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Advances in Smart Antenna Systems for Wireless Communication

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Abstract

Wireless communication is one of the fastest growing fields of communication industry. Cellular phones have shown the drastic exponential growth from the last decade and this growth has reached about one billion mobile phone users worldwide. Certainly, mobile phones have become one of the most importants components of daily life and a critical business tool in all countries. Huge gap between a vision for future wireless communication systems and the current system's performance represents that massive research work has to be carried out to make future communication system vision a reality. In this paper, all most all the types of beamforming and direction of arrival schemes for wireless communications have been presented. This paper also presents the comprehensive study of smart antenna systems, its advancement in recent years and futuristic scope.

Keywords Array antenna · Beamforming · DOA · ESPRIT · Smart antenna · Wireless communication

1 Introduction

From the last few years, provision of service through the wireless communication reached beyond all the expectations. This aspect introduces the most challenging technical issue in wireless networks; the need to increase the efficiency of spectrum. Until recently less importance was given towards the antenna related technology. Indeed self configured or new intelligent and highly efficient technique will be replaced to achieve the requirement of future communication systems.

In practice, no antenna is smart, but an antenna system is smart. Hence a 'smart antenna' combines an array of antenna with signal processing capabilities, which makes it possible to transmit and receive the incoming signals in spacial sensitive and adaptive manner.

Adaptive antennas were first studied by Van Atta [1] and their main applications were only in military. He described adaptive antenna as a "self-phased array". In this array, received signals are reflected back in the direction of incoming signals. Hence they were called as "retro directive array". Later, in 1960, phased-looked loop (PLL) was used in the

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retro directive array to improve its performance [2, 3]. Howell et al. [4] developed side lobe canceller (SLC) to suppress the interferences. It was the first attempt to apply SLC to the adaptive arrays in wireless communication

applications. Later, Applebaum [5, 6] have devised new algorithm for antenna array to suppress the side lobes and to improve the signals-to-noise ratio (SNR). He coined this algorithm as “Howells-Applebaum algorithm.

2 DOA Estimation Methods

Widows et al. in the year 1967 studied the least mean square (LMS) adaptive algorithm for antenna array [7, 8]. They used digital signal processor to process the signal. Digital signal processor makes antenna arrays adaptive and hence they are also called as “smart antennas”.

In the same year, Capon shown the processing of multidimensional maximum likelihood (ML) of a large aperture seismic array (LASA) [9, 10]. This method was able to minimize the output noise power and distortion less signals estimation. Hence this method is popularly called as “minimum variance distortion less response” (MVDR) [9].

Sanudin et al. [11] have developed a capon-like DOA algorithm. This method is designed for directional antenna arrays and is similar to classical capon method. The antenna array gain of this algorithm in maintained at the look-direction. As compared to classical method, Capon-like algorithm gives more accuracy when deployed for directional antenna elements.

Diagonal loading and capon beam forming for signal estimation has been discussed in [12]. A Capon beam former based on doubly constrained array method has been reported in [13]. A continuation of this work is carried out in [14] and it uses ellipsoidal uncertainty test for capon beam former using doubly constrained array for improving the resolution of signal estimation.

Spectral estimation methods proposed in [14], were able to provide higher angular resolution. Hence these algorithms are called as “super resolution algorithms”. These algorithms are basically used for the estimation of DOA and direction of departure (DOD) of incoming signals. One of the earliest techniques for spectral estimation is the Bartlet method [15]. This algorithm is popularly known as Bartlet DOA and it is a beam forming method.

Later linear prediction methods were developed for signal detection and estimation. This method was able to minimize the error between the actual output and output of the Lth antenna. Pezessbki et al. have devised multi-rank MVDR algorithm based on eigen value beam forming in [16]. Balasem et al. [17] analyzed the performance of “least constraint minimum variance” (LCMV) and MVDR. They have conducted an experiment consists of 4-element antenna array at the receiver side of communication systems. Both MVDR and LCMV algorithms are studied under various conditions and the experimental results exhibits that the former outperforms the later in terms of improved system capacity.

Wang et al. [18] have devised a linear adaptive beam former. This method is based on noise plus interference covariance matrix and can be used for the estimation of steering vectors.

Xu et al. [19] have devised semi definite programming relation for constrained LCMV beam forming algorithm which provides robustness to the LCMV beam former in low SNR scenario.

Russian mathematician introduced signal estimation using pisarenko harmonic decomposition (PHD) methods for spectral eatimation of desired signals, this method was able to minimize the mean squared error (MSE) of antenna array output.

Basically, as we know that, DOA methods are of two types (i) Classical and (ii) subspace method. All the spectrum estimation methods discussed above are classical methods. The subspace based methods were introduced in the late 1980s. The most popular subspace method till today is "MUltiple SIgnal Classification" (MUSIC) algorithm. This method was put forth by Schmidt in the year 1986 in [20]. This method provides high resolution signal estimation by isolating signal and noise subspaces.

This is a popular eigen-structure based one of the most studied method. This algorithm has many variants. Some of its most popular variants are: spectral-MUSIC [21] and root-MUSIC.

The complexity of MUSIC algorithm is reduced by forming the covariance matrix without decomposing the eigen values in [22]. Ying and Ng [23] have proposed the estimation of signal using MUSIC-like algorithm which does not require a priori knowledge of number of sources.

Modified MUSIC algorithm (M-MUSIC) for high-resolution DOA estimation is reported in [24]. This method uses double orthogonal-triangular (QR) decomposition instead of singular value decomposition (SVD). This method significantly reduces the computational load as compared to normal MUSIC.

Optimization of MUSIC algorithm for various conditions like element distance in linear array, SNR, and number of snapshots is analyzed in [25]. Zeng et al. [26] have devised l_p-MUSIC DOA estimation algorithm for impulsive noise environment. Becker et al. have devised 2Q-MUSIC DOA algorithm in [27]. This method optimizes the compromise between maximum number of sources and system performance.

Root-MUSIC based multiple input multiple output (MIMO) radar has been reported in [28]. Pseudo-noise re-sampling (PR) based DOA algorithms have been reported recently. Qian et al. have developed "improved unitary root-MUSIC algorithm" in [29] for signal estimation. This method solved the abnormal DOA estimator referred to as outlier.

The experiment results show that this new method can effectively enhance the resolution of estimation at low SNR and small samples. This PR-based unitary root MUSIC is further improved by the same authors in the year 2015. They proposed unitary root-MUSIC (URM) using two-step reliability test (TSRT) for DOA estimation in [30]. This method combines the two well-known techniques, unitary root-MUSIC and conventional beam forming deploying the PR technique. This novel method provides better threshold performance. Hence this TSRT using URM method provides better results and outperforms the conventional root-MUSIC.

Mahdi et al. in the year 2013 have developed iterative root-MUSIC in [31]. The proposed algorithm significantly enhances the estimation quality, especially for small snapshot scenarios. Iterative root-MUSIC algorithm has following important aspects.

