Wireless Pers Commun (2018) 101:875–895
https://doi.org/10.1007/s11277-018-5730-5

On the Performance of Downlink Multiuser Cognitive Radio Inspired Cooperative NOMA

Mona Shokair[1] · Waleed Saad[1,3] · Shady M. Ibraheem[1,2]

Published online: 17 April 2018

Abstract This paper is considered as an application of a centralized control non orthogonal multiple access (NOMA) based cognitive radio network. Here, a base station (BS) sends simultaneously two information signals by employing the superposition coding scheme to two different types of users, i.e., group of near users and one far user. The near users, namely, the secondary users, exchange cooperatively their own received information among themselves ensuring the realization of maximal diversity gain. Besides, they are responsible for relaying information to the far user, namely, the primary user. One potential secondary user is selected to decode and forward the BS information signal to the primary user and the rest of the secondary users to reinforce the reliability, as well as, mitigate the non-decodable messages. Two equivalent cases of a relay (secondary user) selection scheme are proposed. In the first case, the selection aims at maximizing the minimum of the joint secondary to secondary (S to S) and secondary to primary (S to P) channels' coefficients under a certain limit of interference condition. In the second case, the selection aims at maximizing the minimum of the BS to S and S to S paths while a certain quality of service of the primary user is strictly guaranteed. Assuming Rayleigh fading channels, new closed form expressions are derived for the achievable capacity associated with the two information signals. Simulation results reveal the advantage of our proposed schemes over the conventional orthogonal max–min approach and confirm the validity of our analysis.

✉ Shady M. Ibraheem
shadymam@gmail.com

Mona Shokair
mona.sabry@el-eng.menofia.edu.eg

Waleed Saad
waleed.saad@el-eng.menofia.edu.eg

1 Electronic and Electrical Communications Department, Faculty of Electronic Engineering, Menoufia University, Menouf, Egypt

2 Telecom Egypt, Tanta, Egypt

3 Electrical Department, College of Engineering, Shaqra University, Shaqra, Kingdom of Saudi Arabia

Keywords Cooperative relaying system (CRS) · Non-orthogonal multiple access (NOMA) · Cognitive radio

1 Introduction

Non orthogonal multiple access (NOMA) has been considered as a promising technique to enhance the spectral efficiency of the fifth generation (5G) communication networks [1]. Unlike orthogonal multiple access (OMA) techniques, NOMA had been proposed to serve multiple users of multiple communication signals at the same time/frequency/code slot but with different power levels [2, 3]. Then, the separation is carried out by the method of successive interference cancellation (SIC).

The impact of user pairing has been recognized as a concept of hybrid NOMA/OMA design where the authors in [4] compared the sum rate of a dual pairing NOMA based users with orthogonal based ones. In [5] the authors considered the fairness of the users where proper power resource coefficients were allocated fairly. The NOMA technique had been utilized by many transceiver communication systems such as uplink transmissions [6], coordination deployment systems [7] and downlink transmissions [8]. Actually, a closed form expression for the sum rates of NOMA with two BSs was derived in [7] using a coordinated superposition coding.

Meanwhile, cognitive radio networks have been recognized as a useful tool for solving the problem of scarcity of spectral resources and to provide a spectral efficiency by licensed/unlicensed spectrum sharing [9]. In cognitive radio networks, it is permissible to unlicensed secondary users to access the spectrum of licensed primary users without interference with the primary users [10]. NOMA techniques can be involved in cognitive radio networks where the secondary users have the opportunities to share the licensed spectrum even if the primary users are in very poor channel conditions. The key feature lies in firstly decoding the message intended to the primary user by the secondary user. Then, the secondary user can detect its own message by utilizing the appropriate power level and decoding techniques, e.g., SIC [11]. Thus, both the primary and secondary users can be served simultaneously.

Cooperative communication based NOMA has attracted much attention to promote the transmission reliability and improve the system performance. Particularly, the secondary user can serve as potential relay to improve the performance of the primary user via cooperative communication. Various cooperative techniques have been proposed in the literature. For example, in [12] the user with a good channel condition acts as a relay for the user with such a poor one. More recently, NOMA with a dedicated relay was introduced in [13] to avoid overloaded information on relay users. Wireless power harvesting was applied to cooperative NOMA in [14], where the impact of the locations of randomly deployed users was extensively investigated in a downlink single cell system. The scenario of multiple users equipped with multiple antennas was investigated in [15]. Two optimal relay selection strategies based on cooperative NOMA scheme were proposed in [16]. A comparative study between the achievable capacities in cooperative NOMA/OMA and power allocation schemes was depicted in [17]. In [18] the multiple-input–multiple-output MIMO based NOMA system was discussed within multi-antenna networks where the MIMO channels were converted into multiple parallel single channels.

However, in the context of a cooperative NOMA scheme within a cognitive radio network, the primary user may suffer from very poor channel condition or its channel may vary according to the location which requires the paired secondary user/s to adjust the allocated power level of the redirected signal to the primary user, e.g., the power allocation resource is proportional to the channel condition. Moreover, it is desired in the secondary user network to exchange signals via the lowest possible number of possible resources (i.e., time/frequency/code).

Motivated by this fact, in this paper a novel of a downlink cognitive radio network will be proposed where a BS controls multiple secondary users and one primary user. One potential secondary user (e.g., any-casting transmission) is selected to transmit the controlled signals simultaneously to both the other secondary users and the primary user to increase the reliability of both networks.

Different from the contributions [16, 17] and [19] in the literature, the fading coefficient of the primary user within the network dynamically determines the amount of allocated powers and it can be efficiently modeled by a certain QoS threshold condition at the primary user. Therefore, the allocated power resources for both secondary and primary users vary greatly depending on the channel characteristics of both (S to S) and (S to P) paths.

The contributions of the paper can be summarized as follows:

- Two equivalent cases of a secondary user selection scheme are proposed that aim at maximizing the minimum of the joint base-station to secondary (BS to S) and secondary to secondary (S to S) channels' coefficients under a certain condition of secondary to primary (S to P) channel coefficient.
- At high SINR, approximately exact and asymptotic NOMA expressions for the average achievable sum rate of the network will be derived and compared with the existing OMA schemes. Simulation results emphasize the correctness of our derivations and ensure that the proposed system outperforms the conventional OMA by using the lowest possible number of time resources. This paves the way of exploiting more and more saved time resources at successive stages of communications [24].
- It will be shown that the channel characteristic of the BS-S path has a great impact on the performance and it should be optimized to make the use of NOMA scheme.

The reminder of this paper is organized as follows: in Sect. 2 the system model is presented. In Sect. 3, the proposed user selection is investigated and a comprehensive analysis for the average achievable sum rate as an important metric is provided. Simulation results are carried out in Sect. 4 to emphasize the correctness of our theoretical analysis. Finally, Sect. 5 concludes the paper.

2 System and Channel Models

A CR network based NOMA system is considered with one BS, one primary user and a set of $R_{\mathcal{N}}$, $\mathcal{N} \in \{1, 2, \ldots., N\}$, cooperative secondary users which mutually exchange messages among themselves. Particularly, the BS employs the NOMA technique to communicate simultaneously with the primary user and one any-casting secondary user which is selected from the secondary cooperative system to mitigate the non-decodable message to both primary and secondary nodes. As shown in Fig. 1, the selected secondary user acts as a relay to the primary user and both are paired to perform NOMA.

It is assumed that all nodes are equipped with a single antenna and a half duplex mode communication is considered. Furthermore, all communication channels are flat fading and error free estimated Rayleigh channels. It is further assumed that all the nodes are

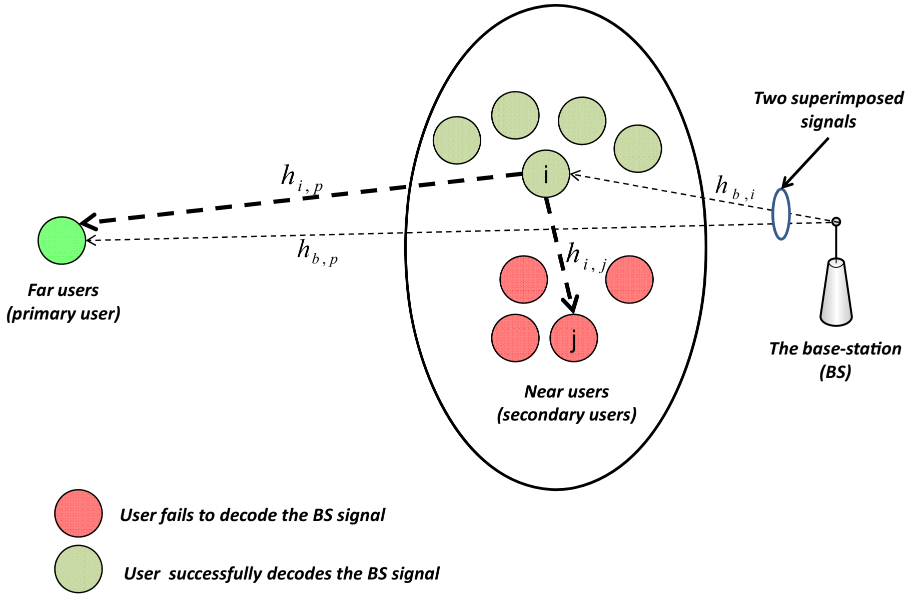

Fig. 1 System and channel model

confirmed to know global channel state information (CSI). Users' channels are ordered according to their QoS requirements.

Two time slots are involved in communications. In the first time slot, the BS broadcasts simultaneously two superimposed signals by employing the superposition code, the transmitted signal can be expressed by [1, 2],

$$B = x_s\sqrt{P_b a_{si}} + x_p\sqrt{P_b a_{sp}} \tag{1}$$

where x_s and x_p are the transmitted symbols of the secondary and primary system, respectively, P_b is the BS limited power, a_{si} and a_{sp} are the power allocation resources and $E\left\{|x_k|^2\right\} = 1, k \in \{s, p\}$. Following the NOMA principle, it is noted that $a_{si} + a_{sp} = 1$, $a_{si} < a_{sp}$, i.e., S_p is allocated with more power to ensure fairness to the downlink users. Thus, the received signal at the secondary user i, R_i, and the primary user p can be respectively expressed as,

$$r_{bi} = h_{bi}B + n_i, \tag{2}$$

$$r_{bp} = h_{bp}B + n_p^{(b)} \tag{3}$$

where h_{bi} is the $BS - R_i$ channel fading coefficient, $n_i \sim CN(0, N_0)$ is the additive white Gaussian noise (AWGN) signal at the secondary user i, h_{bp} is the channel fading coefficient of the $BS - p$ direct path, $n_p^{(b)} \sim CN(0, N_0)$ is the corresponding additive white Gaussian noise (AWGN) signal associated the reception of the BS signal (where the superscript b belongs to the transmit BS signal).

