Wireless Personal Communications (2019) 108:1759–1783
https://doi.org/10.1007/s11277-019-06495-8

A Review on Training and Blind Equalization Algorithms for Wireless Communications

Sheeraz Ahmed[1,2] · **Yousaf Khan**[3] · **Asad Wahab**[2]

Published online: 8 May 2019

Abstract

Every wireless communication system comes with an innate problem of multipath propagation, which results in spreading the resultant symbols on a time scale and thus causes the symbols to overlap and end up in Inter-symbol Interference (ISI). The overall signal is distorted and the receiver is unable to recover the original signal. ISI from the signal must be removed and the signal must be brought back to its original form as it was sent or as close to it as possible; and process of equalization is used in all wireless communication system for this purpose. Two types of equalization processes are common in modern wireless communication systems. Training based equalization requires the sender block of communication system to constantly send a pilot/training signal in order to update the receiver about the original signal. The receiver removes the ISI and extracts the unadulterated signal. The second equalization process is called blind equalization and it does not require any pilot signal. The receiver only needs to know the type of constellation scheme used in modulation and then the original signal is extracted based on that information. In this paper we have thoroughly reviewed four equalization algorithms, two from each type of equalization for 16-QAM constellation and 64-QAM constellation. We came up with constellation diagrams for each equalization algorithm and comparison of BER, residual ISI and MSE for 16-QAM and 64-QAM is done through simulations. In case of LMS and RLS algorithm for 16 QAM, the performance of RLS gets slightly better than LMS at 6 dB, however, at around 12–14 dB and onwards the BER of RLS leads and there is a significantly better BER than LMS. In future, we will compare these algorithms in order to figure out the best algorithms for the current and upcoming 5G and 6G communication technologies.

Keywords Inter-symbol Interference · Communication system · Equalization · QAM

✉ Yousaf Khan
yousafkhalil@gmail.com

[1] Iqra National University, Peshawar, Pakistan

[2] Career Dynamics Research Centre, Peshawar, Pakistan

[3] University of Engineering and Technology, Peshawar, Pakistan

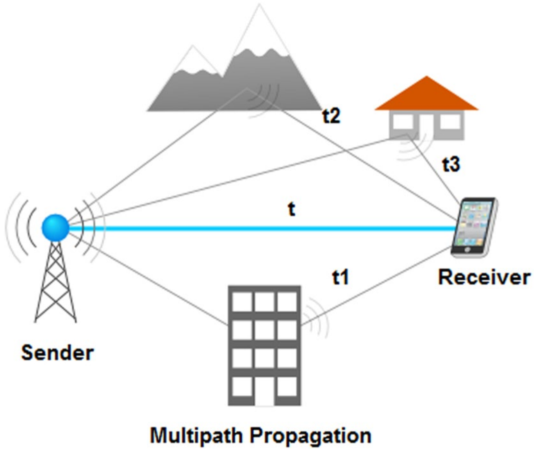

Fig. 1 Representation of a multipath propagation due to environment

1 Introduction

Connectivity is the backbone of modern lifestyle. The pre-existing communication technologies are performing very efficiently and coping with the ever increasing demands of the modern world. However, every technology comes with its share of limitations and issues. Our modern communication systems are no exception to this and they come with their fair share of issues. There are multiple issues that hamper the performance of these communication technologies.

Below are some of the problems that has high impact on the performance of the modern communication system.

(A) Spectrum Scarcity is the foremost problem in latest wireless communication problem. Each mobile operator is assigned a certain spectrum in which it has to contain its communication [1]. The user base of mobile phone users is growing and there is no significant increase in the spectrum. Techniques are constantly devised to house as much users at once as possible; however, the spectrum crunch is inevitable and sooner or later we will have to look around for alternate spectrum.

(B) As efforts are being made to accommodate as many users in a certain spectrum simultaneously, there are a lot of risks related to the mutual interference among the users. This concern will remain so unless there is an expansion of spectrum or new spectrums are explored for communication.

(C) One of the major concerns of the wireless communication system is the multipath propagation. When a signal is sent from the receiver, it is broadcasted all around in each direction. Apart from the direct line of sight arrival, the signal reaches the receiver from various directions after reflections from the obstacles around. This causes the receiver to receiver multiple versions of same signal, each with very slight difference. This results in Inter-symbol Interference (ISI) [2]. Each symbol is spread over a certain period of time and when the second symbol arrives before the first symbol's reflected signals are finished, the two symbols mutually interfere. The ISI is eliminated through the process of equalization and channel estimation. Figure 1 represents multipath propagation due to several external objects around.

Fig. 2 Representation of Inter-Symbol Interference (ISI)

(1) All contemporary communication technology required a good amount of energy in order to function efficiently. While we are yet to go miles in our electricity storage and battery related technologies, the high power consumption has become a serious bottleneck.
(2) Noises like thermal noise, Gaussian noise etc. comes natural with modern communication technologies. This somehow limits our bandwidth and overall performance.
(3) The entire modern communication systems around the world are mainly based on RF spectrum. The electromagnetic waves are known to have direct and indirect ecological effects and hence are not very safe to use in certain specific environments like airplanes and hospitals.
(4) Of all the problems for any modern wireless communication system, multipath propagation poses a serious issue which results in a distorted signal. Since the multipath propagation is a constantly occurring issue with every changing nature, there has been a lot of work done in order to keep the effects of multipath propagation at bay.

Multipath propagation causes ISI in the signal at the receiver ends thus distorting the symbols and resulting in a scrambled signal at the receiver's end. Paper [1–3] states that ISI is a major parameter that effects the performance of the entire wireless communication system. In order for the system to work smoothly, the ISI from the signal must either be completely eliminated or reduced as much as possible. Theoretically, ISI is considered the worse in comparison to other types of noises in the channel. To eliminate or minimize ISI, equalization is used [3] (Fig. 2).

The value of the channel and its impact on the signal is calculated in equalization. Those changes are then separated from the signal received to extract the signal that was originally sent. There are multiple equalization methods used to cope with the multipath propagation and ISI. The two main types of equalization are Data Aided/Training Equalization and Blind Equalization.

Following are some of the major parameters which helps decide about the performance of the modern communication systems. The importance of these parameters increased with the advent of latest technologies in wireless communication.

(1) The rate of convergence of the signal had a very high importance in the equalization process. Since most of the latest wireless technologies involved moving nodes, thus guaranteeing quick convergence of the signal is very important to ensure delay-free communication between the sender and the receiver.
(2) The BER of the communicated signal is also of utmost importance. The lower the BER remains, the better the signal transmission flow would be. Due to the reliance on wireless communication for use of internet, the BER has to be kept at bay in order to use the internet and other communication services. Since multipath results in distortion of signal and cause ISI, it results in increases BER. Hence, whichever equalization process we use, it must ensure a lower BER for smoother performance.
(3) Another major concern is the limitation of bandwidth. Whatever system we implement in order to equalize a given signal, we cannot bear to utilize too much bandwidth in it. The latest communication systems demand a lot of bandwidth and hence we cannot use too much of it on signal processing. Hence we ought to ensure elimination of ISI from the given signal while consuming a negligible amount of bandwidth [4].

There is more than one algorithm for each type of equalization methods. Least Mean Square (LMS) and Recursive Least Square (RLS) are the two major training based algorithms, while Square Contour Algorithm (SCA) and Multi Modulus Algorithm (MMA) are the most famous blind equalization algorithms that are used in modern communication systems [5, 6]. Through this research, we will compare the performance of these algorithms for the advanced 16-QAM and 64-QAM constellation based systems and find out the best algorithm in terms of BER, ISI removal and MSE. This will give us a clear perspective about the algorithm that is best fit for use in latest communication technologies.

ISI and equalization has been under research for many years. But with every progressing generation of communication technology, there is a need to refine the equalization process and come up with better and more fulfilling solution that meets the demands of the day. With the world moving steadily towards 5G, there is a need to refigure the entire equalization process and ensure that the equalization process does not prove to be a bottleneck in the advancement of the technology. We will have a look at some of the earlier work done in the field of equalization in order to eliminate the ISI caused in wireless communication systems due to multi-path propagation.

1.1 Related Work on Inter-symbol Interference

Paper [1] researches equalization process in a communication channel that has a very high ISI value. The high data rate transmission system is used for the equalization model which is a challenging yet practical situation. The given channel has a lot of uncertainties, which are presumed to be in range of polytope with finite set of vertices. Augmentation method is used in this paper through which the filtering error system of equalization system is also assumed to contain polytopic uncertainties. The paper suggested a design for equalizer which has filtering error system I in order to get lowest H-Performance even in uncertain channels. The paper proposes practical designs of models to prove the effectiveness of the method proposed in the paper. Authors of paper [2] have researched statistical distribution in ultra-wideband system for single user in presence of Additive White Gaussian Noise (AWGN) and ISI. The model is then simulated by gathering the data regarding its conditional probability density function in output at receiver block. The paper concludes that the Middleton Class-A distribution model works in a better way for Line-Of-Sight (LOS) system while compromise on performance in Non-LOS models. Paper [3] deals with cognitive radio over a channel with ISI. The paper proposes a strategy for transmission for a secondary user (sender) to improve signal quality of the received signal from the primary user and then transmit it combined using the properties of the channel with ISI. The paper demonstrates the possible of a secondary network to achieve significant output without causes any loss of capacity in the primary network. The model shows that it has better efficiency than the traditional cognitive radio system.

1.2 Related Work on Training/Adaptive Algorithm

Paper [4] has reviewed the adaptive equalization technique for a communication system with ISI factor. The research comes up with a transversal equalizer and few practical models for adaptive equalizers. The research also explains the entire work through mathematical models and figures. In paper [5] the authors compared the performance of RLS, LMS and Zero-Forcing (ZF) algorithms and conclude that the LMS has a better performance in comparison to the ZF algorithms in terms of BER. The RLS ensure reduction of ISI in the signal by eliminating the channel effects, however; its performance can be improved further if we use neural network equalization. The authors in paper [6] have applied the LMS algorithm on two distinct communication systems with different channels. The results of the implementation are then compared. The paper also compares the performance of LMS and CMA algorithms for a random communication test channel. Authors in paper [7] present a Data Reuse Least Mean Square (DR-LMS) algorithm in order to achieve equalization performance with low computational complexity to facilitate practical hardware implementation. The paper demonstrates the effectiveness of the proposed algorithm for shallow water communication, in comparison to LMS and RLS algorithms.

