and for their natural logarithmic transformation. Table 1 provides the summary statistics of data for original series and Table 2 provides the summary statistics of data for their natural logarithmic transformation.[3]

Empirical Results

In this section, we report results of the unit root tests used in the determination of the order of integration of a series. We performed five variants of three different unit root

[3] Data files are available from the corresponding author upon request.

Table 1 Summary statistics (Original Series)

Variables	Time Period	Number of observations	Mean	Standard Deviation	Minimum	Maximum
In Level						
Openness index [a]	1929–2012	84	0.14564	0.07151	0.05071	0. .30714
Exports [a]	1929–2012	84	386.419	560.3179	2	2195.9
Imports [a]	1929–2012	84	493.394	752.014	1.9	2743.1
Exports [a] + Imports [a]	1929–2012	84	879.8131	1310.213	3.9	4939
GDP [a]	1929–2012	84	3839.294	4816.381	57.2	16,244.6
Unemployment rate [b]	1947–2012	66	0.05774	0.01641	0.02910	0.09689
Number of people unemployed [b]	1947–2012	66	6264.621	3038.883	1834	14,825
Number of people employed [b]	1947–2012	66	98,625.92	30,165.08	57,038	146,047
Labor force [b]	1947–2012	66	104,890.5	32,590.28	59,349	154,975
In First Differences						
Openness index [a]	1929–2012	83	0.00233	0.01261	−0.05197	0.03424
Exports [a]	1929–2012	83	26.38554	65.54907	−259.2999	259.7
Imports [a]	1929–2012	83	32.98193	102.3712	−580.3999	386
Exports [a] + Imports [a]	1929–2012	83	59.36747	164.8533	−839.7	645.7
GDP [a]	1929–2012	83	194.4578	229.5375	−302.3994	818.4004
Unemployment rate [b]	1947–2012	65	0.000642	0.01094	−0.02086	.03470
Number of people unemployed [b]	1947–2012	65	156.8462	1143.038	−2178	5341
Number of people employed [b]	1947–2012	65	1314.323	1530.197	−5485	4171
Labor force [b]	1947–2012	65	1471.169	799.3574	−273	3242

Sources: [a] Bureau of Economic Analysis (www.bea.gov)

[b] Bureau of Labor Statistics (www.bls.gov)

tests on each time series: DF test, PP test and Elliot, Rothenberg and Stock DF-GLS test. In each case, the null hypothesis was that the variable under investigation had a unit root, against the alternative that it did not have a unit root. A significant test statistic rejects the null hypothesis that the series has a unit root; thus, significant values indicate the series to be stationary. The results are presented in Table 3 for original series and in Table 4 for their natural logarithmic transformation and the tests were performed sequentially. The top half of Table 3 and Table 4 report the unit root test results for stationarity of the variables in levels. The bottom half of Table 3 and Table 4 report the results for the various tests of unit root on first differences of the variables.

Consider the test results for an openness index and its components. The results in Table 3 indicate that each component in the numerator, exports and imports, is individually $I(d = 1)$, the variable in the denominator, GDP, is $I(d = 1)$, and the resultant openness index is also $I(d = 1)$. Denote exports + imports as x_{1t}, GDP as y_{1t} and

Table 2 Summary statistics (Natural Logarithm)

Variables	Time Period	Number of observations	Mean	Standard Deviation	Minimum	Maximum
In Level						
Openness index [a]	1929–2012	84	−2.04698	0.49639	−2.98155	−1.18044
Exports [a]	1929–2012	84	4.34280	2.15375	0.69314	7.69434
Imports [a]	1929–2012	84	4.37441	2.30452	0.64185	7.91684
Exports [a] + Imports [a]	1929–2012	84	5.05892	2.22732	1.36097	8.50491
GDP [a]	1929–2012	84	7.10591	1.75391	4.04655	9.69551
Unemployment rate [b]	1947–2012	66	−2.89096	0.28332	−3.53684	−2.33414
Number of people unemployed [b]	1947–2012	66	8.61941	0.51716	7.51425	9.60407
Number of people employed [b]	1947–2012	66	11.45075	0.31721	10.95147	11.89168
Labor force [b]	1947–2012	66	11.51038	0.32423	10.99119	11.95102
In First Differences						
Openness index [a]	1929–2012	83	0.01225	0.09802	−0.34438	0.39549
Exports [a]	1929–2012	83	0.07131	0.15646	−0.41689	0.73631
Imports [a]	1929–2012	83	0.07462	0.13497	−0.42285	0.33506
Exports [a] + Imports [a]	1929–2012	83	0.07304	0.12840	−0.39688	0.39374
GDP [a]	1929–2012	83	0.06078	0.06906	−0.26301	0.24907
Unemployment rate [b]	1947–2012	65	0.01121	0.19394	−0.46694	0.64544
Number of people unemployed [b]	1947–2012	65	0.02597	0.19330	−0.47000	0.65536
Number of people employed [b]	1947–2012	65	0.01408	0.01504	−0.03846	0.04287
Labor force [b]	1947–2012	65	0.01476	0.00799	−0.00305	0.03221