The secondary user i decodes the symbol x_p by treating the symbol x_s as a noise, then, cancels it from the superimposed signals using successive interference cancellation (SIC) to acquire the symbol x_s. Thus, the conditions for the secondary user i to be able to successfully decode the two signals, i.e., its own signal and the primary user signal, are given by [21],

$$\frac{1}{2}\log_2\left(1+\gamma_{bi}^{(p)}\right)=\frac{1}{2}\log_2\left(1+\frac{|h_{bi}|^2P_ba_{sp}}{|h_{bi}|^2P_ba_{si}+N_0^2}\right)>R_p, \tag{4}$$

$$\frac{1}{2}\log_2\left(1+\gamma_{bi}^{(s)}\right)=\frac{1}{2}\log_2\left(1+\frac{|h_{bi}|^2P_ba_{si}}{N_0^2}\right)>R_s \tag{5}$$

where $\gamma_{bi}^{(p)}$ and $\gamma_{bi}^{(s)}$ are the signal to interference plus noise ratios (SINRs) of the primary and the secondary users, respectively, experienced at the secondary user i. Here, R_p and R_s represent the target rates of the primary and the secondary users, respectively. The primary user decodes its own signal treating the other as a noise provided that,

$$\frac{1}{2}\log_2\left(1+\gamma_{bp}\right)=\frac{1}{2}\log_2\left(1+\frac{|h_{bp}|^2P_ba_{sp}}{|h_{bp}|^2P_ba_{si}+N_0^2}\right)>R_p. \tag{6}$$

where γ_{bp} is the signal to interference plus noise ratios (SINRs) of the primary user p.

At the second time slot, the secondary user i forwards the re-encoded primary signal to the primary user to reinforce the high priority signal as well as it forwards simultaneously the secondary signal to the secondary system by employing different resource allocation coefficients.[1] The two signals can be obtained separately as,

$$S_{ip}=x_p\sqrt{P_Ra_{ip}}, \tag{7}$$

$$S_{ij}=x_s\sqrt{P_Ra_{ij}}, j\in\mathcal{N}-\{i\}, \tag{8}$$

where S_{ip} is the transmitted signal to the primary user at the secondary user i end, $S_{ij}, j\in\mathcal{N}-\{i\}$, is the transmitted signal to the secondary user j at the secondary user i end, P_R is the secondary user (relay) limited power, a_{ij} and a_{ip} are the power allocation resources.

It is noted that the two separate signals are summed. Thus, the received signal at the secondary user $j, j\in\mathcal{N}-\{i\}$, and the primary user p can be respectively expressed as,

$$r_{ij}=h_{ij}\left(S_{ij}+S_{ip}\right)+n_j, \tag{9}$$

$$r_{ip}=h_{ip}\left(S_{ij}+S_{ip}\right)+n_p^{(i)}, j\in\mathcal{N}-\{i\}, \tag{10}$$

where $h_{ij}, j\in\mathcal{N}-\{i\}$, is the R_i-R_j channel fading coefficient, $n_j\sim CN(0,N_0)$ is the additive white Gaussian noise (AWGN) signal at the secondary user j, h_{ip} is the channel fading coefficient of the R_i-p path, $n_p^{(i)}\sim CN(0,N_0)$ is the corresponding additive white Gaussian noise (AWGN) signal associated the reception of the R_i signal (where the superscript i belongs to the secondary user i transmitted signal).

Recall from the previous concept that the secondary user j decodes the received signal using SIC with an achievable rate no more than $\frac{1}{2}\log_2\left(1+\frac{|h_{ij}|^2P_Ra_{ij}}{N_0^2}\right)$ provided that the primary constraints are satisfied.

[1] It is reasonable to assume that all of the near secondary nodes can decode the BS signals correctly.

In contrast to the previous secondary user i power allocation scheme which implies that, $a_{ij} + a_{ip} = 1$, $a_{ij} < a_{ip}$, a strict target SINR that guarantees the primary user's QoS is introduced exploiting the channel characteristic of the primary and the secondary users. In particular, an upper limit of power allocated by a_{ij} may need to satisfy,

$$\frac{\left|h_{ip}\right|^2 P_R a_{ip}}{\left|h_{ip}\right|^2 P_R a_{ij} + N_0^2} \geq \square,\ j \in \mathcal{N} - \{i\} \tag{11}$$

where $\square$ represents the constraint SINR. Thus, a_{ij} varies as a function of the primary fading coefficient which implies that,

$$a_{ij} = min_i\left(max\left(0, \frac{\left|h_{ip}\right|^2 P_R - N_0^2 \square}{\left|h_{ip}\right|^2 P_R(\square + 1)}\right)\right), \quad j \in \mathcal{N} - \{i\}. \tag{12}$$

Two involved relay selection schemes are proposed in the next section based on the previous condition. The necessary condition for the priority of the primary user to receive the secondary signal can be given as,

$$a_{ip}\left|h_{ip}\right|^2 > a_{ij}\left|h_{ij}\right|^2. \tag{13}$$

In the following section, two equivalent relay selection schemes which are characterized by strict power allocation coefficients are proposed for any-casting cognitive radio system to guarantee an adoptable QoS to the primary user. Then, we focus on the achievable rate analysis as an important performance metric.

3 Relay Selection and Performance Analysis

Different from the work [16, 17], one potential secondary user is selected from the set $R_{\mathcal{N}}$ of secondary users, e.g., relying on the following selection criteria, to play its role in any-casting cognitive radio system where it transmits the signals intended to the primary user and the rest subset of R_j, $j \in \mathcal{N} - \{i\}$, secondary users by dynamically allocating a variable power resource depending on the instantaneous measurement of the primary channel fading coefficient.

Besides, new expressions for secondary user selection based on resource allocation parameters and statistical channels' conditions are derived. Then, the impact of those parameters on the achievable transmission rate will be calculated.

3.1 Relay Selection Schemes

(1) *Statistical max–min the joint end to end (case 1)* This scheme can be carried out by selecting the secondary user, i^*, with the best weakest amongst all the candidates as follows:

$$i^* = \arg max_i\left(min\left(\gamma_{bi}, \gamma_{ip}, \gamma_{ij}\right)\right) \triangleq \arg max_i\left(min\left(|h_{bi}|^2, \left|h_{ip}\right|^2, \left|h_{ij}\right|^2\right)\right), a_{ij} = const.. \tag{14}$$

where γ_{bi}, γ_{ip} and γ_{ij}, are the signal to interference plus noise ratios (SINRs) of the

$BS - R_i$, the $R_i - p$ and the $R_i - R_j$, respectively. Obviously, $|h_{bi}|^2$, $|h_{ip}|^2$ and $|h_{ij}|^2$ are uncorrelated, i.e., statistically independent, and the selection can be simply realized.

(2) *Statistical max–min the joint secondary user phases (case 2)* This scheme can be carried out by selecting the secondary user, i^*, that maximizes the minimum of the joint secondary user statistics under the QoS constraint of primary user in (12), namely, i^* satisfies,

$$\begin{aligned} i^* &= \arg max_i\left(min\left(\gamma_{bi}, \gamma_{ij}\right)\right) \\ &= \arg max_i\left(min\left(\gamma_{bi}^{(s)}, \gamma_{ij}\right)\right) \triangleq \arg max_i\left(min\left(\frac{|h_{bi}|^2 P_b a_{si}}{N_0^2}, \frac{|h_{ij}|^2 P_R a_{ij}}{N_0^2}\right)\right), a_{ij} \\ &= min_i\left(max\left(0, \frac{|h_{ip}|^2 P_R - N_0^2 \square}{|h_{ip}|^2 P_R(\square + 1)}\right)\right), j \in \mathcal{N} - \{i\}. \end{aligned} \tag{15}$$

where at the secondary user i, $R_p > R_s$.

Particularly, we will statistically analyze the second scheme and the first scheme can be obtained simply by a similar analysis. Let $\bar{F}_A(x) = 1 - F_A(x)$ be the complement of the cumulative density function (CDF) of the random variable (R.V.) A. Let us generally define that the fading coefficients $|h_{qr}|^2, q, r \in \{b, i, p, j\}$ satisfy Rayleigh fading conditions and undergo independent exponential distributions with means λ_{qr}, where their CDFs family can be expressed by,

$$F_{|h_{qr}|^2}(x) = 1 - \exp\left(-\Omega_{qr} x\right). \tag{16}$$

where $\Omega_{qr} = \left(\lambda_{qr} \frac{P_s}{N_0}\right)^{-1}, \quad s \in \{b, R\}$.

By further considering, $\mathrm{Z_i} \triangleq \left(min\left(\gamma_{bi}, \gamma_{ij}\right)\right)$, we have the following theorem.

Theorem 1 *The secondary user selection rule according to* (15) *is determined statistically by selecting i that maximizes The CDF of Z_i, is given by,*

$$i^* = \arg max_i(Z_i) = \arg Pr(max_i(Z_i) \leq z) \Rightarrow arg \prod_{i=1}^{N} F_{Z_i}(z). \tag{17}$$

where $\bar{F}_{Z_i}(z) = \bar{F}_{\gamma_{bi}}(z)\bar{F}_{\gamma_{ij}}(z), \bar{F}_{\gamma_{bi}}(z) = \exp\left(-\frac{\Omega_{bi} N_0^2 z}{P_b a_{si}}\right), \bar{F}_{\gamma_{ij}}(z) = \frac{2\Omega_{ij} N_0^2}{P_R} \sqrt{\mathrm{z}\square(\square + 1)\Omega_{ip}\Omega_{ij}}\exp$ $\left(-\frac{N_0^2}{P_R}\left(\square\Omega_{ip} + z(\square + 1)\Omega_{ij}\right)\right) K_1\left(2\frac{N_0^2}{P_R}\sqrt{\mathrm{z}\square(\square + 1)\Omega_{ij}\Omega_{ip}}\right), j \in \mathcal{N} - \{i\}, K_1(.)$ is the first order modified Bessel function of the second kind and z is an arbitrary threshold.

Proof See Appendix 1.