In paper [8], the authors have studied adaptive equalization technology in an effort to minimize the communication errors resulting from multipath effect. The paper takes LMS algorithm methodology in view and test several configurations of it such as trained LMS algorithm, decision-directed algorithm and dispersion minimization algorithm. During the test, Step Size coefficient is varies for each of the algorithm. The paper concludes that decision directed linear equalizer has an improved performance as compared to the rest. The authors of paper [9] have figured out the linear pre-coder for communication system in order to reduce the BER for better performance in a high SNR system using ZF algorithm. The model proposed in the research is devised to ensure elimination of inter-block interference via Cycle-Prefix (CP) and Zero-Padding (ZP). The research concludes that the pre-coder with lowest BER is CP as well as ZP has very low error rate in comparison to the traditional block schemes. In paper [10] the authors considered a wideband system with a transmit array of M-elements. The paper combines the traditional ZF pre-equalization with time reversal. The paper presents a beam-former which equalizes the channel perfectly and the spatial focusing properties of the time reversal method are kept intact. The result is the compared to a pure ZF beam-former and time reversal in other to display the significant BER improvement and low interception probability for a fixed wireless network in an urban environment.

1.3 Related Work on Blind Equalization

The authors in paper [11] has analyzed the performance of CMA, stop-and-go decision directed and Wei Rao's CMA for QAM scheme in a linear band-limited channel based communication system. It is concluded that the impulse response of the cascade of channel and equalizer after convergence is same as ideal channel's impulse response. The research suggests that the stop-and-go algorithm has improved performance in comparison to other in terms of MSE and convergence rate. The authors in paper [12] have worked on the modified form of CMA, i-e MMA and have eliminated the need of a separate phase recovery system of CMA algorithm. The research concludes that the proposed algorithmic model has better data rate with low BER. Paper [13] proposes a blind equalization algorithm to outperform the supervised algorithms without depending on the QAM order. The supervised algorithm generally has a relatively low maladjustment. The paper proposes methods to speed up convergence and provide enough stability to the symbol-based decision algorithm, which is an extension to decision-directed algorithm.

Paper [14] proposed a way to optimize the performance of the ant colony algorithm. For this, better initial weights are found through Back Propagation (BP) algorithm which is then used for the neural network, which results in a better result for the blind equalization. The results of the novel blind equalization algorithm is then compared to the Genetic Algorithm optimization Neural Network Blind Equalization Algorithm (GA-NNBE) and Neural Network Blind Equalization (NNBE). In paper [15], the authors have compared the performance of Sato's Algorithm and Godard based blind algorithm for Pulse Amplitude Modulation (PAM) signal. The paper proposes that if optimum value for tap adjusting coefficient value of Sato's algorithm (α) and the step size (μ) are calculable, it can result in quick convergence. Instead of a fixed values for α and μ, variable values based on the iterations will help in speeding up the convergence and minimizing the maladjustments. Paper [16] proposes a method of blind multi-modulus equalization for equalizer. The paper calculates a cost from cost function as per the Constant Modulus Algorithm (CMA). Then modulus for each region for MMA is then determined after updating the equalizer coefficients according to the cost. When the cost reaches a certain threshold, the equalizer is switched to MMA. MMA has several stages based on the thresholds and the regions keep increasing at each stage. The coefficient of equalizers is updated as per the costs and the modulus of each region is determined for subsequent MMA stage. Equalizer is then switched to the MMA stage when the cost function reaches the threshold for the given stage. The process is iterated until a preset value is achieved.

The authors of paper [17] have worked on a semi-blind and blind equalizer for a channel with fading MIMO system. The paper proposes a channel for each equalizer and suggests algorithms based on the channel capacity. The research concludes that Semi-Blind and Blind Algorithms performance better than training based equalizers in Ricean environment. However, in Releigh channels the training based equalizers work efficiently than the rest of the algorithms. Paper also measures the optimum training size for semi-blind and training algorithms.

1.4 Equalization Techniques in Latest Communication Technologies

Paper [18] proposed a Fractionally Spaced Iterative Block Decision Feedback Equalizer (FS-IBDFE) receiver, for transmission over frequency selective channels, which combines the non-linear processing, FS equalization and frequency domain implementation. The paper has three major contributions (1) extending the existing frequency domain iterative equalization methods while keeping FS in consideration. (2) Minimizing the Mean Square Error (MSE) at detection point through the suggested FD equalizer coefficient. (3) Designing an equalizer with equalization coefficients calculated in closed form with no matrix inversions. In paper [19] the authors have dealt with visible light communication system (VLC), which is one of the latest and upcoming communication technologies. The research is based on the use of extended bandwidth via phosphorescent based white LED. The paper proposes a post-equalization design with two passive and one active equalizer. Using the proposed circuit and blue filtering, 151 MHz of bandwidth is achieved. The system allows an OOK-NRZ transmission and a data rate of up to 340 Mbps. The VLC model used perform efficiently at 43 cm using an LED of 1 W with a BER ranging from 2×10^{-3}. In paper [20] the authors have demonstrated the 3.24 Gbps VLC system with a 512-QAM SC-FDE using an RGB-LED with 10 MHz bandwidth. Three different channels with various wavelengths are used during the experiment and the BER of the channels are carefully noted. It was noted that the BER of all the three channels remained well below the pre-FEC threshold level of 3.8×10^{-3}.

2 Comparative Analysis of Equalization Algorithms

As mentioned earlier, equalization can be divided into two major types, the training based equalization algorithms and the blind equalization algorithms. Here we will study them mathematically and understand how they work.

2.1 Training Based Adaptive Equalization

Training equalization algorithm is based on sending a small piece of pilot signal or data along with the desired signal to the receiver carrying the knowledge about the original signal sent. This piece of signal is called training or pilot signal and helps the receiver in recovering the original signal from the distorted signal received from the channel. The training signal is constantly and periodically sent to update the receiver about the sent data. Training based equalization is considered to be efficient and simpler with better convergence rate and is considered as a perfect solution for systems in which fast fading is desired while having a high Doppler spread and lesser coherence time. The method however comes with a catch. The systems needs to send training signal constantly which may be a small chunk of data but over a longer period of time, it consumes a significant portion of the bandwidth. As per the research, in GSM, roughly 18% of the bandwidth is used in sending the training signal [21–23].

Within the domain of training based equalization there are several algorithms that use the same principle of sending training signal. The most important of these algorithms are the LMS and RLS [24, 25].

Figure 3 shows a communication model for training based adaptive equalization. The model include a feedback system which send the pilot signal to the receiver periodically in an effort to keep the receiver updated about the original signal so it can extract data from the received signal quickly hence ensuring a quick conversion rate.

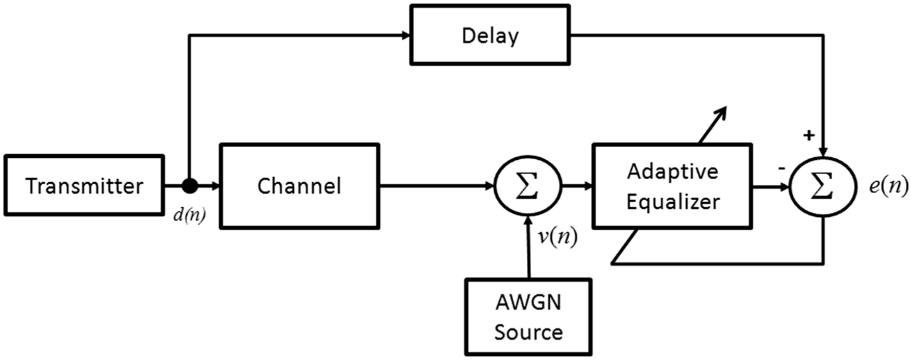

Fig. 3 Communication model for training based adaptive equalization algorithm

2.2 Least Mean Square Equalization

LMS is the abundantly used form of training based equalization algorithm. The method is very efficient when it comes to wireless communication system with non-stationary moving nodes due to its quick convergence. LMS constantly update the equalizer weights during the process of equalization using the stochastic gradient descent. The complex form of LMS equalizer for a complex channel gain can be mathematically written as

$$z(k) = \omega^{\tau}(k)x(k) \tag{1}$$

where z(k) stands for the output of the training based LMS equalizer and is basically the product of the weight of the equalizer and the received signal at the receiver.

$$e(k) = s(k) - z(k) \tag{2}$$

Equation 2 represents error estimation as e(k), and shows s(k) as the desired signal. By removing the equalizer's gain z(k) from the s(k), it would result in the error estimation for our given channel.

Here we need to update the weight of the system every now and then. For which we will use the following mathematical representation.

$$w(k+1) = w(k) + 2\mu e * (k)x(k) \tag{3}$$

In the above equation, the * represents the complex conjugate. The step size is shown as μ which is related to the rate of convergence of any given equalizer. In order to come up with the perfect step size, it takes a very careful and precise measurement and is an extremely lengthy process. To simplify the process, we randomly checked multiple numeral values and came up with an optimal value that ensures the most efficient convergence. This equation is used for updating the tap which is then used to increment the weight of the equalizer periodically until the equalizer provide us with the most efficient convergence rate.

