


A Selected Mapping Technique Using Interleavers for PAPR Reduction in OFDM Systems

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Abstract In this article, we propose a new selected mapping (SLM) technique using interleavers for reducing high peak-to-average-power ratio (PAPR) in orthogonal frequency division multiplexing systems. The proposed technique can be considered as a modified version of the conventional SLM technique where a set of candidate symbol blocks can be generated from the original symbol block by inserting some pre-known block interleavers between the mapper and the phase rotation multipliers. One of the advantages of this approach is that with very low complex block interleavers the PAPR performance of the SLM scheme can be improved without increasing the number of IFFTs. Simulation results show that the proposed algorithm has better PAPR performance than the conventional SLM for a given number of IFFTs with no additional side information.

Keywords OFDM systems · PAPR reduction schemes · SLM technique · Block interleavers

1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier modulation technique that has been adopted in many high data rate wireless communication systems [1]. While this technique has a high spectral efficiency and can sustain high data rates in frequency selective fading radio channels, it has some drawbacks. One of the major problems of OFDM is that its peak-to-average-power ratio (PAPR) is high, which can impair its performance when nonlinear power amplifiers are used.

In the literature, many methods have been proposed for reducing high PAPR in OFDM systems. The selected mapping (SLM) scheme, proposed in [9], has the best PAPR performance among all the PAPR reduction schemes. However, the performance of the

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conventional SLM scheme can be improved by increasing the number of candidate signals at the cost of higher computational complexity. Thus, there is a trade-off between PAPR performance and complexity in the design of SLM.

In this paper, we propose a new PAPR reduction scheme by combining a SLM scheme with interleavers. The proposed scheme can be considered as a modified version of the conventional SLM technique where a set of candidate symbol blocks can be generated from the original symbol block by inserting some pre-known block interleavers between the mapper and the phase rotation multipliers. One of the advantages of this approach is that with very low complex block interleavers the PAPR performance of the algorithm can be improved without increasing the number of IFFTs.

The rest of this presentation is organized as follows. In Sect. 2, the conventional SLM scheme is reviewed. In Sect. 3, a new SLM technique using interleavers is proposed. In Sect. 4, we demonstrate the performance results obtained via computer simulation. And finally, conclusions are given in Sect. 5.

2 The Conventional SLM Scheme

This section presents a brief introduction to the SLM method for PAPR reduction in OFDM systems. The idea of SLM for reducing PAPR was presented in [9] for the first time. It has been shown that the SLM scheme can reduce PAPR without distortion compared with the other existing schemes in the literature, [2].

Figure 1 illustrates a block diagram of an OFDM transmitter with the conventional SLM scheme. In the OFDM transmitter, the incoming bit stream of 1s and 0s is demultiplexed into N parallel bit streams. Then, the N parallel bit streams are independently mapped into complex symbols $X[k]$ resulting in the complex vector

$$\mathbf{X} = [X[0], X[1], \dots, X[N - 1]]^T, \tag{1}$$

where $X[k]$, $n = 0, 1, \dots, N - 1$, are complex symbols in a given constellation, such as QPSK or QAM. The SLM scheme first defines an ensemble of phase vectors, as

$$\mathbf{P}_v = [P_v[0], P_v[1], \dots, P_v[N - 1]]^T, \tag{2}$$

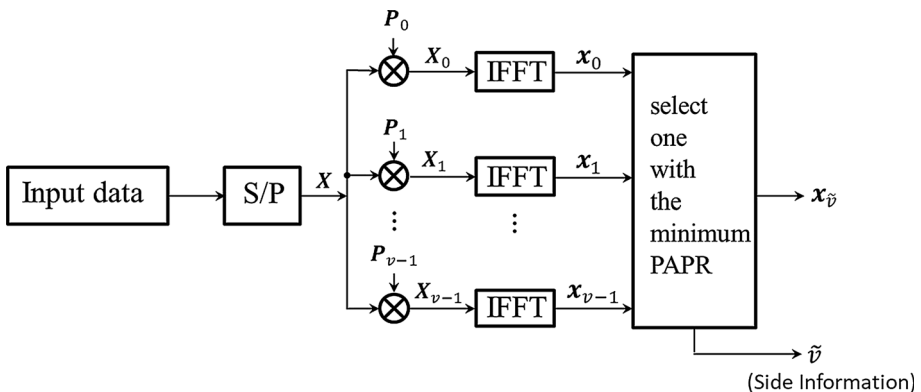


Fig. 1 The block diagram of the conventional SLM scheme

$v = 0, 1, \dots, V - 1$, that are known to both the transmitter and receiver. The entries of \mathbf{P}_v are formed as follows:

