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# An Analytical Model for LoS Probability and Area Transport Efficiency of Millimeter Wave Cellular Network

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**Abstract** Millimeter wave based small cell deployment is a promising technique for the evolution of fifth generation (5G) cellular network. This paper investigates coverage probability of millimeter wave cellular network (MWCN) over Nakagami-*m* fading channel. Stochastic geometry based ITU-R statistical model is used for modeling the blockages in MWCN which accounts line of sight (LoS) propagation characteristics of urban environment. SNR distribution is also considered under nearest-neighbor and furthest-neighbor based receiver association schemes for better representation of coverage probability in noise limited environment. Furthermore, new performance metric, namely, area transport efficiency (ATE) is proposed to capture the impact of the different network parameters on the system performance. Numerical results are evaluated to compare the performance of the MWCN under nearest-neighbor and furthest-neighbor based receiver association schemes. It is seen that height of transmitter plays an important role in the performance of the MWCN. It is also observed that furthest-neighbor scheme outperforms nearest-neighbor scheme for low SNR threshold and higher BS heights. Whereas for high SNR threshold performance of nearest-neighbor scheme is better as compared to furthest-neighbor scheme.

**Keywords** Millimeter wave · Coverage probability · Stochastic geometry · Area transport efficiency

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## 1 Introduction

Fifth generation (5G) cellular network demands high data rate and seamless connectivity due to exponential growth in data demanding wireless devices and multimedia services. Dense deployment of small cell is the promising solution to support the better network coverage and higher data rates in 5G cellular network. Below 3 GHz frequency, band is limited and crowded, therefore, its utilization is congested and expensive for small cell cellular network [1]. Other issues associated with small cell deployment are to provide inexpensive high capacity and reliable backhaul service to all the small base stations (SBSs) [2]. In cellular communication, base stations (BSs) are connected to the backhaul network by fiber cables because of higher bandwidth (BW) capacity. But SBSs are deployed on street light poles, buildings walls or roof where fiber cables access is difficult and expensive. Hence, wireless backhaul is an attractive option for SBSs as it provides a less expensive and reliable alternative to fiber cables [2, 3]. It is observed that coverage performance of millimeter wave (MMW) (e.g., 28, 38 GHz, and E band) are similar to 3 GHz band for small cells [3]. Hence, MMW is a prominent and reliable alternative to 3 GHz band to obtain high data rate and less expensive backhaul service for small cells [1, 4].

The channel capacity of the 5G cellular network may be improved by using the underutilized large transmission BW of MMW [5, 6]. In [7, 8], it is observed that MMW based cellular network (MWCN) is noise limited. Hence, coverage occurs in MWCN when signal to noise ratio (SNR) above the specified threshold. In [9], ray tracing method is used to model the blockages based on the geographical information of the urban area. In [10], random shape theory is proposed to model the blockage.

MWCN is sensitivity to the blockage and rain attenuation [3] hence, propagation characteristics are different from the traditional below 3 GHz band based cellular system. In MWCN, where line of sight (LoS) is an important issue, coverage probability is the basic performance measure. This paper investigates the coverage performance of the MWCN. The impact of the building blockages on system performance is analyzed by incorporating the ITU-R recommended statistical model [11] and stochastic geometry. Steerable directional antenna with narrow beam is used to combat higher path loss and additional environmental losses (rain and oxygen gas absorption) associated with MMW propagation [5, 12]. Area spectral efficiency (ASE) is an important metric to examine the spectral efficiency of the cellular network [13]. It is defined as the product of the spatial throughput and the link spectral efficiency of the network [14]. However, ASE does not consider the transmission distance, a key parameter for cellular network; specially in MWCN where signal strength is sensitive to blockages and distance due to higher path loss. In order to incorporate this relevant and important aspect, transmission distance is included in the definition of the ASE leading to a new metric, namely, area transport efficiency (ATE) for MWCN.

Major contributions of this paper are summarized as follows:

- An analytical framework is developed to model the blockages in urban environment for accounting the LoS propagation characteristics in MWCN using stochastic geometry based ITU-R statistical model.
- SNR distribution and coverage probability expressions are derived under nearest-neighbor receiver association scheme (NNRAS) and furthest neighbor receiver association scheme (FNRAS).
- New performance metric ATE is proposed and derived under NNRAS and FNRAS.

– Impact of the different network parameters on coverage probability and ATE are numerically examined and some useful observations are drawn for MWCN.

The rest of the paper is organized as follows. System model is given in Sect. 2. Section 3 presents the analysis of the LoS probability and coverage probability of the MWCN. ATE is described in Sect. 4. Numerical results are discussed in Sect. 5. Finally, the paper is concluded in Sect. 6.

