

Low Complexity Linear Detection for Uplink Multiuser MIMO SC‑FDMA Systems

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Abstract

In this article, the computational complexity reduction of zero forcing (ZF) and minimum mean square error (MMSE) detection is presented for the uplink multiple input multiple output (MIMO) single carrier frequency division multiple access (SC-FDMA) systems. MIMO SC-FDMA structure with detection of user's data by the well-known multiuser linear detection approaches such as MMSE and ZF often use single input single output detection systems due to its simple detection and signifcant performance. Although, these approaches involve inversion of a matrix computation whose matrix dimension depend on number of subcarriers used in the system, mainly, it can be few thousands. In practical detection, the computational complexity of matrix inversion becomes very high. Also, the complexity of the receiver is raised because of the superposition of all the transmitted signals at each antenna received in the systems. The proposed conjugate gradient approach reduces the higher computational overhead of linear detectors, which updates iteratively the ZF and MMSE solution and avoids the direct computation of matrix inverse operation. The analysis of the proposed algorithm reveals the superior performance and the low complexity detection in spatial multiplexing SC-FDMA system. Simulations have investigated that the computational complexity of the proposed method has been greatly reduced and bit-error-rate performance is closer to matched flter bound.

Keywords ZF · MMSE · MIMO · Spatial multiplexing · SC–FDMA · Conjugate gradient

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1 Introduction

Multiple-input-multiple-output (MIMO) schemes are mandatory nowadays and for the next generation of wireless technologies because of the demand for higher capacity services and reliability in wireless communication system. Massive MIMO and multiuser MIMO techniques have widely attracted to achieve reliability and higher data rates performance in many standards of wireless transmission systems in recent years. Mode of gain in MIMO system is divided into two major categories: spatial diversity gain [\[1–](#page-15-0)[3](#page-15-1)] and spatial multiplexing gain [[4](#page-15-2)[–10\]](#page-15-3). Spatial diversity enables link performance in wireless channels while spatial multiplexing scales up capacity by transporting number of streams in parallel through multiple antennas.

Linear detector algorithms are widely designed to utilize for their low complexity receiver as compared to maximum likelihood (ML) detection [[6\]](#page-15-4). Some of the emerging wireless systems i.e., 3GPP Long Term Evolution (LTE), IEEE 802.11n and IEEE 802.16e prefer linear receivers. The minimum mean square error (MMSE) and/or zero forcing (ZF) linear detectors are too complex to be exploited in systems with large scale antennas. It is very feasible for MIMO with small scale antennas. However, the complexities overhead of ZF and MMSE detection depend on the size of data streams to be detected and order of modulations to be used. The scales up computational complexities of the ZF and/or MMSE detector in some of the aforementioned situations have aimed the design of several other schemes for the MIMO system [\[11\]](#page-16-0). The linear receiver [[12](#page-16-1)], the successive interference cancellation (SIC) method, vertical- bell laboratories layered space time (V-BLAST) approach $[13, 14]$ $[13, 14]$ $[13, 14]$ $[13, 14]$ and decision feedback detection $[15-17]$ $[15-17]$ are techniques that can present attractive trade-ofs between complexity and performance.

Modern cellular communication standards, orthogonal frequency division multiplexing (OFDM) employed with MIMO techniques to achieve higher spectral efficiency. Moreover, one of the key issues of high peak to average power ratio (PAPR) property is in OFDM. Thus, single carrier frequency division multiplexing access (SC-FDMA) system exhibits signifcantly low PAPR than OFDM. The system adapts the low PAPR benefts in the mobile handsets of power amplifier, which increases the power back-off of mobile device equipped with small batteries. SC-FDMA system has been selected as a physical layer technology of LTE and LTE-advanced (LTE-A) standards [\[18](#page-16-6)[–20\]](#page-16-7). The LTE-A is widely recognized as one of the strong candidates for power efficient technology in the Internet of things (IoT). There are two well known subcarrier assigning techniques in SC-FDMA, namely: localized FDMA (LFDMA) and interleaved (IFDMA) [\[18\]](#page-16-6).

Spatial multiplexing SC-FDMA (SM SC-FDMA) scenario achieves excellent capacity gain by adding linearly with multiple number of transmit and receive antennas $[4, 5, 7-10]$ $[4, 5, 7-10]$ $[4, 5, 7-10]$ $[4, 5, 7-10]$ $[4, 5, 7-10]$ $[4, 5, 7-10]$. There are variety of receivers employed to adapt in MIMO SC-FDMA detection such as linear frequency domain ZF, MMSE, and ML receivers [[4](#page-15-2)[–9](#page-15-7)]. Even though, ML detection ofers an optimal bit error rate (BER) performance the complexity increases in the system. The linearly combining of all the transmitted signals at each received antenna is raised in receiver computational overhead. Therefore, an efficient computational complexity reduction receiving technique is needed for SM SC-FDMA transceiver systems. In several practical applications MIMO SC-FDMA system, receiver complexity is a signifcant concern, and a receiver detection system with reasonable complexity is widely preferred. The linear receivers such as ZF and MMSE receiver have addressed this article.