1. It has excellent performance in terms of MSE.
2. The probability of detection is very high.
3. It provides more accurate DOA estimation as compare to classical iterative MUSIC and root-MUSIC algorithm.

CLOSEST method was developed as an alternative method for beam-space MUSIC algorithm in [31]. This method searches far antenna array weights in the noise subspace which are close to steering vectors.

Roy and Kailath in 1989 proposed a new subspace method called "estimation of signal parameters via rotational invariance technique" (ESPRIT) in [32]. The main objective of this method is the use the rotational technique in the signal subspace which is formed by two translational invariance structure based antenna arrays. This method assumes identical antenna arrays called "doublets".

Direction finding using low complexity ESPRIT algorithm has been reported in [33]. Huang et al. have further reduced the complexity of algorithms using signal subspace fitting for signal detection and estimation in [34, 35]

Zing-Qing et al. in the year 2015 continued this work and devised "robust sparse covariance fitting" for wide band signals estimation. This new method outperforms the spacial smoothing MUSIC algorithm and provides better results in non-uniform noisy environment.

2.1 Nyström Based ESPRIT Algorithm

In this method, the complexity of ESPRIT algorithm is reduced by using Nyström theory. DOA estimation using subspace based method uses intensive computations to calculate singular vector decomposition (SVD). Nystrom is the well known mathematician; he has shown that the SVD of sample covariance matrix (SCM) can be computed without actually computing SCM [36].

Figure 1 shows the performance of various DOA estimation methods.

It can be noticed that, among all methods, Nystrom based DOA method exhibits superior performance with reduced complexity.

2.2 Comparison of DOA Estimation Methods

In Tables 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, performances of various popular DoA algorithms are compared.

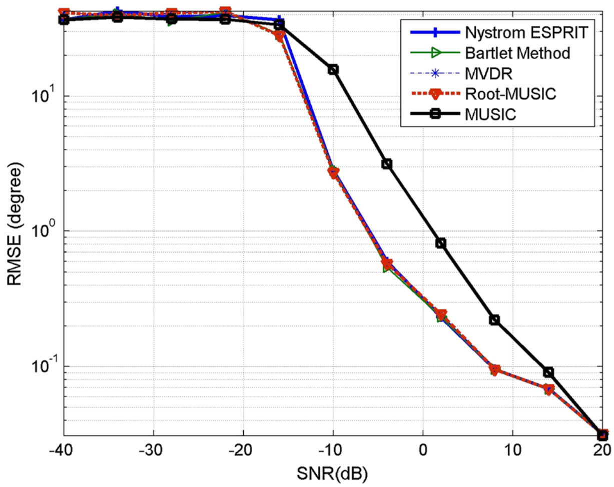

Fig. 1 Performance of various DOA estimation methods

Table 1 Performance of Bartlett method

Algorithm	Property	Observations
Bartlett method	Bias	Biased [15]
	Resolution	It depends on aperture of antenna array [15, 16]
	Sensitivity	Robust to position of antenna elements [16]
	Type of array	General Antenna array [14]

Table 2 Performance of MEM method

Algorithm	Property	Observations
Maximum entropy method (MEM)	Bias	Biased [67]
	Resolution	It is better than Bartlett method and can perform well in lower SNR scenarios [15, 67]

Table 3 Performance of MVDR method

Algorithm	Property	Observations
MVDR method	Bias	Unbiased [16]
	Variance	Its variance is minimum [17]
	Resolution	MVDR > Bartlett [18]
	Type of array	General antenna array [17]

Table 4 Performance of linear prediction method

Algorithm	Property	Observations
Linear prediction method (LP)	Bias	Biased [51]
	Resolution	LP > MVDR [16, 18, 51]
		> Bartlett [15]
		> MEM [14]
	Type of array	Uniform linear array (ULA) [12]
	Performance	It can perform well in lower SNR scenarios [16, 18]
		It can be used for correlated signals

3 Beamforming Algorithms

Various beam forming techniques are studied and compared in this section.

Table 5 Performance of MLM

Algorithm	Property	Observations
MLM	Bias	Unbiased
	Resolution	MLM < LP
		< Bartlett
		< MUSIC
		Less than MUSIC for small samples
	Variance	Not efficient particularly for finite samples
		Less efficient for random and deterministic signals
	Computation	Computation is intensive for large array samples
		Same for random and deterministic signals for large signals [11]
	Performance	It works with single snapshot (one sample) [14]
		It can be used for correlated signals [18]

Table 6 Performance of MUSIC method (element spaced MUSIC)

Algorithm	Property	Observations
Element spaced MUSIC method	Bias	Biased [21]
	Variance	Less than min-norm and ESPRIT methods
		Close to CLOSEST, MLM, FINE [22]
		Variance of weighted MUSIC > unweighted MUSIC [24]
	Resolution	Resolution is limited by bias [24]
	Array	Applicable for general Antenna array
		It is robust for increasing antenna aperture
	Performance	It is a subspace based method and has very good resolution
		Performance of MUSIC > MEM [21]
		> MLM [21]
		> LP [20]
		> MVDR [22]
		Deviates from performance for correlated signals
		Modification of the algorithms can improve the performance and can be applied for correlated signals
	Computation	Requires intensive computations [21]
	Sensitivity	Array calibration is important
		This method is sensitive to array aperture
		Resolution can be improved by preprocessing [22]

3.1 Narrow Band Beam Former

Figure 2 is the narrow band beam former. Here the signal induced on antenna array element are multiplied by complex weight and then the multiplied output are summed together to obtain the array output.

Table 7 Performance of beam spaced MUSIC method

Algorithm	Property	Observations
Beam spaced MUSIC method	Bias	Less than MUSIC [22]
	Variance	Beam spaced MUSIC > MUSIC [26]
	Resolution	Better than min-norm and MUSIC [24]
		Similar to CLOSEST [26]
		If separation between the sources is increased, SNR decreases
	RMS error	It is less than min-norm and ESPRIT [32, 33]
	Computation	Requires less computation as compared to MUSIC [22]
	Sensitivity	Sensitivity is robust as compared to MUSIC [23]

Table 8 Performance of Root-MUSIC method

Algorithm	Property	Observations
Root MUSIC	Variance	Less than min-norm method, ESPRIT [27]
	Resolution	It has better resolution than MUSIC [28]
	Array	Equi-spaced linear array [26]
	RMS Error	It is less than min-norm and ESPRIT [32]
	Performance	Better than MUISC, ESPRIT, min-norm, MEV, MLM and MEM methods [10]

Table 9 Performance of min-norm method

Algorithm	Property	Observations
Min-norm method	Bias	Min norm method < MUSIC [32]
	Resolution	Better than MUSIC and CLOSEST [25]
	Type of array	General antenna array [44]