2.3 Recursive Least Square

RLS algorithm is another type of training based equalization algorithm that is more complex than the LMS. A working RLS equalizer can be represented mathematically as

$$u(k) = \psi_{\lambda}^{-1}(k-1)x(k) \tag{4}$$

Here Ψ is used in order to reduce the complexity of the computation required in the RLS. Ψ represents a diagonal matrix with values of 1, λ, λ2, λ3…. Furthermore, to create a recursion in ψ_{λ}^{-1}, matrix inversion lemma is put to use. The input vector for RLS equalizer can be represented as

$$x(k) = \frac{1}{\lambda + x^{H}(k)u(k)}u(k) \tag{5}$$

Above equation represents the equalizer's gain and as we can see, its value is dependent on λ. Equations 4 and 5 are collectively used to get the gain vector K(k). The λ here is the forgetting factor with its value around 1. The weight factor λ ensures that it provide lesser importance to the earlier values during iterations and more value to the latest numbers coming. As the iterations increases, the initial values are steadily ignored in order to position the equalizer towards perfect convergence.

$$\hat{\mathrm{z}}_{\mathrm{k}-1}(k) = \hat{\mathrm{w}}^{H}(k-1)x(k) \tag{6}$$

Equation 6 represents the RLS equalizer's input signal while the $\hat{\mathrm{z}}_{k-1}(k)$ is the output signal and $\hat{\mathrm{w}}(k)$ represents the updated weights.

Just as we had to estimate the error in LMS, in RLS the error estimation is given by

$$\hat{\mathrm{e}}_{k-1}(\mathrm{k}) = \mathrm{s}^{*}(\mathrm{k}) - \hat{\mathrm{z}}_{\mathrm{k}-1}(k) \tag{7}$$

In above equation the $\hat{\mathrm{e}}_{k-1}(\mathrm{k})$ shows error that is calculated by subtracting the output of the equalizer $\hat{\mathrm{z}}_{k-1}(k)$ from the desired signal $\mathrm{s}^{*}(\mathrm{k})$.

$$\hat{\mathrm{w}}(k) = \hat{\mathrm{w}}(k-1) + K(k)\hat{e}_{k-1}(k) \tag{8}$$

Equation 8 is the equation used to update the taps for the given equalizer. In this equation the error estimated e(k) and Gain K(k) are multiplied to get the tap update for the Kth iteration of the RLS equalizer.

$$\psi_{\lambda}^{-1}(k) = \lambda^{-1}(\psi_{\lambda}^{-1}(k-1) - K(k)\left[x^{H}(k)\psi_{\lambda}^{-1}(k-1)\right] \tag{9}$$

The above equation is used to get the values for updating the ψ_{λ}^{-1}.

2.4 Blind Channel Equalization

Blind channel equalization is another important type of equalization process which is comparatively slower in convergence however, since this method saves a lot of bandwidth and does not require the sender to constantly send training signal, thus saving plenty of bandwidth during the process. This type of equalization is considered perfect for systems where we have multiple receivers receiving signal from a single node. Similarly, for s communication system with static sending and receiving nodes, this method is considered efficient because it saves the precious bandwidth. The receiver blind equalization technique does not need a training signal to extract the original signal from the distorted signal. It only requires the constellation model and mapping technique to figure out the original signal from the received lot of data. This also requires a good amount of calculation and the quality of the signal extract is also slightly low quality in comparison to the training based equalizers. There are several algorithms that apply the blind equalization methods, the most commonly used algorithms are the SCA and MMA [26].

Figure 4 represents a communication model for blind equalization. The model does not have any feedback system and hence does not send any pilot signal like the adaptive algorithm. The model use blind algorithm and with just the knowledge of the

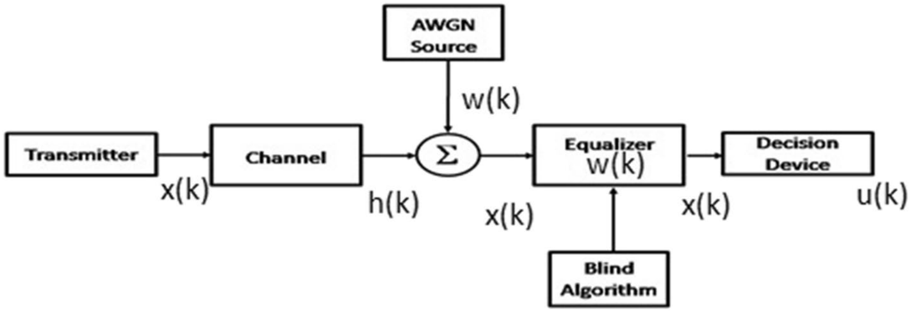

Fig. 4 Wireless communication model for blind equalization algorithm

constellation type of the signal, the equalization perform various iterations in order to extract the received signal.

2.5 Multi-modulus Algorithm

MMA is an improved version of the old CMA algorithm that too was based on blind equalization method. In CMA both the imaginary and real parts of the output of the equalizer had to be separated from each other. Similarly in MMA, the real and imaginary parts are individually represented in order to find out the cost function. Mathematically, we can represent it as

$$J_{MMA} = \boldsymbol{E}\left\{ \left(|z_{kr}|^p - R^p_{MMA}\right)^2 + \left(|z_{ki}|^p - R^p_{MMA}\right)^2 \right\} \tag{10}$$

In above equation, E stands for expectation operator, while the real part is shown by z_{kr} and imaginary part is denoted by z_{ki}. Both the values are calculated for the Kth value of the equalizer's output. 'p' is an integer considered as a calculation necessity for this equation.

$$R^p_{MMA} = \frac{E|s_{kr}|^{2p}}{E\{|s_{kr}|^p\}} = \frac{E|s_{ki}|^{2p}}{E|s_{ki}|^p} \tag{11}$$

In Eq. 11, R^p_{MMA} stands for the Goddard are constant, while the real and imaginary parts are represented by S_{kr} and S_{ki} respectively. The efficiency of the MMA equalizer can be increased by increasing the value of p in the equation, however; it would come at the cost of a very complex calculation. In order to simplify the process and for ease of calculation we used value of p as 2.

Now, to update the MMA equalizer's weight, the following equation is used.

$$e_k = z_{kr}|z_{kr}|^{p-2}\left(|z_{kr}|^p - R^p_{MMA}\right) + jz_{ki}|z_{ki}|^{p-2}\left(|z_{ki}|^p - R^p_{MMA}\right) \tag{12}$$

MMA algorithm has a very efficient way of recovering the original signal from the received distorted signal.

2.6 Square Contour Algorithm

SCA algorithm is also based on blind equalization method and has its roots in the constellation of the signal received. In contrast to CMA and MMA which reduces the dispersion of the equalizer's output considering a circular constellation, the SCA minimizes the output dispersion considering the square constellation as a standard. SCA also recovers the phase shift occurred due to the channel. The cost function of the SCA can be mathematically written as

$$J_{SCA} = E\left\{ \left(\left|z_{kr} + z_{ki}\right|+\left|z_{kr} - z_{ki}\right|\right)^{p} - R^{p}_{SCA}\right)^{2} \right\} \tag{13}$$

In above equation, the out of the SCA equalizer is shown by J, whereas E stands for expectation operator and p has the same function as in MMA. The real part of the Equalizer's output is represented by z_{kr} while z_{ki} stands for imaginary part. R is a mathematical constant used to denote the kind of constellation used by the signal. So we have

$$\left|z_{kr} + z_{ki}\right|+\left|z_{kr} - z_{ki}\right| = 2\max\left\{\left|z_{kr}\right|,\left|z_{ki}\right|\right\} \tag{14}$$

For SCA algorithm, the zero-error contour can mathematically be represented as

$$\max\left\{\left|z_{kr}\right|,\left|z_{ki}\right|\right\} = \frac{R_{SCA}}{2} \tag{15}$$

The above shows a square constellation with a center representing the origin. The error for SCA $e_{k,SCA}$ can be calculated as

$$\begin{aligned} e_{k,SCA} = {} & \left(\left(\left|z_{kr} + z_{ki}\right|+\left|z_{kr} - z_{ki}\right|\right)^{p} - R^{p}_{SCA}\right)\left(\left|z_{kr} + z_{ki}\right|+\left|z_{kr} - z_{ki}\right|\right)^{p-1} \\ & \times \left(\text{signum}\left[z_{kr} + z_{ki}\right](1+j) + \text{signum}\left[z_{kr} - z_{ki}\right](1-j)\right) \end{aligned} \tag{16}$$

here the constellation constant for SCA algorithm is represented by R^{p}_{SCA} and can be mathematically represented as

$$R^{p}_{SCA} = \frac{E\left\{\left(\left|s_{kr} + s_{ki}\right|+\left|s_{kr} - s_{ki}\right|\right)^{p}.Q\right\}}{E(Q)} \tag{17}$$

The above equation carried a new variable Q, which can be represented mathematically as

$$Q = \left(\left|s_{kr} + s_{ki}\right|+\left|s_{kr} - s_{ki}\right|\right)^{p-1}\left(\text{sig}\left[s_{kr} + s_{ki}\right](1+j) + \text{sig}\left[s_{kr} - s_{ki}\right](1-j)\right)s^{*}_{k} \tag{18}$$

here * is the complex conjugate while the real part is denoted by the s_{kr} and the imaginary part of the equation is represented by s_{ki}. Table 1 shows a comparison of the above six identified algorithms on training based and blind channel with their functional efficiency.

3 Simulation, Results and Discussion

Based on all the above-mentioned mathematical values, we simulated all the algorithms through MATLAB and came up with constellation diagrams for each equalization algorithm and comparison of BER, residual ISI and MSE for 16-QAM and 64-QAM. Here are the plots along with analysis of the simulations.