MIMO SC-FDMA system with detection of user data by the MMSE and/or ZF flter is typically simple and efficient $[2, 4, 7-10, 21, 22]$ $[2, 4, 7-10, 21, 22]$ $[2, 4, 7-10, 21, 22]$ $[2, 4, 7-10, 21, 22]$ $[2, 4, 7-10, 21, 22]$ $[2, 4, 7-10, 21, 22]$ $[2, 4, 7-10, 21, 22]$ $[2, 4, 7-10, 21, 22]$ $[2, 4, 7-10, 21, 22]$ $[2, 4, 7-10, 21, 22]$ $[2, 4, 7-10, 21, 22]$ $[2, 4, 7-10, 21, 22]$ $[2, 4, 7-10, 21, 22]$. These approaches involve unaffordable inversion of matrix computation depends on equal number of subcarriers. Also it sufers scales up of complexity of receiver in these systems as large as a few thousands of subcarrier employed in realistic systems. The authors in $[4, 10, 23]$ $[4, 10, 23]$ $[4, 10, 23]$ $[4, 10, 23]$ $[4, 10, 23]$ $[4, 10, 23]$ suggested the reduction of complexity MMSE detector where the interference matrix is approximated by a banded matrix with size of bandwidth (B), which looks for the elements of the matrix outside *B* are considered to have negligible impact. It is noticed that this approximation of the interference matrix has been applied to obtain the performance with low-computational complexity in OFDMA and SC-FDMA systems. This approximation results in critical performance degradation issues. Consequently, the system may not be suitable for practical use. Regularized complexity detection for MIMO SC-FDMA uplink scheme is investigated [[4](#page-15-2)]. An iterative interference cancellation scheme based on the Neumann power series expansion is presented in [\[24,](#page-16-11) [25](#page-16-12)]. It is noted that this system requires additional modifcations by converting the matrix inversion into a series of matrix multiplications and additions, which is able to alleviate the complexity issue. However, the Neumann power series based scheme sufers because of number of users increases. An iterative sequential MMSE detection approach is proposed to reduce the computational complexity for estimating data symbols in large scale MIMO systems [[26](#page-16-13)]. However, its adjacent interference cancellation involves higher delay leading to suffer performance deterioration.

In addition, a low-complexity ZF and/or MMSE detection for OFDMA and/or SC-FDMA uplink system is proposed in [\[27–](#page-16-14)[29](#page-16-15)] by employing a conjugate gradient (CG) method. It computes the ZF and MMSE detector output without being involved in an inverse computation. This approach has a much lower complexity compared to existing techniques and is simple and efficient, while maintaining the optimal result. It is noticed that CG technique is a fast estimation of ZF and MMSE solution and achieves better performance than the direct MMSE solution $[16]$. To the best of our knowledge, this work is the frst to apply the CG algorithm based linear detection especially for SM SC-FDMA uplink system. This study proposes low-complexity ZF and MMSE detection for SM SC-FDMA uplink system. The views of the research are furnished bellow:

- 1. The new CG algorithm based ZF and MMSE detection for SM SC-FDMA is proposed whereas scheme [[29](#page-16-15)] time domain windowing based on OFDMA system, consequently experienced some bandwidth loss.
- 2. The computational complexity of proposed scheme is quite diferent from the one proposed in [\[4](#page-15-2)], which performed only the regularized ZF based MIMO SC-FDMA system.
- 3. By implementing CG algorithm with ZF and MMSE criteria that estimates solution, without requiring matrix inversion operations, which will enable us to realize near optimal performance with low complexity detection. It has a benchmark of lower complexity linear detection for SM SC-FDMA system.
- 4. It can be utilized for a broad range of carrier frequency ofsets (CFOs) and also applicable to both LFDMA and IFDMA system.

The remaining chapter is organized as follows. Section [2](#page-3-0) describes transceiver confguration model for uplink SM SC-FDMA. Section [3](#page-6-0) presents non-iterative linear detection. Section [4](#page-9-0) presents implementation of proposed iterative linear detection. The low complexity analysis and comparisons are described in Sect. [5.](#page-10-0) Simulation results of the SM SC-FDMA detectors are reported in Sect. [6.](#page-11-0) Finally, Sect. [7](#page-13-0) draws the conclusion. Upper and lower case letters represent frequency domain and time domain components, respectively. Superscripts, $^{-1}$, T and * designate an inverse, transpose and conjugation operations, respectively. I_p represents identity matrix of P dimension. *C* indicates the set of complex number. The operators $\| \cdot \|$ and diag{ $.\}$ represent the Euclidean vector norm and diagonal matrix, respectively. The notation "(.)_{*ab*}" and "(.)^{*a*×*b*" represent dimension of *a* × *b* matrix.}