Table 10 Performance of CLOSEST method

Algorithm	Property	Observations
CLOSEST Method	Variance	Variance of CLOSEST method is similar to MUSIC [20]
	Resolution	It is similar to MUSIC and better than min-norm [26]
	Performance	Its performance is excellent in clustered situations [32]
	Sensitivity	An abrupt deterioration in variance and bias is noticed for increased phase error and sensor gain beyond certain value [42]

The components like band bass filters, preamplifiers and so on are not shown in the Figure. The antenna array output y(t) is given by $y(t) = s_i$, where * represent the complex conjugate. The weights of the narrow band beam former is expressed as $w = [w_1, w_2, \ldots w_l]^T$. The input signals impinging on antenna element is expressed as $x(t) = [x_1(t), x_2(t), \ldots$

Table 11 Performance of ESPRIT method

Algorithm	Property	Observations
ESPRIT method	Bias	ESPRIT method is unbiased [32]
	Mean square error	It is less than min-norm method [32]
	Variance	Variance of this method is less than MUSIC algorithm [55]
	Computation	Computation is less than MUSIC algorithm [57]
	Array configuration	It needs doublets
		But calibration is not needed
	Performance	This method is robust than MUSIC algorithm. It cannot handle the correlated sources
		Its mean square error is robust as compared to sensor gain errors [100]

Table 12 Performance of FINE method

Algorithm	Property	Observations
FINE method	Bias	FINE method < MUSIC [24]
	Resolution	Better than MUSIC, and min-norm [97]
	Variance	Less than min-norm method [64]
	Performance	Its performance is good at low signal to noise ratio [67]

Table 13 Comparison of Adaptive beamforming algorithms

Algorithm	Iterations	Complexity	Performance
LMS [96]	80	O(L)	Less complex and low convergence
RLS [81]	15	O(L)	Less efficient in non-stationary environment
SMI [90]	No Iterations	$O(L^3)$	It require large computations
CGM [108]	8	$O(L^2)$	Fast convergence
CMA [98]	70	O(L)	Not stable in dynamic environment
LS-CMA [101]	12	$O(L^3)$	Lower null depth
NLMS [78]	60	O(L)	Not efficient due to normalization
VSSLMS [111]	50	O(L)	Deep null depth
VSSNLMS [112]	50	O(L)	Deep null depth
FLMS [110]	23	$O(L^2)$	Convergence is lower than RLS, CGM, SMI, NLMS
RLMS [81]	12	$O(L^2)$	Convergence is lower than RLS, CGM, SMI
LLMS [110]	25	O(L)	Convergence is lower than RLS, CGM, SMI, RLMS
LMS with SMI [66]	22	$O(L^2)$	It require large computations
KALMAN [65]	20	$O(L^2)$	Lower null depth
Novel LMS [117]	6	O(L)	Less complex and has very high convergence speed as compared any beam forming method

$x_l(t)]^T$. Finally, the output of this structure is expressed as $y(t) = w^H x(t)$, Here the subscripts H and T represent the complex conjugate and transpose respectively.

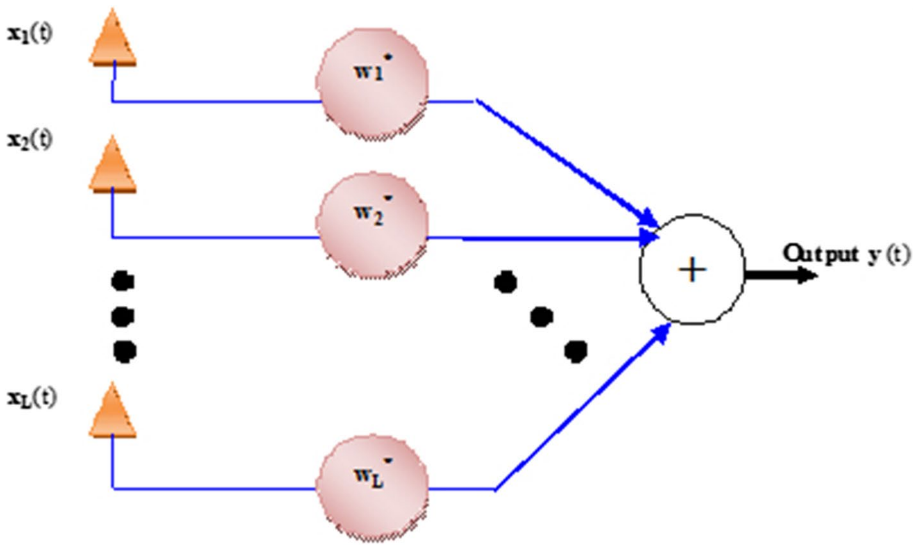

Fig. 2 Narrow band beam former

3.2 Conventional Beam Former

This beam former is a simple beam former; it is also called as delay and sum beam former. The important aspects of this beam former are discussed as below.

1. Conventional beam former's all weights have equal magnitude.
2. In the look direction, it has unity gain.
3. Processor mean output power due to source and source power is same in the look direction.
4. Hence when in the look direction, both the powers are similar, this process is similar to mechanical steering of beam, but the only difference in this method is, the beam is steered electronically by varying the phase shifters. Hence, this method is also called as 'electronic steering'.
5. The concept of conventional beam former can be well understood by considering the Fig. 3, which shows the two element delay and sum beam former.
6. In this method, each waveform is scaled by 0.5 which results in unity gain in the azimuth (θ) direction.
7. Though the conventional beam former gives maximum output signal to noise ratio, this method not effective and suitable when the interferences or jammers are present.

3.3 Null Steering Beam Former

This beam former is used to place the null in the direction of incoming desired signal by cancelling the plane wave. This is done by estimating the direction of arrival of known signal by steering a beam in the source direction and then subtracting its output from

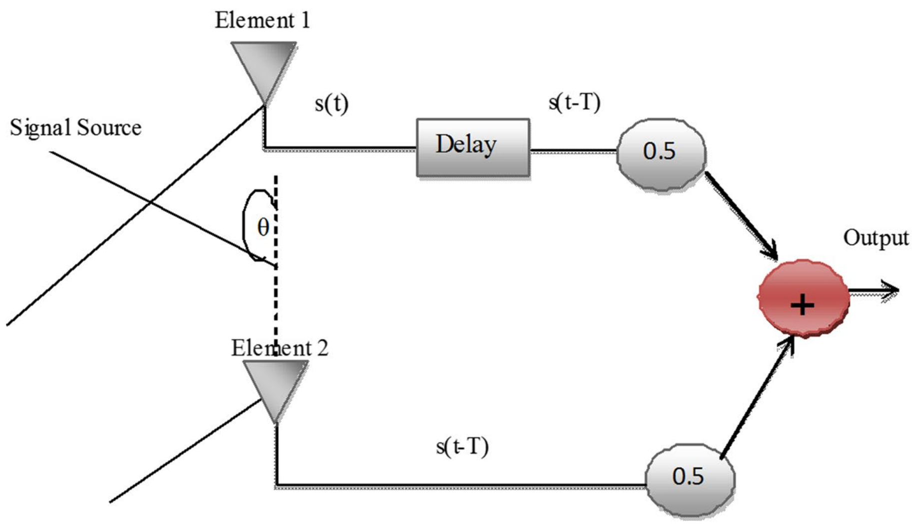

Fig. 3 Two element delay and sum beam former

each antenna element. This method is very effective and useful for mitigating the strong interferences. This procedure can be repeated for cancellation of multiple interferences.