$$P_v[k] = e^{j\phi_v[k]}, \tag{3}$$

where $\phi_v[k] \in [0, 2\pi)$, $v = 0, 1, \dots, V - 1$ and $k = 0, 1, \dots, N - 1$. In general, binary or quaternary elements are randomly used for $P_v[k]$, that is, $P_v[k] \in \{\pm 1\}$ or $P_v[k] \in \{\pm 1, \pm j\}$. As noted, the advantage of using only phase shifts by integer multiples of π or $\pi/2$ is that they can be implemented without any multiplications [9]. In SLM, the component-wise multiplication of the input symbol vector \mathbf{X} with V different phase vectors results in a set of V modified input symbol vectors as

$$\mathbf{X}_v = [X_v[0], X_v[1], \dots, X_v[N - 1]]^T \tag{4}$$

with components

$$X_v[k] = X[k]P_v[k] = X[k]e^{j\phi_v[k]} \tag{5}$$

for $v = 0, 1, \dots, V - 1$ and $k = 0, 1, \dots, N - 1$. Then, transforming the modified input symbol vectors \mathbf{X}_v into the time domain using V blocks of N -point IDFT (or IFFT) yields

$$\mathbf{x}_v = \text{IDFT}[\mathbf{X}_v] \tag{6}$$

where

$$\mathbf{x}_v = [x_v[0], x_v[1], \dots, x_v[N - 1]]^T \tag{7}$$

with components

$$x_v[n] = \frac{1}{N} \sum_{k=0}^{N-1} X_v[k] e^{j2\pi kn/N} \tag{8}$$

for $n = 0, 1, \dots, N - 1$. The peak to average power ratio (PAPR) of all the candidate vectors \mathbf{x}_v are calculated as follows:

$$\Gamma_{\mathbf{x}_v} = \text{PAPR} \{ \mathbf{x}_v \} \triangleq \frac{\max_{0 \leq n \leq N-1} |x_v[n]|^2}{P_{\mathbf{x}_v}} \tag{9}$$

where $P_{\mathbf{x}_v}$, known as the average power of \mathbf{x}_v , is defined as

$$P_{\mathbf{x}_v} = \frac{1}{N} E[\mathbf{x}_v^H \mathbf{x}_v] = \frac{1}{N} \sum_{n=0}^{N-1} E[|x_v[n]|^2] \tag{10}$$

where $E[\cdot]$ denotes the expected value and the superscript H denotes the Hermitian transpose (i.e., the conjugate transpose). Then, among V candidate vectors \mathbf{x}_v , the SLM scheme selects the one with the lowest PAPR, as

$$\tilde{v} = \arg \min_{0 \leq v \leq V-1} \text{PAPR} \{ \mathbf{x}_v \}. \tag{11}$$

The vector $\mathbf{x}_{\tilde{v}}$ is then transmitted along with its index \tilde{v} as side information. The number of bits that are required to be transmitted as side information is $\log_2 V$ for each data block.

Since the PAPR of an OFDM signal is a random variable, the performance (or capability) of any PAPR reduction scheme most commonly is measured by the complementary cumulative distributive function (CCDF) of its PAPR [14], which is defined as the probability that the PAPR of an OFDM block \mathbf{x} exceeds a given threshold γ (usually given in decibels), as

$$\begin{aligned} F_{\Gamma_{\mathbf{x}}}^c(\gamma) &= 1 - F_{\Gamma_{\mathbf{x}}}(\gamma) \\ &= 1 - \Pr [\text{PAPR} \{\mathbf{x}\} \leq \gamma] \\ &= \Pr [\text{PAPR} \{\mathbf{x}\} > \gamma], \end{aligned} \quad (12)$$

where $F_{\Gamma_{\mathbf{x}}}(\gamma) = \Pr [\Gamma_{\mathbf{x}} \leq \gamma]$ is the cumulative distributive function of $\Gamma_{\mathbf{x}}$. The probability in (12) is also called the clipping probability.

