CrossMark

Application of Multifractal Spectrum Calculation Program in Rubber Wear Under High Temperature

Zepeng Wang[1] · **Shunxiao Hu**[1] · **Zhanli Miao**[1] · **Lianxiang Ma**[1]

Abstract Due to the complexity of the rubber wear process, the research work of the wear mechanism is limited to normal temperature, and the mechanism of high temperature wear has not been established. The temperature has great influence on the appearance of wear and the change of the surface characteristics of wear. This paper presents the wear surface morphology of rubber materials at different temperatures. The two-dimensional gray level images of the rubber surface are captured by a 3D measuring laser microscope and are converted to black-and-white binary images. Then, the quantitative analysis of the wear surface is processed using fractal dimension and the multifractal spectrum method. Results show that the wear rubber surface is fractal. Additionally, the roughness and the spectral width values increases with increase in temperature. Furthermore, the fractal dimensions shrink and the homogeneity of the rubber surface worsens. As temperature increases, the corresponding $\Delta\alpha$ and $\Delta f(\alpha)$ increase, resulting in rough wear surface with lower height homogeneity and higher complexity. $\Delta\alpha$ is equal to 0.238, 0.281 and 0.283 at the temperature of 25, 60 and 80 °C, respectively. $\Delta f(\alpha)$ is equal to 0.027, 0.035 and 0.039 at the temperature of 25, 60 and 80 °C, respectively. In conclusion, fractal theory can be used to describe and evaluate the surface morphology and wear characteristics of rubber materials.

✉ Shunxiao Hu
  18563980665@163.com

[1] School of Electromechanical Engineering, Qingdao University of Science and Technology, Qingdao 266061, China

🐾 Springer

1 Introduction

Wear performance is an important index of many rubber products, especially for the tire. It directly affects the safety and life of products. Many scholars have conducted a lot of theoretical and experimental researches on the wear of rubber in order to better understand the wear performance [1]. Also, evaluation system of wear characteristics for rubber composites at room temperature is established. However, there still exists a big difference between the actual wear and normal wear of tires and other rubber products. The reason is the hysteresis heat which causes high temperature in the rolling process of tire and leads to wear. Previous wear evaluation system cannot accurately reflect the high temperature abrasion performance and characteristics [2–4]. Therefore, it is necessary to deeply study wear performance and mechanism of rubber composites at the high temperature.

Akron and DIN abrasion tests were carried out to test wear performance of the rubber composite with different filler (nano-carbon black, alumina, clay and so on), different base material (natural rubber, Styrene Butadiene rubber, silicone rubber, Ethylene Propylene Diene Monomer, etc.), and different test conditions (pressure, surface severity) [5, 6]. Atomic Force Microscope (AFM) and Scanning Electron Microscope (SEM) were employed to investigate the wear surface and further reveal the wear mechanism. The surface abrasion pattern of rubber composite is not always generated. For instance, the wear surface does not produce abrasion pattern at low loads; instead, the wear pattern appears when the load exceeds beyond a critical value. A series of abrasion pattern are observed which are perpendicular to the sliding direction and mutually parallel [7–9].

The above research works only focused on examining the wear surface and did not explore the relationship and laws between the morphology and wear performance.

Fractal geometry is an effective tool to study the surface morphology of rubber composites. The basic theory of fractal geometry is an important branch of nonlinear science, which is widely used to describe and distinguish complicated morphology in nature since its introduction by Mandelbrot in the 1970s. The wear surface of the rubber composites has characteristics of self-similarity and self-affinity. Both local and overall characteristics of rubber surface can be described by multifractal characteristics [10–12].

Fractal theory is mainly applied to study the scaling properties of an irregular shape, a physical quantity or a natural phenomenon. This scaling property can be expressed mathematically by the concept of self-similarity or self-affinity. Despite magnification of the object, this behavior suggests that a fractal object appears statistically similar [13].

Rubber wear surface is similar to wear surface of the cylinder liner and the machined surface. Similarly, it has the fractal characteristics of randomness, self-similarity, self-affinity and multi-scale. Therefore, Fractal theory and method can be used to describe and evaluate the morphology characteristics of rubber wear surface [14].