Null-steering beam former becomes cumbersome as the interference number grows rapidly. The main downside of this method is that, this beam former cannot maximize the noise (uncorrelated) at the antenna array output. The application of null steering beam former is described in [37] to mitigate strong jammers in cellular mobile communication systems. In [38], it is used to minimize the effect of interferences and noise towards other mobiles. A detailed performance analysis of this beam former is discussed in [39].

The knowledge of exact direction interference signal is indeed required in this method. It is also reported that, it does not increase the output signal to noise ratio in certain wireless communication systems (Tables 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13).

3.4 Optimal Beamforming

This beam former over comes the limitations of null steering beam former that is it does not require the exact knowledge of interference direction and also does not require to place the null in the direction of plane waveform for beam steering. This method maximizes the output signals to noise ratio effectively as compared to previous approach.

The array weight of optimum beam-former are computed using NAME. Hence the processor used for this purpose is called as NAME processor [40]. This beam former is also called as maximum length (ML) filter [10]. As discussed in [41], this beamformer does not mitigate the effect of interferences significantly but can enhance the output SNR. This beamformer is often known as optimal combiner in mobile communication literature. In [42], it is shown that, this beam former is used to cancel the multipath interference and also improved the performance of system in mobile communication.

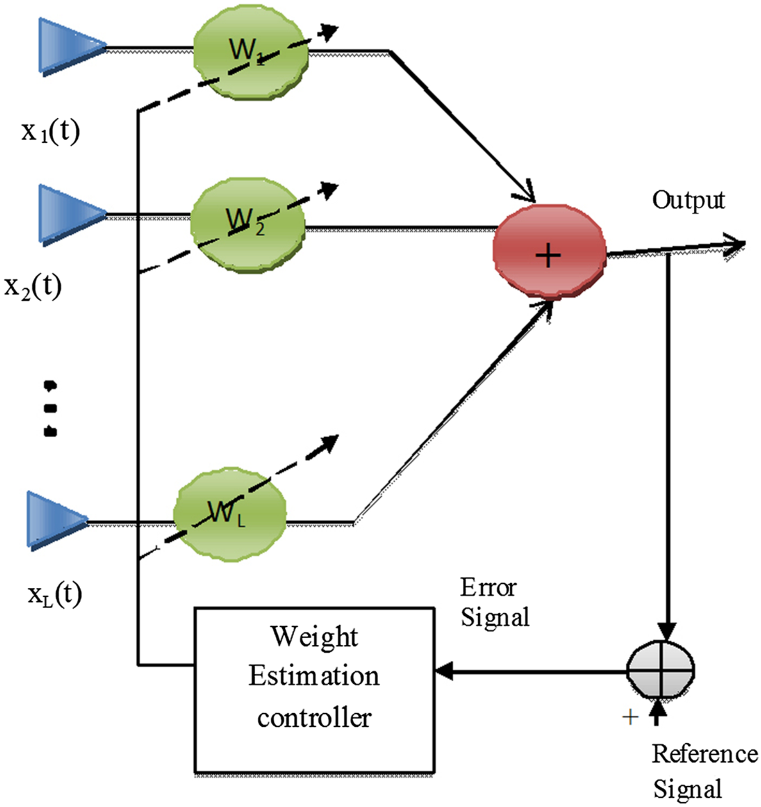

Fig. 4 A narrow-band beam former structure using a reference signal

3.5 Optimization Using Reference Signal

A narrow-band beam former structure using a reference signal for calculating the beam former's weights is shown in Fig. 4 [43].

This method can be used to acquire a weak signal, for this, the narrow band beam formers makes the reference signal to zero in presence of strong interferences. This method cancels the interferences which are strong and then it can cancel the weak signals. In cellular mobile communication systems, synchronization signal can be employed for initial weight calculation followed by the employ of references signal. Hence the detected signal is used as a reference signal. The performance this beam former using a references signal in mobile communication system is presented in [44, 45].

The structure of this processor is shown in Fig. 5. The various names used for beam space processor are:

1. Multi beam antenna.
2. Adaptive-adaptive arrays.
3. Partial adaptive arrays.
4. Pertained processor.
5. Homells-applebaum array.

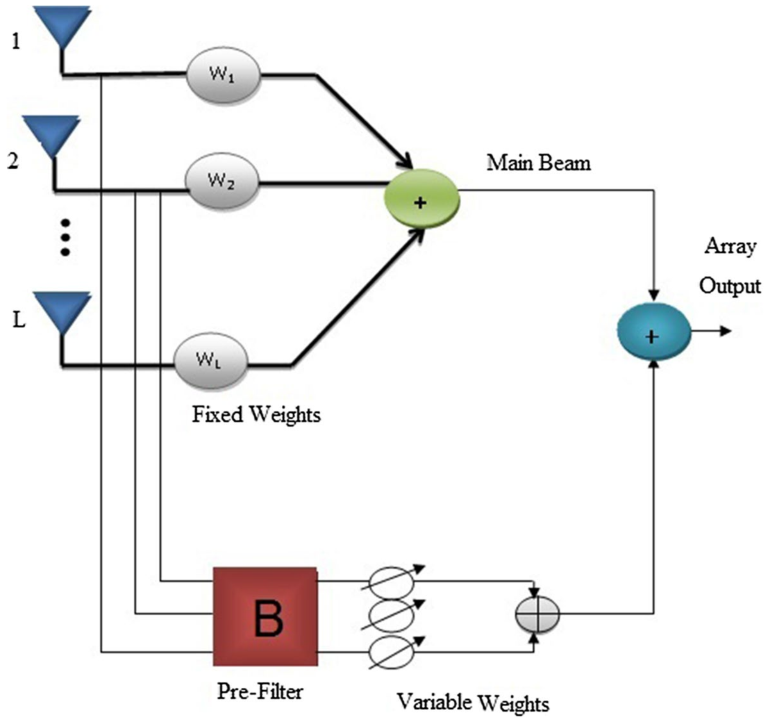

Fig. 5 Only the beam space processor

The beam space processor shown in Fig. 5 can cancel only a single interference. A comparison of beam space processor and element spaced processor is discussed in [46]. It is reported that the performance of beam space processor is superior as compare to element space processor and they provide the same performance in absence of error. Beam space processor based 16-element antenna array is developed in [47].