Sources: [a] Bureau of Economic Analysis (www.bea.gov)

[b] Bureau of Labor Statistics (www.bls.gov)

the openness index as z_{1t}. The expression of openness index can be written as $z_{1t} = \frac{x_{1t}}{y_{1t}}$. Taking the natural log of both sides, the expression becomes $\ln z_{1t} = \ln x_{1t} - \ln y_{1t}$. The results in Table 4 indicate that $\ln x_{1t} \sim I(1)$, $\ln y_{1t} \sim I(1)$ and $\ln z_{1t} \sim I(1)$. This translates into $I(1) \pm I(1) = I(1)$, which appears to not contradict the Granger proposition. Although not reported here, we found a structural break in GDP, exports and imports in 1997. For a further discussion of structural break in US GDP in 1997, see Luitel and Mahar (2015a). Note that the unit root property of the data is inconsistent with the structural break, which implies that the parameters governing the data generating process have changed. Because the current literature treats the unit root property of the data as if not affected by a structural break, the finding of unit root itself becomes questionable.

Next, let us consider the unemployment rate and its components. The results in Table 3 indicate the number of people unemployed, the number of people employed and the labor force. Each individual series is $I(d = 1)$ but the unemployment rate is $I(d = 0)$. Our finding regarding unemployment rate is confirmatory to the finding of Nelson and Plosser (1982), who concluded "that the series (unemployment rate) is

Table 3 Unit root test results of order of integration (Original Series)

	Dickey-Fuller Test		Phillips-Perron Test		DF-GLS Test
Variables	Model 1[§]	Model 2[†]	Model 3[§]	Model 4[†]	Model 5[†]
In Level					
Openness index [a] (Z_{1t})	0.690	−3.148	1.437	−12.772	−1.142
Exports [a]	5.157	2.083	5.356	4.453	0.747
Imports [a]	3.205	0.632	4.376	2.437	0.482
Exports [a] + Imports [a] (x_{1t})	4.029	1.223	4.860	3.441	0.698
GDP [a] (y_{1t})	11.411	2.605	3.188	1.361	0.219
Unemployment rate [b] (Z_{2t})	−2.760***	−3.047	−15.218**	−18.338*	−3.293**
Number of people unemployed [b] (x_{2t})	−1.109	−2.599	−4.043	−16.405	−3.734***
Number of people employed [b]	0.062	−1.862	−0.012	−7.641	−1.816
Labor force [b] (y_{2t})	0.970	−2.287	0.166	−5.594	−1.344
In First Differences					
Openness index [a]	−9.025***	−9.261***	−72.051***	−73.718***	−6.200***
Exports [a]	−7.282***	−9.046***	−67.329***	−76.641***	−6.848***
Imports [a]	−9.119***	−10.623***	−83.560***	−84.439***	−7.623***
Exports [a] + Imports [a]	−8.547***	−10.325***	−79.284***	−82.058***	−7.676***
GDP [a]	−2.502***	−5.217***	−10.925*	−41.094***	−4.757***
Unemployment rate [b]	−7.153***	−7.091***	−48.735***	−48.716***	−6.229***
Number of people unemployed [b]	−5.999***	−5.949***	−40.639***	−40.559***	−5.893***
Number of people employed [b]	−5.237***	−5.203***	−37.007***	−37.154***	−5.141***
Labor force [b]	−3.857***	−3.868**	−24.507***	−25.470**	−2.754

***indicates 1 % significance level, **indicates 5 % significance level and * indicates 10 % significance level.
[§] Model 1 and Model 3: $y_t = \alpha + \delta y_{t-1} + e_t$. [†] Model 2, Model 4 and Model 5: $y_t = \alpha + \beta t + \delta y_{t-1} + e_t$

Sources: [a] Bureau of Economic Analysis (www.bea.gov)

[b] Bureau of Labor Statistics (www.bls.gov)

well described as a stationary process" (p. 152). Furthermore, we denote unemployed as x_{2t}, labor force as y_{2t} and unemployment rate as z_{2t}. The expression of unemployment rate can be written as $z_{2t} = \frac{x_{2t}}{y_{2t}}$. Taking the natural log of both sides, the expression becomes $\ln z_{2t} = \ln x_{2t} - \ln y_{2t}$. The results in Table 4 indicate that $\ln x_{2t} \sim I(1)$, $\ln y_{2t} \sim I(1)$ but $\ln z_{2t} \sim I(0)$. This translates into $I(1) \pm I(1) = I(0)$, that contradicts the Granger proposition (i.e. the sum or difference between two time series variables of different order of integration will always yield another time series variable that will retain the "order of integration property" of the two series that has the higher order of integration).