In the above theorem, we state the statistical conditions for selecting the ith secondary user out of N users to forward the BS signals. If the order statistics are applied [24], $F_{Z_i}(z)$ has to be substituted in the following equation,

$$F_{Z_{i:N}}(z) = \frac{N!}{(i-1)!(N-i)!} \sum_{w=0}^{N-i} \binom{N-i}{w} (-1)^w \frac{\left(F_{Z_i}(z)\right)^{i+w}}{i+w}, \tag{18}$$

where $Z_i, 1 \leq i \leq N$, are independent and identical R.V.s.

3.2 Statistical Achievable Rate

In order to successfully decode the received signals at the secondary and the primary users, certain constraints on transmission rates should not be exceeded. The achievable rate represents an important metric that characterizes those constraints. It is dominated by the channel of the poor condition. The achievable rate associated with the symbol x_s, $C_i = \frac{1}{2}\log_2(1+\gamma_i)$, involves an achievable SINR, γ_i, which can be extracted using SIC at the secondary user i to be in the form of,

$$\gamma_i = min\left(\frac{|h_{bi}|^2 P_b a_{si}}{N_0^2}, \frac{|h_{ij}|^2 P_R a_{ij}}{N_0^2}\right), \quad j \in \mathcal{N} - \{i\}. \tag{19}$$

At the end of the second time slot, the primary user performs maximal ratio combining (MRC) of the received signals. Thus, The achievable rate associated with the symbol x_p, $C_p = \frac{1}{2}\log_2(1+\gamma_p)$, involves an achievable SINR, γ_p,which can be given by,

$$\gamma_p = min\left(\frac{|h_{bi}|^2 P_b a_{sp}}{|h_{bi}|^2 P_b a_{si} + N_0^2}, \frac{|h_{ip}|^2 P_R a_{ip}}{|h_{ip}|^2 P_R a_{ij} + N_0^2} + \frac{|h_{bp}|^2 P_b a_{sp}}{|h_{bp}|^2 P_b a_{si} + N_0^2}\right). \tag{20}$$

To fulfill the system performance analysis, it is necessary to marginally derive statistical formulas for the achievable rates, γ_i and γ_p. The CDF of γ_i follows directly from Theorem 1, and it can be denoted by the R.V. Z_i, i.e. $F_{Z_i}(z) = Pr\left(\left(\gamma_{bi}^{(s)} > z, \gamma_{ij} > z\right)\right) = 1 - \bar{F}_{Z_i}(z)$. Thereafter, The CDF of γ_p will be derived in the following theorem.

Theorem 2 *According to* (20) *the complement of the CDF of Z_p is determined statistically by* (21) *at the top of next page, where* $\mathcal{J}(z) = \int_0^\infty \frac{a_{ip}N_0^2}{P\left(a_{ip}-xa_{ij}\right)^2}\exp\left(-\frac{N_0^2}{P}\left(\frac{\Omega_{ip}x}{\left(a_{ip}-xa_{ij}\right)} + \frac{\Omega_{bp}(z-x)}{\left(a_{sp}-(z-x)a_{si}\right)}\right)\right)dx.$

Proof See Appendix 2.

Under the considered conditions, it is obvious that $\gamma_{ip} = \frac{|h_{ip}|^2 Pa_{ip}}{|h_{ip}|^2 Pa_{ij}+N_0^2}$ is lower bounded by $\square$. Therefore, $\left(min\left(\gamma_{bi}^{(p)}, \gamma_{ip} + \gamma_{bp}\right)\right) \geq \left(min\left(\gamma_{bi}^{(p)}, \square + \gamma_{bp}\right)\right)$ which can be used instead and it represents the worst-case scenario.

Corollary *According to* (20) *the complement of the CDF of* $Z_p = \left(min\left(\gamma_{bi}^{(p)}, \square + \gamma_{bp}\right)\right)$ *is determined statistically*

$$\bar{F}_{Z_p}(z) = \exp\left(-\frac{\Omega_{bi} z N_0^2}{P\left(a_{sp} - z a_{si}\right)}\right) \cdot \times \begin{cases} \exp\left(-\frac{\Omega_{bp} z N_0^2}{P\left(a_{sp} - z a_{si}\right)}\right) + \exp\left(-\frac{\Omega_{ip} z N_0^2}{P\left(a_{ip} - z a_{ij}\right)}\right) - \exp\left(-\frac{\Omega_{ip} z N_0^2}{P\left(a_{ip} - z a_{ij}\right)}\right) \times \exp\left(-\frac{\Omega_{bp} z N_0^2}{P\left(a_{sp} - z a_{si}\right)}\right) \\ + \left(\exp\left(\frac{\Omega_{ip} N_0^2}{P a_{ij}}\right) + \mathcal{J}(z)\right) \times \left(1 - \exp\left(-\frac{\Omega_{ip} z N_0^2}{P\left(a_{ip} - z a_{ij}\right)}\right)\right)\left(1 - \exp\left(-\frac{\Omega_{bp} z N_0^2}{P\left(a_{sp} - z a_{si}\right)}\right)\right), z < \frac{a_{ip}}{a_{ij}} < \frac{a_{sp}}{a_{si}} \\ \exp\left(-\frac{\Omega_{bp} z N_0^2}{P\left(a_{sp} - z a_{si}\right)}\right) + \left(\exp\left(\frac{\Omega_{ip} N_0^2}{P a_{ij}}\right) + \mathcal{J}(z)\right) \times \left(1 - \exp\left(-\frac{\Omega_{bp} z N_0^2}{P\left(a_{sp} - z a_{si}\right)}\right)\right), \frac{a_{ip}}{a_{ij}} < z < \frac{a_{sp}}{a_{si}} \\ \left(\exp\left(\frac{\Omega_{ip} N_0^2}{P a_{ij}}\right) + \mathcal{J}(z)\right), \frac{a_{ip}}{a_{ij}} < \frac{a_{sp}}{a_{si}} < z \end{cases} \tag{21}$$

$$\bar{F}_{Z_p}(z) = \exp\left(-\frac{\Omega_{bi} z N_0^2}{P\left(a_{sp} - z a_{si}\right)}\right) \times \begin{cases} \exp\left(-\frac{\Omega_{bp} z N_0^2}{P\left(a_{sp} - z a_{si}\right)}\right) + \exp\left(-\frac{\Omega_{bp} (z - \mathbb{I}) N_0^2}{P\left(a_{sp} - (z - \mathbb{I}) a_{si}\right)}\right) - \exp\left(-\frac{\Omega_{bp} (z - \mathbb{I}) N_0^2}{P\left(a_{sp} - (z - \mathbb{I}) a_{si}\right)}\right) \\ \times \exp\left(-\frac{\Omega_{bp} z N_0^2}{P\left(a_{sp} - z a_{si}\right)}\right), z < \frac{a_{sp}}{a_{si}} \\ \exp\left(-\frac{\Omega_{bp} (z - \mathbb{I}) N_0^2}{P\left(a_{sp} - (z - \mathbb{I}) a_{si}\right)}\right), \frac{a_{sp}}{a_{si}} < z < \frac{a_{sp}}{a_{si}} + \mathbb{I} \\ 0, \frac{a_{sp}}{a_{si}} + \mathbb{I} < z \end{cases} . \tag{22}$$

by (22) at the top of this page and for simplicity the following equation is used:

$$\bar{F}_{Z_p}(z) = \begin{cases} \exp\left(-\frac{\Omega_{bi} z N_0^2}{P\left(a_{sp} - z a_{si}\right)}\right) \times \exp\left(-\frac{\Omega_{bp} (z - \mathbb{I}) N_0^2}{P\left(a_{sp} - (z - \mathbb{I}) a_{si}\right)}\right), \\ z < \frac{a_{sp}}{a_{si}} + I \\ 0, \frac{a_{sp}}{a_{si}} + I < z \end{cases} . \tag{23}$$

Proof Let $Z_p \triangleq \left(min\left(\gamma_{bi}^{(p)}, \mathbb{I} + \gamma_{bp}\right)\right)$, then, $\bar{F}_{\mathbb{I}+\gamma_{bp}}(z)$ can be formulated as, $\bar{F}_{\mathbb{I}+\gamma_{bp}}(z) = Pr\left(\mathbb{I} + \gamma_{bp} > z\right) = Pr\left(\frac{\left|h_{bp}\right|^2 P a_{sp}}{\left|h_{bp}\right|^2 P a_{si} + N_0^2} > z - \mathbb{I}\right) = \exp\left(-\frac{\Omega_{bp}(z-\mathbb{I})N_0^2}{P\left(a_{sp}-(z-\mathbb{I})a_{si}\right)}\right)$ and (22) follows after simple algebraic manipulations by shifting by $\mathbb{I}$ and by using $Pr\left(\frac{\left|h_{bp}\right|^2 P a_{sp}}{\left|h_{bp}\right|^2 P a_{si} + N_0^2} > z - \mathbb{I}\right) = 1$, if $\frac{a_{sp}}{a_{si}} < z$ and $z < \frac{\left|h_{bp}\right|^2 P a_{sp}}{\left|h_{bp}\right|^2 P a_{si} + N_0^2}$. A simple upper bound of (22) by reference to (23) and the proof is completed.

3.3 Average Achievable Sum Rate Analysis

The average sum rate is an important metric which gives an insightful investigation for the steady-state transmission rate. According to [25], the average achievable rate, i.e., the ergodic rate is defined as,

$$R_{aver} = \int_0^\infty \frac{1}{2}\log_2(1+z)f_\gamma(z)dz = \frac{1}{2\ln 2}\int_0^\infty \frac{\bar{F}_\gamma(z)}{1+z}dz. \tag{24}$$

where $F_\gamma(z) = 1 - \bar{F}_\gamma(z)$ is the CDF of the SINR random variable.

3.4 Primary User

The average achievable rate for the primary user, e.g., associated with the symbol x_p, is complicated by applying the formulas (21) and (22). Thus, (23) is applied instead. At the primary user, the average achievable rate can be calculated as in the following theorem.

Theorem 3 *According to (23) the average achievable rate associated with x_p is given by,*

$$R_{aver,p} \approx \frac{1}{2\ln 2} \times \frac{\pi\left(\frac{a_{sp}}{a_{si}} + \mathbb{1}\right)}{2L} \times$$
$$\sum_{l=1}^{L} \sqrt{1-\theta_l^2} \times \frac{\exp\left(-\left(\frac{\Omega_{bi}\mu_l N_0^2}{P\left(a_{sp}-\mu_l a_{si}\right)} + \frac{\Omega_{bp}(\mu_l-\mathbb{1})N_0^2}{P\left(a_{sp}-(\mu_l-\mathbb{1})a_{si}\right)}\right)\right)}{1+\mu_l} \tag{25}$$

where L is the adopted samples to ensure the approximation, $\theta_l = \cos\left(\frac{2l-1}{2L}\pi\right)$ *and* $\mu_l = \left(\frac{\left(\frac{a_{sp}}{a_{si}}+\mathbb{1}\right)(\theta_l+1)}{2}\right)$.