## 2 System Model

Consider urban outdoor MWCN where antenna is mounted on poles or roof of the buildings. Figure 1 represents the MWCN and relationship among different system parameters. The location of the BSs is distributed by homogeneous spatial Poisson point process (HSPPP) on the two dimensional Euclidean plane [15]. Density of BSs is taken as $\lambda$ with transmitted power $\rho_{BS}$. The BSs that operates in MMW have antenna array with high gain and narrow beam width. Due to the narrow beam width at the transmitter and receiver node, interference is ignored and noise limited system is considered for the analysis [6, 7, 11, 16]. In this work, Nakagami-$m$ fading and path loss are considered to model the channel [5]. Therefore, the received power is given by $\rho_{\mathrm{BS}}\mathcal{G}_0 h d_{BM}^{-\beta}$. Where $\mathcal{G}_0$ is the normalized beam pattern from BS to desired MR [12]. Perfect beam steering toward the MR is assumed. $h$ is the Gamma distributed channel gain and $\beta$ denotes the path loss exponent. $d_{BM}$ is the distance between BS and MR. In this work, MR is associated with a particular BS based on the distance dependent receiver association schemes. These schemes are NNRAS and FNRAS. In NNRAS, MR is associated with its nearest BS. MR selects furthest BS to maximize the throughput within the maximum transmission range of BS in FNRAS. $d_{BM}$, (i.e., $0 < d_{BM} \le \mathcal{D}_{\max}$) is the transmission distance between the BS and MR with maximum transmission range $\mathcal{D}_{\max}$ and in a two dimensional plane its distribution for NNRAS and FNRAS schemes are expressed as, respectively [17, 18]

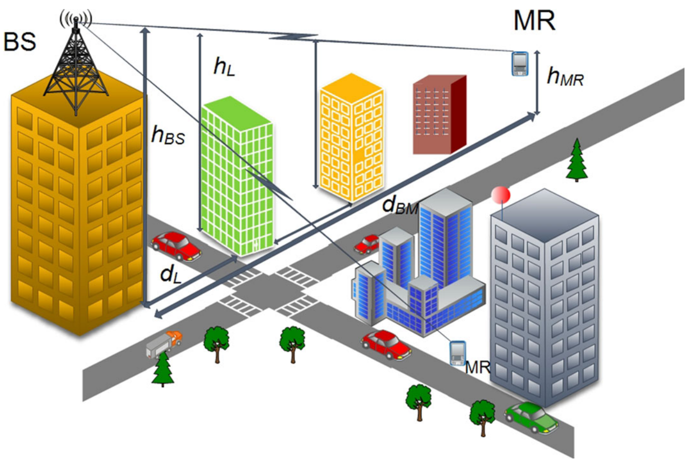



**Fig. 1** Typical urban millimeter wave cellular network system model

$$f^N(d_{\mathrm{BM}}) = 2\pi d_{\mathrm{BM}}\lambda e^{-\pi\lambda d_{\mathrm{BM}}^2} \tag{1}$$

$$f^F(d_{\mathrm{BM}}) = \frac{2\pi\lambda d_{\mathrm{BM}} e^{\pi\lambda d_{\mathrm{BM}}^2}}{e^{\pi\lambda \mathcal{D}_{\max}^2} - 1} \tag{2}$$

It is assumed that all transmitters have the similar transmitting power and MRs have similar sensitivity level. Since system is noise limited so, maximum separation distance between BS and MR is expressed by transmitting power of the BS and MR sensitivity level only. Using standard distance dependent path loss model the maximum transmission range $\mathcal{D}_{\max}$ of the BS is expressed by [19] $\left(\frac{\rho_{BS}}{\eta}\right)^{1/\beta}$. Where $\eta$ denotes the MR sensitivity level.

## 3 Coverage Analysis

In this section, an analytical model is investigated that incorporates the stochastic geometry and ITU-R statistical model [11] to derive the LoS probability of a link. Furthermore, conditional coverage probability and coverage probability of MWCN are calculated.

**Theorem 1** *For NNRAS the probability that the height of a blockage is lower than $h_j^N$ can be expressed as*

$$\mathcal{P}_j^N = 1 - e^{-\frac{(h_j^N)^2}{2\xi^2}}, \tag{3}$$

*where $j \in \{0, \ldots, n_d - 1\}$ and*

$$h_j^N = h_{\mathrm{BS}} - \frac{\pi(2j+1)\sqrt{\lambda}\,(h_{\mathrm{BS}} - h_{\mathrm{MR}})\,\mathrm{erf}\left(\sqrt{\pi}\sqrt{\lambda}\mathcal{D}_{\max}\right)}{2n_{\mathrm{bd}}}.$$

*Proof* Please see "Appendix 1". □

So, the probability of LoS at location $r_j$ under NNRAS is given by

$$\mathcal{P}_{\mathrm{Lj}}^N = \prod_{i=0}^{j} \mathcal{P}_i^N, i \in \{0, \ldots, j\} \tag{4}$$

**Theorem 2** *For FNRAS the probability that the height of a blockage is lower than $h_j^F$ can be expressed as*

$$\mathcal{P}_j^F = 1 - e^{-\frac{(h_j^F)^2}{2\xi^2}}, \tag{5}$$

*where $j \in \{0, \ldots, n_d - 1\}$ and*

$$h_j^F = h_{\mathrm{BS}} - \frac{\pi(2j+1)\sqrt{\lambda}\,(h_{\mathrm{BS}} - h_{\mathrm{MR}})\,\mathrm{erfi}\left(\sqrt{\pi}\sqrt{\lambda}\mathcal{D}_{\max}\right)}{2n_{\mathrm{bd}}\left(e^{\pi\lambda\mathcal{D}_{\max}^2} - 1\right)} \tag{6}$$

*Proof* Please see "Appendix 2". □

Thus, the probability of LoS at location $r_j$ under FNRAS is expressed as

$$\mathcal{P}_{\mathrm{Lj}}^{F} = \prod_{i=0}^{j} \mathcal{P}_{i}^{F}, i \in \{0, \ldots, j\} \tag{7}$$