At the receiver, in order to recover the original symbol vector, first the discrete Fourier transform (DFT) is applied to the received discrete-time signal. Then the receiver has to undo phase rotation based on the priori knowledge. For implementation, the receiver must have the perfect knowledge of all the phase rotation vectors \mathbf{P}_v . After receiving the index \tilde{v} as side information, then the receiver can determine which phase rotation vector has been used at the transmitter. As it will be seen in the simulation results, the PAPR performance of the SLM method can be improved by increasing the number of phase rotation vectors V . However, this performance gain can be obtained at the expense of increasing complexity (due to the number of N -point IDFTs) and reducing data rate (due to side information).

Many researchers have proposed methods such as blind or semi-blind data recovery techniques only based on received data or some pre-known data such as pilots, [3, 4, 8, 10, 15]. In this paper, it is assumed that the receiver knows the table in advance and the transmitter needs to send $\log_2 V$ bits as the side information to the receiver.

Note that the PAPR performance of the SLM method depends on the number of phase rotation vectors V and the design of these vectors.

3 The Proposed PAPR Reduction Scheme

As explained in the previous section, the conventional SLM approach, proposed in [9], is based on the phase rotation of data symbols and which is performed by multiplying of the data (or symbol) frame by random phase vectors. In this section, we propose a new SLM method using interleavers. An advantage of this proposed approach is that it does not require any additional multiplications. So, the complexity of generating the candidates is low, since no extra complex multiplication is added.

In the proposed scheme, some pre-known interleavers are inserted between the mapper and the phase rotation multipliers.

An interleaver is a device that operates on a block of N symbols to reorder or permute them, without repeating or omitting any of the symbols in the block. From a mathematical point of view, an interleaver can be considered as a bijective map that maps every vector to a permuted version of itself. One of the most commonly used interleaver in communication systems is the block (or rectangular) interleaver. A block interleaver of size $N = R \times C$ performs block interleaving by filling a $R \times C$ matrix with the input symbols row by row and reading out the matrix contents column by column. The mapping function for such a rectangular interleaver can be formulated as

$$\pi(k) = kC + \left\lfloor \frac{k}{R} \right\rfloor \text{ mod } N, \tag{13}$$

where $\lfloor \cdot \rfloor$ means the integer value of the argument and $\text{mod } N$ denotes Modulo N .

Here we define a set of I distinct block interleavers $\{\Pi_i\}$ with the following mapping functions:

$$\pi_i(k) = kC_i + \left\lfloor \frac{k}{R_i} \right\rfloor \text{ mod } N, \quad i = 0, 1, \dots, I - 1, \tag{14}$$

where R_i and C_i denote the number of rows and columns of the i th interleaver, respectively. The interleavers $\{\Pi_i\}$ generate I different vectors \mathbf{Y}_i ,

$$\mathbf{Y}_i = [Y_i[0], Y_i[1], \dots, Y_i[N - 1]]^T, \tag{15}$$

by permuting the symbols in the input symbol vector \mathbf{X} , as

$$\begin{aligned} \mathbf{Y}_i &= \Pi_i(\mathbf{X}) \\ &= [X[\pi_i(0)], X[\pi_i(1)], \dots, X[\pi_i(N - 1)]]^T \end{aligned} \tag{16}$$

As a result, the components of the new vectors \mathbf{Y}_i are obtained from the components of \mathbf{X} as follows:

$$Y_i[k] = X[\pi_i(k)] \tag{18}$$

for $i = 0, 1, \dots, I - 1$ and $k = 0, 1, \dots, N - 1$.

Since the DFT and IDFT can be efficiently implemented using fast Fourier transform (FFT) when N is a power of 2, the block interleavers in our design will be selected with the following sizes: $2^{i+1} \times \frac{N}{2^{i+1}}$ for $i = 0, 1, \dots, I - 1$. Then, it can be easily shown that the maximum number of interleavers is constrained to $\log_2 N/2$. In other words, in this design the number of interleavers is chosen such that $I \leq \log_2 N/2$.

Figure 2 illustrates a block diagram of the proposed PAPR reduction scheme.

