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A Cluster-Based Cooperative Spectrum Sensing Strategy to Maximize Achievable Throughput

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Abstract In this paper, a novel cooperative spectrum sensing (CSS) strategy is proposed for cognitive radio networks (CRN) with imperfect reporting channels. This CSS strategy uses simultaneously four techniques to overcome undesirable effects of reporting channels, which are errors and overhead traffic. First, it uses an energy efficient clustering algorithm to maximize the CRN lifetime. Second, in each cluster, an incremental weighing fusion rule is used to improve the accuracy of local sensing performed by secondary users. Third, it selects more reliable improved decisions for sending to the fusion center, to decrease overhead traffic. Fourth, it employs a space–time block code to reduce the probability of errors in reporting channels. We determine the optimal settings of the proposed strategy, such as number of clusters, and their corresponding members by maximizing the achievable throughput of the CRN. Numerical and simulation results will prove the proposed CSS strategy yields the highest throughput for the CRN, while it guarantees the maximum lifetime of CRN, and maximum protection of primary users.

Keywords Cognitive radio network (CRN) · Cooperative spectrum sensing (CSS) · Cooperative communication · Space–time block codes (STBCs) · Imperfect reporting channel · Energy efficiency

1 Introduction

Cognitive radio (CR) has been proposed as a promising solution to improve the current severely underutilized radio spectrum. In this technique, CRs (or secondary users: SUs) opportunistically access the temporarily unused licensed spectrums by finding them

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through spectrum sensing (SS), and without degrading quality of service of the primary users (PUs) [1].

In wireless channels, signals often suffer from shadowing or fading, which may lead to the problem of not being able to correctly detect an active PU. To overcome this problem and increase the probability of correctly detecting a PU, a CSS is proposed in [1], which has three successive stages: *local sensing, reporting, and data fusion.* In local sensing stage, spectrum is sensed by several local SUs. Then, in reporting stage, SS results are sent to a fusion center (FC), and in the last stage, the FC combines them by a specific rule to make a better overall decision.

To perform the above three stages, four great challenges must be considered. First, the reporting channels suffer from noise and fading causing CSS performance is severely degraded [1–3]. In [4, 5], undesired bounds on the sensing performance caused by imperfect reporting channels are discussed. These bounds limit performance of CSS irrespective of local sensing performances achieved by SUs. In [3], these performance limitations are defined in the form of bit error probability walls.

Second, data fusion rule at the FC has a significant impact on CSS. Depending on the type of local SS results that are reported (soft or hard), there are different rules to combine them and taking the final decision [1–3]. But two important points, which should be considered in the design of a fusion rule, are the imperfect reporting channels, and the amount of information available about a CRN (Such as instant SNR).

Third, the overhead time for reporting SS results is directly related to the number of cooperative SUs. When the local sensing time is fixed, increasing the number of cooperative SUs increases reporting time, and reduces the time-slot available for data transmission by each SU. However, in [6, 7], it was shown that the CSS performance is improved by increasing the number of cooperative SUs. So, there is an important tradeoff between the throughput degradation associated with reduced time-slot for data transmission and the throughput gain associated with a successful transmission [5].

Fourth, energy efficiency should be considered for CSS schemes, since a cooperative method and its reporting time significantly affect the lifetime of CRN [8, 9]. The above four challenges have not been considered simultaneously, since they are dependent on each other.

To improve sensing performance caused by errors due to reporting channels, either a cluster-based CSS is used in [10, 11], or a spatial diversity-based CSS is applied [4, 5]. However, inner wireless channels (i.e. intra-cluster channels which are between a SU and each of cluster members) in [10, 11] are considered as error-free. Furthermore, using spatial diversity-based CSS of [4, 5] imposes a lot of overhead and complexity to the CRN, in which the CRN has to switch between two different reporting modes (cooperative and non-cooperative), repeatedly [12].

In the design of fusion rules in [7, 13], reporting channels are considered as error-free. In [2, 14], channels are considered as noisy. But fusion rule in [2] uses all reported decisions, which increases overhead, and in [14], it requires much network information.

In [8, 9], energy efficient CSS schemes are proposed, in which all SUs are not participated in CSS. However, in both schemes, the reporting channels are considered as error-free.

In [15], with the objective of maximizing the achievable throughput of the CRN, a cluster-based CSS is proposed to obtain a proper assignment policy. This policy determines the number of SUs required in each cluster, to cooperate for sensing a PU's channel. However, in [15], reporting channels are considered to be error-free, which is impractical.

Moreover, the main parameter: overhead time for reporting is ignored in the formulation of the achievable throughput.

In this paper, we consider the above four challenges simultaneously for a practical CRN, in which all channels (such as reporting channels and inner channels) suffer from noise and fading. To improve the spectrum efficiency of this CRN, we propose a new cluster-based CSS strategy, with an initial phase and four periodic phases. In initial phase, SUs participating in the CSS are divided into smaller clusters. In Phase 1, each SU senses PU's spectrum independently. In Phase 2, each cluster member improves its local decisions by using its own and other cluster members' decisions. In Phase 3, each cluster separately selects more reliable decisions from its improved decisions, and sends them to the FC by using a cooperative communication. In Phase 4, the FC makes final decision.

The novelties and contributions of this paper are as follows:

- In contrast to previous works (which ignore energy-efficiency of their CSS schemes in the non-error-free environment such as non-cluster-based CSS schemes in [2, 5, 8, 9, 13] and cluster-based CSS schemes in [10, 11, 15]), we propose a clustering algorithm for the CSS strategy to improve its performance, as well as to guarantee maximum the CRN lifetime. This is done by using a method that evenly distributes the total energy consumption of the CRN among clusters.
- Local sensing decisions are improved by using the proposed incremental weighing (IW) fusion rule. This method increases the probability of detecting the PU without increasing overhead traffic. Furthermore, the IW rule adapts itself to inner channels SNR, and so, unlike many conventional methods (such as [3–5]), does not allow inner channels errors causing any undesired bounds on the sensing performance.
- We propose a reliability method, based on IW fusion rule, to selecting and sending more reliable decisions to the FC. By using this method, the overhead traffic is reduced in reporting channels while sensing performance at FC is maintained.
- To overcome fading effects in reporting channels, we employ cooperative communications by using orthogonal distributed space–time block code (ODSTBC) [12]. We chose this method for the following reasons:
  - This method is more bandwidth efficient compared to the other repetition-based methods, such as [16–20], in which data is sent to the FC in different time slots. Also, the method in [16] requires some parity bits for verification of received data at relay users, which add undesirable overhead time. Furthermore, these repetition-based methods are non-energy-efficient since relay users consume more energy compared with those of other users in the network. Thus, relay users will have shorter lifetimes, which is equivalent to a shorter useful lifetime for the network.
  - ODSTBCs can be directly obtained from conventional orthogonal STBCs. So, they have low decoding complexity at the FC [12, 21].
  - As we proved in [12], even when inner channels suffer from noise and fading, it is possible to achieve a better BER with this method compared to those of the non-cooperative methods.
- We derive the detection probability and the false alarm probability for the proposed IW rule at each cluster, and the final fusion rule at the FC. By using them, a closed-form expression for the CRN throughput is derived. Finally, we find the optimal settings of the proposed CSS strategy by maximizing the achievable throughput of the CRN under the constraint of the SUs' interference on the PU.

The remainder of this paper is organized as follows. In Sect. 2, the proposed network model is described. Section 3 presents optimal energy efficient clustering. In Sect. 4, performance, and in Sect. 5, throughput of the proposed CSS strategy is analyzed. In Sect. 6, simulation results are presented to evaluate the proposed CSS strategy. In Sect. 7, conclusions are presented.

2 System Model

In the proposed CSS system, there are S SUs, each equipped with a single antenna, and one FC (Fig. 1). In this CRN, SUs are interested in detecting the presence of PU(s) in a single underutilized frequency band (i.e. the sensing channel). Toward this goal, we propose a cluster-based CSS strategy. This CSS strategy has an initial phase and four periodic phases, which are described in the following sub-sections.

2.1 Initial Phase (Clustering)

As shown in Fig. 1, the clustering algorithm divides N selected SUs, $1 \leq N \leq S$, into M clusters. Number of members belonging to cluster m, $m = 1, 2, \ldots, M$, denoted by n_m. As will been seen in Sect. 3, the optimal energy efficient algorithm determines $\vec{N}_M = \{n_1, n_2, \ldots, n_M\}$ to maximize the CRN lifetime. Moreover, in Sect. 4, we show this flexibility to select N cooperative SUs and M clusters, $1 \leq M \leq N$, helps the CRN to enhance sensing performance.

This initial phase is performed once, but as shown in Fig. 2, the following four phases are repeated in ach time frame.

2.2 Phase 1 (Local Spectrum Sensing)

In local sensing time-slot (T_s), in Fig. 2, each SU performs a local spectrum sensing independently and simultaneously, and then makes a binary local decision.