Tire rubber wear testing is mainly performed at room temperature, which is different from its actual usability temperature. In the actual rolling process, tire temperature continuously rises to 60–80 °C due to the periodic stress and strain hysteresis of rubber. Therefore, in current work, a new wear testing instrument is designed for a more realistic study.

The morphology of rubber wear surface is also analyzed at different temperatures by studying the homogeneity and irregularity. Wear surface morphology of the rubber composites is viewed through an Olympus LEXT OLS4100 3D measuring laser microscope.

The characteristic parameters of multifractal spectrum are then applied to describe the complicated and irregular rubber wear. And spatial phenomenon is defined. The relation between characteristic parameters and wear is established and the fractal characterization of wear rubber surface is realized.

2 Materials and Methods

Fractality can be described briefly as a rough or discrete geometry figure that can be subdivided into infinite parts, and each part maintains certain similarity to the overall structure. Fractal dimension of a straight line, area, volume is respectively 1, 2, 3 in Euclidean geometry. Contrary to the Euclidean geometry, the dimensions of fractal geometry are all fractional. If the topology dimension of an image is D_t, the dimensions of fractal geometry is D, then $D_t < D < D_t + 1$.

In the box-counting method of multifractal theory, the fractal image is covered by square box with side length of ε, and n_{ij} represents the number of pixels of the fractal in the box of (i, j), $\sum n_{ij}$ represents the number of pixels of all fractal numbers. The fractal image probability measure $P_{ij}(\varepsilon)$ in every $\varepsilon \times \varepsilon$ box is defined as,

$$P_{ij}(\varepsilon) = \frac{n_{ij}}{\sum n_{ij}} \tag{1}$$

In a scale-free, self-similar area, the probability measure $P_{ij}(\varepsilon)$ can be written as,

$$P_{ij}(\varepsilon) \propto \varepsilon_{ij}^{\alpha_{ij}} \tag{2}$$

where α_{ij} is the dimension of the fractal image in a small square box, which is called the local fractal dimension also the singularity exponent,

$$\alpha = \lim_{\varepsilon \to 0} \frac{\log P(\varepsilon)}{\log \varepsilon} \tag{3}$$

Then, $\Delta\alpha = \alpha_{max} - \alpha_{min}$ defines the disorder of probability measures in overall fractal area, as well as the complexity of the surface. A larger $\Delta\alpha$ represents a lower surface uniformity of rubber.

Dimension function $f(\alpha)$ is defined as a subset of singularity exponent α,

$$f(\alpha) = -\lim_{\varepsilon \to 0} \frac{\log N_\alpha(\varepsilon)}{\log \varepsilon} \tag{4}$$

The complexity and irregularity of the wear surface is mainly shown by $\Delta f(\alpha) = f(\alpha_{min}) - f(\alpha_{max})$. When $\Delta f(\alpha) > 0$, the multifractal spectrum curve is left-hook-shaped, so the subset of maximum probability numbers are less than the minimal ones. Thus, most of the wear rubber surface is comprised of sharp peaks with the relatively lower height. When $\Delta f(\alpha) < 0$, the multifractal spectrum curve is right-hook-shaped, so the subset of maximum probability numbers is larger than the minimal ones. Hence, most of the wear rubber surface contains very high blunt peaks.

Because of the unique one to one correspondence between α and $f(\alpha)$, there is a dimension spectrum to describe multifractal properties, called the multifractal spectrum.

The sum of q square of $P_{ij}(\varepsilon)$ can be expressed in partition function $\chi_q(\varepsilon)$, and the partition function $\tau(q)$ is related to q as,

$$\tau(q) = \frac{\ln \chi_q(\varepsilon)}{\ln \varepsilon}(\varepsilon \to 0) \tag{5}$$

where $\tau(q)$ is called mass index; q is called weighting factor.