## 3.1 Conditional Coverage Probability

**Lemma 1** *The conditional coverage probability of MWCN under NNRAS is given by*

$$\mathcal{P}_{c}^{N} = \int_{0}^{\mathcal{D}_{\max}} \sum_{n=1}^{m} (-1)^{(n+1)} \binom{m}{n} e^{-\frac{n\mathcal{T} v \mathcal{N}_0 d_{\mathrm{BM}}^{\beta}}{\mathcal{G}_0 \rho_{\mathrm{BS}}}} f^{N}(d_{\mathrm{BM}}) dd_{\mathrm{BM}}, \tag{8}$$

*where $f^N(d_{BM})$ is the corresponding pdf of the transmission distance between transmitter and receiver. m is the fading parameter, $v = m(m!)^{-1/m}$.*

*Proof* As given in "Appendix 3". □

*Special case* For $\beta = 2$ and $\mathcal{G}_0 = 1$, and using (1), (8) for NNRAS $\mathcal{P}_c^N$ may be simplified as

$$\mathcal{P}_{c}^{N} = \sum_{n=1}^{m} (-1)^{(n+1)} \binom{m}{n} \frac{\pi \lambda \rho_{\mathrm{BS}} \left(1 - e^{-\mathcal{D}_{\max}^{2} \left(\frac{v n \mathcal{N}_0 \mathcal{T}}{\rho_{\mathrm{BS}}} + \pi \lambda\right)}\right)}{\pi \lambda \rho_{\mathrm{BS}} + v n \mathcal{N}_0 \mathcal{T}} \tag{9}$$

**Lemma 2** *The conditional coverage probability of MWCN under FNRAS is given by*

$$\mathcal{P}_{c}^{F} = \int_{0}^{\mathcal{D}_{\max}} \sum_{n=1}^{m} (-1)^{(n+1)} \binom{m}{n} e^{-\frac{n\mathcal{T} v \mathcal{N}_0 d_{\mathrm{BM}}^{\beta}}{\mathcal{G}_0 \rho_{\mathrm{BS}}}} f^{F}(d_{\mathrm{BM}}) dd_{\mathrm{BM}}, \tag{10}$$

*where $f^F(d_{BM})$ is the corresponding pdf of the transmission distance between transmitter and receiver.*

*Proof* Proof of Lemma 2 is similar to proof of Lemma 3. □

*Special case* For $\beta = 2$ and $\mathcal{G}_0 = 1$, and using (2), (10) for FNRAS $\mathcal{P}_c^F$ may be simplified as

$$\mathcal{P}_{c}^{F} = \sum_{n=1}^{m} (-1)^{(n+1)} \binom{m}{n} \frac{\pi \lambda \rho_{\mathrm{BS}} \left(1 - e^{\mathcal{D}_{\max}^{2} \left(\pi \lambda - \frac{v n \mathcal{N}_0 \mathcal{T}}{\rho_{\mathrm{BS}}}\right)}\right)}{\left(e^{\pi \lambda \mathcal{D}_{\max}^{2}} - 1\right)(\pi \lambda \rho_{\mathrm{BS}} - v n \mathcal{N}_0 \mathcal{T})} \tag{11}$$

## 3.2 Coverage Probability

A BS can be either LoS or non LoS (NLoS) to the typical MR. In this work, only LoS link is considered and NLoS link is ignored to calculate the coverage probability of MWCN. Therefore, the coverage probability can be expressed as [20]

$$\mathcal{P}_{cov}^{k} = \mathcal{P}_{\mathrm{Lj}}^{k}\mathcal{P}_{c}^{k}, \tag{12}$$

where $k \in \{N, F\}$. $\mathcal{P}_{\mathrm{Lj}}$ is the corresponding LoS probability and $\mathcal{P}_c$ is the corresponding conditional coverage probability for the link.

## 4 Area Transport Efficiency

The ATE of the MWCN is investigated in this section. ATE specifies, that for a given SNR threshold, how efficiently the information bits can travel towards its receiver. In this work, transmission distance is a random variable and its distribution depends on the receiver association schemes (described in Sect. 2). It is assumed that SNR threshold and BS density are fixed. Mathematically, it is defined as the product of spatial throughput, distance between BS and MR, and the link spectral efficiency $\mathcal{S}$. Hence, it is expressed as

$$ATE_k = \lambda\mathcal{S}\mathbb{E}\left[d_{BM}\mathcal{P}_{cov}^{k}\right] \tag{13}$$

where $\mathcal{S} = log_2(1+\mathcal{T})$ and spatial throughput is given by $\lambda\mathcal{P}_{cov}^{k}$ [13]. The unit of the ATE is given by $bit.m/s.Hz.m^2$. ATE of the MWCN for NNRAS is given by the following lemma

**Lemma 3** *ATE of MWCN for a given SNR threshold under NNRAS is expressed by*

$$ATE_N = \int_0^{\mathcal{D}_{\max}} \sum_{n=1}^{m} (-1)^{(n+1)} \binom{m}{n} d_{BM}\mathcal{P}_{\mathrm{Lj}}^{N} e^{-\frac{n\mathcal{T}v\mathcal{N}_0 d_{\mathrm{BM}}^{\beta}}{\rho_{\mathrm{BS}}\mathcal{G}_0}} f^N(d_{\mathrm{BM}})dd_{\mathrm{BM}}, \tag{14}$$

*where $f^N(d_{\mathrm{BM}})$ and $\mathcal{P}_{\mathrm{Lj}}^{N}$ are given by (1) and (4), respectively.*

*Proof* Following the same steps as in proof of Lemma 1, as given in "Appendix 3". □