Furthermore, in a peer review process, a reviewer made comments that there are many different unit root tests and cointegration tests, and that they do not always give the same answer is unfortunate. The original DF test, which allows for no lags in the autoregression, would be an awful test to use for almost all macroeconomic data that displays serial correlation in first differences. Also, the PP testing framework has been shown in multiple Monte Carlo studies to have poor size and power properties.

Table 4 Unit root test results of order of integration (Natural Logarithm)

	Dickey-Fuller Test		Phillips-Perron Test		DF-GLS Test
Variables	Model 1[§]	Model 2[†]	Model 3[§]	Model 4[†]	Model 5[†]
In Level					
Openness index [a] (lnz_{1t})	−0.082	−4.359***	−0.399	−23.351**	−1.993
Exports [a]	0.442	−4.357***	0.208	−26.697**	−2.439
Imports [a]	0.730	−4.933***	0.340	−23.133**	−2.153
Exports [a] + Imports [a] (lnx_{1t})	0.726	−4.827***	0.316	−24.368**	−1.946
GDP [a] (lny_{1t})	0.133	−2.476	−0.023	−16.886	−3.116**
Unemployment rate [b] (lnz_{2t})	−2.892***	−3.198*	−15.499**	−19.037*	−3.152**
Number of people unemployed [b] (lnx_{2t})	−1.597*	−3.054	−4.156	−17.259*	−2.967*
Number of people employed [b]	−1.213	−0.102	−0.486	−1.335	−1.425
Labor force [b] (lny_{2t})	−2.004**	1.417	−0.424	0.552	−1.022
In First Differences					
Openness index [a]	−7.188***	−7.188***	−53.416***	−54.400***	−5.069***
Exports [a]	−6.118***	−6.058***	−39.954***	−40.103***	−4.381***
Imports [a]	−7.611***	−7.561***	−55.704***	−56.759***	−2.990*
Exports [a] + Imports [a]	−6.496***	−6.424***	−40.739***	−41.091***	−3.322**
GDP [a]	−4.746***	−4.694***	−27.543***	−26.705**	−3.408**
Unemployment rate [b]	−7.430***	−7.368***	−49.901***	−49.917***	−6.357***
Number of people unemployed [b]	−7.414***	−7.354***	−49.901***	−49.961***	−6.262***
Number of people employed [b]	−6.081***	−6.114***	−45.196***	−45.417***	−5.720***
Labor force [b]	−3.838***	−3.965**	−24.912***	−26.648***	−2.809

***indicates 1 % significance level, **indicates 5 % significance level and * indicates 10 % significance level.
[§] Model 1 and Model 3: $y_t = \alpha + \delta y_{t-1} + e_t$. [†] Model 2, Model 4 and Model 5: $y_t = \alpha + \beta t + \delta y_{t-1} + e_t$

Sources: [a] Bureau of Economic Analysis (www.bea.gov)

[b] Bureau of Labor Statistics (www.bls.gov)

Comments such as these give the impression that the DF-GLS test for unit root was to be preferred over DF, ADF or PP tests. However, our results indicate that the DF-GLS test is even less reliable.[4] Consider the DF-GLS test results for the first difference of number of people employed, the first difference of number of people unemployed and the first difference of labor force as reported in Table 3 (Model 5). The results in the table indicate that the first difference of the number of people employed ~$I(0)$, the first difference of number of people unemployed ~$I(0)$, but the first difference of labor force ~$I(1)$.[5] This translates into $I(0) \pm I(0) = I(1)$, which is clearly a violation of the Granger proposition both in meaning and in intent.

[4] The anomalies that arise from the use of panel unit root tests are taken up separately.

[5] According to the Bureau of Labor Statistics (BLS), which reports labor force statistics at the state level as well as at the federal level for the United States, labor force is the sum of employed and unemployed. It follows that the first difference of labor force is literally equal to the sum of the first difference of number of people employed and the first difference of number of people unemployed.