Where mass index $\tau(q)$ can be written in the form,

$$\alpha = \frac{d\tau(q)}{dq} \tag{6}$$

From the Legendre conversion of $\tau(q)$ and q, $f(\alpha)$ is written as,

$$f(\alpha) = \alpha q - \tau(q) \tag{7}$$

In this paper, multifractal spectrum of the wear rubber surface is obtained through previous processes. The gray level images were covered by small squares with the 1/2, 1/4, 1/8, 1/16, 1/32, 1/64, 1/128 of the wear surface length, and the corresponding probability $P_{ij}(\varepsilon)$ with weighting factor $q = [-20, 20]$ are then calculated using a Matlab program written for fractal dimension and multifractal spectrum analysis.

The experimental setup includes rubber mixer (type XSM-500), roll mill (type BL-6175-BL), plate vulcanizing apparatus (type HS-100T-FTMO-2PT), 3D measuring laser microscope (type LEXT OLS4100) and the new type wear testing instrument.

The new type wear testing instrument is designed for simulating the high temperature rolling friction process of the tire rubber. The new wear testing instrument is composed of an adjustable-speed motor, a speed controller, a sandglass support, a sand funnel, a grinding wheel and support, a counterbalance, a load lever with scale, heating wires with voltage regulator, thermocouples and temperature controller, a slip ring, an adjusting wheel and plate, a screw, a lock nut and adjusting nut, and a base.

This newly designed instrument can process rubber wear testing under different temperatures, loads, speeds, and angles. It can provide a more realistic and accurate description of the tire wear process and different aspects in practical situations.

The tread formula of all-steel radial tire is following: Natural rubber 100 parts, N330 carbon black 37.3 parts, White carbon black 15 parts, Zinc oxide 3.6 parts, Stearic acid 2 parts, Plasticizer A 2 parts, Antioxidant RD 1.5 parts, Antioxidant 6PPD 2 parts, Silane coupling agent 3 parts, N-(oxidiethylene)-2-benzothiazolyl sulfonamide 1.5 parts, Sulfur 1 part.

Rubber samples are prepared by the means of plate vulcanizing apparatus. To prepare rubber strip specimen, rubber compound is placed in the rheometer mold cavity for 15 min at temperature 150 °C, and pressure of 15 MPa.

The strips are then convolved and attached to a special rubber wheel before drying them in an oven for firm bonding.

Next, the wear testing is performed on pre-polished strips for one hour using a new testing instrument, and then strip samples are cleared for weighing using a balance with the accuracy of 0.001 g.

The imaging process is carried out with OLS4100 3D measuring laser microscope which is equipped with a dual confocal scanning system. The dual confocal scanning can remove the light spots in a fuzzy image and captures planes with same height. Due to these particular qualities, this microscope can be used to capture precise images from samples of materials with different reflective characteristics. Also, the dual confocal scanning generated by round light holes guarantees consistency in different directions.

With the OLS4100 laser microscope, a precise 3D image of the wear surface with a resolution of 10 nm in z-axis and an x–y plane resolution of 0.12 μm can be obtained in a noncontact manner by a laser of short-wavelength i.e. 405 nm.

The steps followed in gray scale image acquisition are as under:

1. The microscope control software is started to finish the initialization of system automatically.
2. The Z axis coarse adjusting knob is unlocked. The 20 times objective lens and zoom $1\times$ is selected in visible light mode. The coarse image is adjusted based on the Z axis.
3. The image to a clear state is tuned and adjusted to fine focus. Magnification is switched to 20 times before starting observation and measurement.
4. Laser mode is activated and fine scanning mode is selected. Because sample is uneven, the highest position (upper limit) and the lowest position of the sample (the lower limit) is properly set.
5. After the 'get' button is pressed, OLS4100 automatically starts taking photos. It took about 10 s to finish photo shooting process photo. Then, software automatically popped up pictures of samples and scale is generated.
6. Saving image.

The imaging principle of the microscope ensures the coaxial of light and excludes the interference of other light sources.

The gray value of the treat gray image has relatively fixed height. The 2D gray level images of the rubber wear surface captured by OLS4100 are shown in Fig. 1. These images record the morphology of the rubber wear surface at the temperature of 25, 60 and 80 °C, then converted to black-and-white binary images according to the gray level.