2 Uplink Spatial Multiplexing SC‑FDMA Transmission Model

The uplink confguration of SM SC-FDMA transceiver is shown in Fig. [1.](#page-3-1) It consists of the base station (BS) with N_r receive antennas and user equipment (UE) with N_t transmit antennas. One of the MIMO schemes is spatial multiplexing that maximizes the data rate of wireless transmission scenarios. Each user's data streams are mapped to a complex constellation symbols (e.g., QPSK or 16-QAM). SM SC-FDMA scheme converts complex mapped symbols into separate signal based on the size of transmitting antennas. The resulting complex sequence *s* is converted into frequency domain through an *M*-point DFT. The frequency domain symbol at the *m*th subcarrier for the *k*th $(k=1,..., K)$ user can be expressed as follows [[30](#page-16-17)]:

$$
X^{k}(m) = \sum_{n=0}^{M-1} s^{k}(n) e^{\frac{-j2\pi nm}{M}} \quad 0 \le m \le M-1
$$
 (1)

The frequency domain symbols are then mapped to the subcarrier mapping methods. After performing an *N*-point inverse DFT (IDFT) to the subcarrier mapped sequence. The timedomain IDFT sequence is expressed as follows:

$$
x_q^k(n) = \sum_{m=0}^{N-1} \tilde{X}_q^k(m) e^{\frac{j2\pi nm}{N}} \quad 0 \le n \le N-1, \quad q = 0, 1, \dots, N_t
$$
 (2)

Fig. 1 The transreceiver stucture of SM SC-FDMA systems

where $\tilde{X}^k_p(m)$ is the *m*th sample at *q*th the antenna of the *k*th user after the assignment operation. Now, $N_t = N_r = 2$ transmit and receive antennas are considered. Before transmission in each block, a cyclic prefix (CP), which is the last part of the block data of $x^k(n)$, is inserted in front of that block to eliminate the inter-block interference.

The assumption is that the receiver has precise synchronization, the received SM SC-FDMA symbol at the N_r receiving antennas, after discarding of CP, can be described as follows [\[10](#page-15-3)]:

$$
y = \sum_{k=1}^{K} \tilde{\Phi}^{k} \bar{h}^{k} x^{k} + z
$$
 (3)

Then, (3) (3) is expressed in matrix structure $[4]$ $[4]$ as:

$$
y = \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T, \quad x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T, \quad z = \begin{bmatrix} z_1 & z_2 \end{bmatrix}^T \text{ and } \bar{h} = \begin{bmatrix} \bar{h}_{11} & \bar{h}_{12} \\ \bar{h}_{21} & \bar{h}_{22} \end{bmatrix}
$$
 (4)

where $y_p \in C^{N \times 1}$, $x_q \in C^{N \times 1}$ and $z_p \in C^{N \times 1}$ denote the received symbol column vector of the *p*th receive antenna, the transmitted symbol column vector of the *q*th transmit antenna and the additive white Gaussian noise (AWGN) column vector of the *p*th receive antenna, respectively. $\tilde{\Phi}_k$ denotes the diagonal CFO $N \times N$ matrix modeled as $\tilde{\Phi}_k = diag \left\{ 1, e^{\frac{j2\pi k_k}{N}}, \dots, e^{\frac{j2\pi (N-1)\epsilon_k}{N}} \right\}.$ $\varepsilon_k = \frac{\Delta f_k}{f_{sub}}$ is the normalized CFO of the kth user, Δf_k represents the CFO between the *k*th user and BS receiver and *fsub* denotes subcarrier spacing and $\bar{h}_{pq}^k \in C^{N \times N}$ denotes channel circulant matrix. It shows the MIMO channels between qth transmit and *p*th receive antenna of the *k*th user which are expressed as follows [[7,](#page-15-6) [10,](#page-15-3) [23](#page-16-10)]:

$$
\bar{h}_{pq}^{k} = \begin{bmatrix}\nh_{pq}^{k}(0) & 0 & \cdots & h_{pq}^{k}(L-1) & h_{pq}^{k}(1) \\
h_{pq}^{k}(1) & h_{pq}^{k}(0) & \ddots & 0 & h_{pq}^{k}(2) \\
\vdots & h_{pq}^{k}(1) & \ddots & \ddots & \vdots \\
h_{pq}^{k}(L-1) & \vdots & \ddots & \ddots & \ddots \\
0 & h_{pq}^{k}(L-1) & \cdots & \ddots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots \\
0 & 0 & \cdots & h_{pq}^{k}(1) & h_{pq}^{k}(0)\n\end{bmatrix}
$$
\n(5)

It is a frequency domain diagonal channel matrix, obtained through DFT computation of the circulant element of h_{pq} , whose diagonal elements are channel frequency response; i.e., $\bar{H}^k_{pq}(m) = diag\{\sum_{n=0}^{\lfloor \frac{n}{2} \rfloor} h^k_{pq}(n) e^{\frac{-j2n\pi m}{N}}\}\ 0 \leq m \leq N-1$, where $h_{pq} \in C^1$ denotes the column vector of channel impulse response from *p*th receive antenna to *q*th transmit antenna. The circulant matrix is efectively diagonalized by the way of the fast Fourier transform (FFT) matrix operator F_N with its inverse F_N^H . It is formulated as follows [[7\]](#page-15-6):

$$
\bar{H}^k_{pq}(m) = F_N \tilde{\Lambda}^k_{pq}(m) F^H_N
$$
\n⁽⁶⁾

where $\tilde{\Lambda}_{pq}^k$ is an $N \times N$ diagonal matrix whose elements consist of FFT of the circulant sequence of H^k _n *p q*.