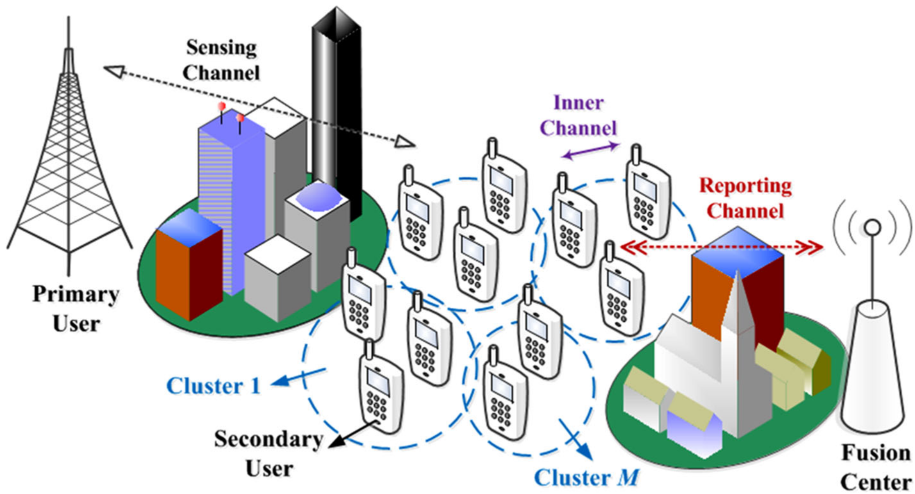

Fig. 1 A CR network in fading environments

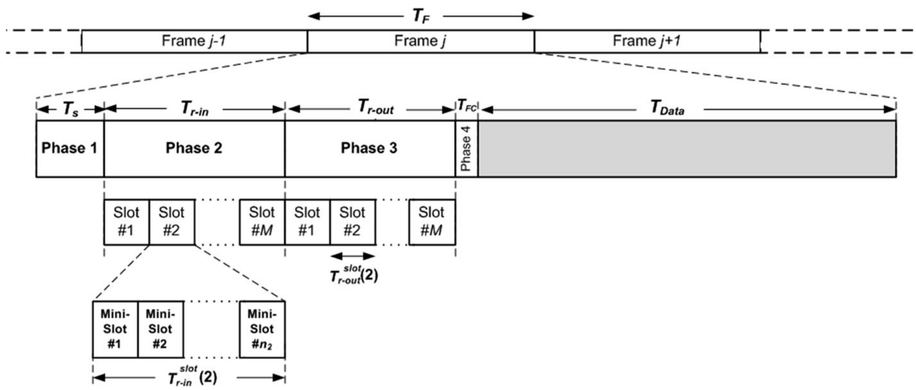

Fig. 2 Frame structure of the periodic phases in the proposed CSS strategy

2.3 Phase 2 (Cluster-Based Decisions Improving)

This phase is composed of M time-slots, where time-slot m ($m = 1, 2, \ldots, M$), has time duration $T^{slot}_{r-in}(m)$, consisting of n_m mini time-slots (Fig. 2). The process performed by cluster m is as follows:

For $m = 1{:}\,M,$

SU 1 in cluster m broadcasts its binary decision using the first mini time-slot.

For $k = 2{:}\,n_m,$

SU k uses an IW fusion rule to make a new binary decision based on its own decision and the decisions received from $k-1$ previous SUs. Then, its broadcasts its new binary decision using mini time-slot k.

End,

End.

This proposed IW fusion rule will be explained later in Sect. 4.2 using (20) and (21). At the end of Phase 2, all SUs in a cluster are aware of each other's updated decisions.

2.4 Phase 3 (Cluster-Based Decisions Reporting)

This phase is composed of M time-slots, where time-slot m ($m = 1, 2, \ldots, M$) has time duration $T^{slot}_{r-out}(m)$ (Fig. 2). The process performed by cluster $m, m = 1, 2, \ldots, M$ is as follows:

1. Based on a proposed reliability method, described in Sect. 4.3, each SU selects l_m of n_m decisions, which can include its own decision and decisions received in Phase 2.

2. Cluster members act like a virtual antenna array, and each SU makes a *bit sequence* of l_m selected decisions.
3. Each SU simultaneously encodes and transmits its *bit sequence* to the FC, according to a distinct column of a matrix called OSTBC matrix [12]. This matrix is designed using the method presented in [22].

2.5 Phase 4 (Decisions Combining)

In this phase, which takes T_{FC} seconds, the FC combines the improved decisions received from all clusters to make a final decision. If the FC decides the PU is absent in the sensing channel, SUs proceeds to transmit data in the rest of the frame (T_{Data}). Otherwise, they must stay quiet and wait until the next time frame to do the CSS again.

As explained in this section, to combat fading in reporting channels, we employ a spatial diversity by using a cooperative communication (ODSTBC). The cooperative communication has two steps: data exchange and cooperative transmission [12, 21], which are equivalent to Phases 2 and 3 in the proposed CSS. But it should be noted that both reporting channels (i.e. the channels between a cluster member and the FC), and inner channels affect the achievable diversity gain and sensing performance. Therefore, in the time of Phase 2, we improve local sensing decisions by using the proposed IW fusion rule. This IW fusion rule depends on the SNR of inner channels. As will be explained in Sect. 4.2, this fusion rule will greatly reduce the destructive effects of errors caused by inner channels.

3 Optimal Energy Efficient Clustering for Proposed CSS

In this section, by analyzing energy consumption in the proposed CSS, we show why we selected the clustering approach for the CSS. Then, we formulate our optimization problem to maximize useful lifetime of the CRN, and evenly distribute the energy consumption of the proposed CSS among clusters. By solving this optimization, we find the optimum clustering algorithm for the proposed CSS.

If there is no clustering, N SUs perform the CSS, and C of S SUs can participate in the reporting to the FC by using cooperative communications. By increasing C, cooperative diversity order increases, and bit error rate (BER) of reporting channels decrease. As a result, the CSS performance can be improved. However, by increasing C, total time duration T_{r-out}, of transmissions between SUs and the FC increases non-linearly, and a longer T_{r-out} left a shorter T_{Data} (Fig. 2), which result in a lower achievable throughput of CRN. So, selecting the value of C, is an important issue since it makes a tradeoff between diversity and throughput. As a result, $C = N$ is not always an optimum solution to maximize CRN performance. On the other side, there is another problem, when $C \neq N$. In this case, C SUs, which report to the FC, consume more energy compare with other SUs, which don't report. Thus, when C SUs run out of energy, the network loses its full coverage and optimum performance, which means the CRN will have a shorter useful lifetime.

Considering the above two challenges, we use clustering approach in the proposed CSS, which has been explained in Sect. 1. In addition to degrees of freedom N, the proposed clustering approach provides two degrees of freedom:

1. The number of clusters (M), and
2. Clustering algorithm.

We use M to determine the diversity order within each cluster (i.e. act like C). As will be described in Sect. 5, optimal value for this parameter depends on the maximization of achievable throughput.

The clustering algorithm plays an important role in determining a relationship between N cooperative SUs, and M clusters, with n_m SUs in cluster m ($m = 1, 2, \ldots, M$). In the rest of this section, we use a clustering algorithm to evenly distribute the energy consumption among clusters, which increases the lifetime of the CRN. Toward this goal, n_m should be determined in such a way to minimize the mean square difference of energy consumption (MSDE) between any two arbitrary clusters. So, the energy optimization problem can be modeled as follows

$$\begin{gathered}\min_{\vec{N}_M=\{n_1,n_2,\ldots,n_M\}} MSDE\left(\vec{N}_M\right) = \frac{2}{M(M-1)}\sum_{i=1}^{M-1}\sum_{m=i+1}^{M}\left[E_i(n_i) - E_m(n_m)\right]^2 \\ s.t. \sum_{m=1}^{M} n_m = N,\end{gathered} \tag{1}$$

where $E_m(n_m)$ is the average energy consumption of cluster m with n_m SUs, which consists of three main components as follows

$$E_m(n_m) = \frac{1}{n_m}\left(n_m P_s T_s + E_{in,m}(n_m) + E_{r,m}(n_m)\right). \tag{2}$$

In (2), P_s is the power consumption of each SU in local sensing time-slot (Phase 1), $E_{in,m}(.)$ is the energy consumption of inner communications among SUs of cluster m in Phase 2, and $E_{r,m}(.)$ is the energy consumption of reporting decisions by cluster m to the FC in Phase 3. The last two energy functions are related to the number of SU in the cluster, n_m, which are given by

$$E_{in,m}(n_m) = P_{in,m}\left(SNR_{in}^m\right) \times T_{r-in}^{slot}(m) = \frac{P_{in,m}\left(SNR_{in}^m\right)n_m}{R_b}, \tag{3}$$

$$E_{r,m}(n_m) = P_{r,m}\left(SNR_r^m\right) \times T_{r-out}^{slot}(m) = \frac{P_{r,m}\left(SNR_r^m\right)l_m}{R_b^{eff}(n_m)}, \tag{4}$$

where $P_{in,m}\left(SNR_{in}^m\right)$ and $P_{r,m}\left(SNR_r^m\right)$ are the total average power consumptions in Phases 2 and 3, whose values, respectively, depend on the average SNR of inner and reporting channels of cluster m (i.e. SNR_{in}^m and SNR_r^m), l_m is total number of bits transmitted from cluster m to the FC, which will be defined in (33), R_b is SUs bit rate, and $R_b^{eff}(n_m)$ is the effective bit rate of the ODSTBC transmission.