Proof From (24), $R_{aver} = \frac{1}{2\ln 2}\int_0^\infty \frac{\bar{F}_\gamma(z)}{1+z}dz = \frac{1}{2\ln 2}\int_0^{\frac{a_{sp}}{a_{si}}+\mathbb{1}} \frac{\exp\left(-\frac{\Omega_{bi}zN_0^2}{P\left(a_{sp}-za_{si}\right)}\right)\times\exp\left(-\frac{\Omega_{bp}(z-\mathbb{1})N_0^2}{P\left(a_{sp}-(z-\mathbb{1})a_{si}\right)}\right)}{1+z}dz.$

The above integral is mathematically intractable, Gauss–Chebyshev quadrature [24, 8.8.12] is applied to find an approximation. The integral yields,

$$\int_0^{\frac{a_{sp}}{a_{si}}+\mathbb{1}} \frac{\exp\left(-\frac{\Omega_{bi}zN_0^2}{P\left(a_{sp}-za_{si}\right)}\right) \times \exp\left(-\frac{\Omega_{bp}(z-\mathbb{1})N_0^2}{P\left(a_{sp}-(z-\mathbb{1})a_{si}\right)}\right)}{1+z}dz$$
$$\approx \frac{\pi\left(\frac{a_{sp}}{a_{si}}+\mathbb{1}\right)}{2L}\sum_{l=1}^{L}\sqrt{1-\theta_l^2}\,\frac{\exp\left(-\frac{\Omega_{bi}\mu_l N_0^2}{P\left(a_{sp}-\mu_l a_{si}\right)}\right) \times \exp\left(-\frac{\Omega_{bp}(\mu_l-\mathbb{1})N_0^2}{P\left(a_{sp}-(\mu_l-\mathbb{1})a_{si}\right)}\right)}{1+\mu_l}$$

where θ_l and μ_l are defined in (25) which completes the proof.

3.5 Secondary User

The average achievable rate for the secondary user i, e.g., associated with the symbol x_s, may be calculated approximately as in the following theorem.

Theorem 4 *According to* (17) *and* (19), *the average achievable rate associated with x_s is given by,*

$$R_{aver,s} \approx \frac{1}{2\ln 2} \times 2\beta \times \exp\left(-\frac{N_0^2}{P}\left(\mathbb{I}\Omega_{ip}\right)\right) \times \left[\frac{E_1(0)}{2\beta}\exp\left(\frac{\beta^2}{2\alpha}\right)W_{0,\frac{1}{2}}\left(\frac{\beta^2}{\alpha}\right) - \sum_{l=1}^{L} A_l \times \frac{1}{\sqrt{B_l}} \times \frac{\exp(-\alpha B_l) \times K_1\left(2\beta \times \sqrt{B_l}\right)}{1+B_l}\right] \tag{26}$$

where L is the adopted samples to ensure the approximation, $W_{\mu,\upsilon}(.)$ *is the Whittaker function,* $E_1(.)$ *is the exponential integral function,* $\alpha = \frac{N_0^2}{P}\left(\frac{\Omega_{bi}}{a_{si}} + (\mathbb{I}+1)\Omega_{ij}\right)$, $\beta = \frac{N_0^2}{P}\sqrt{\mathbb{I}(\mathbb{I}+1)\Omega_{ij}\Omega_{ip}}$, $A_l = \frac{\pi^2 \sin\left(\frac{2l-1}{2L}\pi\right)}{4L\cos^2\left(\frac{\pi}{4}\times\cos\left(\frac{2l-1}{2L}\pi\right)+\frac{\pi}{4}\right)}$ *and* $B_l = \tan\left(\frac{\pi}{4} \times \cos\left(\frac{2l-1}{2L}\pi\right) + \frac{\pi}{4}\right)$.

Proof From $\bar{F}_{Z_i}(z)$ in (17), $R_{aver} = \frac{1}{2\ln 2}\int_0^\infty \frac{\bar{F}_\gamma(z)}{1+z}dz = \frac{1}{2\ln 2} \times \frac{2\Omega_{ij}N_0^2}{P}\sqrt{\frac{\mathbb{I}(\mathbb{I}+1)\Omega_{ip}}{\Omega_{ij}}} \times \exp\left(-\frac{N_0^2}{P}\left(\mathbb{I}\Omega_{ip}\right)\right) \times \int_0^\infty \frac{\exp\left(-\frac{N_0^2}{P}z\left(\frac{\Omega_{bi}}{a_{si}}+(\mathbb{I}+1)\Omega_{ij}\right)\right)\times\sqrt{z}\times K_1\left(2\frac{N_0^2}{P}\sqrt{z\mathbb{I}(\mathbb{I}+1)\Omega_{ij}\Omega_{ip}}\right)}{1+z}dz.$ Putting $\theta = \sqrt{z}$, the above integral becomes as,

$$2 \times \int_0^\infty \frac{\exp\left(-\frac{N_0^2}{P}\theta^2\left(\frac{\Omega_{bi}}{a_{si}} + (\mathbb{I}+1)\Omega_{ij}\right)\right) \times \theta^2 \times K_1\left(2\frac{N_0^2}{P}\sqrt{\mathbb{I}(\mathbb{I}+1)\Omega_{ij}\Omega_{ip}} \times \theta\right)}{1+\theta^2}d\theta$$

$$= \underbrace{\int_0^\infty 2\exp\left(-\frac{N_0^2}{P}\theta^2\left(\frac{\Omega_{bi}}{a_{si}} + (\mathbb{I}+1)\Omega_{ij}\right)\right) \times K_1\left(2\frac{N_0^2}{P}\sqrt{\mathbb{I}(\mathbb{I}+1)\Omega_{ij}\Omega_{ip}} \times \theta\right)d\theta}_{\mathcal{J}_1}$$

$$\underbrace{-\int_0^\infty 2\frac{\exp\left(-\frac{N_0^2}{P}\theta^2\left(\frac{\Omega_{bi}}{a_{si}} + (\mathbb{I}+1)\Omega_{ij}\right)\right) \times K_1\left(2\frac{N_0^2}{P}\sqrt{\mathbb{I}(\mathbb{I}+1)\Omega_{ij}\Omega_{ip}} \times \theta\right)}{1+\theta^2}d\theta}_{\mathcal{J}_2}.$$

$\mathcal{J}_1$ can be solved by using [22, 6.643.3], $\mu = 0$ and $\upsilon = \frac{1}{2}$, after putting $\theta = \sqrt{x}$. Therefore, $\mathcal{J}_1$ can be expressed as,

$$\mathcal{J}_1 = \frac{\Gamma(1)\Gamma(0)}{2\beta}\exp\left(\frac{\beta^2}{2\alpha}\right)W_{0,\frac{1}{2}}\left(\frac{\beta^2}{\alpha}\right)$$

where $W_{\mu,\upsilon}(.)$ is the Whittaker function, β and α are defined in (26).

The integral $\mathcal{J}_2$ is a challenge, numerical analysis is used to provide a formula that is very closed to the exact one. Applying Gauss–Chebyshev quadrature, The integral $\mathcal{J}_2$ can be expressed by putting $\theta = \sqrt{\tan x}$ as,

$\mathcal{J}_2 = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\tan x}} \times \frac{\exp\left(-\alpha\left(\sqrt{\tan x}\right)^2\right)\times K_1\left(2\beta\times\sqrt{\tan x}\right)}{1+\tan x} \times \sec^2 x\, dx \approx \sum_{l=1}^{L} A_l \times \frac{1}{\sqrt{B_l}} \times \frac{\exp(-\alpha B_l)\times K_1\left(2\beta\times\sqrt{B_l}\right)}{1+B_l}$, where A_l and B_l are defined in (26). The proof is completed.

It should be noted that the integral $\mathcal{J}_1$ may lead to an indeterminate value, and therefore, it is required to be approximated, e.g., by limits, which is worth nothing. Alternatively, $\mathcal{J}_1$ and $\mathcal{J}_2$ can be approximated at once using Gauss–Chebyshev quadrature and the average achievable rate in (26) can be rewritten as,

$$
\begin{aligned}
R_{aver,s} &= \frac{1}{2\ln 2}\int_{0}^{\infty}\frac{\bar{F}_{\gamma}(z)}{1+z}dz \\
&\approx \frac{1}{2\ln 2}\times 2\beta\times\exp\left(-\frac{N_0^2}{P}\left(\mathbb{I}\Omega_{ip}\right)\right)\times\sum_{l=1}^{L}A_l\times\sqrt{B_l} \\
&\quad\times\frac{\exp(-\alpha B_l)\times K_1\left(2\beta\times\sqrt{B_l}\right)}{1+B_l}.
\end{aligned}
\tag{27}
$$

Generally, the average achievable rates associated with x_s and x_p are still complex formulas. Hence, the asymptotic average achievable rates will be studied as approximations at high SINR. This represents a main agent that identifies and controls the network.

Let $\frac{N_0^2}{P}\ll 1$, then, by using the series expansion of Bessel functions for small values of x which is given by,

$$
xK_1(x) = 1+\frac{x^2}{2}\ln\frac{x}{2}-\frac{x^2}{4}C
\tag{28}
$$

where $C=\psi(1)+\psi(2)$ and $\psi(.)$ is the Psi function defined in [23], it is evident that the asymptotic average achievable rate associated with x_s can be obtained by applying $2\beta\sqrt{z}K_1(2\beta\sqrt{z})\approx 1, \beta\to 0$, as follows,

$$
\begin{aligned}
R_{aver,s}^{\infty} &\approx \frac{1}{2\ln 2}\times\exp\left(-\frac{N_0^2}{P}\left(\mathbb{I}\Omega_{ip}\right)\right)\times\int_{0}^{\infty}\frac{e^{-\alpha z}\times 2\beta\sqrt{z}\times K_1(2\beta\sqrt{z})}{1+z}dz \\
&= \frac{1}{2\ln 2}\times\exp\left(-\frac{N_0^2}{P}\left(\mathbb{I}\Omega_{ip}\right)\right)\int_{0}^{\infty}\frac{e^{-\alpha z}}{1+z}dz \\
&= -\exp(\alpha)Ei(-\alpha)
\end{aligned}
\tag{29}
$$

where the last term follows from [22, 3.352.4] and is the exponential integral.