The received frequency domain signal after DFT is

$$
Y = \sum_{k=1}^{K} \Phi^k \Lambda^k X^k + Z \tag{7}
$$

Then, (7) can be written in matrix form as $[10]$

$$
Y = \begin{bmatrix} Y_1 & Y_2 \end{bmatrix}^T, \quad X = \begin{bmatrix} X_1 & X_2 \end{bmatrix}^T, \quad Z = \begin{bmatrix} Z_1 & Z_2 \end{bmatrix}^T \text{ and } \Lambda^k = \begin{bmatrix} \tilde{\Lambda}^k_1 & \tilde{\Lambda}^k_{12} \\ \tilde{\Lambda}^k_{21} & \tilde{\Lambda}^{k2}_{22} \end{bmatrix} \tag{8}
$$

where Y_p , X_q and Z_p are the Fourier transforms of y_p , x_q and z_p , respectively. Φ_k denotes the frequency component of circulant CFO matrix. An individual detection is carried out by each user from the receiver side.

In order to detect user data, the frequency domain detection and demapping are carried out as follows $[8]$:

$$
\bar{r}^u = [A^u]^T \prod^u Y \tag{9}
$$

where $[A^u]^T$ denotes M \times N demapping matrix of *uth* user. Π^u denotes the $N \times N$ diagonal MMSE equalization frequency domain matrix of *uth*
 $\Pi^u = (H^u)^H[H^u(H^u)^H + \frac{\sigma_n^2}{\sigma_z^2}I_N]^{-1}$ where $\frac{\sigma_n^2}{\sigma_z^2}$ is variance of noise to signal ratio. user *i.e.*,

The received signal of uth $(k = 1, ..., K)$ user's after separation is modeled as follows $[23]$:

$$
\tilde{r}^{\mu} = [A^{\mu}]^{T} \Pi^{\mu} (\Phi_{\mu} H^{\mu} A^{\mu} F_{M} x^{\mu} + \sum_{k=1, k \neq \mu}^{K} \Phi_{k} H^{k} A^{k} F_{M} x^{k} + Z^{k})
$$
(10)

Further, the M-point IDFT, the uth user detection output can be expressed as:

$$
\hat{x}^{u} = F_{M}^{H} \tilde{r}^{u}
$$
\n
$$
= F_{M}^{H} [A^{u}]^{T} \Pi^{u} [\Phi_{u}]_{1,1} H^{u} A^{u} F_{M} x^{u} + F_{M}^{H} [A^{u}]^{T} \Pi^{u} (\Phi_{u} - [\Phi_{u}]_{1,1} I_{N}) H^{u} A^{u} F_{M} x^{u}
$$
\n
$$
+ F_{M}^{H} [A^{u}]^{T} \Pi^{u} \sum_{k=1, k \neq u}^{K} \Phi_{k} H^{k} A^{k} F_{M} x^{k} + F_{M}^{H} [A^{u}]^{T} \Pi^{u} Z
$$
\n
$$
= \underbrace{\underbrace{\qquad \qquad }_{III}}_{III}
$$
\n
$$
(11)
$$

From the Eq. (11) , it is denoted that the *Ist* term is transmitted with *uth* user's transmitted information symbol, the *II*nd term, inter-symbol-interference (ISI), the *III*rd term, multiple access interference (MAI) and the *IVth* term, AWGN, respectively. Further, from the receiver extract, the original transmitted information can be obtained through demodulation and detection operations.

3 Conventional Multiuser Detections

3.1 Non‑iterative Linear Detection Systems

Firstly, ZF and MMSE criterions associate with non-iterative detection methods are investigated. After performing subcarrier demapping with multiuser interference cancellation, the frequency domain linear detection is formulated to equalize the received signal. The estimate of transmitted signal of *n*th subcarrier by ZF/MMSE equalization can be shown as [\[6](#page-15-4), [7](#page-15-6), [9](#page-15-7), [12,](#page-16-1) [31\]](#page-16-18):

$$
\hat{X}_{ZF}(n) = H^H(n)[H(n)H^H(n)]^{-1}Y
$$
\n(12)

and perform frequency domain linear MMSE equalization,described as [\[8](#page-15-9), [23](#page-16-10), [32\]](#page-16-19):

$$
\hat{X}_{MMSE}(n) = H^{H}(n) \left[H(n)H^{H}(n) + \frac{\sigma_{n}^{2}}{\sigma_{s}^{2}} I_{N} \right]^{-1} Y
$$
\n(13)

To reduce interference between subcarriers linear equalization can be employed. However, the interference cancellations based on ZF and MMSE approach perform an $N \times N$ matrix inversion when N becomes high in many applications. Thus, high computational difculty occurs. To elevate the complexity crisis, an iterative procedure for CG approach to obtain the result of the linear equalizer coefficients without any matrix inversion is involved. The frequency domain equalized signal, \ddot{X} , is transformed to time component equivalent via M-point IDFT. Finally, as per the decoding procedure the transmitted signal is retrieved.