Suppose that T_{coh} is the coherence time of the fading reporting channels, and $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. So, each group of $F = \lfloor R_b T_{coh} \rfloor$ bits sent to the FC, consists of $\vartheta \times n_m$ training bits, which are used to obtain channel state information [5]. As a result, $R_b^{eff}(n_m)$ in (4) can be represented by

$$R_b^{eff}(n_m) = \frac{F - \vartheta n_m}{F} \times R_{STBC}(n_m) \times R_b, \tag{5}$$

where $R_{STBC}(.)$ is code rate, which can be expressed as follows for the complex modulations [5, 22]

$$R_{STBC}(n_m) = \begin{cases} \dfrac{n_m + 2}{2n_m}, & n_m \in even, \\ \dfrac{n_m + 3}{2(n_m + 1)}, & n_m \in odd. \end{cases} \tag{6}$$

Now, by using these definitions, we present Theorem 1, which gives the optimal clustering algorithm.

Theorem 1 T*he solution of the optimization problem given by* (1)*, determine the optimal number of SUs in each cluster,* $\hat{n}_m$*, which can have a lower and upper bounds as follows*

$$N - \sum_{\substack{q=1 \\ q \neq m}}^{M} \left\lceil E_q^{-1}(E_m(\hat{n}_m + 1)) \right\rceil \leq \hat{n}_m \leq N - \sum_{\substack{q=1 \\ q \neq m}}^{M} \lfloor E_q^{-1}(E_m(\hat{n}_m - 1)) \rfloor, \quad 1 \leq m \leq M, \tag{7}$$

where $\lceil x \rceil$ *denotes the smallest integer bigger than or equal to x, and* $E_q^{-1}(.)$ *is the inverse of the total energy consumption in* (2).

Proof Proof of (7) is presented in "Appendix 1".

Suppose SUs sparsely dispersed in the CRN, and its dimensions are small as compared to the distances between each SU and the FC. So, we can assume the average SNR per bit in reporting channels of all clusters are approximately the same, and denoted by SNR_r [2, 14]. Furthermore, a clustering algorithm assigns each node to a cluster whose cluster center is closer to the node. Hence, it is a reasonable assumption that the average SNR per bit in each inner channel of a cluster is approximately the same, which denoted by SNR_{in}. By using these assumptions, we present Proposition 1, which gives a simple version of (7).

Proposition 1 *If SNR of inner channels, as well as SNR of reporting channels are respectively almost the same, then the optimal number of SUs in each cluster,* $\hat{n}_m$*, in* (7) *can be simplified as follows*

$$\hat{n}_m = \begin{cases} n + 1, & 1 \leq m \leq r, \\ n, & r + 1 \leq m \leq M, \end{cases} \tag{8}$$

where

$$n = \lfloor \frac{N}{M} \rfloor, \quad r = N - nM \tag{9}$$

Proof By these assumptions, we can infer that $P_{in,m}$ and $P_{r,m}$ are approximately the same for all clusters ($P_{in}(SNR_{in}) = P_{in,m}\left(SNR_{in}^m\right), P_r(SNR_r) = P_{r,m}\left(SNR_r^m\right)$ for $\forall m$). Therefore, by using (3) and (4), we have

$$E_m(\hat{n}_m) = E_q(\hat{n}_m), \quad 1 \leq m \leq M, 1 \leq q \leq M. \tag{10}$$

$$E_q^{-1}(E_m(\hat{n}_m)) = E_q^{-1}\left(E_q(\hat{n}_m)\right) = \hat{n}_m. \tag{11}$$

By using (11), we can rewrite (7) as

$$\frac{N-(M-1)}{M} \leq \hat{n}_m \leq \frac{N+(M-1)}{M}, \quad 1 \leq m \leq M. \tag{12}$$

Since $\hat{n}_m$ can only be a positive integer, we have

$$\lceil \frac{N}{M} - \frac{(M-1)}{M} \rceil \leq \hat{n}_m \leq \lfloor \frac{N}{M} + \frac{(M-1)}{M} \rfloor, \quad 1 \leq m \leq M. \tag{13}$$

Substituting (9) into (13), we have

$$\lceil n + \frac{r-(M-1)}{M} \rceil \leq \hat{n}_m \leq \lfloor n + \frac{r+(M-1)}{M} \rfloor, \quad 1 \leq m \leq M. \tag{14}$$

From (9), it's clear that $0 \leq r \leq M-1$. As a result, $\hat{n}_m$ can only be one of two values, n or $n+1$. Now, we need to determine how many clusters containing n and $n+1$ SUs. If $M-J$ and J, respectively, denoting the number of clusters that have n and $n+1$ SUs, the total number of SUs will be calculated as follows:

$$\sum_{m=1}^{M} \hat{n}_m = (M-J)n + J(n+1) = Mn + J = N. \tag{15}$$

Looking at (9), we find that $J = r$. Therefore, r and $M-r$ clusters have $n+1$ and n SUs, respectively, and we get the same description as (8) that we wanted. According to (8) and (9), when N is an integer multiples of M, the numbers of SUs in each cluster are the same; otherwise the maximum difference between the two clusters will be equal to 1. Therefore, as expected, the optimal clustering algorithm uses a uniform distribution for the number of SUs, to achieve an evenly distribution of energy consumption. Thereby, achieving the maximum effective life-time is guaranteed.

4 Performance Analysis of the Proposed CSS

In this section, the phases described in Sect. 2 for the proposed CSS strategy, are analytically studied in the following sub-sections.

4.1 Local Spectrum Sensing

In this phase, each SU evaluates a decision statistic based on the received signal from the sensing channel with bandwidth W_s, over the sensing time-slot T_s. Then, SU k $(1 \leq k \leq n_m)$ in cluster m $(1 \leq m \leq M)$ compares this decision statistic with a common detection threshold (ψ_E) to make a one-bit hard local decision d_k^m. It's notable that $d_k^m = 1$ corresponds to the hypothesis H_1 (a PU signal is present) and $d_k^m = 0$ corresponds to the hypothesis H_0 (a PU signal is absent).

We assume that SUs experience a same average SNR on the sensing channel, which denoted by SNR_s [2, 5, 6, 14]. Therefore, we have

$$\Pr\{d_k^m = 1|H_0\} = P_f^{Local}, \quad \Pr\{d_k^m = 0|H_0\} = 1 - P_f^{Local}, \tag{16}$$

$$\Pr\{d_k^m = 1|H_1\} = P_d^{Local}, \quad \Pr\{d_k^m = 0|H_1\} = 1 - P_d^{Local}, \tag{17}$$

where P_d^{Local} and P_f^{Local} are the local probabilities of detection, and false alarm. These probabilities for energy detection method on Rayleigh fading channel can be expressed as [6]

$$P_f^{Local} = \frac{\Gamma(u, \psi_E/2)}{\Gamma(u)}, \tag{18}$$

$$P_d^{Local} = \frac{\Gamma\left(u-1, \frac{\psi_E}{2}\right)}{\Gamma(u-1)} + e^{\frac{-\psi_E}{2(1+uSNR_s)}}\left(1+\frac{1}{uSNR_s}\right)^{u-1}\left[1-\frac{\Gamma\left(u-1, \frac{u\psi_E SNR_s}{2(1+uSNR_s)}\right)}{\Gamma(u-1)}\right], \tag{19}$$

where $\Gamma(.)$ is gamma function, $\Gamma(.,.)$ is a complementary incomplete gamma function, and $u = \lfloor W_s T_s \rfloor$ is the time-bandwidth product.

4.2 Cluster-Based Decisions Improving

In this phase, we are looking to simultaneously achieve three main goals for each cluster:

1. All SUs in a cluster are aware of each other's decisions, which is a prerequisite step in a cooperative communication [5, 12, 21].
2. SUs improve and modify their local decisions based on other SUs' decisions.
3. These decisions improvement is done in a way that the BER of inner channels has minimal impact on it.

Therefore, as described in Sect. 2, we proposed a process in which SUs share their decisions simultaneously in order to improve their decisions. In this sub-section, this process is analyzed mathematically for cluster m as follows.

As explained in Sect. 2, this process starts from SU 1 and ends by SU n_m in n_m similar steps (i.e. time-slots). In step k, SU k after receiving $k-1$ decisions of the $k-1$ previous SUs, computes soft valued decision statistics $\Lambda_{IW}(m,k)$ as

$$\Lambda_{IW}(m,k) = w_m \sum_{i=1}^{k-1} \hat{x}_{i,k}^m + d_k^m, \tag{20}$$

where $\hat{x}_{i,k}$ is the decoded binary decision by SU k based on the binary decision x_i^m transmitted by SU i, and w_m is the weight assigned to $k-1$ decoded binary decisions.Then, SU k makes the final decision as

$$x_k^m = \begin{cases} 1, & \Lambda_{IW}(m,k) \geq \psi_{IW}(m,k), \\ 0, & \Lambda_{IW}(m,k) < \psi_{IW}(m,k), \end{cases} \tag{21}$$

where $\psi_{IW}(m,k)$ is the fusion threshold of SU k. We call the proposed fusion rule in (20) and (21) as the *incremental weighing* (IW) fusion rule, as by increasing SU members from 1 to n_m, number of elements of its decision statistic increases linearly. We set $\psi_{IW}(m,1) = 1$ for $\forall m$. So, from (20) and (21), we have $x_1^m = d_1^m$ for $\forall m$.