Table 1 Comparison of six identified algorithms

S. no	Name of algorithm	Working mechanism	Functional efficiency
1.	Training based adaptive equalization	Training-based	Better convergence rate
2.	Least mean square equalization	Training-based	Quick convergence
3.	Recursive least square	Training-based	Better gain
4.	Blind channel equalization	Blind-channel based	Bandwidth saving
5.	Multi-modulus algorithm	Blind-channel based	Reduces dispersion
6.	Square contour algorithm	Blind-channel based	Minimum dispersion

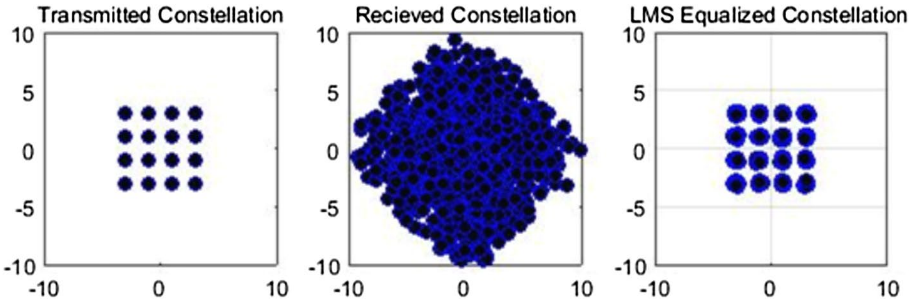

Fig. 5 Transmitted, received and equalized signal for LMS algorithm in 16 QAM constellation

3.1 16-QAM Constellation Diagrams of Transmitted, Received and Equalized Signals

In the plots above we can see the transmitted 16 QAM signals on the left side for LMS, RLS, MMA and SCA algorithms respectively. The left side representations are the ideal signal that is being transmitted. In order to avoid a consistent complexity in calculation, we are using the baseband signal without the use of carrier signal (Figs. 5, 6, 7, 8).

In the central figure representations, we can see that the received signals are not only distorted with symbols overlapping each other, but also the signals have received significant phase shifts. The distortion in the symbols is due to the AWGN in the channel while the phase-shifts are mainly because of the channel's parameters.

The right side of the graphs shows the recovered signals after equalization process in each of the algorithm, in which the channel-caused phase shift has been reversed, while the signal is recovered to the best possible shape.

As we can clearly see the recovery of the signal in both the training based algorithm is better in comparison to the blind algorithms because of the fact that the training algorithms are constantly updated with a pilot training signals and hence the equalizer can easily equalize the signal and receive a better result, although this comes at the cost of the bandwidth that is consumed due to constant transmission of the training signal. In modern communication systems, bandwidth consumptions is a serious issue and that's where the blind algorithms jumps into rescue. The recovery of the blind algorithms may not be as good as the training based algorithms but they save a lot of bandwidth due to lack of training signals. The bandwidth is saved at the cost of time delay and complexity of calculation,

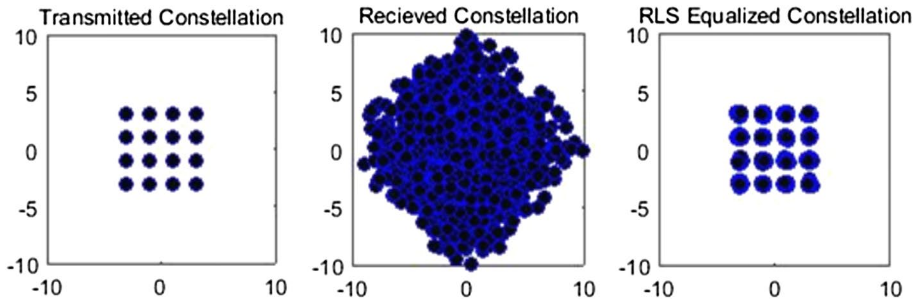

Fig. 6 Transmitted, received and equalized signal for RLS algorithm in 16-QAM constellation

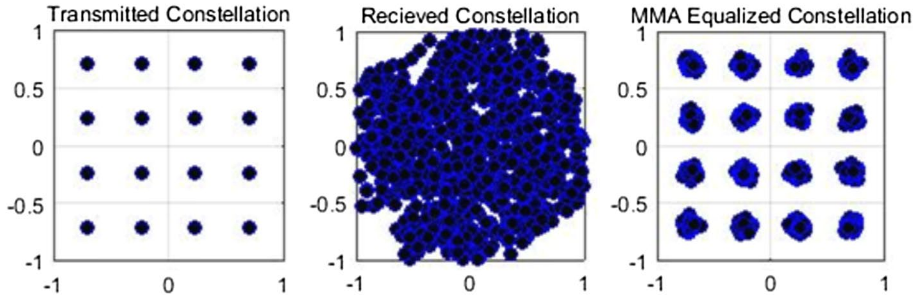

Fig. 7 Transmitted, received and equalized signal for MMA algorithm in 16 QAM constellation

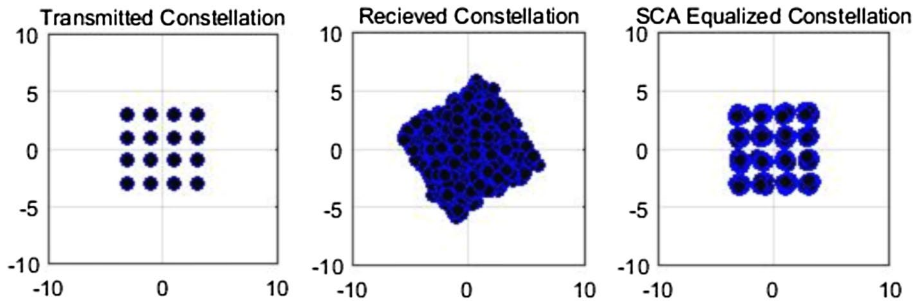

Fig. 8 Transmitted, received and equalized signal for SCA algorithm in 16 QAM constellation

however in communication systems where a slight delay can be ignored and we can enough energy and resourced to conduct complex calculations the blind algorithms works very efficiently.

In mobile communication nodes as in case of 4G and latest, the training based algorithms works well because there cannot be any delay due to the nature of communication incurred in the given system. The blind algorithms on the other hands can well be used in communication technologies where the nodes are static and there is little to no motion as in case of WiFi, where complexity and delay can be handled in order to save the precious bandwidth from the burden of training signals.

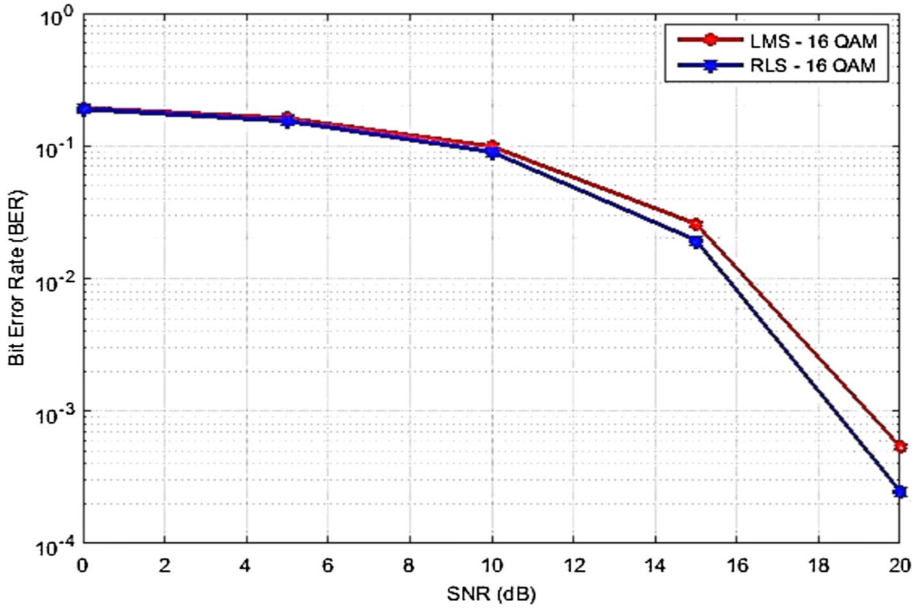

Fig. 9 BER comparison of LMS and RLS algorithm for a 16-QAM constellation

3.2 Bit Error Rate (BER) Comparison of Training and Blind Equalization Algorithms for 16-QAM and 64-QAM

In order to better understand the difference between the LMS and RLS algorithm, we need to check the BER for both of the algorithms to figure out which one would yield better result for a certain intensity of transmitted signal in 16-QAM. In BER comparison, we have to focus on the SNR value of the transmitted signal at which the BER of one algorithm is lower than its counterpart. In case of LMS and RLS algorithm for 16 QAM, the performance of RLS gets slightly better than LMS at 6 dB, however, at around 12–14 dB and onwards the BER of RLS leads and there is a significantly better BER than LMS (Fig. 9).

We can clearly see the RLS has a better performance but it comes with a catch. RLS is a complex algorithm in comparison to LMS and hence the better BER value comes at a cost of complex calculations and circuitry which turns into cost during practical implementation. LMS, which is comparatively simpler algorithm, requires less complex calculations, hence simpler circuitry and low cost of implementation (Fig. 10).

In MMA and SCA, the BER performance of MMA is significantly better than the SCA and as we increase the SNR of the signal, the performance get even better as the difference between the two increases even more. Hence we can safely say that for a 16 QAM constellation, the performance of MMA is better than SCA in terms of bit error rate for any given SNR.

In the plot in Fig. 11, the values of the plots remained the same until 20 dB, afterwards the RLS shows improvement in terms of BER and as we move further, the performance of the RLS starts improving further as the BER significantly reduces. At 25 dB the difference is clearly visible and if we go beyond that the performance will keep improving. So it can be implied that for signals with higher intensity RLS is a better choice while for low intensity signals, LMS should be opted for due to its low complexity.