3.2 MIMO SC‑FDMA System with Combined Regularized Linear Detection

The regularized [\[4](#page-15-2), [10\]](#page-15-3) linear ZF receiver overcomes the direct matrix inverse, utilized in the existing linear detection. It carries out through in two step procedure, the frst step is to detect received signal with low complexity by implementing the banded structure of the interference matrix and a regularization sequence is appended in the equalization process to reduce the noise amplifcation in the step second.

(i) In this step MIMO signals for frst user data is estranged, inter-antenna-interference is minimized by applying the matrix *w* and expressed as follows [[4,](#page-15-2) [10\]](#page-15-3):

$$
\hat{x} = w_1^k \tilde{r} \tag{14}
$$

where \tilde{r} is obtained by expression ([10](#page-15-3)). w_1 can be described as [\[4](#page-15-2), 10]:

$$
w_1^k = \begin{bmatrix} I_N & -\Delta_2^k \\ -\Delta_1^k & I_N \end{bmatrix}
$$

where $\Delta_1^k = \Phi_{21}^k (\Phi_{11}^k)^{-1}$ and $\Delta_2^k = \Phi_{12}^k (\Phi_{22}^k)^{-1}$

(ii) In this step ISI and MAI are minimized by implementing matrix w_2^k as [[4,](#page-15-2) [10\]](#page-15-3) by

$$
\hat{x} = w_1^k w_2^k \tilde{r}_d^k = w_2^k (w_1^k \tilde{r}_d^k \Phi^k) X^k + w_1^k w_2^k Z \n= w_2^k \left[\begin{array}{cc} \Theta_1^k & 0_{N \times N} \\ 0_{N \times N} & \Theta_2^k \end{array} \right] X^k + w_2^k w_1^k Z
$$
\n(15)

where w_1 and w_2 are the inter-antenna interference and inter symbol interference, respectively. $\Theta_1^k = \Phi_{11}^k - \Delta_2^k \Phi_{21}^k$ and $\Theta_2^k = \Phi_{22}^k - \Delta_1^k \Phi_{12}^k$. It is noted that w_2^k can be constructed as follows [\[4,](#page-15-2) [10](#page-15-3)]:

$$
w_2^k = \begin{bmatrix} (\Theta_1^k)^{-1} & 0_{N \times N} \\ 0_{N \times N} & (\Theta_2^k)^{-1} \end{bmatrix}
$$
 (16)

To reduce complexity problems in ZF scheme, the authors proposed as α is a regularization constraint, which adapts to elevate the noise amplifcation and the MAI problems. Note down that $\alpha = 0$ in the ZF equalizer in (16). It is significant to notice that the matrices Φ_{11}^k , Φ_{12}^k , Φ_{21}^k and Φ_{22}^k can be approximated by banded matrices. The MMSE detection output for (15) can be derived as $[4, 10]$ $[4, 10]$ $[4, 10]$ $[4, 10]$:

$$
w^{k} = \begin{bmatrix} (H_1^{kH} H_1^{k} + \alpha I_N)^{-1} H_1^{kH} & 0_{N \times N} \\ 0_{N \times N} & (H_2^{kH} H_2^{k} + \alpha I_N)^{-1} H_2^{kH} \end{bmatrix}
$$
(17)

The optimal rate of α is related to signals -to- noise ratio (SNR).

3.3 A Sub‑optimal ML Estimation for MIMO SC‑FDMA

Following [[6](#page-15-4)], the ML detection generates effective channel response between the transmitted and received signals. Before the Euclidean distance metric of each symbol is evaluated, the decision line distance metric is applied to receive signal by the selective ML post detection processing scheme. Consequently, MMSE detection process of each received symbol is introduced to ML criterion. Then, it performs the removal of multiple transmits streams interference of the received signal. The overall received signal can be described [[6\]](#page-15-4) as:

$$
y = H_{\text{eff}}x + z.\tag{18}
$$

This scheme can be performed as MMSE

$$
\hat{x} = [H_{\text{eff}}^H H_{\text{eff}} + SNR I_N]^{-1} H_{\text{eff}}^H y \tag{19}
$$

where H_{eff} is the effective ML channel estimation.

To improve MMSE system performance, a closer of unreliable symbols are carried out of the *MNq* estimated MMSE symbols in *x̂*. The error probability of the *i*th symbol can be expressed as in Euclidean distance metric

$$
\gamma_i = \min_{x_i^{(j)} \in \{\hat{x}_i\}^C} \left\| y - H x_i^{(j)} \right\|^2 \tag{20}
$$

where $\{x_i\}^C$ is a subset of symbol constellation.