*Special case*: For $\beta = 2$ and $\mathcal{G}_0 = 1$, and using (1), (14) for NNRAS $ATE_N$ may be expressed as

$$\begin{aligned} ATE_N = &\sum_{n=1}^{m} (-1)^{(n+1)} \binom{m}{n} \frac{1}{2}\pi\lambda^2\mathcal{S}\rho_{\mathrm{BS}}\mathcal{P}_{\mathrm{Lj}}^{N} \\ &\times \left( \frac{\sqrt{\pi}\sqrt{\rho_{\mathrm{BS}}}\mathrm{erf}\left(\frac{\mathcal{D}_{\max}\sqrt{\pi\lambda\rho_{\mathrm{BS}}+vn\mathcal{N}_0\mathcal{T}}}{\sqrt{\rho_{\mathrm{BS}}}}\right)}{(\pi\lambda\rho_{\mathrm{BS}}+vn\mathcal{N}_0\mathcal{T})^{3/2}} - \frac{2\mathcal{D}_{\max}e^{-\mathcal{D}_{\max}^2\left(\frac{vn\mathcal{N}_0\mathcal{T}}{\rho_{\mathrm{BS}}}+\pi\lambda\right)}}{\pi\lambda\rho_{\mathrm{BS}}+vn\mathcal{N}_0\mathcal{T}} \right) \end{aligned} \tag{15}$$

**Lemma 4** *ATE of MWCN for a given SNR threshold under FNRAS is given by*

$$ATE_F = \int_0^{\mathcal{D}_{\max}} \sum_{n=1}^{m} (-1)^{(n+1)} \binom{m}{n} d_{BM}\mathcal{P}_{\mathrm{Lj}}^{F} e^{-\frac{n\mathcal{T}v\mathcal{N}_0 d_{\mathrm{BM}}^{\beta}}{\mathcal{G}_0\rho_{\mathrm{BS}}}} f^F(d_{\mathrm{BM}})dd_{\mathrm{BM}}, \tag{16}$$

*where $f^F(d_{\mathrm{BM}})$ and $\mathcal{P}_{\mathrm{Lj}}^{F}$ are given by (2) and (7), respectively.*

*Proof* Follows from the proof of Lemma 3 by considering transmission distance distribution $f^F(d_{BM})$. □

*Special case* For $\beta = 2$ and $\mathcal{G}_0 = 1$, and using (2), (16) for FNRAS $ATE_F$ may be expressed as

$$ATE_F = \sum_{n=1}^{m} (-1)^{(n+1)} \binom{m}{n} \pi \lambda^2 \mathcal{S} \rho_{\mathrm{BS}} \mathcal{P}_{\mathrm{Lj}}^{F} \times \frac{\left( \frac{\sqrt{\pi}\sqrt{\rho_{\mathrm{BS}}}\,\mathrm{erf}\left(\frac{\mathcal{D}_{\max}\sqrt{v n \mathcal{N}_0 \mathcal{T} - \pi\lambda\rho_{\mathrm{BS}}}}{\sqrt{\rho_{\mathrm{BS}}}}\right)}{(v n \mathcal{N}_0 \mathcal{T} - \pi\lambda\rho_{\mathrm{BS}})^{3/2}} - \frac{2\mathcal{D}_{\max} e^{\mathcal{D}_{\max}^2 \left(\pi\lambda - \frac{v n \mathcal{N}_0 \mathcal{T}}{\rho_{\mathrm{BS}}}\right)}}{v n \mathcal{N}_0 \mathcal{T} - \pi\lambda\rho_{\mathrm{BS}}} \right)}{2\left(e^{\pi\lambda\mathcal{D}_{\max}^2} - 1\right)} \tag{17}$$

## 5 Results and Discussions

In this section, numerical results of coverage probability and ATE are investigated based on the analytical expressions derived in the previously sections for NNRAS and FNRAS. Summary of notations and values of parameters used are given in Table 1. By default system parameters are taken as given in Table 1 and are used to calculate the numerical results, unless otherwise stated. In Fig. 2, conditional coverage probabilities are compared using (9) and (11) for different values of noise power under NNRAS and

**Table 1** Notation summary

| Notation | Description | Values |
|---|---|---|
| $\zeta$ | Density of building blockages | $750 \times 10^{-6}\mathrm{m}^{-2}$ |
| $\delta$ | Ratio between the blockage areas to overall urban area | 0.15 |
| $\xi$ | Variable which explain the Rayleigh distributed height of the blockages | 7.63 |
| $\lambda$ | Density of BS | $2 \times 10^{-5}\mathrm{m}^{-2}$ |
| $\rho_{BS}$ | Transmitted power | 10 mw |
| $\eta$ | MR sensitivity level | −70 dBm. |
| $\beta$ | Path loss exponent | 2 |
| $d_{BM}$ | Transmission distance between the BS and MR | NA |
| $\mathcal{D}_{\max}$ | Maximum transmission range | NA |
| $f^N(d_{BM})$,$f^F(d_{BM})$ | Distribution of distance for NNRAS and FNRAS schemes, respectively | NA |
| $h$ | Gamma distributed channel gain | NA |
| $h_{BS}$ | Height of the blockages. | 60 m |
| $h_{MR}$ | Height of the MR | 2 m |
| $\mathcal{N}_0$ | Noise power | −80 dBm |
| $n_d$ | Number of blockages crossed by the transmitted beam | NA. |
| $n_{bd}$ | Expected number of blockages per unit distance | NA |
| $m$ | Nakagami-$m$ fading parameter | 3 |
| $\mathcal{P}_c$ | Conditional coverage probability | NA |
| $\mathcal{P}_{cov}$ | Coverage probability | NA |
| $\mathcal{T}$ | SNR threshold | 0 dB |