3.6 Broad Band Beam Forming

The performance of this structure (or beam former) deteriorates, if the band width of the signal increases. Hence for processing of such broad band signals, Time delay line (TDL) structure can be used which is shown in Fig. 6 [48]. It consist of steering delays, tapped delay line and weights. The array can be steering in the look direction (∅) by using steering delays. This steering delay and the TDL structure is a finite impulse response (FIR) structure.

Usefulness of broad-band beam former cellular mobile communication is studied in [49]. A large delay spread in a CDMA and TDMA to overcome multipath `fading in mobile communication is presented in [50]. This structure is usually called as direct form of realization or an element space process. Partitioned realization of this structure is studied in [51] and it is shown in Fig. 7. The upper part of this structure is a conventional broad beam former. The lower part of the structure consists of matrix pre-filter.

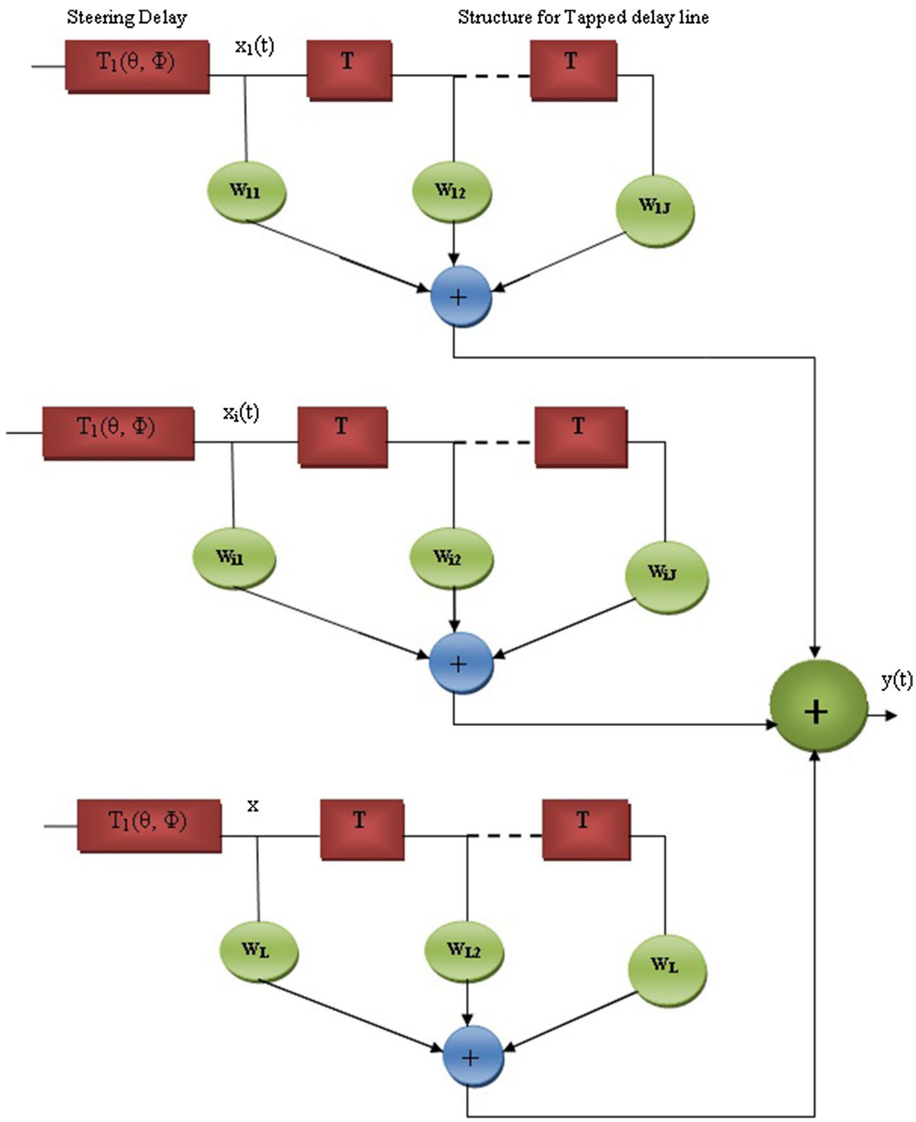

Fig. 6 Time delay line (TDL) structure

Normal broad-band and partitioned processor shown in Fig. 7 have same degree of freedom. The alternative methods other than TDL structure for broad-band antenna array have been proposed. They are:

1. Maximizing SNR by the use of adaptive nonlinear schemes.
2. Davis beam former for increasing the convergence speed.
3. Weighted chebyshev method.
4. Correlation transformation method.

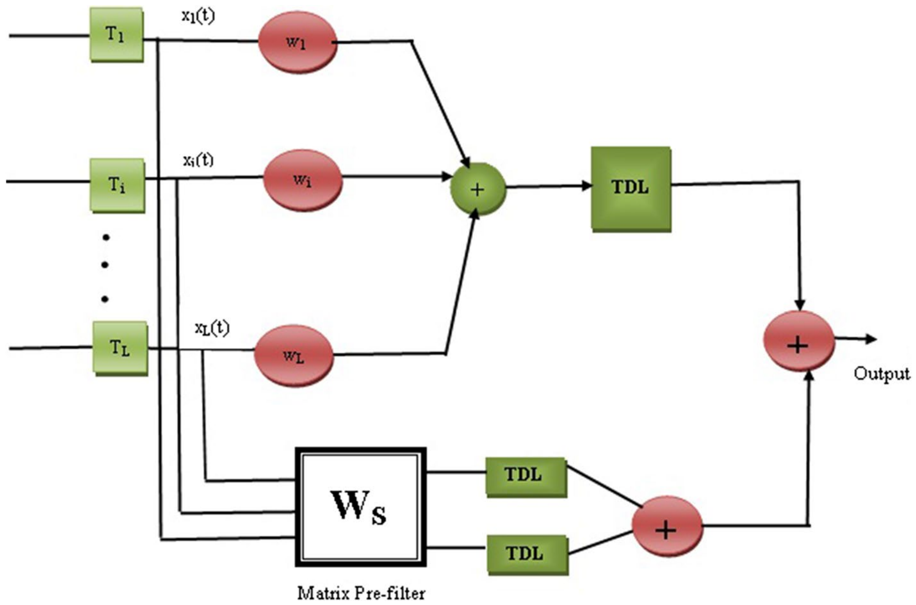

Fig. 7 Normal broad-band and partitioned processor

5. Optimum filter.
6. Broad beam former using a master and slave processor.
7. Wide band interference rejection using a composite system.

Figure 8 shows the structure of element-space broad band processor. It consists of fast fusion transform (FFT) blocks at the input site and inverse (FFT) at the output side. Broad band time domain signals of each antenna element are converted into frequency domain using FFT. The narrow- band processor process each frequency signal and the all weights are summed to produces the output. This frequency domain output is converted back into time- domain using inverse FFT.