In addition to what is reported above, Table 5 presents a holistic view of violation of the Granger proposition. As noted in the introduction, Table 5 in essence summarizes the clarifications of the Granger proposition provided by Hendry (1986), Engle and Granger (1987), Cuthbertson et al. (1992), and Royal Swedish Academy of Science (2003). For ease of exposition, we only consider the values d = 0 and d = 1.

Note that the above outcomes are collectively exhaustive. Not surprisingly, allowing researchers to selectively pick one unit root test over another means that any outcome becomes possible. Thus, the finding of cointegration between two variables cannot be a special case as implied or emphasized in the literature. To put it in a nutshell, there is no uniqueness in the original Granger proposition.

Summary and Conclusion

The method of cointegration analysis for modeling nonstationary economic time series variables has become a dominant paradigm in empirical economic research. In our review of the development of the concept of cointegration, we identified that the most important proposition of two integrated series was that if $x_t \sim I(d_x)$, $y_t \sim I(d_y)$ then $z_t = bx_t + cy_t \sim I(\max(d_x, d_y))$ (Granger 1981, page 126). In this research, we show this proposition, not highlighted previously, to be false for two reasons: First, it is not necessarily true that the sum or difference between two time series variables of different order of integration will always yield another time series variable nor will it retain the "order of integration property" of the two series that has the higher order of integration. Second, as shown in Table 5, there was no uniqueness in the proposition to begin with. Thus, the finding of cointegration between two variables

Table 5 Algebra of two integrated series. Based on proposition if $x_t \sim I(d_x)$, $y_t \sim I(d_y)$, then $z_t = bx_t + cy_t \sim I(\max(d_x, d_y))$ (Granger 1981, page 126).

Cases	Outcome	Interpretation
1. $d_x = 0$, $d_y = 1$	$I(0) \pm I(1) = I(1)$	The sum or difference between a stationary series and nonstationary series yields a nonstationary series.
2. $d_x = 1$, $d_y = 0$	$I(1) \pm I(0) = I(1)$	The sum or difference between a nonstationary series and stationary series yields a nonstationary series.
3. $d_x = 1$, $d_y = 1$	$I(1) \pm I(1) = I(1)$	The sum or difference between a nonstationary series and a nonstationary series yields a nonstationary series.
4. $d_x = 1$, $d_y = 1$	$I(1) \pm I(1) = I(0)$	The sum or difference between a nonstationary series and a nonstationary series yields a stationary series.
5. $d_x = 0$, $d_y = 0$	$I(0) \pm I(0) = I(0)$	The sum or difference between two stationary series yields a stationary series. This outcome is analogous to case 3 above for nonstationary series, so it is essentially a trivial case.
6. $d_x = 0$, $d_y = 0$	$I(0) \pm I(0) = I(1)$	The sum or difference between two stationary series yields a nonstationary series. Although this outcome is explicitly ruled out by the Granger proposition, in practice this outcome is also possible if one is to selectively pick one unit root test over another.

does not reveal any special relationship as implied or emphasized in the extant literature. The importance of the violation of Granger's key proposition is further discussed in Luitel and Mahar (2015b).

In conclusion, the failure of cointegration analysis, particularly, in the context of financial econometrics, was previously discussed in Moosa (2011). Due to the existence of disconnection between what the existing literature predicts and what actually goes on in the economy, we reassessed the major proposition of the unit root and cointegration literature in an effort to explain apparently unbelievable observed real world phenomena, and contributed to the existing literature by bringing knowledge of its failure to the attention of a general audience. Arguably, one of the most powerful uses of cointegration is in forecasting. On the one hand, vector error correction (VEC) models have been shown to outperform vector auto regression (VAR) models in many forecasting applications. Proponents of the unit root and cointegration argue this fact to be prima facie evidence of the value of the concept of cointegration. On the other hand, in the last decade or so, the nowcasting method has been on the rise in the economic literature and has gone far beyond the point of just being a "special analysis" technique. To consolidate such diverse economic thinking, our research indicates a need for a comparative analysis of forecasting, backcasting and nowcasting econometrics methods. We believe this is a fruitful area that needs more investigation and we leave it for future research endeavors.

Acknowledgments The research was presented at the 85th Annual Meetings of the Southern Economic Association, November 21-23, 2015, New Orleans, USA and the 49th conference of the Canadian Economics Association during Thursday, May 28, 2015 - Sunday, May 31, 2015, in Toronto, Canada. We thank Afshin Amiraslany, Murshed Chowdhury, Brandon Mackinnon, Mariana Saenz and session participants in the above conferences for their helpful comments and suggestions. We also thank three anonymous referees for their comments and suggestions.

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