3 Results and Discussion

Fractal dimension of the wear rubber surface at different temperatures is calculated by a Matlab program. The geometric arithmetic average deviation R_a of 39 positions at three different temperatures is measured by OLS4100 3D measuring laser microscope. The fitting curve between the geometric arithmetic average deviation R_a and the corresponding fractal dimensions illustrate the roughness of the wear surface of rubber composite materials.
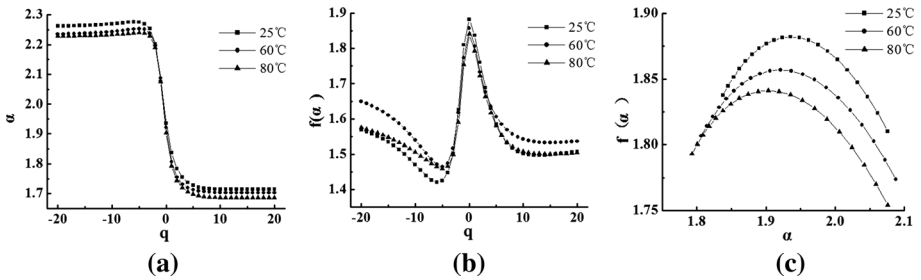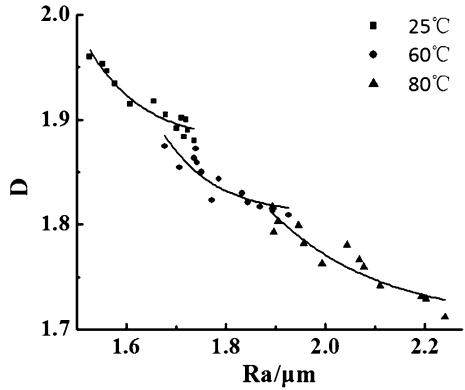
Figure 2 depicts that the smaller is the fractal dimension, the poor is the homogeneity of the rubber wear surface and vice versa.

The multifractal spectrum q–α, q–$f(\alpha)$ and α–$f(\alpha)$ is created from black-and-white binary images using a Matlab program, shown in Fig. 3.
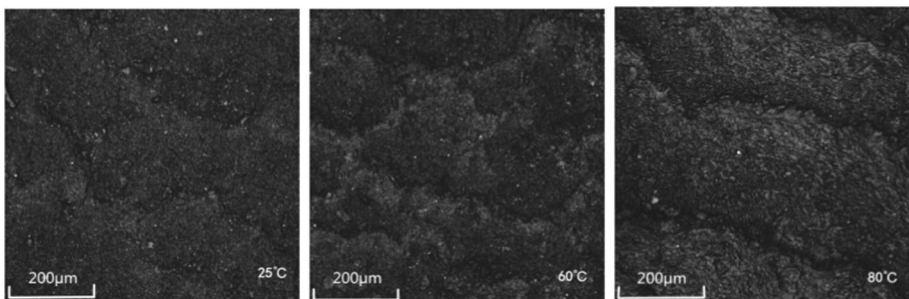
Fig. 1 2D microscope images of wear rubber surface

Fig. 2 R_a–D curve at different temperatures

Fig. 3 Multifractal curves. **a** q–α curves, **b** q–$f(\alpha)$ curves, **c** α–$f(\alpha)$ curves

From Fig. 3a, b, it can be seen that the rate of change of α and $f(\alpha)$ approaches 0 as the weighting factor q is close to -20 or 20. Hence, $q = [-20, 20]$ can meet the multifractal requirements of the gray level images. Moreover, α decreases steadily with the increase of q, which illustrates that the figure is multifractal. From Fig. 3c, it can be found that the multifractal spectrums are convex functions of α while the α–$f(\alpha)$ curves are left-hook-shaped.