A study in [52] reports the frequency domain beam-forming which is well suited for implementation of VLSI and its sensitivity is less for the co-efficient quantization.

3.7 Eigen Structure Method

Eigen structure method is subspace method in which the signal space and the noise space are separated using eigen value of matrix R. This fact is introduced for beam forming in a various ways in [53]. If ‘M’ is the number of sources, the largest ‘M’ eigen values corresponding to this are known as signal eigen vector. If ‘L’ is the number of antenna element than (L–M) eigen values are known as noise eigen vector.

Application of eigen structure scheme for beam forming in cellular mobile communication is discussed in [54].

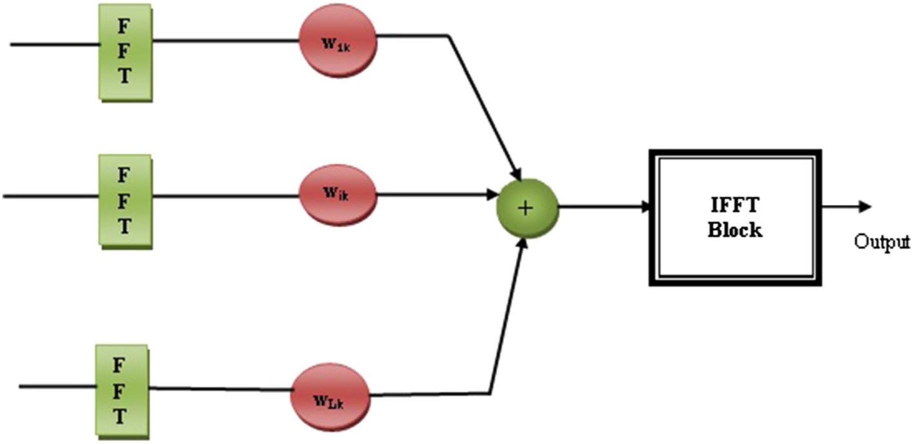

Fig. 8 Structure of element-space broad band processor

3.8 Digital Beamforming

In this beamforming, analog to digital converters (ADC) are used. In this method, weights of each antenna element are sampled and then stored. All these sampled signals are added together using summer to produce beams. This process incorporates required delay. The time domain continuous signals can be reconstructed back, if the sampling rates are at least double or more. This is known as "Nyquist-criterion" [55]. This requires high speed ADCs, large bandwidth etc. [56].

In [57, 58], digital interpolation method is introduced to overcome the necessity of high sampling rates. A study in [59] shows the trade-off between memory, hardware composite and system performance. A new technique called extrapolation is introduced in [60] to improve the beam pattern of large antenna array using digital beam forming. A detailed information extrapolation method for signals is reported in [61]. The application of digital beam forming for mobile satellite communication schemes reported in [62].

4 Adaptive Beamforming Algorithms

The shape of radiation pattern of smart antenna is controlled through the use of algorithms. It is based on certain criteria, that may be enhancing the signal to interference ratio (SIR), minimizing MSE, minimizing the variance beam steering towards signal of interest (SOI), tracking a moving emitter and mitigating the interference signals. These algorithms can be implemented electronically via analog devices. The implementations will be easier and accurate using digital signal processing. An antenna radiation pattern also called as beam is formed by using digital signal process, the method is known as digital beam forming. Figure 9 shows the analog (a) and digital beamforming (b) structures.

When digital signal processing algorithm is adaptive then they are referred to as adaptive algorithm. The adaptive beam forming is the sub category of digital beam

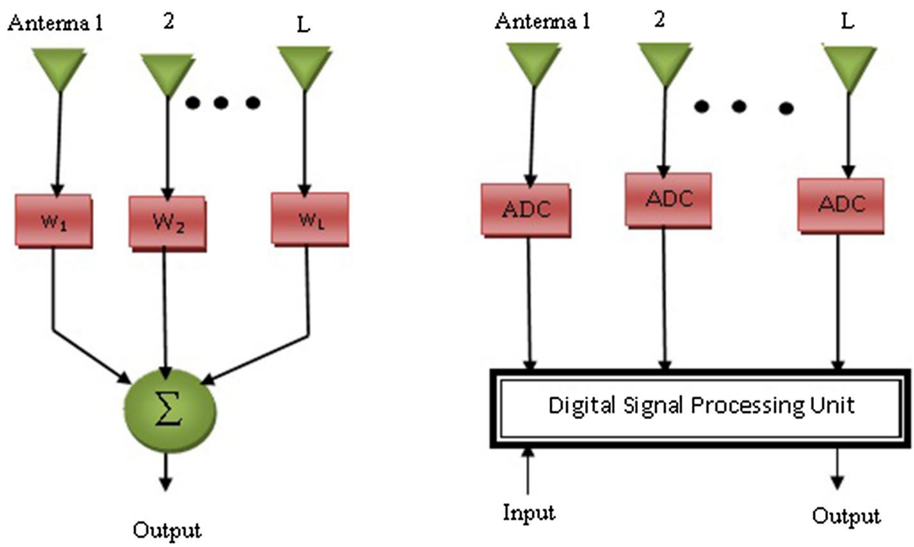

Fig. 9 Beamforming structures **a** analog, **b** digital

forming. Digital beam forming technique has been used to sonar systems [63], radar systems [64] and communication system [65].

The main advantage of this method is that, array weighting and phase-shifting can be done on the digital signal which is the output of ADC rather than in hardware. In the digital beam forming method no effort is made to shift the antenna element phase, Hence this method cannot be strictly referred as electronic beam steering. Adaptive beam forming method is more effective and useful than the digital beam forming. Until recently radar system rely on traditional electronic scanning technologies. Very recently adaptive beam forming technologies are used in radar systems [66]. Many popular examples of adaptive beam forming algorithm are:

1. Least mean square (LMS)
2. Sample matrix inversion (SMI)
3. Recursive least squares (RLS) algorithms
4. constant modulus algorithms (CMA)
5. conjugate gradient (CG) method
6. Waveform diversity algorithms

4.1 Least Mean Square Algorithm (LMS)

The application of this method to estimate the complex weight of an antenna array is widespread. When the complex weights of this method are constraints at each iteration, then this algorithm is called as constrained LMS. If the complex weights are not constrained at each iteration then this algorithm is called unconstrained LMS algorithm.