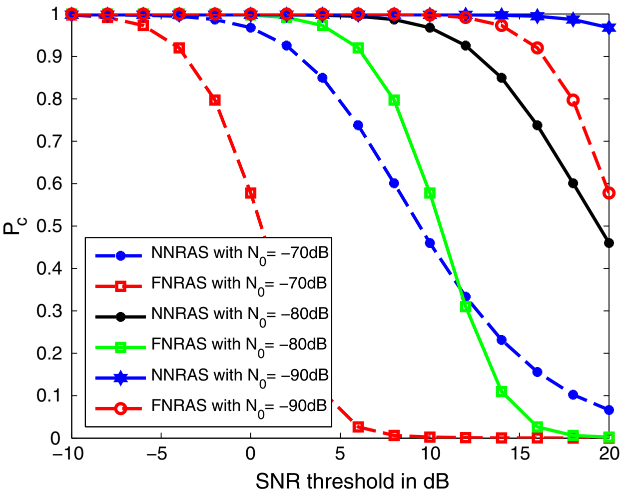
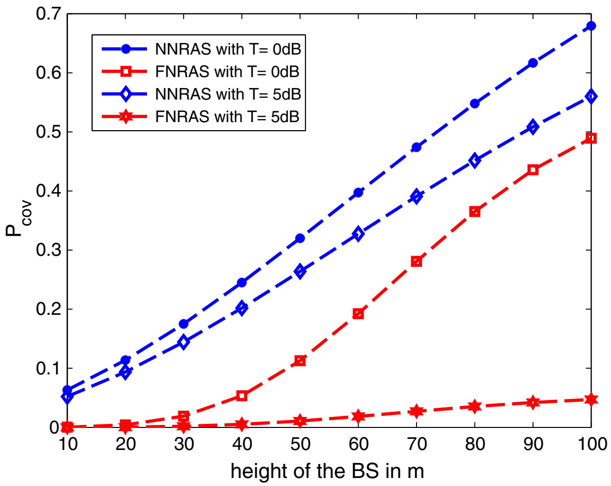



**Fig. 2** Conditional coverage probability for MWCN under NNRAS and FNRAS

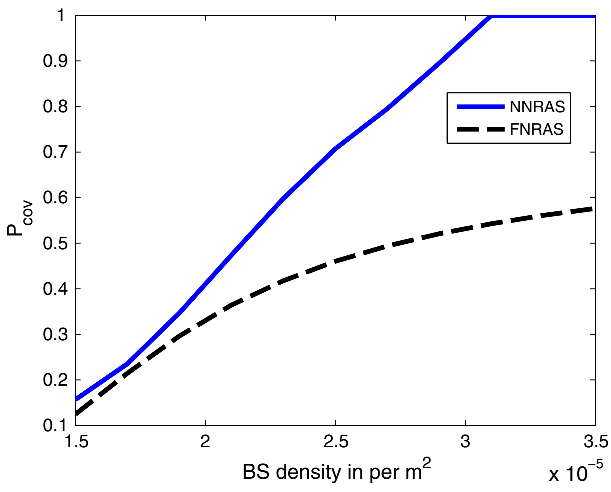
FNRAS, respectively. As expected, for both the schemes the conditional coverage performances degrades with increasing values of noise power. It is observed that for low noise power (i.e., -90 dB) both the schemes give similar performance for the low SNR threshold. Whereas NNRAS gives better performance at high SNR thresholds. As the noise power decreases, communication links become robust for the low SNR threshold. Thus, coverage event is likely to occur for both the schemes. The coverage probability $\mathcal{P}_{cov}$ with respect to the height of the BS $h_{BS}$ is plotted using (12) for different receiver association schemes in Fig. 3. As expected $\mathcal{P}_{cov}$ decreases for the increasing values of the SNR threshold. It can be seen that as the height of the transmitter increases the coverage of the network also increases because higher values of the BS height increases the LoS probability for the typical link. The coverage performance is better for the NNRAS as compared to FNRAS. This is due to the receiver association scheme (as explained in Sect. 2). In NNRAS, MR is