Multifractal spectrum width $\Delta\alpha$ gives the size of probability distribution of the entire fractal structure. In other words, multifractal spectrum width $\Delta\alpha$ reflects the size of the probability distribution range of gray values. High uneven gray value probability distributions result in wider multifractal spectrum curve α–$f(\alpha)$ and increased $\Delta\alpha$, due to more dispersive surface height distributions However, more uniform gray value probability distributions result in narrower multifractal spectrum curve α–$f(\alpha)$ and smaller $\Delta\alpha$, due to more concentrated surface height distributions. if $\Delta\alpha = 0$, the probability distribution is uniform, and the surface height is constant. $\Delta\alpha$ represents the smoothness of rubber wear surface. The larger is $\Delta\alpha$, the poor is the distribution uniformity, and the high is rubber surface wear. $\Delta\alpha$ is maximum at 80 °C as shown in Fig. 3c, and therefore the rubber wear surface is highly coarse. The second remarkably high value of $\Delta\alpha$ is obtained at 60 °C, followed by the value at 25 °C. Because $\Delta f(\alpha) > 0$, the multifractal spectrum curve is left-hook-shaped, the subset of maximum probability numbers is less than the minimal ones. Thus, most of the wear surface of rubber is comprised of relatively lower spikes.

In addition, different temperatures process curves with different opening areas corresponds to different $\Delta\alpha$.

Table 1 Key values of α–f(α) multifractal spectrum

Temperature (°C)	α_{min}	α_{max}	$\Delta\alpha$	$f(\alpha_{min})$	$f(\alpha_{max})$	$\Delta f(\alpha)$
25	1.838	2.076	0.238	1.838	1.810	0.027
60	1.806	2.088	0.281	1.806	1.771	0.035
80	1.793	2.076	0.283	1.793	1.754	0.039

25℃ 60℃ 80℃
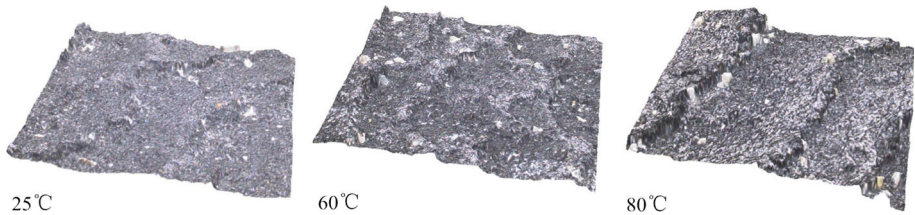
Fig. 4 3D microscope gray images of rubber wear surface

The key values of α and f(α) are extracted from α–f(α) curves using a Matlab program and the results are displayed in Table 1.

And, the 3D microscope gray images of wear rubber surfaces are shown in Fig. 4. It can be seen that, the values in Table 1 match up with the surface morphology characteristics in Fig. 4. As temperature increases, the corresponding $\Delta\alpha$ and $\Delta f(\alpha)$ increase, resulting in rough wear surface with lower height homogeneity and higher complexity.

From the multifractal spectrum curve shown in Fig. 3c, $\Delta\alpha$ is equal to 0.238, 0.281 and 0.283 at the temperature of 25, 60 and 80 °C, respectively. This implies that at 25 °C, rubber surface wear is highly fine. This analytical result is consistent with the 3D morphology measured by 3D laser microscope shown in Fig. 4.

4 Conclusions

The rubber wear surface has obvious fractal characteristics. Spectrum width $\Delta\alpha$ and spectrum change $\Delta f(\alpha)$ are obtained from multifractal spectrum of gray image using multifractal theory, and they are used to estimate the height uniformity and roughness of rubber wear surface. Comparing with the traditionally and empirically quantitative surface wear analysis method, the multifractal spectrum analysis is more convenient and accurate for rubber wear investigation.

With increase in temperature, the geometric arithmetic average deviation Ra increases, and fractal dimensions decrease. Moreover, the multifractal $\Delta\alpha$ and $\Delta f(\alpha)$ increase, indicating the homogeneity of wear is worsen on the rubber surface with higher complexity.

This research established a comprehensive description of the height homogeneity and complexity of wear rubber surface under different temperatures based on multifractal theory. It achieved an effective method for the study of surface morphology of rubber composite materials.