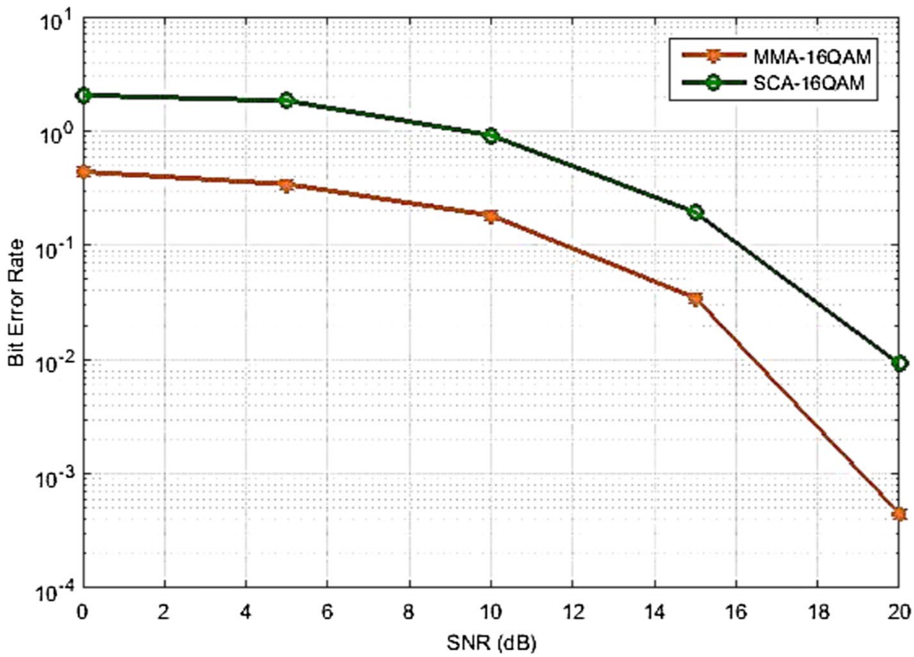

Fig. 10 BER comparison of MMA and SCA algorithm for 16-QAM constellation

There is a clear difference between MMA and SCA when it comes to BER as in Fig. 12 when it comes to 64 QAM. Performance of MMA is comparatively better than SCA with low BER for any value of SNR. There is a static difference between the two algorithms till 25 dB, after which the gap start increasing and around 30db, the significant improvement in the performance of MMA is distinctively visible. Hence we can assume that the performance of MMA is better in 64-QAM scheme.

3.3 ISI Residual Comparison of LMS and RLS for 16 QAM and 64 QAM

In modern communication systems, the convergence rate of the algorithm is important because the systems are time sensitive and slight delay can cause serious troubles related to communication. When the receiver or sender is on move in a communication system, the BER needs to be contained as quickly as possible to avoid distortion in the signal.

For a 16 QAM constellation, the simulation results in Fig. 13 show that the LMS was slightly quicker in its convergence and required lesser iterations to retain a stabilized residual ISI value. The RLS in comparison, required slightly more iterations but better signal with less residual ISI. Just like the case of BER, the LMS being the simpler algorithm provides quicker solution but with a slight compromise on ISI residual at the receiver's end. LMS is better for environments that demand quick conversion with lesser complicated computation requirement and low cost, while RLS is good for cases where the convergence rate can be compromised but the ISI residual in a signal has to be kept at bay.

For residual ISI comparison in the blind equalization algorithm for 16 QAM, the difference between the two is very significant as in Fig. 14. The SCA, as can be seen in the

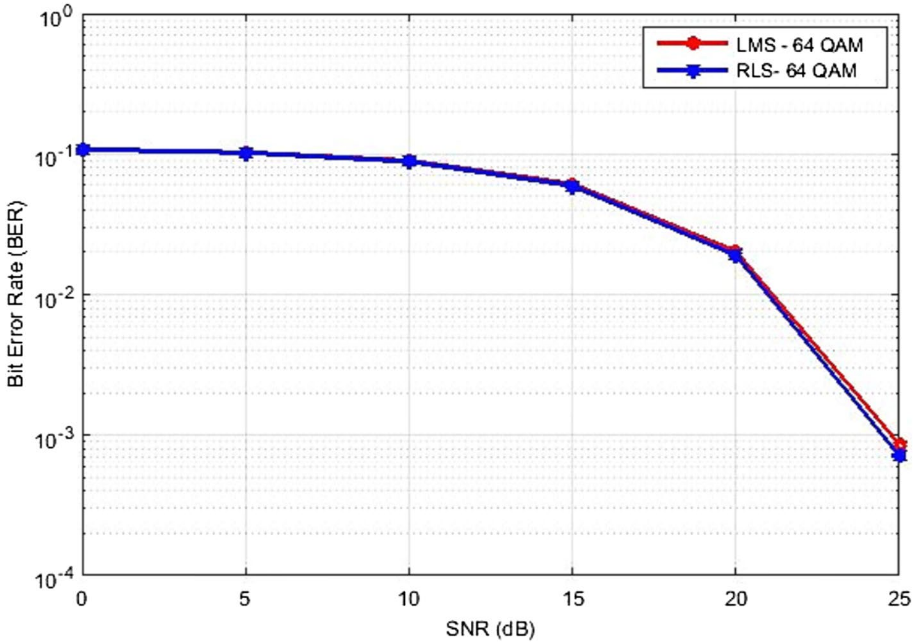

Fig. 11 BER comparison of LMS and RLS algorithm for a 64-QAM constellation

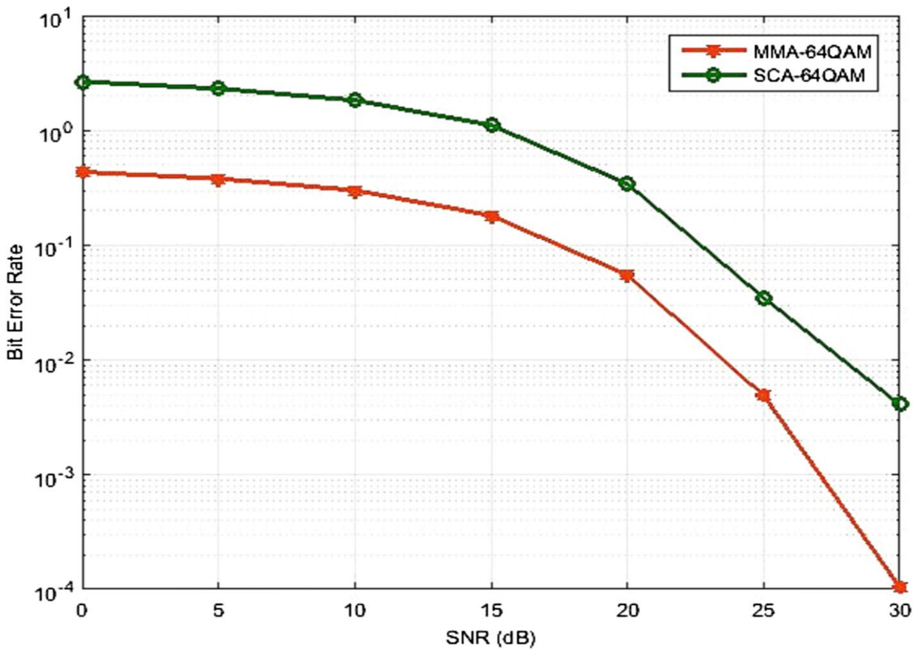

Fig. 12 BER comparison of MMA and SCA algorithm for a 64 QAM constellation

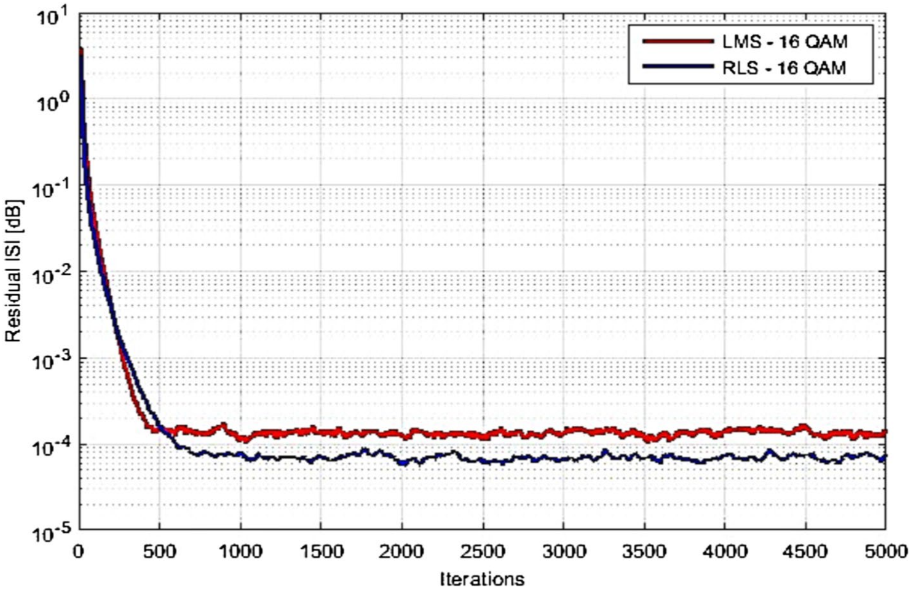

Fig. 13 ISI residual comparison of LMS and RLS algorithm for a 16 QAM constellation

diagram has a very immediate convergence to a stable residual ISI value. At around 1500 iterations, the SCA reached its stable residual ISI value. The MMA went through several ups and downs and required around 7000 iterations before stabilizing. Hence we can conclude that for a 16 QAM signal in blind equalization, the MMA, though takes more time in convergence provide better results for residual ISI, while the SCA has a quick convergence but the resulting signal at the receiver has a high residual ISI component.

The RLS depicts a better convergence value in comparison to LMS as in Fig. 15 when used for 64 QAM. At 700 iterations RLS stabilizes its value while for LMS the stabilization starts around 1000 iterations. Hence we can deduce that for any two fast moving nodes, RLS will have a better performance due to quick convergence. However, the nodes must be able to perform the complex calculations required for RLS to work smoothly.

In Fig. 16, it is visible that the SCA has a quicker convergence and took only 400 iterations before the system was able to extract the original signal from the received signal. In comparison to that, the MMA took around 7000 iterations to stabilize at a certain value. However, the signal from MMA has comparatively very little ISI residual and hence we can conclude that there is a compromise between convergence and ISI in this case. MMA provides better ISI removal which comes at the cost of more iterations and hence increased delay. While SCA converged quickly but does not eliminate ISI in a better way like MMA.