Following similar steps, $R_{aver,p}\approx\frac{1}{2\ln 2}\times\int_{0}^{\frac{a_{sp}}{a_{si}}}\frac{\exp\left(-\frac{\Omega_{bp}(z-\mathbb{I})N_0^2}{P\left(a_{sp}-(z-\mathbb{I})a_{si}\right)}\right)}{1+z}dz$, by considering (23) for $\frac{N_0^2}{P}\ll 1$ and letting $\frac{|h_{bi}|^2P_b a_{sp}}{|h_{bi}|^2P_b a_{si}+N_0^2}\triangleq\frac{a_{sp}}{a_{si}}$ and $\exp\left(-\frac{\Omega_{bp}(z-\mathbb{I})N_0^2}{P\left(a_{sp}-(z-\mathbb{I})a_{si}\right)}\right)\approx 1-\frac{\Omega_{bp}(z-\mathbb{I})N_0^2}{P\left(a_{sp}-(z-\mathbb{I})a_{si}\right)}$. Then, a simplified expression for the asymptotic average achievable rate associated with x_p can be obtained as,

$$
\begin{aligned}
R_{aver,p}^{\infty} &\approx \frac{1}{2\ln 2}\times\int_{0}^{\frac{a_{sp}}{a_{si}}}\frac{1-\frac{\Omega_{bp}(z-\mathbb{I})N_0^2}{P\left(a_{sp}-(z-\mathbb{I})a_{si}\right)}}{1+z}dz \\
&= \frac{1}{2}\log_2\left(1+\frac{a_{sp}}{a_{si}}\right)-\int_{0}^{\frac{a_{sp}}{a_{si}}}\frac{\Omega_{bp}(z-\mathbb{I})N_0^2}{P\left(a_{sp}-(z-\mathbb{I})a_{si}\right)(1+z)}dz \overset{a}{=} \frac{1}{2}\left(1-\frac{N_0^2\Omega_{bp}}{P}A\right) \\
&\quad\times\log_2\left(1+\frac{a_{sp}}{a_{si}}\right)-\frac{1}{2}\frac{N_0^2\Omega_{bp}}{P}B\log_2\frac{\mathbb{I}a_{si}}{a_{sp}+\mathbb{I}a_{si}}
\end{aligned}
\tag{30}
$$

where (a) follows by applying the method of undetermined coefficients as, $\frac{(z-\mathbb{I})}{\left(a_{sp}+\mathbb{I}a_{si}-za_{si}\right)(1+z)}=\frac{A}{(1+z)}+\frac{B}{\left(a_{sp}+\mathbb{I}a_{si}-za_{si}\right)}, A=\frac{-(\mathbb{I}+1)}{\left(a_{sp}+(\mathbb{I}+1)a_{si}\right)}$ and $B=1-\frac{(\mathbb{I}+1)a_{si}}{\left(a_{sp}+(\mathbb{I}+1)a_{si}\right)}$. It is obvious that the asymptotic average achievable rate associated with x_p is bounded by $\frac{1}{2}\log_2\left(1+\frac{a_{sp}}{a_{si}}\right)$, as $\frac{N_0^2}{P}\rightarrow 0$.

Combining (29) and (30), the asymptotic average achievable rate can simply be given by,

$$R^{\infty}_{sum} = R^{\infty}_{aver,s} + R^{\infty}_{aver,p}. \tag{31}$$

For performance comparison, the asymptotic average achievable rate of the conventional OMA cooperative system is analyzed.

In this case, the BS is not able to transmit more than one signal per time slot and each user decodes orthogonally its own received signal only.

Consider the scenario where the BS transmits the symbol x_s at the first time slot and the symbol x_p at the second time slot. In the second time slot, it is noted that the secondary user i relays the x_s signal to the secondary user $j, j\in\mathcal{N}-\{i\}$, and simultaneously the BS transmits the x_p signal which causes interference at the secondary user $j, j\in\mathcal{N}-\{i\}$.

In the considered OMA cooperative system, the achievable sum rate is a lack of the relayed x_p resource and there exists an undesirable interference at the secondary user $j, j\in\mathcal{N}-\{i\}$, as well. This reflects the benefits of using the NOMA system.

In fact, if the same power allocation coefficients is used in the OMA system, the OMA average achievable rate associated with x_p can be given by,

$$\begin{aligned} R_{averOMA,p} &= \frac{1}{2\ln 2}\int_0^{\infty}\frac{\bar{F}_{\gamma_{OMA,p}}(z)}{1+z}dz = \int_0^{\infty}\frac{e^{-\frac{\Omega_{sp}}{a_{sp}}z}}{1+z}dz \\ &= -\,exp\left(\frac{\Omega_{sp}}{a_{sp}}\right)Ei\left(-\frac{\Omega_{sp}}{a_{sp}}\right) \end{aligned} \tag{32}$$

where the fading coefficient $\Omega_{sp}=\left(\lambda_{sp}\frac{P}{N_0}\right)^{-1}$. The asymptotic expression is obtained using the approximation $Ei(-x)\approx \Upsilon+\ln x$, Υ is he Euler constant [22, 8.212.1] for small x. Therefore, (32) can be modified as,

$$R_{averOMA,p} = \frac{1}{2\ln 2}\left(-\Upsilon+\left(\frac{\Omega_{sp}}{a_{sp}}\right)\times\ln\frac{a_{sp}}{\Omega_{sp}}\right). \tag{33}$$

Accordingly, the achievable rate associated with x_s involves an achievable SINR, $\gamma_{OMA,s}$, which can be given by,

$$\gamma_{OMA,s} = min\left(\frac{|h_{bi}|^2Pa_{si}}{N_0^2},\frac{\frac{|h_{ij}|^2Pa_{ij}}{N_0^2}}{\frac{|h_{sj}|^2Pa_{sp}}{N_0^2}}\right),\quad j\in\mathcal{N}-\{i\}. \tag{34}$$

Thus, the CDF of $\gamma_{OMA,s}$ is $F_{\gamma_{OMA,s}}(z)=1-\bar{F}_{\gamma_{OMA,s}}(z)$, where $\bar{F}_{\gamma_{OMA,s}}(z)=Pr\left(\frac{|h_{bi}|^2Pa_{si}}{N_0^2}>z,\frac{\frac{|h_{ij}|^2Pa_{ij}}{N_0^2}}{\frac{|h_{sj}|^2Pa_{sp}}{N_0^2}}>z\right)=Pr\left(\frac{|h_{bi}|^2Pa_{si}}{N_0^2}>z\right)\times Pr\left(\frac{|h_{ij}|^2a_{ij}}{|h_{sj}|^2a_{sp}}>z\right)=\exp\left(-\frac{\Omega_{bi}N_0^2z}{Pa_{si}}\right)\times\frac{\frac{\Omega_{ij}}{a_{ij}}z}{\frac{\Omega_{ij}}{a_{ij}}z+\frac{\Omega_{sj}}{a_{sp}}}$. By substitution, the OMA average achievable rate associated with x_s can be expressed as,

$$
\begin{aligned}
R_{averOMA,s} &= \frac{1}{2\ln 2}\int_0^\infty \frac{\bar{F}_{\gamma_{OMA,s}}(z)}{1+z}dz \\
&= \frac{1}{2\ln 2}\times\int_0^\infty \frac{\exp(-\in z)\times\frac{z}{z+\kappa}}{1+z}dz\,b \\
&= -\frac{1}{2\ln 2}\left(\left(\frac{\kappa}{\kappa-1}+1\right)\exp(\in)Ei(-\in)-\frac{\kappa}{\kappa-1}\exp(\kappa\in)Ei(-\kappa\in)\right)
\end{aligned}
\tag{35}
$$

where $\in=\frac{\Omega_{bi}N_0^2}{P_b a_{si}}$, $\kappa=\frac{\Omega_{sj}a_{ij}}{a_{sp}\Omega_{ij}}$ and (b) : follows by applying the method of undetermined coefficients.

Again, the asymptotic achievable rate associated with x_s will be attained as,

$$
R_{averOMA,s} = -\frac{1}{2\ln 2}\left(\left(\frac{\kappa}{\kappa-1}+1\right)\times(\in)\times(\Upsilon+\ln\in)-\frac{\kappa}{\kappa-1}\times(\kappa\in)\times(\Upsilon+\ln\kappa\in)\right).
\tag{36}
$$

Combining (32), (34) together and (33), (35) together, the average and the asymptotic average achievable sum rates can be easily extracted, respectively.

At high SINR, it should be noted that the average sum rates of both NOMA and OMA schemes can be improved by a scale of $\frac{\sigma}{2}\times\ln\frac{N_0^2}{P}$, where σ is an arbitrary factor. However, in NOMA scheme the factor σ is more stringent than the case of OMA scheme which explains from the previous equations how the NOMA based system outperforms the OMA one. Besides, the scaling factor can be doubled in the NOMA scheme if the BS transmits another resource at the second time slot.

4 Simulation Results

In this section, we verify that the analytical derivations are consistent with the system simulation, then, we study the impact of the system parameters on the average achievable sum rate. This is confirmed by carrying out Monte Carlo simulations with 10^6 experimental trials. A proper co-ordination for the position of nodes is selected to justify the simulation results where all the coefficients, Ω_{qr}, are modeled as, $\Omega_{qr}=\left(\frac{1}{d_{q,r}}\right)^{\delta}$, $\delta=2$, where δ is the path loss exponent and $d_{q,r}$ is the distance between the node q and the node r.

The proposed NOMA scheme is compared with OMA conventional scheme where the benefit of applying NOMA scheme is clarified in the cooperative downlink system. The comparison is carried out against three different parameters, i.e., namely, the transmit SINR, $\frac{P}{N_0^2}$, the power allocation resource a_{si} and the channel coefficient Ω_{bi}. It is noted that, when we keep on the allocated power coefficient to the primary user dynamically proportional to its channel fading coefficient in order to achieve a certain QoS at the primary user, the average achievable sum rate is no longer dependent on, e.g., or limited by, the relayed path to the primary user. Therefore, the achievable rate of the primary user can be at least better than or equal the individual rate of the primary user OMA case. Meanwhile, we have the opportunity to select the secondary user that maximizes the achievable rate of the any-casting signal. Thus, an expected increase in the average achievable sum rate of NOMA scheme can be obtained.