Due to inner channels errors, the decisions received by SU k may differ from those transmitted from the k-1 SUs, which can have a devastating impact on sensing performance. Therefore, the weight assigned to $k-1$ decoded binary decisions, w_m, in (20) is calculated using SNR of inner channels, SNR_{in}^m, as follows

$$w_m = \frac{SNR_{in}^m}{SNR_{in}^m + \alpha}. \tag{22}$$

In above, α is a positive constant, which controls the influence of SNR_{in}^{m} on w_m. As seen in (22), higher weights are assigned to those decoded decisions received with higher SNRs since such decisions are more reliable. Furthermore, as expected, w_m tends to one as the SNR_{in}^{m} tends to infinity and all error-free decisions have a same weight. Therefore, using the proposed IW fusion rule, we can achieve all three goals at this phase.

Theorem 2 *The probabilities of detection ($P_d^{IW}(m,k)$) and false-alarm ($P_f^{IW}(m,k)$) of the proposed IW fusion rule, are given as*

$$P_f^{IW}(m,k) = \Pr\{x_k^m = 1|H_0\} = \sum_{a=0}^{2^{k-1}-1}\left(1 - F_k\left(\vec{A}_k^a\middle|H_0\right)\right)\Pr\left\{\vec{X}_k^m = \vec{A}_k^a\middle|H_0\right\}, \tag{23}$$

$$P_d^{IW}(m,k) = \Pr\{x_k^m = 1|H_1\} = \sum_{a=0}^{2^{k-1}-1}\left(1 - F_k\left(\vec{A}_k^a\middle|H_1\right)\right)\Pr\left\{\vec{X}_k^m = \vec{A}_k^a\middle|H_1\right\}, \tag{24}$$

where $\vec{X}_k^m = \left\{\hat{x}_{1,k}^m, \hat{x}_{2,k}^m, \ldots, \hat{x}_{k-1,k}^m\right\}$ is the vector of decoded decisions received from $k-1$ SUs; $\boldsymbol{\Pi}_{\mathrm{k}}$ is the set of all possible values of a $k-1$ bit binary vector, like $\vec{X}_k^m$; the cardinality of $\boldsymbol{\Pi}_{\mathrm{k}}$ is 2^{k-1}; and $\vec{A}_k^a$, $\vec{B}_k^b$ and $\vec{C}_k^c$ are the ath, bth, and cth entries in $\boldsymbol{\Pi}_{\mathrm{k}}$, which is corresponding to the $k-1$ bit vector equivalent to the decimal values a, b, and c, respectively.

In (23) and (24), $\Pr\left\{\vec{X}_k^m = \vec{A}_k^a\middle|H_y\right\}$, $y \in \{0,1\}$ is the probability mass function (pmf) of $\vec{X}_k^m$, which is given by

$$\begin{aligned}\Pr\left\{\vec{X}_k^m = \vec{A}_k^a\middle|H_y\right\} = \sum_{b=0}^{2^{k-1}-1}\Bigg(&\prod_{j=1}^{k-1} p_j\left(\vec{B}_j^b\middle|H_y\right)^{B_j^b}\left(1 - p_j\left(\vec{B}_j^b\middle|H_y\right)\right)^{1-B_j^b} \\ &\times (P_{e1})^{\left|A_j^a - B_j^b\right|}(1 - P_{e1})^{1-\left|A_j^a - B_j^b\right|}\Bigg).\end{aligned} \tag{25}$$

In above, A_j^a is the bit corresponding to bit j in vector $\vec{A}_k^a$, and auxiliary functions $p_j\left(\vec{B}_j^b\middle|H_y\right)$ is defined as follows

$$p_j\left(\vec{B}_j^b\middle|H_y\right) = \begin{cases}\sum\limits_{c=0}^{2^{j-1}-1}\left\{\left(1 - F_j\left(\vec{C}_j^c\middle|H_y\right)\right) \times (P_{e2})^{\sum_{i=1}^{j-1}\left|B_i^b - C_i^c\right|}(1 - P_{e2})^{j-1-\sum_{i=1}^{j-1}\left|B_i^b - C_i^c\right|}\right\}, & j \neq 1, \\ 1 - F_j\left(\vec{B}_j^b\middle|H_y\right), & j = 1.\end{cases} \tag{26}$$

In (25), $P_{e1} = P_{e2} = P_e^{in}\left(SNR_{in}^m\right)$ which $P_e^{in}\left(SNR_{in}^m\right)$ denotes the BER of inner channel, and $F_j\left(\vec{A}_j^a\middle|H_y\right)$ is cumulative distribution function (CDF) of random variable d_j^m which can be computed as follows

$$
\begin{aligned}
F_j\left(\vec{A}_j^a \middle| H_y\right) &= \Pr\left\{ d_j^m < \psi_{IW}(m,j) - w_m \sum_{i=1}^{j-1} A_i^a \middle| H_y \right\}, \\
&= \begin{cases} 1, & 1 < \psi_{IW}(m,j) - w_m \sum_{i=1}^{j-1} A_i^a, \\ \Pr\left\{ d_j^m = 0 | H_y \right\}, & 0 < \psi_{IW}(m,j) - w_m \sum_{i=1}^{j-1} A_i^a \le 1, \\ 0, & \psi_{IW}(m,j) - w_m \sum_{i=1}^{j-1} A_i^a \le 0. \end{cases}
\end{aligned}
\tag{27}
$$

Proof Proof of (23) and (24) are presented in "Appendix 2".

Moreover, this IW fusion rule doesn't make any limit on sensing, although there are errors in inner channels. This means if we choose appropriate values for α and $\psi_{IW}(m,k)$, then there is no lower bound for $P_f^{IW}(.,.)$ or upper bound for $P_d^{IW}(.,.)$ due to inner channel errors (unlike other fusion rules such as *K-out-of-N* [3–5]). The following propositions prove the above discussion.

Proposition 2 *If for a specific* SNR_{in}^m*, we have*

$$
\psi_{IW}(m,k) > (k-1)\left(\frac{SNR_{in}^m}{SNR_{in}^m + \alpha}\right), \tag{28}
$$

and if $P_f^{Local} \to 0$*, then* $P_f^{IW}(m,k) \to 0$.

Proof When $P_f^{Local} \to 0$, (27) can be simplified to

$$
1 - F_k\left(\vec{A}_k^a \middle| H_0\right) = \begin{cases} 0, & \psi_{IW}(m,k) - w_m \sum_{i=1}^{k-1} A_i^a > 0, \\ 1, & \psi_{IW}(m,k) - w_m \sum_{i=1}^{k-1} A_i^a \le 0. \end{cases} \tag{29}
$$

Furthermore, for $\forall \vec{A}_k^a$ we know

$$
\psi_{IW}(m,k) - w_m(k-1) \le \psi_{IW}(m,k) - w_m \sum_{i=1}^{k-1} A_i^a \le \psi_k^m. \tag{30}
$$

So, if $\psi_{IW}(m,k) > w_m(k-1)$, then $\psi_{IW}(m,k) - w_m \sum_{i=1}^{k-1} A_i^a > 0$ for $\forall \vec{A}_k^a$, and $1 - F_k\left(\vec{A}_k^a \middle| H_0\right) = 0$ for $\forall \vec{A}_k^a$. Therefore, by looking at (23), $P_f^{IW}(m,k) \to 0$. Defining w_m as given by (22), then $\psi_{IW}(m,k) > w_m(k-1)$ is given by (28).

Proposition 3 *If we have*

$$
\psi_{IW}(m,k) \le 1, \tag{31}
$$

and if $P_d^{Local} \to 1$*, then* $P_d^{IW}(m,k) \to 1$.

Proof When $P_d^{Local} \to 1$, (27) can be simplified to

$$1 - F_k\left(\vec{A}_k^a \middle| H_0\right) = \begin{cases} 0, & \psi_{IW}(m,k) - w_m \sum_{i=1}^{k-1} A_i^a > 1, \\ 1, & \psi_{IW}(m,k) - w_m \sum_{i=1}^{k-1} A_i^a \le 1. \end{cases} \tag{32}$$

So, looking at (24) and (30), we find that if $\psi_{IW}(m,k) \le 1$, then $\psi_{IW}(m,k) - w_m \sum_{i=1}^{k-1} A_i^a \le 1$ for $\forall \vec{A}_k^a$, and $P_d^{IW}(m,k) \to 1$.