3.4 Mean Square Error (MSE) Comparison of LMS and RLS for 16-QAM and 64 QAM

For a 16 QAM signal, the LMS and RLS have nearly the same result when we compare their Mean Square Error as in Fig. 17. The LMS converges quickly due to its simplicity of implementation at around 300 iterations. The RLS comparatively takes around 500

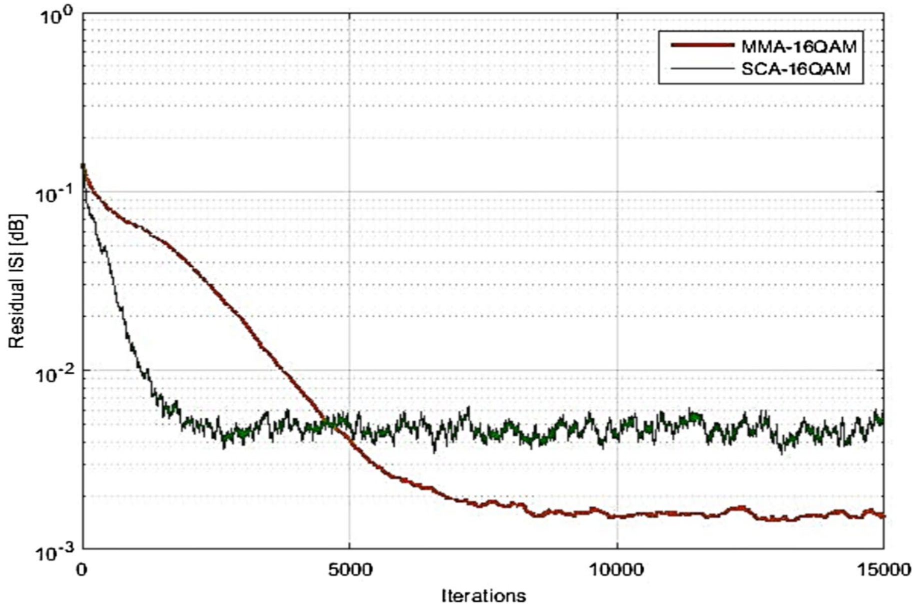

Fig. 14 ISI residual comparison of MMA and SCA algorithm for a 16 QAM constellation

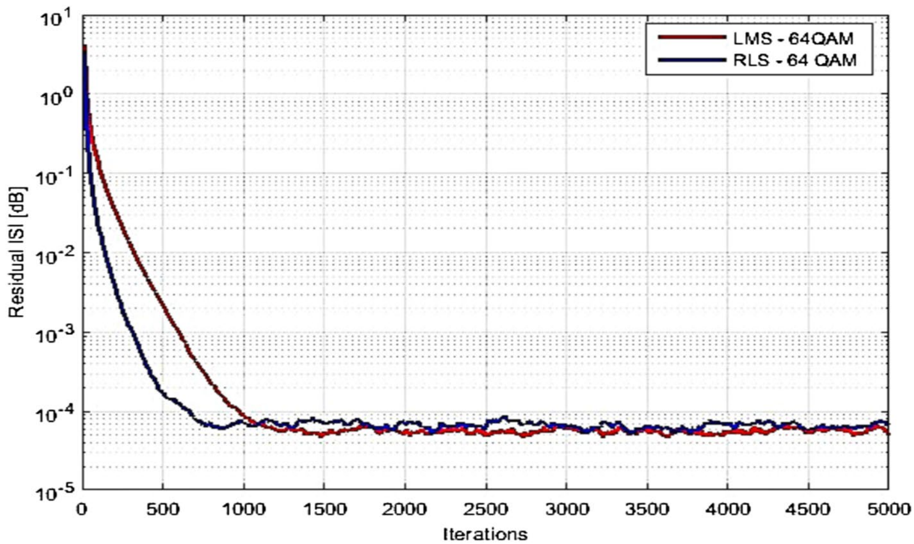

Fig. 15 ISI residual comparison of LMS and RLS algorithm for a 64 QAM constellation

iteration to stabilize. While the variation remains high even in the stable signal, however, the RLS carries lower MSE value than the LMS but at the cost of complexity and higher cost.

In 16 QAM constellation signal, the MMA and SCA has very distinctive results in terms of Mean Square Error (MSE) as in Fig. 18. The MMS requires very little iteration

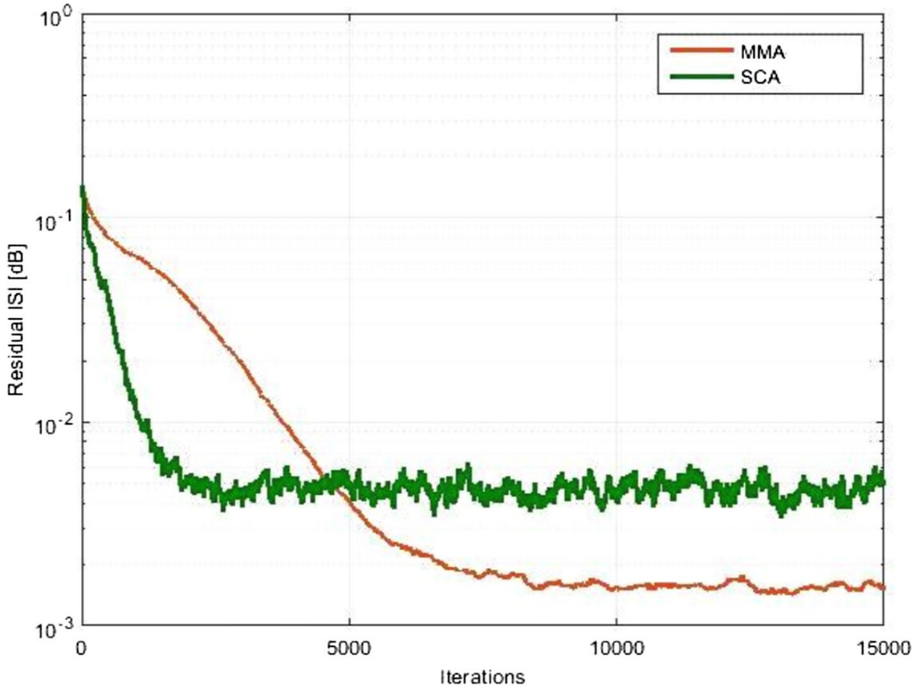

Fig. 16 ISI residual comparison of MMA and SCA algorithm for a 64 QAM constellation

to affix itself to a certain stable MSE value. The SCA which is a complex blind equalization algorithm takes around 2000 iterations to reach the stability required for a smooth transfer of signal. The SCA gives much better MSE value and hence the quality of the signal at the receiver's end is way better than the MMA. For fast moving devices in latest 4G/LTE, there is a need of quick convergence as the nodes cannot bear the delay required for SCA which give a better MSE, so MMA is a better option as it converges quickly. However, for fixed nodes in communication technology, lower MSE can be a better option at the cost of slight delay as once the signal get stabilized, the devices will not move and hence the signal will remain stable.

The MSE performance of LMS and RLS has apparently very little difference except in the initial convergence as shown in Fig. 19 for 64 QAM. We can see that the RLS took at least 600 iterations before it stabilized, while LMS took around 750 iterations before it stabilized at a certain position. Hence, for any non-stationary node, RLS provide better convergence than LMS, however, the innate requirement of complex calculations for RLS must be fulfilled.

The plots in Fig. 20 for MSE comparison between MMA and SCA for 64-QAM scheme shows that the MMA has a very high convergence rate in comparison to SCA, however, the performance of SCA is far better in terms of the MSE. It took MMA around 200 iterations to stabilize, while SCA required 8000 iterations in order to stabilize. Generally we can conclude that SCA should be preferred despite the number of iterations it takes to stabilize, because once it's stabilized it distinctively outperform MMA in comparison of MSE.