As seen in Fig. 2, the interleavers have been inserted between the mapper and the phase rotation multipliers. Thus, I new candidate vectors \mathbf{Y}_i are generated from the input symbol vector X by going through I interleavers, $\mathbf{Y}_i = \Pi_i(\mathbf{X})$. Then, the output of each interleaver is multiplied by a phase rotation factor \mathbf{P}_v and then applied as an input to an $N - \text{point}$ IFFT. Finally, among the output signal vectors of the IFFTs, the algorithm selects the

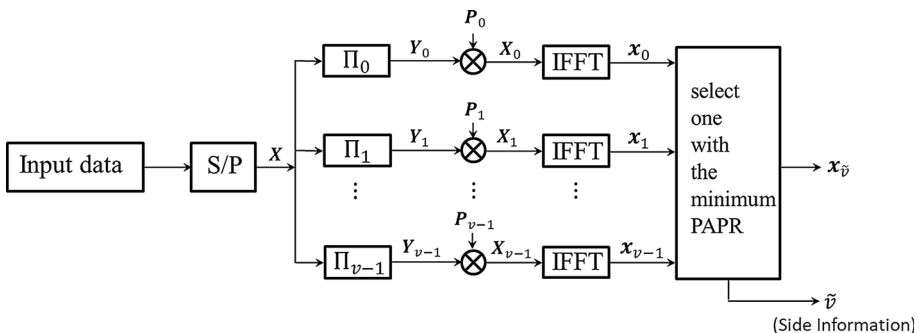


Fig. 2 The block diagram of the proposed PAPR reduction scheme

signal vector with the lowest PAPR and transmits it along with its index as side information. Note that in this design the number of interleavers and phase rotation multipliers can be different.

At the receiver, in order to recover the original symbol vector, first the FFT is applied to the received discrete-time signal. Then the receiver must undo the operations of phase rotation and interleaving based on the priori knowledge. For implementation, the receiver must have the knowledge of all the phase rotation vectors \mathbf{P}_v and the structure of all the interleavers. After receiving the index \tilde{v} as side information, then the receiver can determine which phase rotation vector and interleaver have been used at the transmitter.

4 Simulation Results

In this section, the computer simulation is performed for the proposed PAPR reduction schemes and the conventional SLM scheme. Even the simulation results for the system without any PAPR reduction scheme (denoted by original) have been included. The phase rotation factors are randomly taken from $\{+1, -1\}$ (denoted by type I) and $\{\pm 1, \pm j\}$ (denoted by type II), respectively. In all the simulations, the OFDM system has 128 or 256 subcarriers ($N = 128$ or $N = 256$) with QPSK or 16-QAM constellation.

Figures 3 and 4 show the PAPR performances of the convention SLM scheme (type I and type II) and the proposed SLM scheme (denoted by p-slm) with $V = 4$, QPSK constellation, and $N = 128$ and $N = 256$, respectively. Compared with the conventional SLM scheme, the proposed scheme can improve the PAPR reduction performance by inserting very low complex block interleavers. Moreover, with the increase of N , the PAPR reduction performance becomes more obvious.

Figures 5 and 6 show the PAPR performances of the conventional SLM scheme (type I and type II) and the proposed SLM scheme with 16-QAM constellation, $N = 256$, and $V = 4$ and $V = 6$, respectively. As seen from the figures, increasing the number of

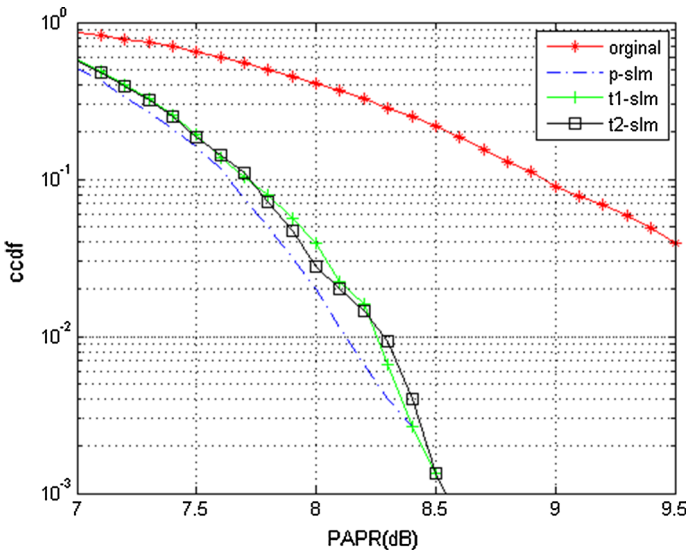


Fig. 3 Comparison of CCDFs for the proposed scheme and the conventional SLM with $V = 4$ and $N = 128$