The LMS algorithm is shown in Fig. 10 is a gradient base method. The convergence rate of LMS algorithm is completely depends on the step size (μ) parameter. If it is too small, convergence rate of the algorithm will be too slow and it result the over damped case. If

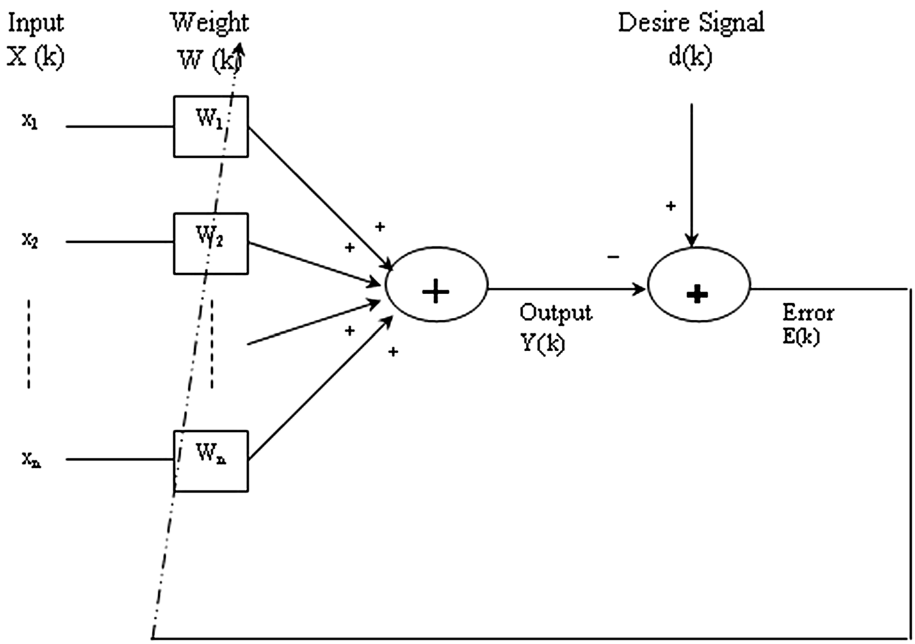

Fig. 10 The LMS beamformer

the 'μ' is too large, we will have under damped case. Hence the convergence rate of an algorithm is very important aspect and its practical important for cellular communication is reported in [67].

In this paper, it is explained how LMS algorithm deviates from its performance when the signal conditions are changing rapidly. The steady-state and transient behavior of this algorithm and MSE are interesting and important aspect of LMS method and are studied in detail in [67].

The standard LMS algorithm uses fixed step size. In [68] it is shown that the step-size is varied during each iteration to improve the convergence speed by minimizing the MSE. Other methods to increase the convergence speed such that cancelling interferences one at a time [69] another method called blocking processing is used in [70]. The convergence speed can be increased for broad band signal by implementing LMS algorithm in frequency domain. This concept is studied in detailed in [71]. It reduces the computational complexity of algorithm.

The sign LMS algorithm is developed in [72] in which the error signal between the reference signal and antenna array output is replaced completely by its sign. Hence this method is called as sign LMS and its computational complexity is less than the normal LMS algorithm. Practical application of LMS algorithm for cellular communications using an antenna array are studied for indoor-radio systems in [73], mobile communication systems in [74] and satellite-to-satellite systems in [75].

4.1.1 Normalized LMS (NLMS) Algorithm

In this algorithm, for each iteration, it uses a data-dependent step-size. The step-size of this algorithm at *n*th iteration is given by $\mu(n) = \mu/x^H(n)x(n)$, Here the value of μ is a constant.

The performance analysis of NLMS algorithm using different types to increase the convergence speed and other parameter are studied in [76, 77]. Standard LMS algorithm requires the calculation of eigen values for covariance matrix. This is avoided in NLMS algorithm. NLMS has better convergence speed as compared to normal LMS algorithm. Normalizing makes it less sensitive as compared LMS algorithm. Application of NLMS algorithm for mobile communications is reported in [78]. The following are the implementation issues found in LMS algorithm

1. Convergence rate.
2. Fluctuations in antenna array weights.
3. Miss-adjustment noise.

Unbiased impulse response based least mean square algorithm has been devised and the performance is analyzed in [79]. Performance of RLS and LMS are analyzed and a new method which combines both algorithm called RLS adaptive beam former has been devised in [80]. RLS algorithm has better convergence rate as compared to LMS algorithm but its computational complexity for multiple input multiple output (MIMO) is large. The variable step size (VSS) LMS algorithm is discussed in [81].

The simulation results validate the effectiveness of algorithm and have better performance than the conventional LMS and RLS algorithm. The use of MIMO antenna in mobile communication technology gives the significant performance improvement. This is the key requirement for 4G and beyond communication system. The channel estimation using LMS and RLS algorithm for 4G MIMO orthogonal frequency division multiplexing (OFDM) [81] has been devised.

A Robust LMS algorithm for interference suppression in mobile communication has been reported in [82, 83]. A VSS-LMS algorithm using novel array structure has been devised in [84]. This method is quite simple to implement and has better convergence rate as compare to standard LMS algorithm. Furthermore, this scheme can resolve narrow band signals impinging on antenna array at the end fire direction.

A new-approach called MIR-LMS algorithm for an adaptive array applicable for high speed mobile communication has been reported in [85, 86] in the year 2013 and 2014.

A re-weighted l_1-norm penalized scheme for LMS algorithm is presented in [87] in the year 2014. This method is studied in detail for estimation of sparse channel and its significance for wireless communication application is discussed. The simulation result clarifies that the re-weighted l_1-norm penalized scheme presented for LMS algorithm shows better performance as compared to standard LMS algorithm.

4.2 Sample Matrix Inversion (SMI)

One of the downside of LMS adaptive beam forming algorithms is it has to undergo many iterations until satisfactory convergence is achieved. If the characteristics of the signals are changing rapidly, the standard LMS algorithm cannot track the desired signal satisfactorily. These limitations can be circumvented by the use of sample matrix inversion method [88]. This algorithm is also called as a direct matrix inversion (DMI). This method calculates the antenna array weights using 'K' samples.

Applications of this method to calculate the optimum weights of an antenna array in mobile communication is discussed in many research papers including [89–91]. It is used for vehicular mobile communication and also used for mobile satellite communications systems [92].

4.3 Recursive Least Square (RLS) Algorithm

The low convergence rate of standard LMS method is solved using recursive least square algorithms in [93]. In this method, the gain matrix for nth iteration is used in place of gradient step-size μ. The RLS method significantly minimizes the mean square error (MSE).

The convergence rate of this scheme dependents on eigen value of correlation matrix. Comparison of RLS, LMS and other few gradient-base methods represent that the RLS algorithm is efficient than LMS [94]. A study of RLS, SMI and LMS methods in mobile communication in [95] reports that the RLS algorithm out performs the later two schemes in flat-fading channels. Practical uses of RLS scheme for reverse link of cellular mobile communication based on CDMA system is reported in [96].

4.4 Constant Modulus Algorithm (CMA)

This algorithm uses a gradient-based method. Many beam forming algorithms in array signal processing tries to minimize the error between the antenna array output and a reference signal. The reference signal used in algorithm is usually a training sequence signal which trains the desired signal or the adaptive antenna array using a priori knowledge of arriving signals.