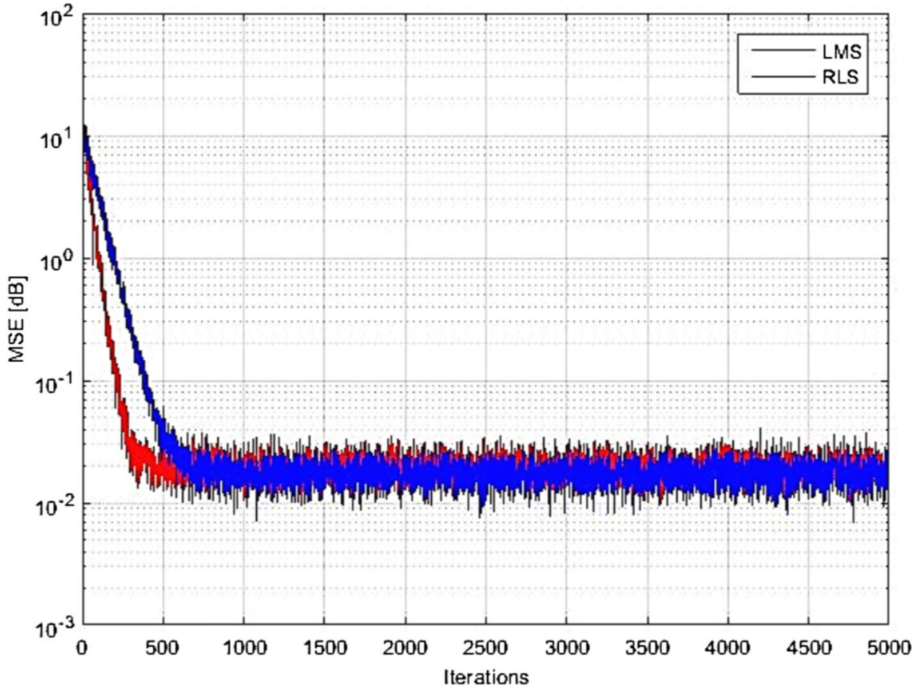

Fig. 17 MSE comparison of LMS and RLS algorithm for a 16 QAM constellation

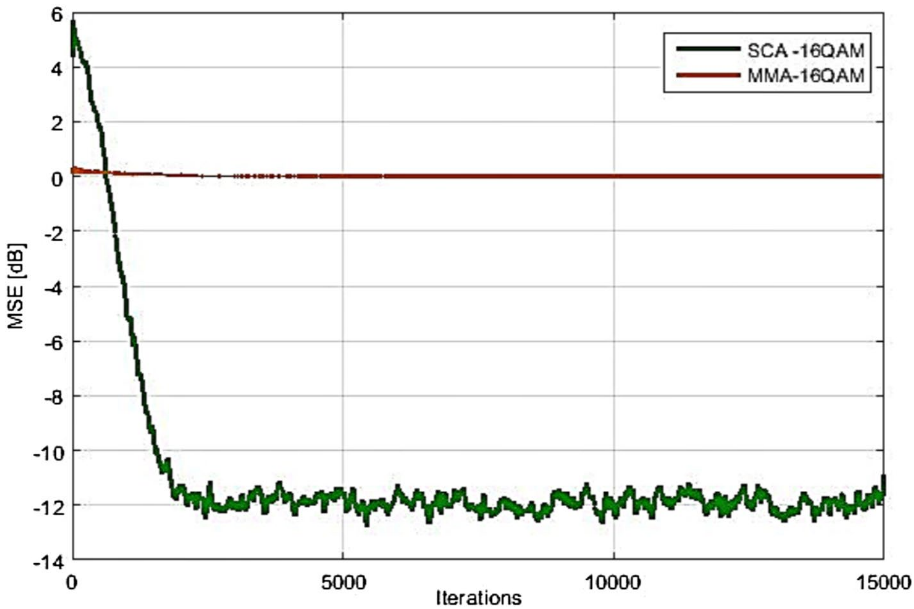

Fig. 18 MSE comparison of MMA and SCA algorithm for a 16 QAM constellation

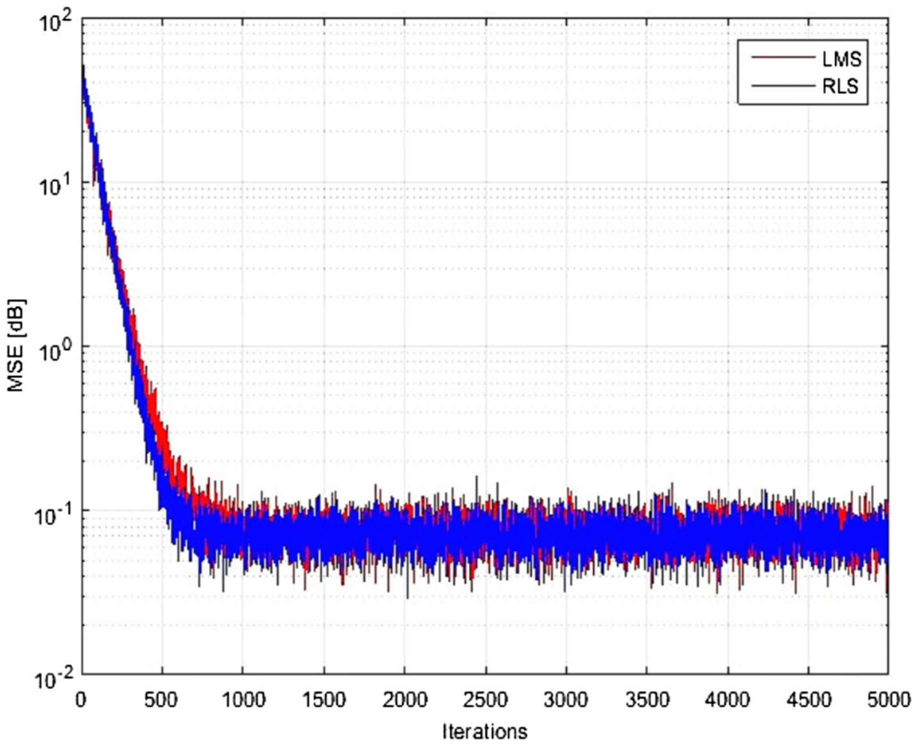

Fig. 19 MSE comparison of LMS and RLS algorithm for a 64 QAM constellation

3.5 64-QAM Constellation Diagrams of Transmitted, Received and Equalized Signals

In the plots above, we can see 64-QAM constellation diagrams four under-comparison equalization algorithms. In each of the constellation, the left side represents the original transmitted signal to the receiver, the central figure represents the distorted and phase shifted signal due to the channel values and the AWGN present in the medium. This results in a distorted and shifted signal at the receiver with ISI. Now in order to extract the original signal, we applied each of the four algorithms and the figure at right in each of the plots is the extracted signal after the application of the respective algorithms (Figs. 21, 22, 23, 24).

4 Conclusion

Every modern wireless technology requires to fulfill the demand of ensuring smooth and flawless communication for bother stationary and non-stationary communication nodes. As ISI is one of the major issues in both the cases, the comparison of the algorithms in this research can help identify the types of algorithms to be used for any certain specific environment. In our research, we found that in training equalization method, the LMS is simple in implementation and ensure quick convergence and comes at low cost. In comparison,

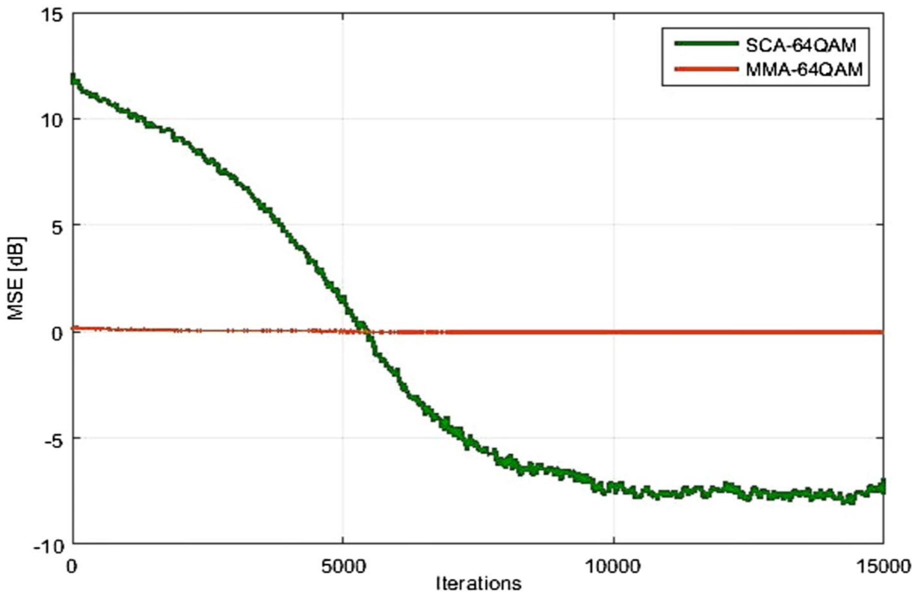

Fig. 20 MSE comparison of MMA and SCA algorithm for a 64 QAM constellation

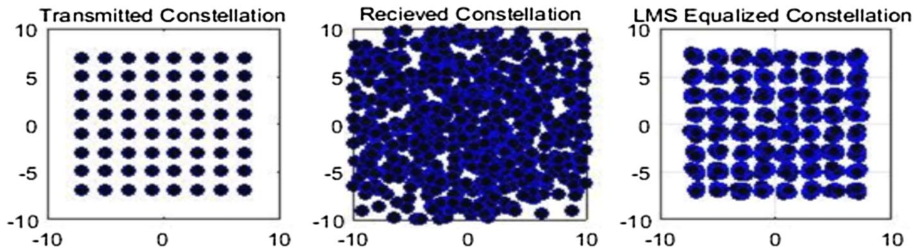

Fig. 21 Transmitted, received and equalized signal for LMS algorithm in 64 QAM constellation

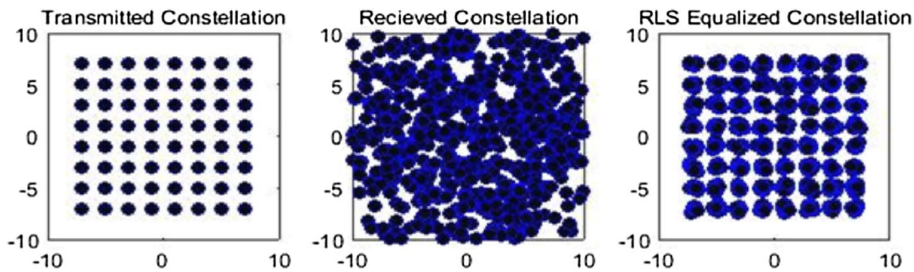

Fig. 22 Transmitted, received and equalized signal for RLS algorithm in 64 QAM constellation

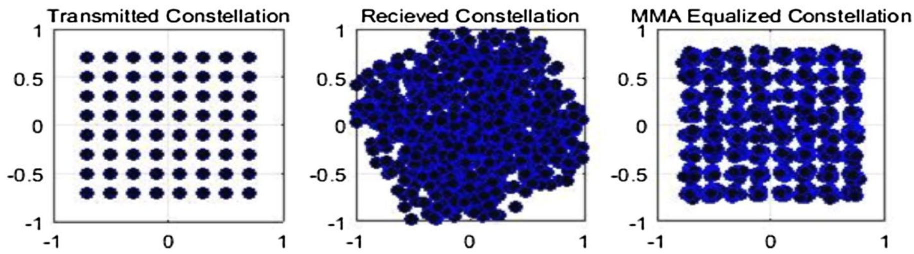

Fig. 23 Transmitted, received and equalized signal for MMA algorithm in 64 QAM constellation

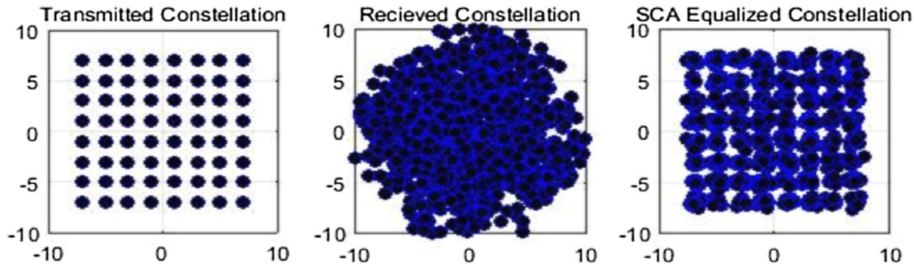

Fig. 24 Transmitted, received and equalized signal for SCA algorithm in 64 QAM constellation

the RLS requires complex calculations but has better performance in terms of residual ISI, MSE and BER. However, due the complexity involved RLS can be a foremost choice for stationary nodes of a wireless system. In blind equalization method, MMA equalization algorithm has comparatively better performance than its counterpart SCA. SCA is a complex algorithm and at times provides better performance, but generally MMA proves to be a much versatile and efficient algorithm. To conclude the research, we can say that these algorithms should be used in a case-to-case basis depending on the requirement of any given environment. Alternatively, a hybrid equalizer can also be used which can shift between different algorithms based on the requirement of the communication and hence ensure smooth communication.