**Fig. 3** Coverage probability for MWCN with $\mathcal{N}_0 = -70\,\text{dBm}$



associated with the closest BS, so probability of the LoS is higher compared to the FNRAS. Consequently, better coverage performance is given by the NNRAS scheme for given system parameters. Figure 4 depicts the effect of the BS density on coverage performance for NNRAS and FNRAS. Density of the BS considerably increases the coverage performance of the network for both the schemes. Higher density of BS decreases the distance between BS and MR, so, less number of blockages obstructs the signal and coverage performance increases with density. Furthermore, the system is noise limited, so increasing density does not increase the interference. Hence, it does not degrade the coverage performance due to the density of the BSs unlike in conventional cellular system. Figure 4 also reveals that NNRAS performance is better as compared to FNRAS because MR is associated with the closest BS in NNRAS. Figure 5 illustrates the relationship between the SNR threshold and ATE using (15) and (17) for NNRAS and FNRAS, respectively, with different BS heights. It can be seen that up to certain SNR threshold, ATE increases then it starts decreasing with further increase in SNR threshold for both NNRAS and FNRAS. Since communication link is robust for the low SNR threshold, therefore, ATE improves with increase in SNR threshold for both the schemes. It is interesting to note that large SNR threshold corresponds to high link spectral efficiency, but it also decreases the coverage performance (probability). Therefore, ATE decreases with higher SNR threshold. Figure 5 also reveals that for low SNR threshold and increasing BS height, FNRAS outperforms NNRAS whereas for high SNR threshold NNRAS performance is better as compared to FNRAS. This is because more BS height also increases the LoS probability of the desired link so the BSs more likely to be associated with the MR in FNRAS. But higher SNR threshold decreases the conditional coverage performance and in NNRAS scheme MR is associated with the nearest BS so its performance is better as compared to FNRAS. Figure 6 investigates the effect of the BS density on ATE performance with different BS heights. Density of the BS considerably improves the ATE performance for both the schemes. This is because more density of BS implies lower load on the network which improves the ATE performance of the MWCN. It is observed that after certain density the rate of improvement is less for NNRAS. Since ATE depends on the coverage probability and transmission distance of the BSs also [(13), Sect. 4]. Therefore, increasing density of the BS reduces the transmission distance between BS and MR for NNRAS

**Fig. 4** Coverage probability for MWCN with different BS densities



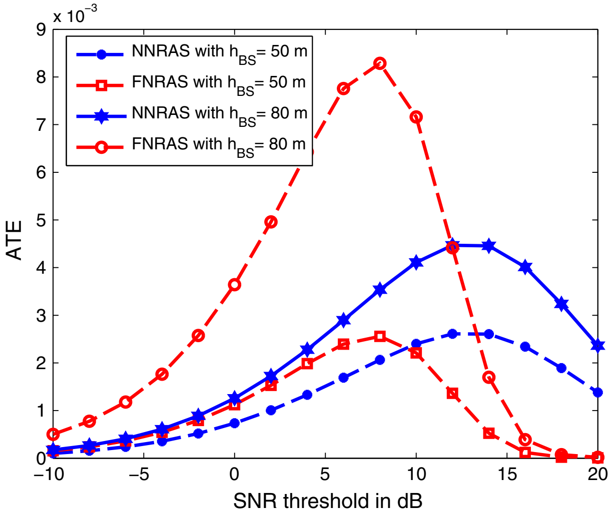



**Fig. 5** ATE for MWCN with different SNR threshold

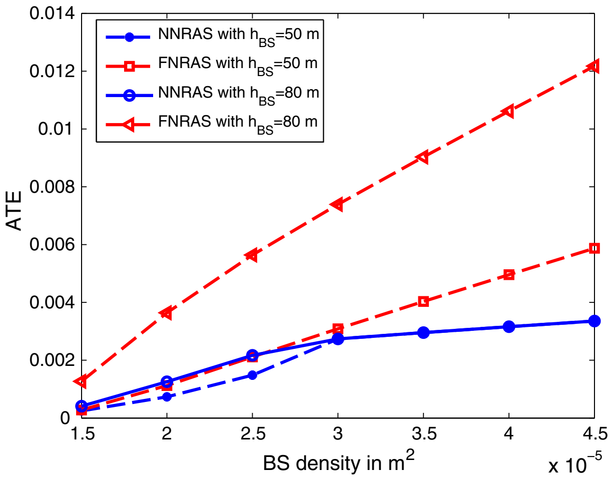



**Fig. 6** ATE for MWCN with different densities

because in this scheme, a typical MR is associated with the nearest receivers. Consequently, MR is easily associated with BS due to the large density with small distance. This is because of the less transmission distance, improvement in ATE is less.

## 6 Conclusion

In this paper, stochastic geometry based statistical model is proposed to characterize the blockages for the MWCN. Coverage probability is calculated for NNRAS and FNRAS. It is observed that height and density of the BS increases the coverage performance of the MWCN in the noise limited environment. A new performance metric ATE is proposed to consider the effect of the distance between BS and MR and density on the network performance. Numerical results show that FNRAS performance is better for low SNR threshold and more BS height as compared to NNRAS. Whereas NNRAS performance is better for high SNR threshold. Finally, it may be concluded that dense deployment of BSs is a promising solution for coverage and capacity improvement in MWCN.

## Appendix 1: Proof of the Theorem 1

The LoS probability of a link is given as [11]

$$\mathcal{P}_{\text{LoS}} = \prod_{n=1}^{n_d} \mathbb{P}(h_n < h_L), \tag{18}$$

where $n_d$ is the number of blockages crossed by the transmitted beam and $h_n$ is the height of the blockages. The height of the beam $h_L$ at the blockage is given by

$$h_L = h_{\text{BS}} - \frac{d_L(h_{\text{BS}} - h_{\text{MR}})}{d_{\text{BM}}}, \tag{19}$$

where $h_{\text{BS}}$ and $h_{\text{MR}}$ are heights of the BS and MR, respectively. $d_L$ and $d_{BM}$ are the distances between BS and blockage and BS to MR, respectively.