The reliabilities of symbols \hat{x}_v , $v = 1, 2, ..., MN_t$ can be estimated as:

$$
\gamma_{\hat{x},\nu} = \min_{\{x \in \{x_i\}^C\} - \{\hat{x}_{\nu}\}} \left\| y - \sum_{l \neq \nu} h_{\text{eff},l} \hat{x}_{\nu} - h_{\text{eff},\nu} x \right\|^2 \tag{21}
$$

where $h_{eff,l}$ is the *l*th column of H_{eff} . A new set of unacceptable symbols is then identified to further enhance performance of the system when the ML estimated symbols are combined into the computation of reliabilities. This technique is restricted, mainly due to imperfect interference cancellation. ML detection is achieved better performance but high computational complexity when the modulation order grows or number of antennas increases.

3.4 An Iterative Sequential Detection

An iterative linear sequential detection scheme is presented to reduce complexity for large scale MIMO systems [[26](#page-16-13)]. The detected solution for ZF algorithm based MIMO detection can be expressed $[26]$ as

$$
\hat{X}_{ZF} = Q[(H^H H)^{-1} H^H Y] = Q[a^{-1}b]
$$
\n(22)

where Q[.] is the quantizer, $b = H^H Y$ and $a^{-1} = H^H H$ is the Gram matrix. The estimated solution for MMSE algorithm based MIMO detection can be described [\[26\]](#page-16-13) as

$$
\hat{X}_{MMSE} = Q[(H^H H + SNR I)^{-1} H^H Y] = Q[a^{-1} b]
$$
\n(23)

where Q[.] is the quantizer, $b = H^H Y$ and $a^{-1} = G + SNR I$ where $G = H^H H$ is the Gram matrix. The corresponding symbol of each user is estimated and updated sequential fashion while nulling the interference from all the other users at every iteration process. For the removal of interference from all other users, the estimated signal of *i*th user can be expressed [[26](#page-16-13)] as

$$
\hat{y} = y - \sum_{k=1, k \neq i}^{K} h_k \mu_t(k)
$$
\n(24)

To compute the symbol related to the ith user, *t*th iterative process the sequence \hat{y} as

$$
\mu_{t+1}(i) = \frac{h_i^T}{\|h_i\|^2} \hat{y} = \frac{1}{\|h_i\|^2} (h_i^T y - h_i^T \sum_{k=1, k \neq i}^K h_k \mu_t(k))
$$
\n(25)

$$
\mu_{t+1}(i) = \frac{1}{\|h_i\|^2} (h_i^T y - \sum_{k=1}^K h_i^T h_k \mu_t(k) + \frac{h_i^T h_i}{\|h_i\|^2} \mu_t(i)
$$
\n(26)

where h_i is the *i*th column of the matrix H , equation (b) can be modified as

$$
\mu_{t+1}(i) = \mu_t(i) + \frac{1}{G(i,i)}(b(i) - \sum_{k=1}^{K} G(i,k)\,\mu_t(k))\tag{27}
$$

where G(i, k) is the (i, k)th element of matrix G, b(i) is the *i*th element of the vector b. Linear iterative sequential detection approach achieved better performance. However, its

adjacent interference cancellation consumes higher delay leading to afect performance degradation.

4 Implementation of Proposed Iterative Linear Detection

An efficient implementation of low complexity algorithm based on linear detection method is presented for SM SC-FDMA system. The linear system equations with positive defnite and Hermitian coefficient matrices are suitable to implement CG algorithm [\[33\]](#page-16-20).

This idea was frst suggested in [\[28](#page-16-21), [29](#page-16-15)], where authors low complexity ZF and MMSE solution obtain through the unique arrangement of the interference matrix while the similar structure (with few slight modifcations) realized great interest to this article as well [\[34](#page-16-22)]. To estimate various users transmitted symbols associate CG method with ZF and MMSE principle, CG formulate to obtain a solution of the term [\(12](#page-6-1)) is suggested [\[28](#page-16-21), [29](#page-16-15)] and modifed as:

$$
Q\hat{X}_{ZF} = d \tag{28}
$$

where $Q = H^HH$ and $d = H^HY$. MMSE criterion implementing with CG algorithm is to evaluate a solution of expression [\(13\)](#page-6-2), is formulated as:

$$
Q\hat{X}_{MMSE} = d \tag{29}
$$

where $Q = H^H H + \frac{\sigma_n^2}{\sigma_c^2} I_N$ and $d = H^H Y$. An algorithm *I* describes the Eqs. ([28](#page-9-1)) and ([29](#page-9-2)) in terms of CG that evaluates the linear detection solution. δ denotes the tolerance of the output, subscript *l* denotes the iteration number. Residual vectors are considered as $\gamma_{(l)}$ and $\rho_{(l)}$, which lead search direction in the *l*th direction. The preconditioning matrix is formulated as *W*, which is assumed as identity matrix, this is introduced to speed up the convergence of the CG operation. The CG scheme may require number of iterations (*Nit*) to obtain an exact solution [\[34\]](#page-16-22). The spreading of the eigenvalue underlying matrix decides required number of iterations.