Most of the radar and wireless communication signals are phase or frequency modulated. Some examples of frequency and phase modulated signals are FM, FSK, PSK and QAM. In all modulation techniques, the amplitude of the signal is constant. Thus signals are said to have constant modulus (CM) or magnitude. In fading channels, if there exists a multipath terms, the signal received may have all multipath terms. Hence the amplitude variation will occur on the magnitude of signal by the channels. Constant modulus property will be destroyed by the frequency selective channels. If we know the incoming signal has a CM, we can easily device techniques that equalize or restore the original signal amplitude.

The property of CM was investigated in [97]. This CM property was used to device blind equalization algorithms and used in 2D data communication systems. The cost function used by Dominique Godard is called a dispersion function. Thus the weight of the CMA updates by minimizing the Godard cost function.

The characteristics of blind beam fading based digital land mobile communication system is studied in [98]. This method was used for the implementation of hardware in [99].

The BER performance for mobile communication which uses high speed transmission is reported in [99].

The beam-space CMA adaptive array is able to cancel the interferences coming from angle which are different from look-direction. CMA is effective for signals such as quadrature pulse shift keying (QPSK) but not suitable for CDMA system because it require power control. The use of CMA scheme to blindly isolate the co-channels frequency modulated signals in cellular mobile communications is reported in [100, 101].

4.4.1 Least Square Constant Modulus Algorithm (LS-CMA)

Slow convergence rate is one of the severe downside of CMA algorithm. This limits its application in environments where the characteristics of signal are changing rapidly. The standard CMA scheme uses steepest descent method similar to LMS algorithm.

A faster blind beam forming algorithm was devised in [102, 103]. This new method used non-linear least squares and is also called as Gauss scheme. This algorithm is known as "least-square constant modules algorithm" (LS-CMA). It is also referred to as an "autoregressive estimator" [103].

It is observed that the LS-CMA method gives better performance for suppressing the interferences and multipath terms than the normal CMA algorithm. The chief advantage of this algorithm is high convergence rage. It can converge hundred times faster as compare to normal CMA algorithm [104].

4.5 Conjugate Gradient Method (CGM)

The convergence rate of algorithms can be accelerated by the use of new method called conjugate gradient. This method was studied first in 1952 in [105]. The main goal of this method is to find the optimum solution by selecting perpendicular (conjugate) paths for every iteration. CGM results orthogonal search which yields faster convergence rate. Later this CGM is studied in detail and in [106], this method has also been referred to as "accelerated gradient approach (AG)". Modified and improved version of CGM for predicting array weights has been reported in [107]. As compared to previous adaptive beamforming algorithm, CGM provides faster convergence [108–111]. The performance of communication system using CGM is better as compared to RLS, SMI, and LMS algorithms [112–116].

4.6 Novel LMS Algorithm

We know that the LMS is algorithm well known for its low complexity, fast tracking capabilities and it is less prone to numerical errors. It requires only O(L) computations to calculate array weighs. The use of LMS algorithm in wireless communication applications is wide spread. It is used in many fields including, cellular communication and surveillances etc.

Novel LMS adaptive beamformer [117] produces satisfactory output for only 6 iterations. Hence this method has fastest convergence rate as compared to any beamforming algorithm in literature. Figure 11 shows the comparison of novel LMS algorithm with other standard forms.

4.6.1 Calculation of Improvement Factor

Let No. of iterations required for Standard LMS = ξ_L
Let No. of iterations required for Novel LMS = ξ_N

We know that LMS algorithm requires at least 80 iterations.
Hence improvement factor can be calculated as:

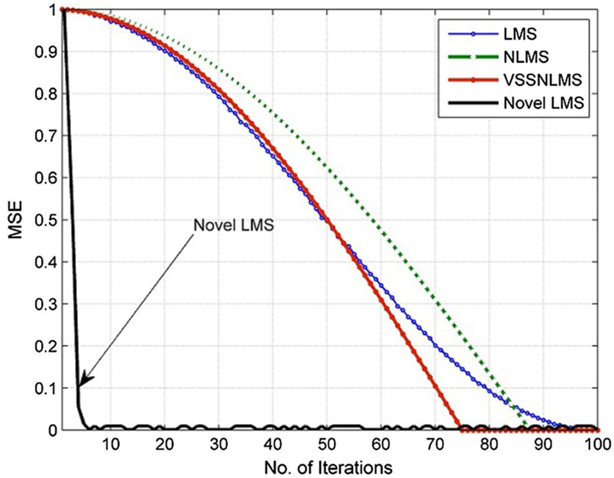

Fig. 11 Comparison of various adaptive beamforming algorithm

$$\text{Improvement factor} = \left(\frac{\xi_L - \xi_N}{\xi_L}\right) \times 100$$
$$= \left(\frac{80 - 6}{80}\right) \times 100 = 92.5\%$$

Hence this method is introduced to overcome the problem of slow convergence rate produced by the LMS algorithm and its variants.

5 Conclusions and Futuristic Scope

In this paper various beamforming and DOA methods are presented. This paper discusses beamfomring and DOA algorithms from early investigations to till recent years. This paper helps lot of research scholars and beginners to understand array signal processing principles applicable to mobile communication. It compares almost all the possible methods existing in the literature.

The main conclusions of this research are as follows

The RLS algorithm uses the eigen spread of array correlation matrix to compute antenna array weights as a result of this, the accuracy and convergence rate of this scheme is low and affected by eigen spread value.

The conjugate gradient method doesn't use eigen spread of the signal correlation matrix. It computes the antenna array weights by using conjugate (orthogonal) for every new iteration. Hence the convergence rate of this method is more as compared to LMS, SMI and RLS schemes.

The CGM adaptive beamforming method exhibits superior performance among classical beamfomers in terms of accuracy, speed and robustness.

The smart antenna system using conjugate gradient method can provide enhanced system capacity and it can mitigate the effect of interference by forming very narrow beam in the look direction.

Novel LMS algorithm shows the improved convergence rate as compared to any variant of LMS algorithm. It requires only 6 iterations to converge MSE to zero. Hence due to very low complexity and fast convergence, this method can be used in practical mobile communication applications. This can be used for 5G and beyond mobile communications.

Nystrom based ESPRIT algorithm is a new method which computes SVD without computing sample covariance matrix. Classical ESPRIT require $O(L^3) + O(L^2K)$ flops to compute SVD of SCM whereas the Nystrom based ESPRIT algorithm requires only $O(LM^2 + LM)$ flops. This makes algorithm efficient especially when the antenna elements or snapshots are too large.

Some of the futuristic scopes are discussed as below.

1. It will be very much useful, if the computational complexity of algorithms is implemented using DSP.
2. Proposal of single snapshot root-MUSIC and ESPRIT algorithms using UCA to have 360 degree azimuth coverage can give better resolutions as compared to exiting ULA configurations.
3. The proposal of beamforming and DOA estimation algorithms for MIMO, massive MIMO and new phased array plus MIMO systems can be very much useful for future wireless communications.

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