The general geometric statistics of building blockages in urban area is described by the three parameters $\delta$, $\zeta$, and $\xi$ [9]. $\delta$ denotes the ratio between the blockage areas to the overall urban area. $\zeta$ denotes the average number of blockages per unit area. $\xi$ is a variable which explain the Rayleigh distributed height of the blockages and is given by [11]

$$f(h_n) = \frac{h_n e^{-\frac{h_n^2}{2\xi^2}}}{\xi^2} \tag{20}$$

The expected number of blockages passed (by MMW beam) per unit distance $n_{\text{bd}}$ is expressed as $n_{\text{bd}} = \sqrt{\delta\zeta}$. Therefore, using (1) the number of blockages for a path of $d_{BM}$ under NNRAS can be expressed as

$$\begin{aligned} n_d &= \int_0^{\mathcal{D}_{\max}} n_{\text{bd}} d_{\text{BM}} f^N(d_{\text{BM}})\, dd_{\text{BM}} \\ &= \int_0^{\mathcal{D}_{\max}} 2\pi n_{bd} \lambda d_{\text{BM}}^2 e^{-\pi\lambda d_{\text{BM}}^2}\, dd_{\text{BM}} \end{aligned} \tag{21}$$

$$= \frac{n_{\text{bd}}}{2} \left( \frac{\text{erf}\left(\sqrt{\pi}\sqrt{\lambda}\mathcal{D}_{\max}\right)}{\sqrt{\lambda}} - 2\mathcal{D}_{\max} e^{-\pi\lambda\mathcal{D}_{\max}^2} \right), \tag{22}$$

where erf(.) is error function. It is assumed that blockages are uniformly spaced between the BS and MR [11]. The blockages distances are given by

$$r_j = \left(j + \frac{1}{2}\right) \Delta_{\text{MR}}, j \in \{0, \ldots, n_d - 1\}, \tag{23}$$

where $\Delta_{\text{MR}}$ is given by

$$\begin{aligned}\Delta_{\mathrm{MR}} &= \int_0^{\mathcal{D}_{\max}} \frac{d_{\mathrm{BM}} f^N(d_{\mathrm{BM}})}{n_d}\, dd_{\mathrm{BM}} \\ &= \int_0^{\mathcal{D}_{\max}} \frac{2\pi\lambda d_{\mathrm{BM}}^2 e^{-\pi\lambda d_{\mathrm{BM}}^2}}{n_d}\, dd_{\mathrm{BM}}\end{aligned} \tag{24}$$

$$= \frac{1}{2n_{\mathrm{d}}}\left(\frac{\operatorname{erf}\left(\sqrt{\pi}\sqrt{\lambda}\mathcal{D}_{\max}\right)}{\sqrt{\lambda}} - 2\mathcal{D}_{\max} e^{-\pi\lambda\mathcal{D}_{\max}^2}\right) \tag{25}$$

Using (22) and (25), (23) may be written as

$$r_j = \frac{j+\frac{1}{2}}{n_{\mathrm{bd}}} \tag{26}$$

So, using (19) at each $r_j$ the height of the LoS beam is given by

$$\begin{aligned}h_j^N &= h_{\mathrm{BS}} - \int_0^{\mathcal{D}_{\max}} \frac{r_j(h_{\mathrm{BS}} - h_{\mathrm{MR}})}{d_{\mathrm{BM}}} f^N(d_{\mathrm{BM}}) dd_{\mathrm{BM}} \\ &= h_{\mathrm{BS}} - \int_0^{\mathcal{D}_{\max}} \frac{r_j(h_{\mathrm{BS}} - h_{\mathrm{BM}})}{d_{\mathrm{BM}}} 2\pi\lambda d_{\mathrm{BM}} e^{-\pi\lambda d_{\mathrm{BM}}^2} dd_{\mathrm{BM}}\end{aligned} \tag{27}$$

$$= h_{\mathrm{BS}} - \frac{\pi(2j+1)\sqrt{\lambda}(h_{\mathrm{BS}} - h_{\mathrm{MR}})\operatorname{erf}\left(\sqrt{\pi}\sqrt{\lambda}\mathcal{D}_{\max}\right)}{2n_{\mathrm{bd}}} \tag{28}$$

Thus, the probability that height of the blockage is lower than $h_j^N$ can be written as

$$\mathcal{P}_j^N = \int_0^{h_j^N} f(h_n) dh_n \tag{29}$$

$$\mathcal{P}_j^N = 1 - e^{-\frac{\left(h_j^N\right)^2}{2\xi^2}}, \tag{30}$$

where $f(h_n)$ is given by (20) and $h_j^N$ is given by (28) for NNRAS.

## Appendix 2: Proof of the Theorem 2

Steps similar to (18)–(22) are as used for NNRAS, then, number of blockages for a path of length for $d_{BM}$ under FNRAS may be expressed as

$$\begin{aligned}n_d &= \int_0^{\mathcal{D}_{\max}} n_{\mathrm{bd}} d_{\mathrm{BM}} f^F(d_{\mathrm{BM}})\, dd_{\mathrm{BM}} \\ &= \int_0^{\mathcal{D}_{\max}} n_{bd} \frac{2\pi\lambda d_{\mathrm{BM}}^2 e^{\pi\lambda d_{\mathrm{BM}}^2}}{e^{\pi\lambda\mathcal{D}_{\max}^2} - 1}\, dd_{\mathrm{BM}}\end{aligned} \tag{31}$$

$$= \frac{n_{\mathrm{bd}}\left(\frac{\mathrm{erfi}\left(\sqrt{\pi}\sqrt{\lambda}\mathcal{D}_{\max}\right)}{\sqrt{\lambda}} - 2\mathcal{D}_{\max} e^{\pi\lambda\mathcal{D}_{\max}^2}\right)}{2 - 2e^{\pi\lambda\mathcal{D}_{\max}^2}}, \tag{32}$$

where $\mathrm{erfi}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{t^2} dt$ is imaginary error function. Again, it is assumed that blockages are uniformly spaced between the BS and MR. The blockages distances are given by