Algorithm I. complexity reduction CG scheme based on ZF/MMSE detection

$$
\gamma^{(0)} = d - QX^{(0)}
$$
\n
$$
\rho^{(0)} = W^{-1}\gamma^{(0)}
$$
\nWhile $\|\gamma^{(l)}\| > \delta \|\gamma^{(0)}\|$ do\n
$$
\xi_{(l)} = \frac{\gamma^{H(l)}W^{-1}\gamma^{(l)}}{\rho^{H(l)}Q\rho^{(l)}}
$$
\n
$$
X^{(l+1)} = X^{(l)} + \xi^{(l)}\rho^{(l)}
$$
\n
$$
\gamma^{(l+1)} = \gamma^{(l)} - \xi^{(l)}Q\rho^{(l)}
$$
\n
$$
B^{(l+l)} = \frac{\gamma^{H(l+1)}W^{-1}\gamma^{(l+1)}}{\gamma^{H(l)}W^{-1}\gamma^{(l)}}
$$
\n
$$
\rho^{(l+1)} = W^{-1}\gamma^{(l+1)} + B^{(l+1)}\rho^{(l)}
$$
\nwhile

end

5 Complexity Analysis

The new scheme carries out numerous matrix computations for instance, matrix inverse, addition, division and multiplications. It adapts CG algorithm to enable convergence fast to estimate MMSE and ZF solution iteratively. To alleviate the cause of inter carrier interference (ICI), a direct practice (expressions [12](#page-6-1) and [13](#page-6-2)) based on the ZF and MMSE detection, which involves $O(N^3)$ computational to invert a size of N ICI matrix for each user. The ML detection compares the received signal vector of size $N \times 1$ with all the 2^{vN} users, requiring in a complexity of $O(N \times 2^{vN})$, where v is the size of constellation, which is high as compared to existing approaches. But, it is not suitable for practical scenario. In practical SC-FDMA schemes specifcally 3GPP LTE, which requires num-ber of subcarrier from 128 to 2048 [[19\]](#page-16-23). A computationally attractive design of CG method is utilized to drastically reduce the receiver overheads and to sustain signifcant performance. A CG algorithm is applicable to calculate the efective solution in this article as a benchmark. Table [1](#page-10-1) Summarizes complexity comparison of the proposed CG method and other existing scenarios. In this scenario, K , N , N_r , B and N_{it} assert number of user, number of subcarriers, number of transmit antenna, bandwidth of banded matrix and number of iterations, respectively.

The main objective of CG algorithm is to carry out the solution for an inverse matrix computation. An amount of evaluation overheads for computing Q^{-1} and $\hat{X} = Q^{-1}d$ require $O(NK^3)$ and O (N^2) operations. Hence, direct ZF (expression [12](#page-6-1)) resulting complexity is $O(NK^2)$ and direct MMSE (expression [13](#page-6-2)) is evaluating the order of $O(N(N + K^2))$. The complex computations of the frequency and domain conversions are estimated by well-known FFT/IFFT algorithms. An *N*-point FFT/IFFT involves (*N*/2) $\frac{1}{2} \log_2 N$ complex multiplication. The complexity of the new system is analyzed and contrasted with direct ZF and MMSE, regularized linear detection [\[4](#page-15-2)] and sub-optimal ML receiver [[6\]](#page-15-4) are reported in Fig. [2.](#page-11-1) The complexity of CG algorithm under linear detections scheme is much superior to the conventional counterparts. Also the proposed ZF computation complexity result is much lower than that of proposed MMSE complexity solution.

Detection system	Number of complex multiplications
Direct ZF equalization	$\frac{1}{3}N^3 + \frac{KN}{2} \log_2 N + 2N^2K^2 + KN_t$
Regularized ZF detection [4]	$\frac{1}{2}N^3B^2 + 2N(2B + 1) + \frac{KN}{2}\log_2 N + KN_t$
ZF Sequential detection $[26]$	$2NK + \frac{KN}{2} \log_2 N + NK^2 + KN_t$
Proposed ZF scheme	$N_{it}(KN \log_2 N + KN_t) + \frac{KN}{4} \log_2 N$
Direct MMSE equalization	$\frac{5}{6}N^3 + N(N + K^2) + \frac{KN}{2} \log_2 N + KN_t$
Regularized MMSE detection [4]	$\frac{1}{2}N^3 + 2N^2(2B + 1) + \frac{KN}{2}\log_2 N + KN_t(B + 1)$
Sub-optimal ML receiver [6]	$\frac{5}{3}N^3 + 2N^2KN_t(N \times 2^{\nu N}) + \frac{KN}{2}\log_2 N$
MMSE sequential detection $[26]$	$3K^2N^3 + N(N + K^2) + \frac{KN}{2} \log_2 N + KN_t$
Proposed MMSE scheme	$N_{ii}(KN \log_2 N + 5KN + 2KN_t) + \frac{KN}{4} \log_2 N$

Table 1 Computational complexity comparison for diferent detection systems

Fig. 2 Complexity evaluations between proposed and diferent detection schemes

6 Simulation Results

The proposed linear detection's system performance is evaluated through computer simulations and performance is compared with some recent existing approaches. This study verifes both localized and interleaved subcarriers allocations scenario for uplink SM SC-FDMA transceiver system. During simulations, performance of the proposed CG algorithm is examined through Monte Carlo process.