4.3 Cluster-Based Decisions Reporting

In this phase, first, SUs in each cluster select last l_m of n_m improved decisions, where

$$l_m = \left\lfloor n_m - (w_m)^{1/3}(n_m - 1) \right\rfloor. \tag{33}$$

Then, these SUs employ cooperative transmission using ODSTBC, and transmit selected decisions, $x_k^m (n_m - l_m + 1 \le k \le n_m)$ to the FC.

In (33), by increasing SNR_{in}^m (i.e. when $w_m \to 1$), SUs send less number of decisions to the FC, since these decisions have sufficient reliable knowledge about other decisions.

As shown in [12], ODSTBC BER depends on the number of cooperative SUs, n_m, SNR of inner channels, and SNR of reporting channels. A closed-form expression for the upper bound of ODSTBC BER is derived in [12], which we use it as $P_e^r\left(n_m, SNR_{in}^m, SNR_r^m\right)$ in the rest of this paper.

4.4 Decisions Combining

In the final phase, the FC, after receiving selected improved decisions from M independent clusters, decides which hypothesis is more likely to be true, which is denoted by z_{FC}. This is done by using a fusion rule to construct a decision statistic, Λ_{FC}, and then it is compared with a threshold.

The optimal fusion rule at the FC for the proposed CSS strategy is the sum of log-likelihood ratios (LLRs) of the received decisions from each cluster [3, 13], i.e.,

$$\Lambda_{FC}^{OPT} = \sum_{m=1}^{M} log \left[\frac{\Pr\left\{ \vec{X}_{FC}^m \middle| H_1 \right\}}{\Pr\left\{ \vec{X}_{FC}^m \middle| H_0 \right\}} \right]. \tag{34}$$

In above, $\vec{X}_{FC}^m = \left\{ \tilde{x}_{n_m - l_m + 1}^m, \tilde{x}_{n_m - l_m + 2}^m, \ldots, \tilde{x}_{n_m}^m \right\}$ is the vector of decoded decisions $\tilde{x}_k^m$ by the FC, which is based on the selected binary decision x_k^m transmitted by cluster m. Looking at (58) and (59), we see that the pmf of $\vec{X}_{FC}^m$, $\Pr\left\{ \vec{X}_{FC}^m \middle| H_y \right\}$ for $y \in \{0,1\}$, is the same as the pmf of $\vec{X}_k^m$ in (25), with the exception that $k = l_m + 1$ and $P_{e1} = P_e^r\left(n_m, SNR_{in}^m, SNR_r^m\right)$. Therefore, $\Pr\left\{ \vec{X}_{FC}^m \middle| H_y \right\}$ in (34) is given by

$$\begin{aligned} \Pr\left\{ \vec{X}_{FC}^m = \vec{A}_k^a \middle| H_y \right\} = \sum_{b=0}^{2^{l_m}-1} \Bigg(\prod_{j=1}^{l_m} p_j\left(\vec{B}_{l_m+1}^b \middle| H_y \right)^{B_j^b} \left(1 - p_j\left(\vec{B}_{l_m+1}^b \middle| H_y \right) \right)^{1-B_j^b} \\ \times (P_{e1})^{\left|A_j^a - B_j^b\right|} (1 - P_{e1})^{1-\left|A_j^a - B_j^b\right|} \Bigg), \end{aligned} \tag{35}$$

where $p_j\left(\vec{\boldsymbol{B}}^b_{l_m+1}\middle|H_y\right)$ is defined in (26).

As seen in (35) and (26), the optimal fusion rule requires average SNR of inner and reporting channels, i.e. SNR^m_{in}, SNR^m_r, and the local sensing performance indices, i.e., the P^{Local}_d and P^{Local}_f, for all clusters, which may be unavailable. Furthermore, it has a non-linear complex form, since according to Eq. (20), the improved decisions of SUs in each cluster are correlated. Therefore, we use a general fusion rule for the FC, which is easy to be implemented, and it reduces the complexity of the throughput optimization problem in the next section. This fusion rule is OR-rule, which is computed as

$$\Lambda_{FC} = \sum_{m=1}^{M} \sum_{k=n_m-l_m+1}^{n_m} \tilde{x}^m_k, \tag{36}$$

$$z_{FC} = \begin{cases} 1, & \Lambda_{FC} \geq 1, \\ 0, & \Lambda_{FC} = 0. \end{cases} \tag{37}$$

Considering independent clusters, we can evaluate the false alarm probability, P^{FC}_f, and the detection probability, P^{FC}_d, at the FC as

$$P^{FC}_f = \Pr\{z_{FC} = 1|H_0\} = 1 - \prod_{m=1}^{M} \Pr\left\{\vec{\boldsymbol{X}}^m_{FC} = \vec{\mathbb{O}}_{l_m+1}\middle|H_0\right\}, \tag{38}$$

$$P^{FC}_d = \Pr\{z_{FC} = 1|H_1\} = 1 - \prod_{m=1}^{M} \Pr\left\{\vec{\boldsymbol{X}}^m_{FC} = \vec{\mathbb{O}}_{l_m+1}\middle|H_1\right\}, \tag{39}$$

where $\vec{\mathbb{O}}_{l_m+1}$ is a vector of l_m zeros, and $\Pr\left\{\vec{\boldsymbol{X}}^m_{FC} = \vec{\mathbb{O}}_{l_m+1}\middle|H_0\right\}$ is defined in (35).

5 Throughputs of the Proposed CSS Strategy

As mentioned in previous sections, the values of N, M, α, and $\psi_{IW}(m,k)$ affect the performance of the proposed CSS. So, we should complete the proposed CSS strategy by providing a way to calculate the optimal values of its parameters. Toward this goal, several criterions can be considered. Since a CR is originally designed to improve the spectrum efficiency, maximizing the throughput of a CRN is one of more practical interest [5, 8, 9, 11, 15]. Therefore, our object is to find the optimal settings of the proposed CSS by maximizing the throughput of the CRN under the constraints of protecting PUs from SUs.

Let C_1 and C_0 denote the throughput of the CRN with and without existence of a PU, respectively. Hence, as seen in Fig. 2, the average achievable throughput by CRN is [5, 13]

$$\begin{aligned} R = &\left(1 - P^{FC}_f\right)\left(1 - \frac{T_s + T_{FC} + \sum_{m=1}^{M}\left\{T^{slot}_{r-in}(m) + T^{slot}_{r-out}(m)\right\}}{T_F}\right) C_0 P(H_0) \\ &+ \left(1 - P^{FC}_d\right)\left(1 - \frac{T_s + T_{FC} + \sum_{m=1}^{M}\left\{T^{slot}_{r-in}(m) + T^{slot}_{r-out}(m)\right\}}{T_F}\right) C_1(1 - P(H_0)), \end{aligned} \tag{40}$$

where $P(H_0)$ is the probability that PU's signal is absent. Since due to the interference $C_0 \gg C_1$, the first term in the right hand side of (40) dominates the achievable throughput [5, 13]. Therefore, the normalized achievable throughput can be approximated by

$$\bar{R}(N, M, \alpha, \psi_{IW}(m,k), \psi_E) = \left(1 - P_f^{FC}\right)\left(1 - \frac{T_s + T_{FC} + \sum_{m=1}^{M}\left\{T_{r-in}^{slot}(m) + T_{r-out}^{slot}(m)\right\}}{T_F}\right). \tag{41}$$

Furthermore, to simplify the following optimization problem, we consider $\psi_{IW} = \psi_{IW}(m,k)$ for $\forall m, k$. So, the optimization problem can be modeled as

$$\begin{array}{ll} \max\limits_{N,M,\alpha,\psi_{IW},\psi_E} & \bar{R}(N, M, \alpha, \psi_{IW}, \psi_E), \\ & P_d^{FC}(N, M, \alpha, \psi_{IW}, \psi_E) \geq P_{d,T}, \\ s.t. & 1 \leq N \leq S, \\ & 1 \leq M \leq N, \end{array} \tag{42}$$

where $P_{d,T}$ denotes the target for detection probability so that the PU is sufficiently protected.

The optimization problem in (42) is non-convex, and in order to solve it, we use an exhaustive search. Since N and M are integers, and, as we show in the following, α and ψ_{IW} have discrete search space, the search space is limited.

Using $I_j = \sum_{i=1}^{j-1} A_i^a$, we can simplify (27) as

$$F_j\left(\vec{A}_j^a \middle| H_y\right) = \begin{cases} 1, & I_j < L_2, \\ \Pr\left\{d_j^m = 0 | H_y\right\}, & L_2 \leq I_j < L_1, \\ 0, & L_1 \leq I_j, \end{cases} \tag{43}$$

where $L_1 = \psi_{IW}/w_m$ and $L_2 = (\psi_{IW} - 1)/w_m$. Since I_j is an integer, one, the smallest integer, is a reasonable step-size for L_1 and L_2 to change $F_j\left(\vec{A}_j^a \middle| H_y\right)$ (i.e. P_f^{FC} and P_d^{FC}). Therefore, the search space of α and ψ_{IW}, which is equivalent to L_1 and L_2, are discrete.