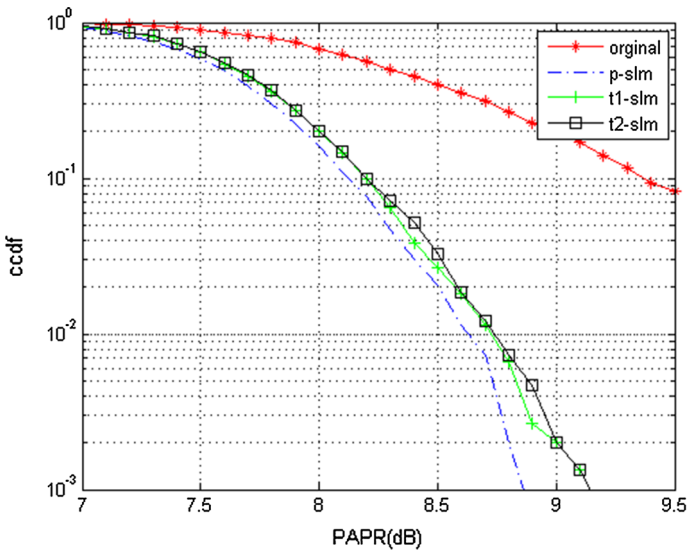


Fig. 4 Comparison of CCDFs for the proposed scheme and the conventional SLM with $V = 4$ and $N = 256$

interleavers can improve the PAPR reduction compared with the conventional SLM schemes due to more randomness among candidate sequences.

Figures 7 and 8 show the PAPR performances of the conventional SLM scheme (type I and type II) and the proposed SLM scheme with $V = 4$, $N = 128$, and 16-QAM and QPSK constellations, respectively. From these two figures, it is clear that the type of constellation mapping does not affect the PAPR reduction performance of our proposed scheme.

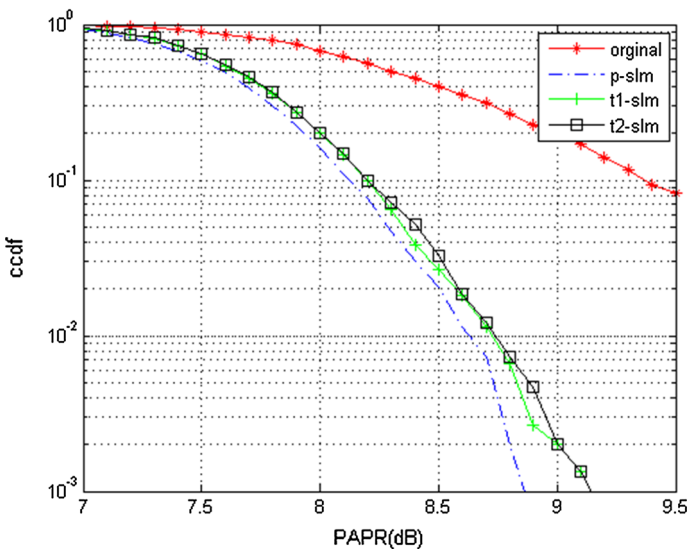


Fig. 5 Comparison of CCDFs for the proposed scheme and the conventional SLM with $V = 6$ and $N = 256$

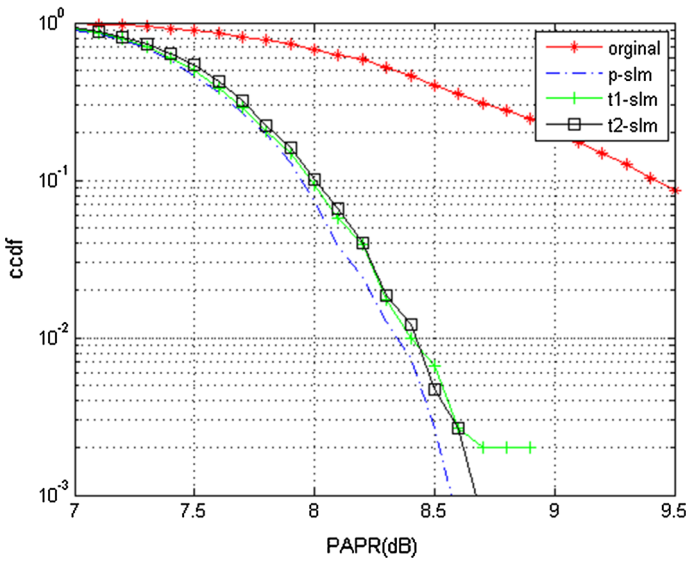


Fig. 6 Comparison of CCDFs for the proposed scheme and the conventional SLM with $V = 4$ and $N = 256$