The Extended vehicular A channel is tested by the fading channel model with uniform power delay profle having 9 taps. The simulation parameters are followed as in the 3GPP LTE technical specifcations release 10 [[35](#page-16-24), [36](#page-16-25)]. In the uplink SM SC-FDMA scenario, 2×2 MIMO antenna model is employed between the UE and BS. Modulation scheme of 16-QAM and QPSK are used to verify efectiveness of the proposed approach for simulations. The system is considered to have $N = 1024$ subcarriers, which assume bandwidth of 20 MHz. The number of users in this scenario is $K = 8$ and number of subcarriers per user is $M = 128$. CP is 1/8 of symbol duration. The CFO of users is presented at random in the range $[-0.4, 0.4]$. Fading coefficients for each path are employed as independent and identically distributed Gaussian coefficients. Consequently, a rate of 1/3 convolutional code is used to encode information bits with a constraint length of 7, and the 32 iteration equalization results are revealed.

Figure [3](#page-12-0) demonstrates the BER performance of the proposed iterative ZF and MMSE detector and diferent detectors for localized subcarrier mapping SM SC-FDMA system with QPSK modulation technique. It reveals that the proposed ZF detection outperforms existing ZF receivers about 1 dB and the proposed MMSE detection outperforms conventional MMSE approaches about 1.5 dB at 10^{-3} BER. Figure [4](#page-13-1) presents the BER performance of the proposed ZF and MMSE detection and some recent receivers for localized subcarrier allocation SM SC-FDMA scheme with the 16-QAM modulation technique. Particularly, while BER is at 10−3, the proposed ZF detector superior performance to existing ZF receivers about 2 dB with 16-QAM and the proposed MMSE detector's better performance than conventional MMSE approaches about 1.5 dB with 16-QAM. From these results, the systems realize both MMSE and ZF receiver with only 32 iteration (N it $=$ 32) converges the detection result.

Figure [5](#page-14-0) shows the BER performance of the proposed multiuser ZF and MMSE receiver for interleaved subcarrier mapping SM SC-FDMA scenario. In QPSK modulation, the proposed ZF detection outperforms the existing ZF receivers about 2 dB and the proposed MMSE detection outperforms the conventional MMSE approaches about 2 dB at the BER of 10−3. Finally, the performance of the proposed ZF and MMSE detector for interleaved subcarrier mapping with the 16-QAM modulation technique is shown in Fig. [6.](#page-15-10) This clearly shows that the proposed ZF detection outperforms the existing ZF detections about 1 dB and the proposed MMSE detection outperforms the conventional MMSE approaches

Fig. 3 BER performance for uplink SM SC-FDMA scheme with proposed and conventional detection scheme

Fig. 4 BER performance for uplink SM SC-FDMA scheme with proposed and conventional detection scheme

about 2 dB at the BER of 10^{-3} . From these results, the systems realize both MMSE and ZF detection under iterative CG algorithm. It converges the detection result at Nit $=32$. It is noticed that the proposed system superior to conventional detection even at low complexity. It is revealed that the proposed iterative CG based linear detection method is well suited to the SM SC-FDMA system.

7 Conclusion

In this article, low-complexity signal detection scheme for the uplink SM SC-FDMA transceiver is investigated. The new CG algorithm is an attractive design of both ZF and MMSE detection scenarios when employed uplink communication at the base station, the linear filter coefficients are carried out, it notably reduced the computational complexity

Fig. 5 BER performance for uplink SM SC-FDMA scheme with proposed and conventional detection scheme

of the detection system. Since there is no complicated complex matrix inversion of calculation and solving equations in proposed method, while sustaining low complexity. Both ZF and MMSE equalizations are thoroughly verifed various scenarios and a number of solutions are generated. The proposed scenario utilized diferent subcarrier allocation to users which achieve better performance than existing approaches. Simulation results for uplink SM SC-FDMA shows the efectiveness of the proposed detection approach. Finally, the complexity of the new approach is far less than direct MMSE and ZF equalization. New scheme provides much superior performance at a reduced complexity than the conventional approaches. It is also clear that this scheme performance is shown to be closer to MFB only serve few iterations of the detection.

Fig. 6 BER performance for uplink SM SC-FDMA scheme with proposed and conventional detection scheme

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