6 Simulation Results

In this section, we evaluate the proposed CSS strategy using simulations and assuming reporting and inner channels are Rayleigh flat fading. Other related CRN parameters are defined in Table 1.

In the first step, we evaluate our proposed clustering algorithm to verify the evenly distribution of energy consumption. In simulations, $N = 14$ SUs in the CRN and $SNR_{in} = SNR_i = 10dB$, and we divide them into three clusters. Accordingly, Fig. 3 shows the MSDE for different values of n_1 and n_2, ($n_3 = N - n_1 - n_2$), that are equivalent to different

Table 1 CRN parameters

$S = 14$	$T_F = 20\,\text{ms}$
Modulation: QPSK	$T_s = 0.025\,\text{ms}$
$R_b = 10\,\text{kbps}$	$T_{FC} = 0.015\,\text{ms}$
$P(H_0) = 0.75$	$T_{coh} = 51\,\text{ms}$
$SNR_s = 3\,\text{dB}$	$P_s = 0.1\,\text{W}$
$W_s = 200\,\text{kHz}$	$P_r(10) = 0.51\,\text{W}$
$\vartheta = 2$	$P_{in}(10) = 0.17\,\text{W}$

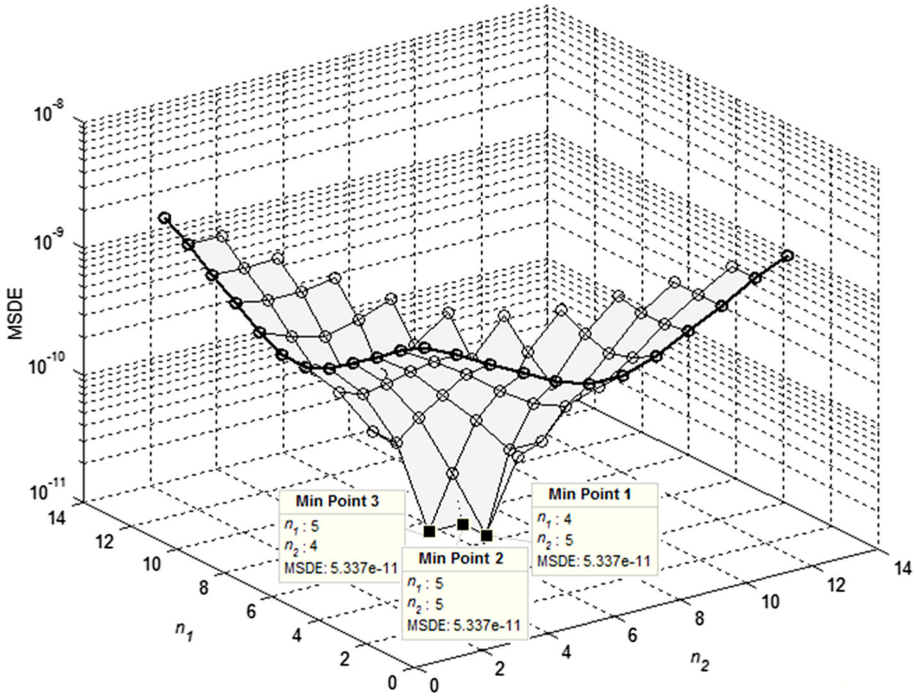

Fig. 3 MSDE versus different values of n_1 and n_2 $(N = 14, M = 3,\ SNR_{in} = SNR_r = 10\,\mathrm{dB})$

Table 2 Optimal numbers of SUs in each cluster based on simulation results $(N = 14, M = 3,\ SNR_{in} = SNR_r = 10\,\mathrm{dB})$

	$\hat{n}_1$	$\hat{n}_2$	$\hat{n}_3$
Minimum point 1, in Fig. 3	5	4	5
Minimum point 2, in Fig. 3	5	5	4
Minimum point 3, in Fig. 3	4	5	5

clustering algorithms. As seen in Fig. 3, the least MSDE, which is equivalent to the best evenly distribution of energy consumption, occurs at 3 points, given in Table 2. On the other hand, using (8), the optimal number of SUs in each cluster can be expressed as follows

$$\hat{n}_m = \begin{cases} 5, & 1 \le m \le 2, \\ 4, & m = 3. \end{cases} \tag{44}$$

Since the ordering of clusters is not important here, these results are equivalent. Thus, the clustering algorithm given in Theorem 1, introduce an energy efficient algorithm.

In the second step, we compare minimum values of $P_f^{IW}(m,k)$ subject to $P_d^{IW}(m,k) \ge 0.9$ achieved by the proposed IW fusion rule with various hard combining fusion rules and no-improvement case ($w_m = 0$). Without loss of generality, we choose $\psi_{IW}(m,k) = 0.98$ for $\forall k$ in the IW fusion rule, which needs to know only the value of SNR_{in}^m, and OR $(w_m = 1, \psi_{IW}(m,k) = 1)$, AND $(w_m = 1, \psi_{IW}(m,k) = k)$, and MAJORITY

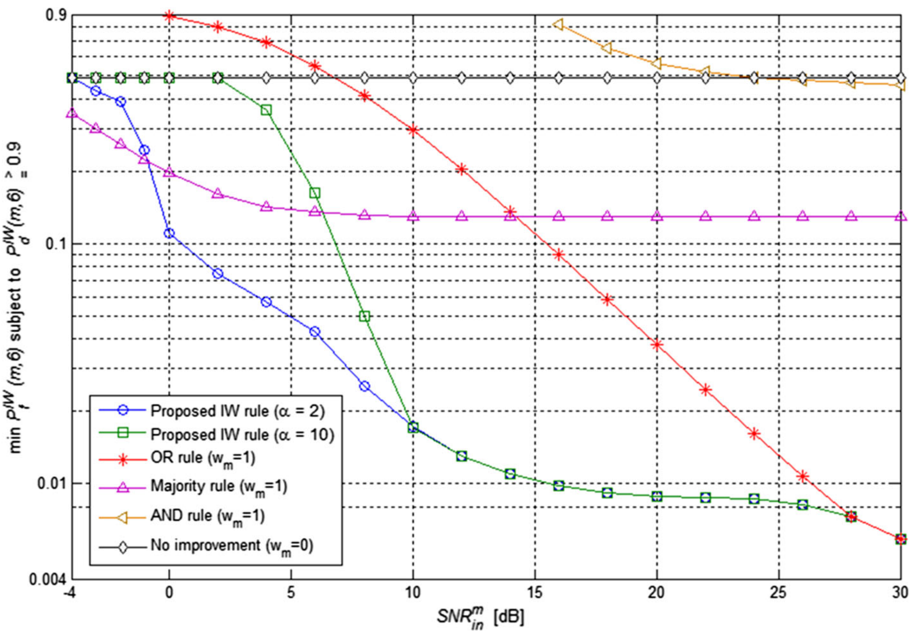

Fig. 4 Minimum $P_f^{IW}(m,6)$ subject to $P_d^{IW}(m,6) \geq 0.9$ achieved by various SNR_{in}^{m} over Rayleigh faded inner channel (SU 6)

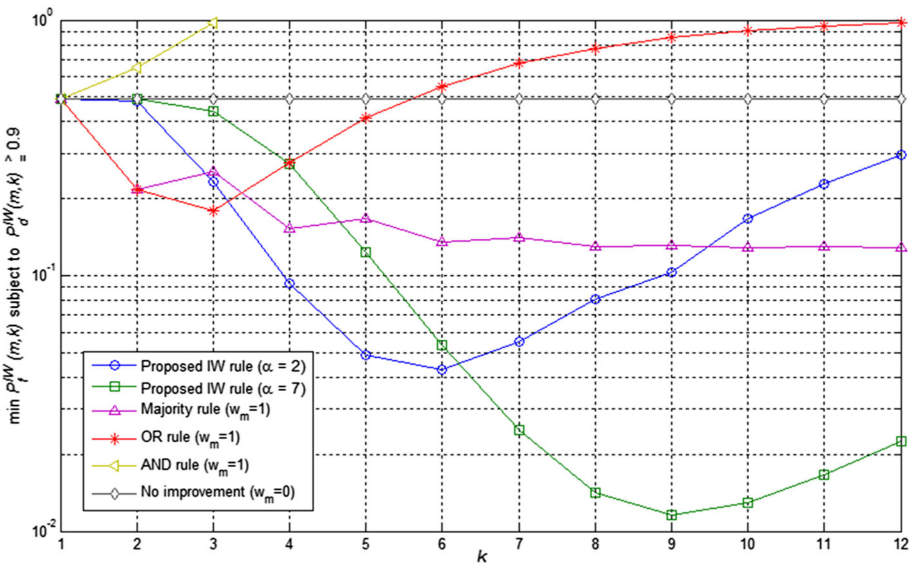

Fig. 5 Minimum $P_f^{IW}(m,k)$ subject to $P_d^{IW}(m,k) \geq 0.9$ achieved by SU k in the cluster m over Rayleigh faded inner channel ($SNR_{in}^{m} = 6\,\text{dB}$)

($w_m = 1, \psi_{IW}(m,k) = \lceil k/2 \rceil$) fusion rules as references. Figure 4 shows this comparison for SU 6 in cluster m for different SNR_{in}^{m} values. As seen, by selecting an appropriate value for α, the proposed IW fusion rule shows a better performance at low to high SNRs. At very low SNR, the proposed IW fusion rule is similar to no-improvement case ($w_m \to 0$), where the MAJORITY fusion rule shows good sensing performance. At very high SNR, the proposed IW fusion rule is similar to the OR fusion rule ($w_m \to 1$).