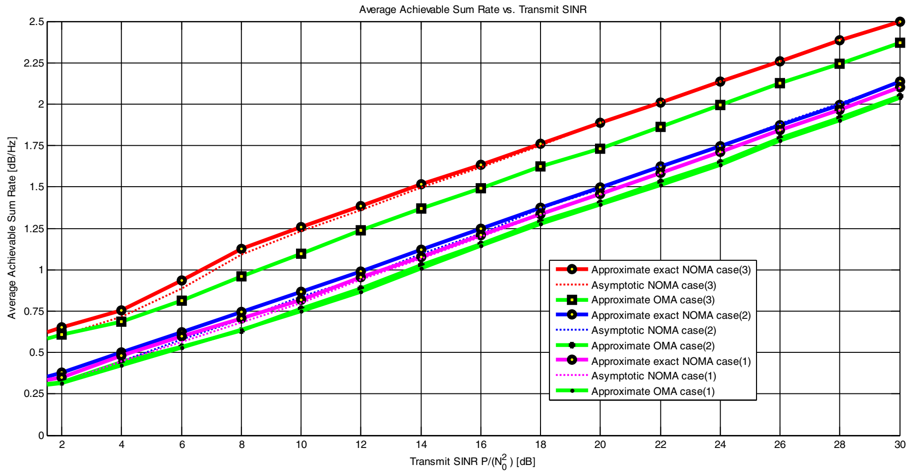

Fig. 2 Theoretical and simulation results of the average achievable sum rate versus the transmit SINR for both NOMA and conventional OMA schemes

From the previous analysis, one can find that the value of Ω_{bi} has a significant effect on the achievable sum rate performance especially when $\Omega_{bi} \ll \Omega_{ip}$. This is reasonable since it increases the effectiveness of exploiting the NOMA scheme, Three different cases of channel coefficients are considered, for $\Omega_{sp} = 1$, (1) $\Omega_{bi} = \Omega_{ip} = 2$, (2) $\Omega_{bi} = 2, \Omega_{ip} = 10$, (3) $\Omega_{bi} = 10$ and $\Omega_{ip} = 2$.

Figure 2 illustrates the relation between the average achievable sum rate and the actual transmit SINR, $\frac{P}{N_0^2}$, at $a_{sp} = 0.8$, $a_{si} = 0.2$, a_{ij} and a_{ip} depends on $\mathbb{I} = 0.5$. In particular, the average achievable sum rates of the NOMA schemes, i.e., R_{sum} and R_{sum}^{∞}, are compared with the sum of individual rates of the OMA schemes, i.e., $\Re_{sum}$ and $\Re_{sum}\, at\ \frac{N_0^2}{P} \to 0$, where $\Re_{sum} = \left(R_{averOMA,s} + R_{averOMA,p}\right)$, at both the primary and secondary users. There exists a successful agreement between the theoretical and simulation points (denoted by square, circle and star yellow bubbles) which justifies our derivations. For high transmit SINR, the asymptotic expressions coincide with the exact ones. It is observed that the average achievable sum rate of the proposed NOMA scheme always outperforms the OMA conventional one due to the additional amount of relayed power resources and/or the existence of undesirable interference in OMA scheme recovered by SIC in NOMA scheme. In the proposed NOMA, a remarkable gain can be significantly observed over OMA scheme especially when $\Omega_{bi} \ll \Omega_{ip}$ [or equivalently, $\frac{\Omega_{bi}}{\Omega_{ip}} \ll 1$ (case 3)] than the other cases at high SINR.

Exclusive search for optimal values of the power resource coefficient a_{si} offers a remarkable gain for the average achievable sum rate which depends inherently on the channel coefficients h_{bi} for a specific user i as depicted in Fig. 3. At $\frac{P}{N_0^2} = 30\,\text{dB}$, this figure indicates a tight local maximum value for the achievable sum rate which represents an optimal adjustment of the transmit power allocation coefficient a_{si}. The gain can achieve a significant enhancement by increasing both of $\frac{P}{N_0^2}$ and/or $\frac{\Omega_{bi}}{\Omega_{ip}}$. Due to space limitation, the analysis of finding out the optimal values of a_{si} is beyond our interest. It can be obtained analytically by solving for a_{si} that maximizes R_{sum}. Again, the analytical results are perfectly closed to the simulation points.

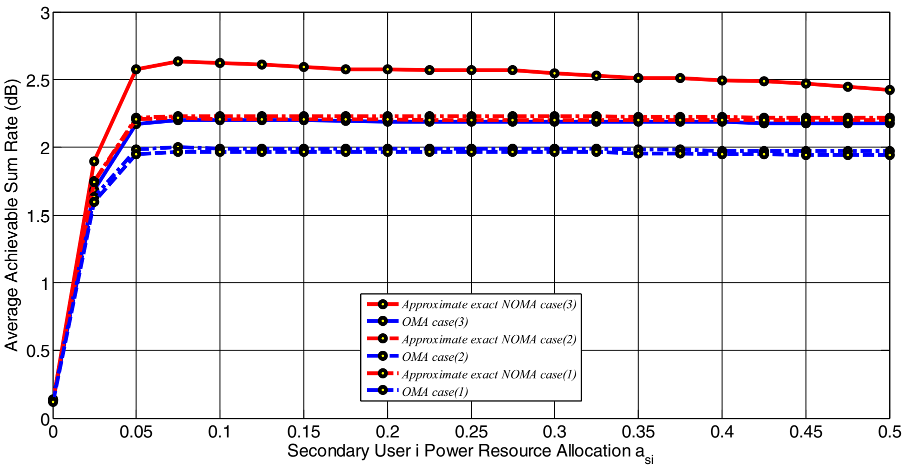

Fig. 3 Performance comparison of the average achievable sum rate versus the secondary user i power resource, a_{si} for NOMA/OMA schemes

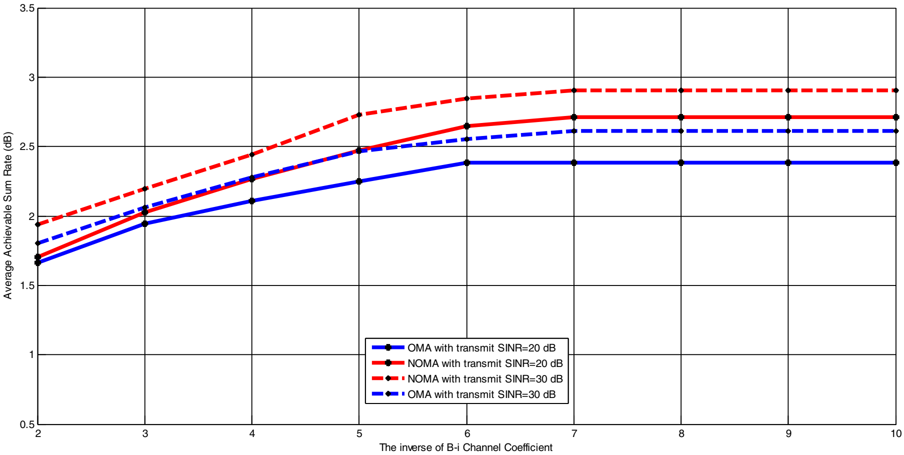

Fig. 4 Average sum rate versus the inverse channel fading coefficient, i.e., $1/\Omega_{bi}$, of the base station $BS \rightarrow i$ (secondary user i) link

In Fig. 4, for fixed $\Omega_{ip} = 2$ and $\Omega_{sp} = 2$, the inverse of B-i channel coefficient, i.e., $1/\Omega_{bi}$, is examined for two different transmit levels of $\frac{P}{N_0^2}$, i.e., $\frac{P}{N_0^2} = 20$ and 30 dB. It is turned out that the performance will be noticeably improved compared with the conventional OMA scheme as Ω_{bi} decreases.

In our proposed NOMA scheme, the reliability of the received signals at both the primary and secondary users can be enhanced when the BS transmits another resource at the second time slot.

5 Conclusions

In this paper, two equivalent cooperative relay selection schemes have been proposed for a downlink controlled cognitive radio based NOMA system. By exploiting the channel characteristics, new closed form expressions for the average achievable sum rate have been derived and compared with the existing OMA schemes at high SINR. Via simulation results, the benefit of employing the NOMA scheme in the considered downlink system has been investigated where employing the NOMA scheme acquires higher average achievable sum rate, saves the time resources and maximizes the network throughput.

Appendix 1

Let $\frac{|h_{ip}|^2 P_R - N_0^2\mathbb{I}}{|h_{ip}|^2 P_R(\mathbb{I}+1)} > 0$, then, the R.V. $\mathrm{Z_i}$ can be expressed by substitution as, $\mathrm{Z_i} \triangleq \left(min\left(\gamma_{bi}^{(s)}, \gamma_{ij}\right)\right) = min\left(\frac{|h_{bi}|^2 P_b a_{si}}{N_0^2}, \frac{|h_{ij}|^2 P_R}{N_0^2} \frac{|h_{ip}|^2 P_R - N_0^2\mathbb{I}}{|h_{ip}|^2 P_R(\mathbb{I}+1)}\right)$. Thus, we have to derive the CDF of $\gamma_{bi}^{(s)}$ and γ_{ij}, separately.

Let $X_i = \frac{|h_{ip}|^2 P_R - N_0^2\mathbb{I}}{|h_{ip}|^2 P_R(\mathbb{I}+1)}$, then, the CDF of X_i will take the form, $Pr\left(\frac{|h_{ip}|^2 P_R - N_0^2\mathbb{I}}{|h_{ip}|^2 P_R(\mathbb{I}+1)} \leq x\right) \Rightarrow Pr\left(|h_{ip}|^2 P_R - N_0^2\mathbb{I} \leq \left(|h_{ip}|^2 P_R(\mathbb{I}+1)\right)x\right) \Rightarrow Pr\left(|h_{ip}|^2 \leq \frac{N_0^2\mathbb{I}}{P_R(1-x((\mathbb{I}+1)))}\right)$. Then, such that $(1 - x((\mathbb{I}+1))) > 0$, the CDF is, $F_{X_i}\left(\frac{N_0^2\mathbb{I}}{P_R(1-x((\mathbb{I}+1)))}\right) = 1 - \exp\left(-\frac{\Omega_{ip} N_0^2\mathbb{I}}{P_R(1-x((\mathbb{I}+1)))}\right)$.

Let $Y_{ij} = \frac{|h_{ij}|^2 P_R}{N_0^2}$, then, the probability density function (PDF) of Y_i can be expressed as, $f_{Y_{ij}}(y) = \frac{N_0^2}{P_R\Omega_{ij}} \exp\left(-\frac{N_0^2}{P_R\Omega_{ij}} y\right)$.