$$r_j = \left(j + \frac{1}{2}\right)\Delta_{\mathrm{MR}}, j \in \{0, \ldots, n_d - 1\}, \tag{33}$$

where $\Delta_{\mathrm{MR}}$ is given by

$$\begin{aligned} \Delta_{\mathrm{MR}} &= \int_0^{\mathcal{D}_{\max}} \frac{d_{\mathrm{BM}} f^F(d_{\mathrm{BM}})}{n_d}\, dd_{\mathrm{BM}} \\ &= \int_0^{\mathcal{D}_{\max}} \frac{2\pi\lambda d_{\mathrm{BM}}^2 e^{\pi\lambda d_{\mathrm{BM}}^2}}{n_d\left(e^{\pi\lambda\mathcal{D}_{\max}^2} - 1\right)}\, dd_{\mathrm{BM}} \end{aligned} \tag{34}$$

$$= \frac{\frac{\mathrm{erfi}\left(\sqrt{\pi}\sqrt{\lambda}\mathcal{D}_{\max}\right)}{\sqrt{\lambda}} - 2\mathcal{D}_{\max} e^{\pi\lambda\mathcal{D}_{\max}^2}}{2n_d - 2n_d e^{\pi\lambda\mathcal{D}_{\max}^2}} \tag{35}$$

Using (32) and (35), (33) may be written as

$$r_j = \frac{j + \frac{1}{2}}{n_{\mathrm{bd}}} \tag{36}$$

Similar to NNRAS, using (19) at each $r_j$ the height of the LoS beam is given by

$$h_j^F = h_{\mathrm{BS}} - \int_0^{\mathcal{D}_{\max}} \frac{r_j(h_{\mathrm{BS}} - h_{\mathrm{MR}})}{d_{\mathrm{BM}}} f^F(d_{\mathrm{BM}}) dd_{\mathrm{BM}} \tag{37}$$

$$= h_{\mathrm{BS}} - \int_0^{\mathcal{D}_{\max}} \frac{r_j(h_{\mathrm{BS}} - h_{\mathrm{BM}})}{d_{\mathrm{BM}}} \frac{2\pi\lambda d_{\mathrm{BM}} e^{\pi\lambda d_{\mathrm{BM}}^2}}{e^{\pi\lambda\mathcal{D}_{\max}^2} - 1} dd_{\mathrm{BM}} \tag{38}$$

$$h_j^F = h_{\mathrm{BS}} - \frac{\pi(2j+1)\sqrt{\lambda}(h_{\mathrm{BS}} - h_{\mathrm{MR}})\mathrm{erfi}\left(\sqrt{\pi}\sqrt{\lambda}\mathcal{D}_{\max}\right)}{2n_{\mathrm{bd}}\left(e^{\pi\lambda\mathcal{D}_{\max}^2} - 1\right)}. \tag{39}$$

Hence, the probability that height of the blockage is lower than $h_j^F$ can be written as

$$\mathcal{P}_j = \int_0^{h_j^F} f(h_n) dh_n \tag{40}$$

$$\mathcal{P}_j = 1 - e^{-\frac{\left(h_j^F\right)^2}{2\xi^2}}, \tag{41}$$

where $f(h_n)$ is given by (20) and $h_j^F$ is given by (38) for FNRAS.

## Appendix 3: Proof of the Lemma 1

Coverage probability conditioned on nearest distance from the typical MR located at the origin is derived as [15]

$$\mathcal{P}_c^N = \int_0^{\mathcal{D}_{\max}} \mathbb{P}[\mathrm{SNR}(\mathrm{d_{BM}}) > \mathcal{T}] f^N(d_{\mathrm{BM}}) dd_{\mathrm{BM}} \tag{42}$$

$$= \int_0^{\mathcal{D}_{\max}} \mathbb{P}\left[h > \frac{\mathcal{T}\mathcal{N}_0 d_{\mathrm{BM}}^{\beta}}{\mathcal{G}_0 \rho_{\mathrm{BS}}}\right] f^N(d_{\mathrm{BM}}) dd_{\mathrm{BM}}, \tag{43}$$

where $\mathcal{T}$ is the SNR threshold, $h$ is the Gamma distributed channel gain, and $\mathcal{N}_0$ denotes the noise power. According to the [21] the tight upper bound of the Gamma random variable with parameter $N$ is given as $\mathbb{P}[x < \mu] < [1 - e^{-q\mu}]^N$, where $q = N(N!)^{-1/N}$. So, (42) can be approximated as

$$\mathcal{P}_c^N \approx \int_0^{\mathcal{D}_{\max}} 1 - \left(1 - e^{-\frac{\mathcal{T} v \mathcal{N}_0 d_{\mathrm{BM}}^{\beta}}{\mathcal{G}_0 \rho_{\mathrm{BS}}}}\right)^m f^N(d_{\mathrm{BM}}) dd_{\mathrm{BM}} \tag{44}$$

$$= \int_0^{\mathcal{D}_{\max}} \sum_{n=1}^{m} (-1)^{(n+1)} \binom{m}{n} e^{-\frac{n\mathcal{T} v \mathcal{N}_0 d_{\mathrm{BM}}^{\beta}}{\mathcal{G}_0 \rho_{\mathrm{BS}}}} f^N(d_{\mathrm{BM}}) dd_{\mathrm{BM}}, \tag{45}$$

where $m$ is the fading parameter, $v = m(m!)^{-1/m}$ and (44) is obtained from the Binomial theorem.

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