Figure 5 shows the above comparison as sensing improvement at SU k in cluster m for $SNR_{in}^{m} = 6\,\mathrm{dB}$. As seen, the proposed CSS strategy outperforms all fusion rules for $k > 3$. For $k \leq 3$, OR-rule shows better performance, which can be seen as a especial case of the proposed IW fusion rule, when $\alpha \to 0$ and $\psi_{IW}(m,k) = 1$.

In the final step, we compare our CSS with three baseline schemes. The first scheme, called S1 has $w_m = 0$, in which the CRN doesn't use IW fusion rule and its reliability method for the selection of decisions. In the second scheme, S2, the cooperative SUs report to the FC using a direct transmission with no diversity gain ($N = M$), which is the same as usual CSS with OR-rule. The third scheme, S3, has fixed $M = 1$, in which SUs don't participate in the proposed clustering. The optimal settings for the proposed strategy and these three schemes are obtained by solving the optimization problem given by Eq. (42). As mentioned in Fig. 5 of [15], for one PU channel (one sensing channel), the proposed CSS doesn't use clustering approach, and its sensing performance is the same as those of usual CSS with OR-rule at the FC. On the other hand, as described in Sect. 2, we assume a CRN with one sensing channel. Therefore, with one sensing channel and imperfect reporting channels, the CSS scheme in [15] is the same as S2 scheme.

Figures 6 and 7 show the maximum normalized achievable throughput of our proposed strategy and these schemes in scenarios with average SNR of inner and reporting channels for $P_{d,T} = 0.9$. When errors in inner channels are worse than reporting channels (so that approximately when $SNR_r > SNR_{in}$), cooperative communication methods are inefficient, and the optimum method of reporting is direct transmission (non-cooperative) to the FC. As seen in Figs. 6 and 7, and as expected, in this condition, the proposed strategy sets the number of clusters equal to the number of cooperative SUs, and its performance is the same as S2. However, in practical environments, the errors in inner channels are often better than reporting channels (i.e. $SNR_r < SNR_{in}$). In this condition, the performance of proposed CSS strategy is much better than S2, which is used by default (Figs. 6, 7). Furthermore, in this condition ($SNR_r < SNR_{in}$), the importance of using the proposed IW rule and its reliability method in the CSS can be seen more clearly. As seen in Figs. 6 and 7, CRN with S1 scheme achieves less throughput compared to the proposed CSS strategy, which this difference increases by increasing SNR_{in} since, in S1 scheme, all local decisions are sent to the FC without any improvement. However, in proposed strategy, by increasing SNR_{in}, without losing sensing efficiency, the less number of decisions in a shorter overhead time are sent to the FC.

In addition, as seen in Figs. 6 and 7, the performance of S3 is very poor in low values for SNR_{in} since, as we proved in [12], when SNR_{in} decreases in one cluster, the ODSTBC BER increases. To overcome this problem, number of SUs in the corresponding cluster must be reduced. However, when $M = 1$, this reduction reduces the number of cooperative SUs, N, causing severe degradation in S3 performance. Therefore, we use clustering in the proposed strategy to benefit diversity-throughput tradeoff. Furthermore, this clustering is energy-efficient that guarantees the maximum life-time of the CRN. As a result, we see that the proposed CSS strategy outperforms the three mentioned schemes at all SNRs. For example in Fig. 7b, the average of this throughput improvement is more than 17% compared to scheme S2.

Tables 3, 4, 5 and 6 show optimal values for key design parameters, which are the number of cooperative SUs, N_{opt}, number of clusters, M_{opt}, and IW fusion parameters,

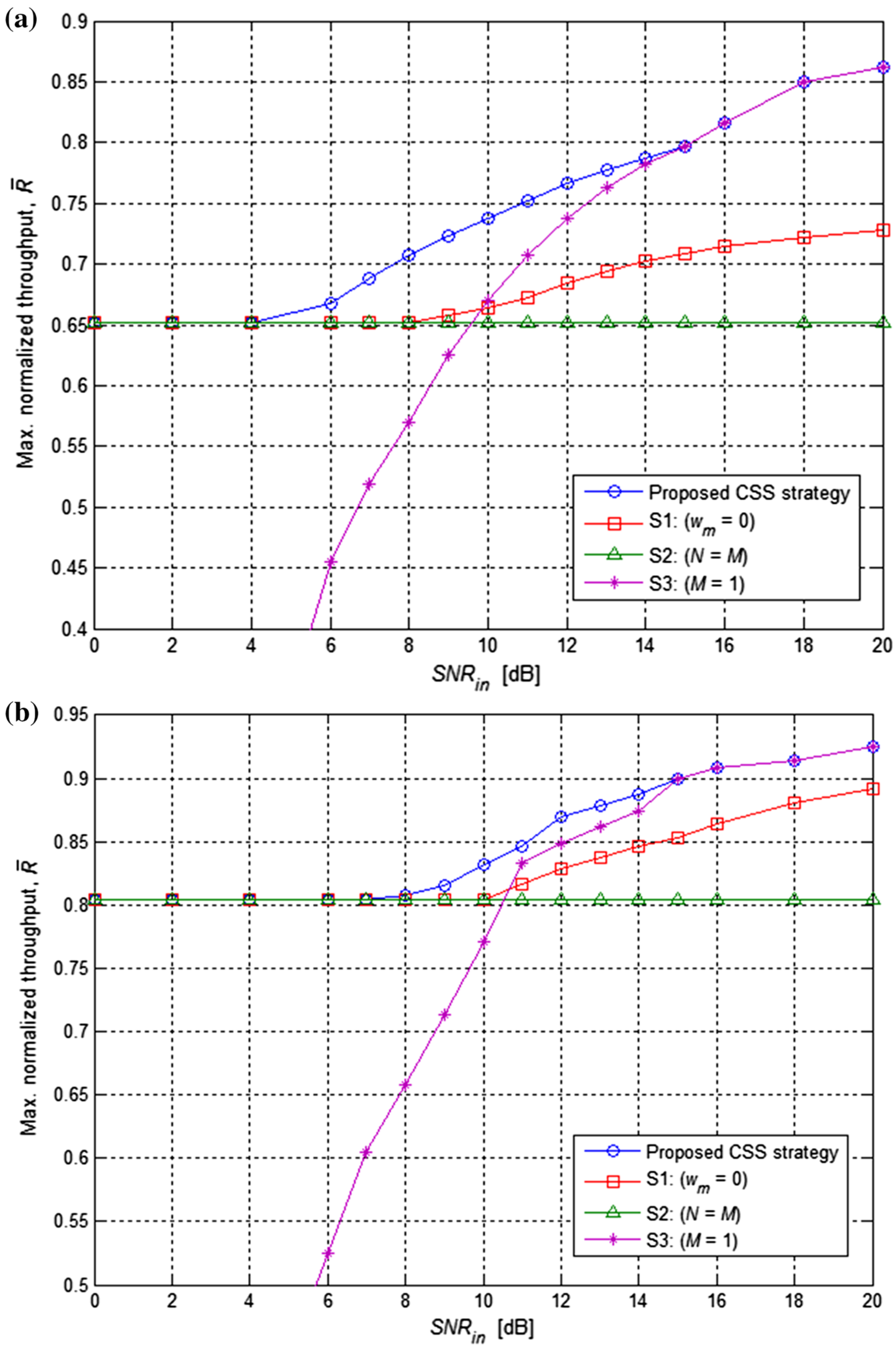

Fig. 6 Maximum normalized achievable throughput versus SNR_{in} for different CSS strategies and $P_{d,T} = 0.9$, **a** $SNR_r = 2\,\text{dB}$, **b** $SNR_r = 7\,\text{dB}$