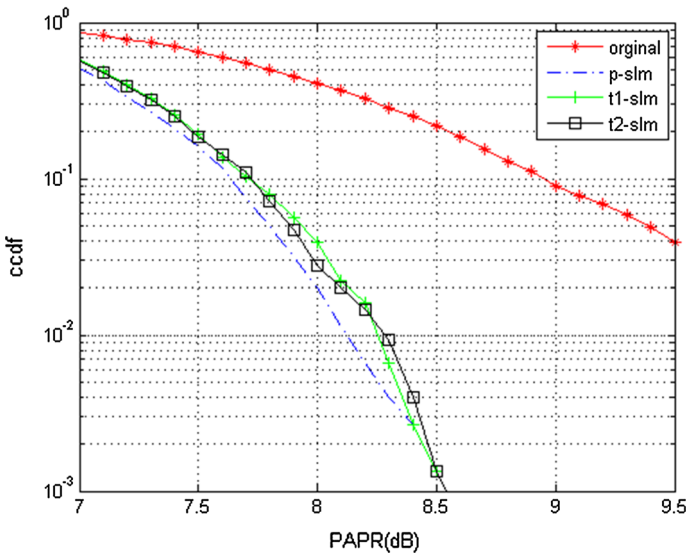


Fig. 7 Comparison of CCDFs for the proposed scheme and the conventional SLM with 16-QAM modulation

In addition, since our proposed SLM scheme does not change the main structure of the conventional SLM scheme, it can be expected that other modified SLM schemes, such as the algorithms proposed in [5–7, 11–13], can be combined or added to our proposed SLM scheme without any major changes. This shows the flexibility of our proposed SLM. Moreover, for simplicity, in the proposed scheme, we considered only block interleavers. However, with the development of digital signal processing” (DSP), more complex interleavers can be added to improve the PAPR reduction performance without adding more complexity.

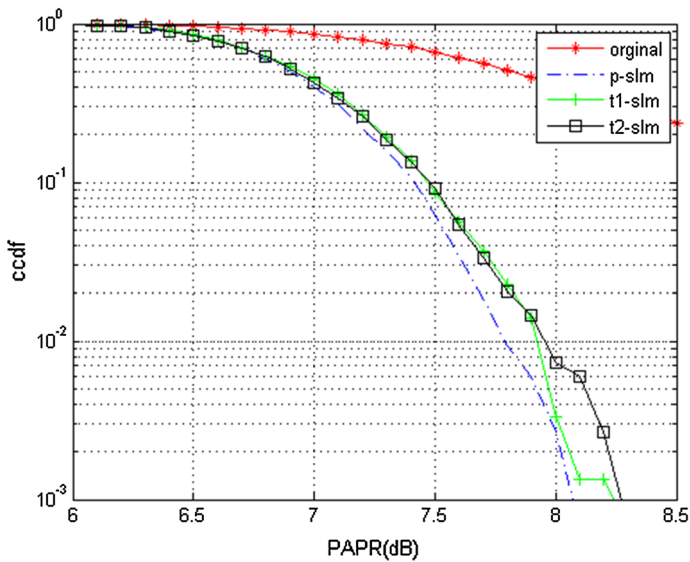


Fig. 8 Comparison of CCDFs for the proposed scheme and the conventional SLM with QPSK modulation

5 Conclusion

In this paper, we have proposed a low complex PAPR reduction scheme for OFDM systems. The proposed algorithm is a modified version of SLM that applies some preset interleavers to randomize the input symbol vector before phase rotation. Using interleavers can increase the freedom in generating more candidate sequences which can improve the PAPR performance of SLM. Simulation results show that the proposed PAPR reduction scheme can outperform the conventional SLM scheme for a given number of IFFTs with no additional side information.

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