References

1. Zhang, H., Shi, Y., & Mehr, A. S. (2012). Robust equalisation for inter symbol interference communication channels. *IET Signal Processing, 6*(2), 73–78.
2. Chung, G. C., Thwin, S. S., & Alias, M. Y. (2013). Statistical distribution of UWB signals in the presence of inter-symbol interference. In *2013 IEEE conference on sustainable utilization and development in engineering and technology (CSUDET)* (pp. 60–62). IEEE.
3. Chang, W., & Jung, B. C. (2014). Optimal transmission strategy without capacity loss at a primary user in cognitive radio networks over inter-symbol interference channels. *IEEE Communications Letters, 18*(3), 411–414.
4. Mishra, G., & Rai, M. (2018). A comparative study on characteristics and properties of adaptive algorithms applied to noise cancellation techniques. In *2018 international conference on computational and characterization techniques in engineering & sciences (CCTES)* (pp. 258–261). IEEE.
5. Talluri, G. R., & Baghini, M. S. (2019). A novel dual loop adaptive equalizer based on spectrum balancing technique. *Microelectronics Journal*, *87*, 127–135.

6. Moshirian, S., Ghadami, S., & Havaei, M. (2012). Blind channel equalization. arXiv preprint arXiv:1208.2205.
7. Tong, F., Benson, B., Li, Y., & Kastner, R. (2010). Channel equalization based on data reuse LMS algorithm for shallow water acoustic communication. In *2010 IEEE international conference on sensor networks, ubiquitous, and trustworthy computing (SUTC)* (pp. 95–98). IEEE.
8. Akaneme, E. S. A., & Onoh, G. N. (2015). Algorithms for evaluating adaptive equalization in wireless communication systems. *European Journal of Engineering and Technology*, *3*(2), 54–61.
9. Ding, Y., Davidson, T. N., Luo, Z. Q., & Wong, K. M. (2003). Minimum BER block precoders for zero-forcing equalization. *IEEE Transactions on Signal Processing, 51*(9), 2410–2423.
10. Kyritsi, P., Stoica, P., Papanicolaou, G., Eggers, P., & Oprea, A. (2005). Time reversal and zero-forcing equalization for fixed wireless access channels. In *Conference record of the thirty-ninth Asilomar conference on signals, systems and computers* (pp. 1297–1301).
11. Wahab, A., Ahmed, S., Javed, N., & Khan, A. (2016). Performance analysis of training and blind equalization algorithms for wireless communication using 64-QAM. In *Proceedings of 2nd international multi-disciplinary conference* (Vol. 19, p. 20).
12. Arivukkarasu, S., & Malar, R. (2015). Multi modulus blind equalizations for quadrature amplitude modulation. *International Journal of Innovative Research in Computer and Communication Engineering, 3*(3), 2301–2305.
13. Mendes Filho, J., Silva, M. T. M., & Miranda, M. D. (2011). A family of algorithms for blind equalization of QAM signals. In *ICASSP* (pp. 3388–3391).
14. Geng, Y., Zhang, L., Sun, Y., Zhang, Y., Yang, N., & Wu, J. (2016). Research on ant colony algorithm optimization neural network weights blind equalization algorithm. *International Journal of Security and its Applications*, *10*(2), 95–104.
15. Suthendran, K., & Arivoli, T. (2014). Performance comparison of blind equalization algorithms for wireless communication. *International Journal of Computer Applications*, *85*(13), 1–6.
16. Ram Babu, T., & Kumar, P. R. (2010). Blind equalization using constant modulus algorithm and multimodulus algorithm in wireless communication systems. *International Journal of Computer Applications, 1*(3), 50–55.
17. Kavitha, V., & Sharma, V. (2005). Comparison of training, blind and semi blind equalizers in MIMO fading systems using capacity as measure. In *IEEE international conference on acoustics, speech, and signal processing, 2005. Proceedings (ICASSP'05)* (Vol. 3, pp. iii–589). IEEE.
18. Benvenuto, N., Ciccotosto, S., & Tomasin, S. (2015). Iterative block fractionally spaced nonlinear equalization for wideband channels. *IEEE Wireless Communications Letters, 4*(5), 489–492.
19. Li, H., Chen, X., Huang, B., Tang, D., & Chen, H. (2014). High bandwidth visible light communications based on a post-equalization circuit. *IEEE Photonics Technology Letters, 26*(2), 119–122.
20. Wang, Y., Li, R., Wang, Y., & Zhang, Z. (2014). 3.25-Gbps visible light communication system based on single carrier frequency domain equalization utilizing an RGB LED. In *Optical fiber communication conference* (pp. Th1F-1). Optical Society of America.
21. Chapin, J., & Lehr, W. (2011). Mobile broadband growth, spectrum scarcity, and sustainable competition. In *TPRC*.
22. Proakis, J. G. (2001). *Intersymbol interference in digital communication systems*. Hoboken: Wiley.
23. Jalali, S. (2012). *Wireless channel equalization in digital communication systems*. CGU theses & dissertations. Paper 42. http://scholarship.claremont.edu/cgu_etd/42.
24. Kavitha, V., & Sharma, V. (2005). Comparison of training, blind and semi blind equalizers in MIMO fading systems using capacity as measure. In *IEEE international conference on acoustics, speech, and signal processing, 2005. Proceedings (ICASSP'05)* (Vol. 3, pp. iii–589). IEEE.
25. Silva, N. A., & Pinto, A. N. (2017). Role of amplifiers gain on the achievable information rate of M-ary PSK and QAM constellations. *Optics Communications, 383,* 215–222.
26. Malik, G., & Sappal, A. S. (2011). Adaptive equalization algorithms: An overview. *International Journal of Advanced Computer Science and Applications*, *2*(3), 63–67.

Dr. Sheeraz Ahmed did his graduation in Electrical Engineering (Communication Group) from University of Engineering and Technology, Peshawar, Pakistan with Honors in 1997. He then did his Masters in Mathematics from University of Peshawar in 1999. In 2006, he got enrolled in MSCS program in City University of Science and Technology, Peshawar and completed his MS program with major in Computer Networks. He then got enrolled in second MS in Electrical Engineering in UET Peshawar and his thesis title was “Design and configuration of Electronic Ballast Circuits”. He then got enrolled in Ph.D. program in Electrical Engineering in COMSATS Islamabad in Spring 2012 and completed his Doctoral degree in Spring 2015. During his stay in Ph.D. program, he published a total of 17 publications both in IF journals of international repute and IEEE conferences. His thesis title was “Towards Cooperative Routing in Underwater and Body Area Wireless Sensor Networks”. Dr. Sheeraz has been engaged in serving various universities of Peshawar first as lecturer, then as Assistant Professor and active researcher. He has been serving for 1 year as Dean Faculty of Engineering and Technology in Gomal University, D.I.Khan, Pakistan in 2017. Currently he is serving as Associate Professor in the Department of Computer Science, Iqra National University, Peshawar. Till date, he has 83 publications to his credit with more than 10 IF journals, 12 non-IF journals and rest IEEE conferences. He has the honor to be the first researcher to establish a Research Academy in Pakistan by the name of Career Dynamics Research Centre. His research interest areas include Underwater Sensor Networks, Body Area Networks, Vehicular Adhoc Networks, Smart Homes, Smart Grid etc.

Dr. Yousaf Khan is serving as Associate Professor/Chairman at Department of Electrical Engineering-University of Engineering and Technology Campus Kohat. His research interests are Passive Optical Networks, Radio over Fiber (RoF) and Next Generation Optical Networks. He graduated in Electrical Engineering from N-W.F.P University of Engineering and Technology Peshawar in 2000 with distinction. He received his master degree in Electrical Engineering from N-W.F.P University of Engineering and Technology Peshawar in 2003. He did MBA in Human Resource Management from University of Peshawar in 2007 and got his Ph.D. degree in Optical Engineering from Beijing University of Posts and Telecommunication-China in 2013. He has published over 35 research papers in scientific journal related to Passive optical Networks. He is frequent speaker and has delivered featured presentations at numerous events at national and international forums. He has also worked as Director QEC at Iqra National University Peshawar where he has arranged number of trainings on Outcome Based Education (OBE), Self Assessment Reports (SAR) and Institutional Performance Evaluation (IPE).Currently he is convener of OBE implementation team at UET Campus Kohat.

Asad Wahab is Ph.D. research scholar at Iqra National University Peshawar, Pakistan and project advisor at Carrier Dynamics Peshawar. He research interests are sensor networks and optical communication. He has several impact factor journal publications. He is associated with academia since decade and remained part of some key projects in his research area.