References

1. Dong, C., Yuan, C., Bai, X., et al. (2014). Study on wear behaviour and wear model of nitrile butadiene rubber under water lubricated conditions. *Rsc Advances*, *4*(36), 19034–19042.
2. Wu, Y. P., Zhou, Y., & Li, J. L. (2016). A comparative study on wear behavior and mechanism of styrene butadiene rubber under dry and wet conditions. *Wear*, *356–357*, 1–8.
3. Vieira, T., Ferreira, R. P., & Kuchiishi, A. K. (2015). Evaluation of friction mechanisms and wear rates on rubber tire materials by low-cost laboratory tests. *Wear*, *328–329*, 556–562.
4. Shen, M. X., Dong, F., & Zhang, Z. X. (2016). Effect of abrasive size on friction and wear characteristics of nitrile butadiene rubber (NBR) in two-body abrasion. *Tribology International*, *103*, 1–11.
5. Wu, Y., Zhou, Y., & Li, J. (2016). Influence of fillers dispersion on friction and wear performance of solution styrene butadiene rubber composites. *Journal of Applied Polymer Science*, *133*(26), 43589.
6. Grosch, K. A. (2007). Rubber friction and its relation to tire traction. *Rubber Chemistry and Technology*, *80*(3), 379–411.
7. Heinz, M., & Grosch, K. A. (2007). A laboratory method to comprehensively evaluate abrasion, traction and rolling resistance of tire tread compounds. *Rubber Chemistry and Technology*, *80*(4), 580–607.
8. Gent, A. N., & Walter, J. D. (2005). *The pneumatic tire*. Washington DC: NHTSA.
9. Grosch, K. A., & Heinz, M. (2003). The use of laboratory tester 100 for the evaluation of tire tread properties. In *Third european conference on constitutive models for rubber* (pp. 15–17). London, Great Britain.
10. Du, G. X., & Ning, X. X. (2008). Multifractal properties of Chinese stock market in Shanghai. *Physica A*, *387*(1), 261–269.
11. Ge, S. R., & Zhu, H. (2005). *Fractal of tribology*. Beijing: Machinery Industry Press.
12. Quan Shuhai. (2002). *Research on fractal character of machined surface topography based on surface grayscale image*. WuHan University of Technology.
13. Zhu, H., & Ji, C. C. (2011). *Fractal theory and its application*. Science Press.
14. Li, P., Hu, K. L., & Wang, B. H. (2004). Design and application about computing program of material multifractal spectrum[J]. *Journal of Nanjing University of Aeronautics and Astronautics*, *36*(1), 77–81.

Zepeng Wang 1976.2, Ph.D., associate professor, master tutor, deputy director of the Office of Academic Affairs of Qingdao University of Science and Technology. He has published more than 30 academic papers, of which EI contains 12 articles; presided over one National Natural Science Foundation, one major national science and technology support program project, one provincial vertical issues, two horizontal issues; participated in 12 vertical courses, including three National Natural Science Foundation of China; editor of four books and Licensed 4 utility model patents. "Special giant tireless model renovation of complete sets of technology, technology and equipment," won the China Federation of Logistics and Purchasing Science and Technology Progress Award. His current research interests include Tire rubber thermal conductivity mechanism, Numerical analysis of mechanics and thermal field of automotive tires, Solar refrigeration technology research.

Shunxiao Hu 1991.07.12, Master of Qingdao University of Science and Technology, under the tutelage of Associate Professor Wang Zepeng. His current research interests is rubber wear in the process of engineering thermal physics issues. Participated in one National Natural Science Fund, one Natural Science Foundation of Shandong Province.

Zhanli Miao born in May 1978, associate professor. Her current research interests is Working substance thermal properties and refrigeration and air conditioning system performance.

Lianxiang Ma born on 1962. 08, Principal of Qingdao University of Science and Technology, Professor, doctoral tutor, member of Chinese Society of Engineering Thermophysics and Chinese Mechanical Engineering Society. His current research interests include thermodynamic properties of high scores of materials, enhanced heat transfer technology and equipment, the numerical calculation of temperature field, low-grade energy recycling, efficient heat exchanger development.