The CDF of $\gamma_{ij} \triangleq X_i Y_{ij}$, $j \in \mathcal{N} - \{i\}$, can be calculated using the integral, $F_{\gamma_{ij}}(z) = \int_0^\infty F_{X_i}\left(\frac{z}{x}\right) f_{Y_{ij}}(x)dx$, as, $F_{\gamma_{ij}}(z) = 1 - \int_0^\infty \exp\left(-\frac{\Omega_{ip} N_0^2\mathbb{I}}{P_R\left(1-\frac{z}{x}((\mathbb{I}+1))\right)}\right) \frac{N_0^2}{P_R\Omega_{ij}} \exp\left(-\frac{N_0^2}{P_R\Omega_{ij}} x\right) dx = 1 - \int_{z((\mathbb{I}+1))}^\infty \exp\left(-\frac{\Omega_{ip} N_0^2\mathbb{I}x}{P_R(x-z((\mathbb{I}+1)))}\right) \frac{N_0^2}{P_R\Omega_{ij}} \exp\left(-\frac{N_0^2}{P_R\Omega_{ij}} x\right) dx$, such that $x > z((\mathbb{I}+1))$, $\int_0^\infty \frac{N_0^2}{P_R\Omega_{ij}} \exp\left(-\frac{N_0^2}{P_R\Omega_{ij}} x\right) dx = 1$. Putting $\theta = x - z((\mathbb{I}+1))$, the CDF of γ_{ij} becomes, $F_{\gamma_{ij}}(z) = 1 - \frac{\Omega_{ij} N_0^2}{P_R} \exp\left(-\frac{N_0^2}{P_R}\left(\mathbb{I}\Omega_{ip} + z(\mathbb{I}+1)\Omega_{ij}\right)\right) \times \int_0^\infty \exp\left(-\frac{N_0^2}{P_R}\left(\Omega_{ij}\theta + \frac{\Omega_{ip} z\mathbb{I}((\mathbb{I}+1))}{\theta}\right)\right) d\theta$.

Thus, $\bar{F}_{\gamma_{ij}}(z)$ follows after some algebraic manipulations and by using [22, 3.471.9]. Similarly, the CDF of $\gamma_{bi}^{(s)}$ can be expressed as,

$F_{\gamma_{bi}^{(s)}}(z) = 1 - \exp\left(-\frac{\Omega_{bi} N_0^2 z}{P_b a_{si}}\right) = 1 - \bar{F}_{\gamma_{bi}^{(s)}}(z)$. Consequently, the CDF of $\mathrm{Z_i} \triangleq \left(min\left(\gamma_{bi}, \gamma_{ij}\right)\right)$ is obtained as,

$$F_{Z_i}(z) = Pr\left(\left(\gamma_{bi}^{(s)} > z, \gamma_{ij} > z\right)\right) = 1 - \bar{F}_{Z_i}(z) = 1 - \bar{F}_{\gamma_{bi}^{(s)}}(z)\bar{F}_{\gamma_{ij}}(z).$$

Finally, by applying, $Pr(max_i(Z_i) \leq z) \Rightarrow max_i(F_{Z_1}(z), F_{Z_2}(z), \ldots F_{Z_N}(z)) = \prod_{i=1}^N F_{Z_i}(z)$, (17) follows and the proof is completed.

Appendix 2

From (20), let $Z_p \triangleq \left(min\left(\gamma_{bi}^{(p)}, \gamma_{ip} + \gamma_{bp}\right)\right) = min\left(\frac{|h_{bi}|^2 P_b a_{sp}}{|h_{bi}|^2 P_b a_{si} + N_0^2}, \frac{|h_{ip}|^2 P_R a_{ip}}{|h_{ip}|^2 P_R a_{ij} + N_0^2} + \frac{|h_{bp}|^2 P_b a_{sp}}{|h_{bp}|^2 P_b a_{si} + N_0^2}\right)$, then, the CDF of the R.V. Z_p can be expressed as, $F_{Z_p}(z_1, z_2) = 1 - \bar{F}_{Z_p}(z_1, z_2)$ $= 1 - Pr\left(\left(\gamma_{bi}^{(p)} > z_1, \gamma_{ip} + \gamma_{bp} > z_2\right)\right) = 1 - \bar{F}_{\gamma_{bi}^{(p)}}(z_1)\bar{F}_{\gamma_{ip}+\gamma_{bp}}(z_2)$, where $\left\{\left\{\gamma_{bi}^{(p)}\right\}\right.$ and $\left.\left\{\gamma_{ip} + \gamma_{bp}\right\}\right\}$ are two disjoint events, z_1 and z_2 are two distinct predefined SINRs. When the communication requires one desired or predefined SINR level, $z = z_1 = z_2$, which leads to $\bar{F}_{Z_p}(z) = \bar{F}_{\gamma_{bi}^{(p)}}(z)\bar{F}_{\gamma_{ip}+\gamma_{bp}}(z)$. Thus, we have to derive the CDF of $\gamma_{bi}^{(p)}$ and $\gamma_{ip} + \gamma_{bp}$, separately.

Let $P_b = P_R = P$, $\bar{F}_{\gamma_{bi}^{(p)}}(z)$ can be obtained simply as,

$$\bar{F}_{\gamma_{bi}^{(p)}}(z) = Pr\left(\gamma_{bi}^{(p)} > z\right) = Pr\left(\frac{|h_{bi}|^2 P a_{sp}}{|h_{bi}|^2 P a_{si} + N_0^2} > z\right) \Rightarrow \bar{F}_{\gamma_{bi}^{(p)}}(z)$$
$$= Pr\left(|h_{bi}|^2 > \frac{zN_0^2}{P(a_{sp} - za_{si})}\right) = \exp\left(-\frac{\Omega_{bi} z N_0^2}{P(a_{sp} - za_{si})}\right)$$

Since $\gamma_{ip}\ and\ \gamma_{bp}$ are two disjoint random variables and $\bar{F}_{\gamma_{ip}+\gamma_{bp}}(z) = Pr\left(\frac{|h_{ip}|^2 P a_{ip}}{|h_{ip}|^2 P a_{ij} + N_0^2} + \frac{|h_{bp}|^2 P a_{sp}}{|h_{bp}|^2 P a_{si} + N_0^2} > z\right)$, the CDF $\bar{F}_{\gamma_{ip}+\gamma_{bp}}(z)$ can be determined using the integral, $\bar{F}_{\gamma_{ip}+\gamma_{bp}}(z) = 1 - \int\limits_0^\infty F_{\gamma_{bp}}(z - x) f_{\gamma_{ip}}(z)dx.$

By following similar steps as $\bar{F}_{\gamma_{bi}^{(p)}}(z)$, $\bar{F}_{\gamma_{bp}}(z)$ and $\bar{F}_{\gamma_{ip}}(z)$ will take the forms of, $\bar{F}_{\gamma_{bp}}(z) = \exp\left(-\frac{\Omega_{bp} z N_0^2}{P(a_{sp} - za_{si})}\right)$ and $\bar{F}_{\gamma_{ip}}(z) = \exp\left(-\frac{\Omega_{ip} z N_0^2}{P(a_{ip} - za_{ij})}\right)$, respectively.

Taking the derivative of $F_{\gamma_{ip}}(z) = 1 - \bar{F}_{\gamma_{ip}}(z)$, the PDF can be expressed as, $f_{\gamma_{ip}}(z)$ $= \frac{a_{ip} N_0^2}{P(a_{ip} - za_{ij})^2} \exp\left(-\frac{\Omega_{ip} z N_0^2}{P(a_{ip} - za_{ij})}\right)$. $\bar{F}_{\gamma_{ip}+\gamma_{bp}}(z) = 1 - \int\limits_0^\infty F_{\gamma_{bp}}(z - x) f_{\gamma_{ip}}(z)dx = \exp\left(\frac{\Omega_{ip} N_0^2}{P a_{ij}}\right)$

$$+ \underbrace{\int\limits_0^\infty \frac{a_{ip} N_0^2}{P(a_{ip} - xa_{ij})^2} \exp\left(-\frac{N_0^2}{P}\left(\frac{\Omega_{ip} x}{(a_{ip} - xa_{ij})} + \frac{\Omega_{bp}(z - x)}{(a_{sp} - (z - x)a_{si})}\right)\right)dx.}_{\mathcal{J}(z)}$$

The integral $\mathcal{J}(z)$ can be solved for certain z by numerical software programming or it can be approximated by expanding exp(.) for $\frac{N_0^2}{P} \ll 1$.

For arbitrary $z \in [0, \infty]$, since $\frac{|h_{ip}|^2 P a_{ip}}{|h_{ip}|^2 P a_{ij} + N_0^2} \le \frac{a_{ip}}{a_{ij}}$, $\frac{|h_{bp}|^2 P a_{sp}}{|h_{bp}|^2 P a_{si} + N_0^2} \le \frac{a_{sp}}{a_{si}}$ and noting that $Pr\left(\left(\gamma_{ip} + \gamma_{bp}\right) = z\right) = 0$, i.e., for continuous R.V.s, $\bar{F}_{\gamma_{ip}+\gamma_{bp}}(z)$ can be obtained by the following three cases:

I. $z < \frac{a_{ip}}{a_{ij}} < \frac{a_{sp}}{a_{si}}$

In such a case, if $\frac{|h_{bp}|^2 Pa_{sp}}{|h_{bp}|^2 Pa_{si}+N_0^2} > z$ or $\frac{|h_{ip}|^2 Pa_{ip}}{|h_{ip}|^2 Pa_{ij}+N_0^2} > z$, then, $Pr\left(\frac{|h_{ip}|^2 Pa_{ip}}{|h_{ip}|^2 Pa_{ij}+N_0^2} + \frac{|h_{bp}|^2 Pa_{sp}}{|h_{bp}|^2 Pa_{si}+N_0^2} > z\right) = 1$. Thus, $\bar{F}_{\gamma_{ip}+\gamma_{bp}}(z) = Pr\left(\frac{|h_{bp}|^2 Pa_{sp}}{|h_{bp}|^2 Pa_{si}+N_0^2} > z\right) + Pr\left(\frac{|h_{bp}|^2 Pa_{sp}}{|h_{bp}|^2 Pa_{si}+N_0^2} < z\right)$ $Pr\left(\frac{|h_{ip}|^2 Pa_{ip}}{|h_{ip}|^2 Pa_{ij}+N_0^2} > z\right) + Pr\left(\frac{|h_{ip}|^2 Pa_{ip}}{|h_{ip}|^2 Pa_{ij}+N_0^2} < z\right) \times Pr\left(\frac{|h_{bp}|^2 Pa_{sp}}{|h_{bp}|^2 Pa_{si}+N_0^2} < z\right) \times Pr\left(\frac{|h_{ip}|^2 Pa_{ip}}{|h_{ip}|^2 Pa_{ij}+N_0^2} + \frac{|h_{bp}|^2 Pa_{sp}}{|h_{bp}|^2 Pa_{si}+N_0^2} > z\right) = \exp\left(-\frac{\Omega_{bp} z N_0^2}{P(a_{sp}-za_{si})}\right) + \exp\left(-\frac{\Omega_{ip} z N_0^2}{P(a_{ip}-za_{ij})}\right) - \exp\left(-\frac{\Omega_{ip} z N_0^2}{P(a_{ip}-za_{ij})}\right) \times$ $\exp\left(-\frac{\Omega_{bp} z N_0^2}{P(a_{sp}-za_{si})}\right) + \left(\exp\left(\frac{\Omega_{ip} N_0^2}{Pa_{ij}}\right) + \mathcal{J}(z)\right) \times \left(1 - \exp\left(-\frac{\Omega_{ip} z N_0^2}{P(a_{ip}-za_{ij})}\right) - \exp\left(-\frac{\Omega_{bp} z N_0^2}{P(a_{sp}-za_{si})}\right)\right.$ $\left. + \exp\left(-\frac{\Omega_{ip} z N_0^2}{P(a_{ip}-za_{ij})}\right) \times \exp\left(-\frac{\Omega_{bp} z N_0^2}{P(a_{sp}-za_{si})}\right)\right)$

II. $\frac{a_{ip}}{a_{ij}} < z < \frac{a_{sp}}{a_{si}}$

$$\begin{aligned}
\bar{F}_{\gamma_{ip}+\gamma_{bp}}(z) &= \Pr\left(\frac{|h_{bp}|^2 Pa_{sp}}{|h_{bp}|^2 Pa_{si}+N_0^2} > z\right) + \Pr\left(\frac{|h_{bp}|^2 Pa_{sp}}{|h_{bp}|^2 Pa_{si}+N_0^2} < z\right) \\
&\quad \times \Pr\left(\frac{|h_{ip}|^2 Pa_{ip}}{|h_{ip}|^2 Pa_{ij}+N_0^2} + \frac{|h_{bp}|^2 Pa_{sp}}{|h_{bp}|^2 Pa_{si}+N_0^2} > z\right) \\
&= \exp\left(-\frac{\Omega_{bp} z N_0^2}{P(a_{sp}-za_{si})}\right) + \left(\exp\left(\frac{\Omega_{ip} N_0^2}{Pa_{ij}}\right) + \mathcal{J}(z)\right) \\
&\quad - \left(\exp\left(\frac{\Omega_{ip} N_0^2}{Pa_{ij}}\right) + \mathcal{J}(z)\right) \times \exp\left(-\frac{\Omega_{bp} z N_0^2}{P(a_{sp}-za_{si})}\right)
\end{aligned}$$

III. $\frac{a_{ip}}{a_{ij}} < \frac{a_{sp}}{a_{si}} < z$

$\bar{F}_{\gamma_{ip}+\gamma_{bp}}(z) = Pr\left(\frac{|h_{ip}|^2 Pa_{ip}}{|h_{ip}|^2 Pa_{ij}+N_0^2} + \frac{|h_{bp}|^2 Pa_{sp}}{|h_{bp}|^2 Pa_{si}+N_0^2} > z\right) = \left(\exp\left(\frac{\Omega_{ip} N_0^2}{Pa_{ij}}\right) + \mathcal{J}(z)\right)$.

The proof is completed.

References

1. Ding, G. Z., Yang, Z., Fan, P., & Poor, H. V. (2014). On the performance of non-orthogonal multiple access in 5G systems with randomly deployed users. *IEEE Signal Processing Letters, 21*(12), 1501–1505.
2. Saito, Y., Benjebbour, A., Kishiyama, Y., & Nakamura, T. (2013). System level performance evaluation of downlink non-orthogonal multiple access (NOMA). In *Proceedings of IEEE annual symposium on personal indoor mobile radio communications (PIMRC)*, London, U.K., (pp. 611–615).
3. Saito, Y., Kishiyama, Y., Benjebbour, A., Nakamura, T., Li, A., & Higuchi, K. (2013). Non-orthogonal multiple access (NOMA) for cellular future radio access. In *Proceedings of IEEE vehicle technology conference*, Dresden, Germany.
4. Ding, Z., Fan, P., & Poor, V. (2015). Impact of user pairing on 5G non-orthogonal multiple access downlink transmissions. *IEEE Transactions on Vehicle Technology* 99, 1.

5. Timotheou, S., & Krikidis, I. (2015). Fairness for non-orthogonal multiple access in 5G systems. *IEEE Signal Processing Letters, 22*(10), 1647–1651.
6. Al-Imari, M., Xiao, P., Imran, M., & Tafazolli, R. (2014). Uplink non-orthogonal multiple access for 5G wireless networks. In *Proceedings on 11th international symposium wireless communication systems (ISWCS)*, (pp. 781–785).
7. Choi, J. (2014). Non-orthogonal multiple access in downlink coordinated two point systems. *IEEE Communications Letters, 18*(2), 313–316.
8. Wei, Z., Yuan, J., Ng, D.W.K., Elkashlan, M., & Ding, Z. (2016). A survey of downlink non-orthogonal multiple access for 5G wireless communication networks. arXiv:1609.01856v1 [cs.IT].
9. Goldsmith, A., Jafar, S. A., Maric, I., & Srinivasa, S. (2009). Breaking spectrum gridlock with cognitive radios: an information theoretic perspective. *Proceedings of the IEEE, 97*(5), 894–914.
10. Seyfi, Mehdi, Muhaidat, Sami, & Liang, Jie. (2013). Relay selection in cognitive radio networks with interference constraints. *IET Communications, 7*(10), 922–930.
11. Liu, Y., Ding, Z., Elkashlan, M., & Yuan, J. (2016). Non-orthogonal multiple access in large-scale underlay cognitive radio networks. *IEEE Transactions on Vehicular Technology, 65*(12), 10152–10157.
12. Ding, Z., Peng, M., & Poor, H. V. (2015). Cooperative non-orthogonal multiple access in 5G systems. *IEEE Communications Letters, 19*(8), 1462–1465.
13. Kim, J.-B., & Lee, I.-H. (2015). Non-orthogonal multiple access in coordinated direct and relay transmission. *IEEE Communications Letters, 19*(11), 2037–2040.
14. Liu, Y., Ding, Z., Elkashlan, M., & Poor, H. V. (2016). Cooperative non-orthogonal multiple access with simultaneous wireless information and power transfer. *IEEE Journal on Selected Areas in Communications, 34*(4), 938–953.
15. Men, J., & Ge, J. (2015). Non-orthogonal multiple access for multiple-antenna relaying networks. *IEEE Communications Letters, 19*(10), 1686–1689.
16. Ding, Z., Dai, H., & Poor, H. V. (2016). Relay selection for cooperative NOMA. *IEEE Wireless Communication Letters, 5*(4), 416–419.
17. Kim, J.-B., & Lee, I.-H. (2015). Capacity analysis of cooperative relaying systems using non-orthogonal multiple access. *IEEE Communications Letters, 19*(11), 1949–1952.
18. Ding, Z., Adachi, F., & Poor, H. V. (2016). The application of MIMO to non-orthogonal multiple access. *IEEE Transactions on Wireless Communications, 15*(1), 537–552.
19. Xu, M., Ji, F., & Wen, M. (2016). Novel receiver design for the cooperative relaying system with non-orthogonal multiple access. *IEEE Communications Letters, 20*(8), 1679–1682.
20. Zhang, J., et al. (2017). *Performance analysis of relay assisted cooperative non-orthogonal multiple access systems*. Submitted to IEEE Wireless Communication Letters.
21. Xu, P., Ding, Z., Dai, X., & Poor, H. (2015). *NOMA: An information theoretic perspective*," [Online]. http://arxiv.org/abs/1504.07751.
22. Gradshteyn, I. S., & Ryzhik, I. M. (2007). *Table of integrals, series, and products* (7th ed.). Cambridge: Academic Press.
23. David, H. A., & Nagaraja, H. N. (2003). *Order statistics* (3rd ed.). New Jeresy: Wiley.
24. Hildebrand, E. (1987). *Introduction to numerical analysis*. New York, NY: Dover.
25. Sun, Q., Han, S., Chin-Lin, I., & Pan, Z. (2015). On the ergodic capacity of MIMO NOMA systems. *IEEE Wireless Communications Letters, 4*(4), 405–408.

Mona Shokair received the B.E. and M.E. degrees in electronics engineering from El-Menoufia University, El-Menoufia, Egypt, in 1993 and 1997, respectively. She received Ph.D. from Kyushu University, Japan, 2005. She was lecturer in El-Menoufia University from 2005 to 2010. She was Associated Professor in El-Menoufia University from 2011 to 2015. Since 2016, she received a Professor degree. Now, she is a head of Electrical and Electronic Communication Department of El-Menoufia University. She received VTS chapter IEEE award from Japan in 2003. Also, she received with co-authors the best paper in NRSC in Egypt 2016 and 2017, respectively. She published more than 130 papers until Jan. 2018. Now her current research is in OFDM system, WIMAX system, cognitive radios, D2D Communication and Wireless Sensor Network.

Waleed Saad has received his BSc (Hons), M.Sc. and Ph.D. degrees from the Faculty of Electronic Engineering Menoufia University, Menouf, Egypt, in 2004, 2008 and 2013, respectively. He joined the teaching staff of the Department of Electronics and Electrical Communications of the same faculty since 2014. In 2005 and 2008, he worked as a demonstrator and assistant lecturer in the same faculty, respectively. He is a co-author of many papers in national and international conference proceedings and journals. His research areas of interest include mobile communication systems, computer networks, cognitive radio networks, D2D communication, OFDM systems, interference cancellation, resource allocations, PAPR reduction, physical and MAC layers design, and implementation of digital communication systems using FPGA.

Shady M. Ibraheem received the B.S. degree in electronics and communications engineering from Tanta University, Egypt, 2003. Since then, he has worked as a maintenance engineer in Telecom Egypt for 15 years. Currently, he is looking forward to the M.S. degree at Menoufia University. His main research interests include channel coding and cognitive radio networks.