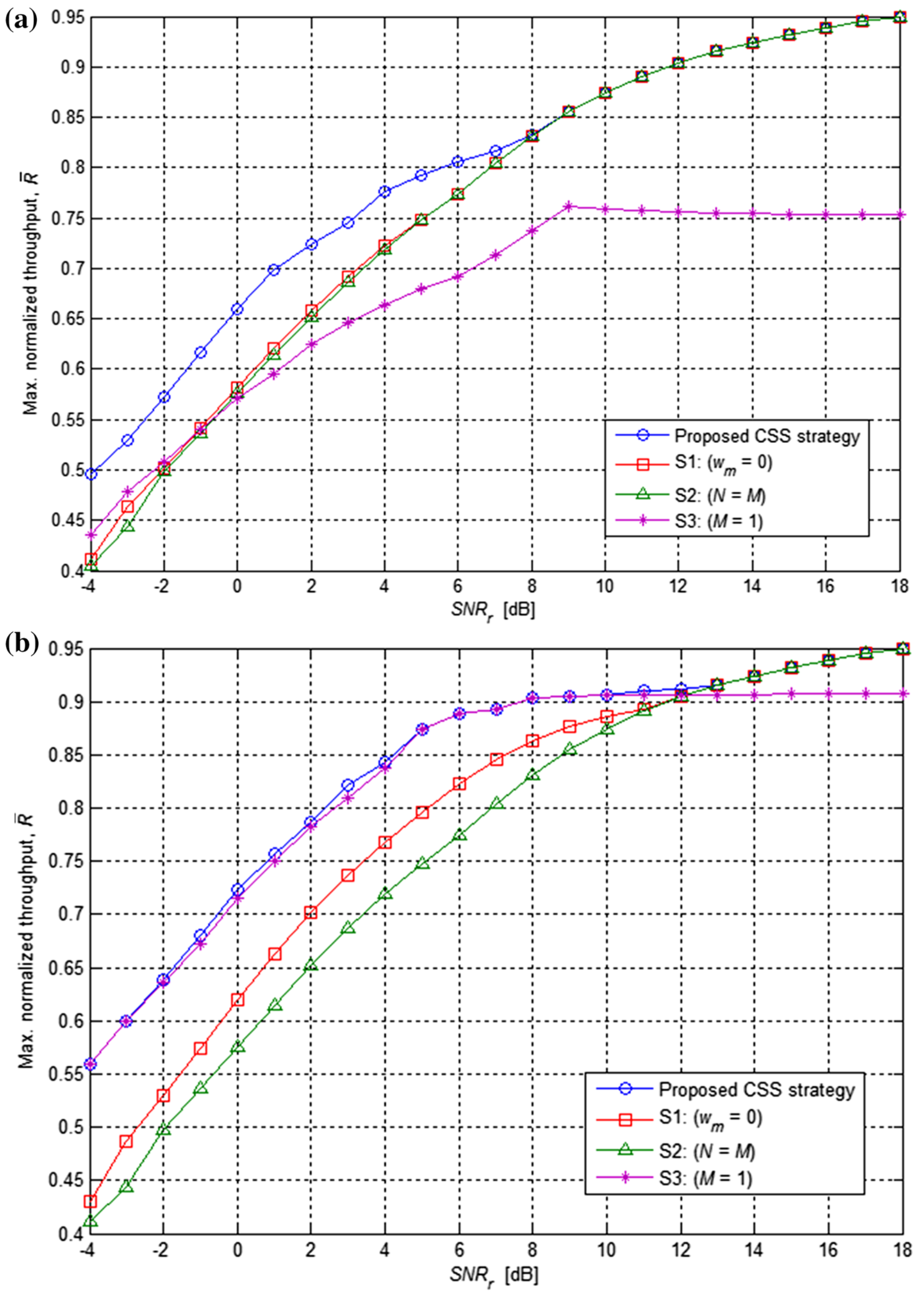

Fig. 7 Maximum normalized achievable throughput versus SNR_r for different CSS strategies and $P_{d,T} = 0.9$, **a** $SNR_{in} = 9\,\text{dB}$, **b** $SNR_{in} = 14\,\text{dB}$

$\alpha_{opt}, \psi_{IW,opt}$, for different SNRs of inner and reporting channels, and $P_{d,T} = 0.9$. It is notable that when $N_{opt} = M_{opt}$, the proposed CSS strategy uses non-cooperative transmission. So, there are no optimal values for α_{opt} and $\psi_{IW,opt}$ in Tables 5 and 6.

Table 3 Optimal number of cooperative SUs, N_{opt}, for the proposed CSS strategy ($P_{d,T} = 0.9$)

		SNR_r (dB)						
		−4	−1	2	5	7	10	12
SNR_{in} (dB)	0	3	3	3	3	4	4	4
	3	3	3	3	3	4	4	4
	7	5	4	4	4	4	4	4
	9	5	4	4	4	3	4	4
	11	8	8	8	7	4	4	4
	14	6	9	8	5	4	4	4
	16	6	6	5	6	5	4	4

Table 4 Optimal number of clusters, M_{opt}, for the proposed CSS strategy ($P_{d,T} = 0.9$)

		SNR_r (dB)						
		−4	−1	2	5	7	10	12
SNR_{in} (dB)	0	3	3	3	3	4	4	4
	3	3	3	3	3	4	4	4
	7	2	2	2	2	4	4	4
	9	2	2	2	2	2	4	4
	11	2	2	2	2	2	4	4
	14	1	2	2	1	2	2	2
	16	1	1	1	1	1	2	2

Table 5 Optimal IW fusion rule parameter, α_{opt}, for the proposed CSS strategy ($P_{d,T} = 0.9$)

		SNR_r (dB)						
		−4	−1	2	5	7	10	12
SNR_{in} (dB)	0	–	–	–	–	–	–	–
	3	–	–	–	–	–	–	–
	7	0.451	0.451	0.451	0.451	–	–	–
	9	0.715	0.715	0.715	0.715	0.715	–	–
	11	13.722	13.722	13.722	13.722	1.133	–	–
	14	27.380	27.380	27.380	27.380	2.261	2.261	2.261
	16	43.394	43.394	3.583	43.394	43.394	3.583	3.583

Figure 8 shows the maximum normalized achievable throughput of our proposed strategy and the mentioned three schemes versus target probability of detection, $P_{d,T}$. As seen, regardless of $P_{d,T}$, by using the proposed CSS strategy, we can achieve highest throughput for the CRN; so that the average of the throughput improvement is more than 11% compared to scheme S2 (the proposed CSS scheme in [15]).

Table 6 Optimal threshold of IW fusion rule, $\psi_{IW,opt}$ for the proposed CSS strategy ($P_{d,T} = 0.9$)

		SNR_r (dB)						
		−4	−1	2	5	7	10	12
SNR_{in} (dB)	0	–	–	–	–	–	–	–
	3	–	–	–	–	–	–	–
	7	0.0917	0.0917	0.0917	0.0917	–	–	–
	9	0.0917	0.0917	0.0917	0.0917	0.0917	–	–
	11	0.5263	0.5263	0.5263	0.5263	0.0917	–	–
	14	0.5263	0.5263	0.5263	0.5263	0.0917	0.0917	0.0917
	16	0.5263	0.5263	0.0917	0.5263	0.5263	0.0917	0.0917

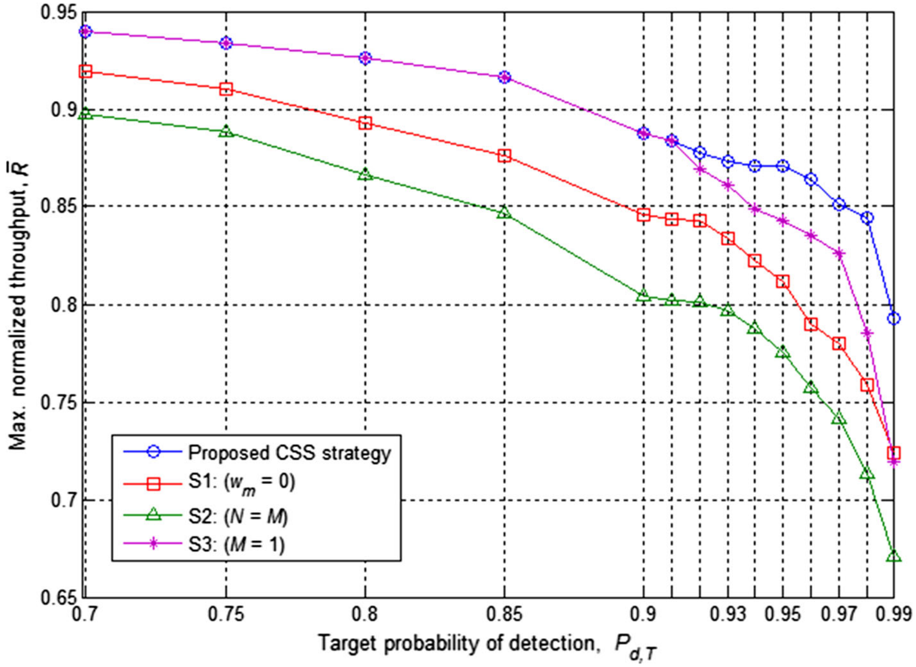

Fig. 8 Maximum normalized achievable throughput versus $P_{d,T}$ for different CSS strategies ($SNR_r = 7\,\text{dB},\ SNR_{in} = 14\,\text{dB}$)

7 Conclusions

In this paper, we consider the adverse effects of the reporting stage in a CSS, such as error and overhead time, which has not been fully studied in most of the literatures. Based on this, we proposed a new cluster-based CSS for a CRN to maximize CRN's performance (CRN's throughput) in a practical environment, as well as, to guarantee maximum CRN lifetime. Throughput maximization is done by using four techniques: an optimal clustering algorithm, a new IW fusion rule, a reliable method to select decisions, and a cooperative transmission based on ODSTBC. Simulation results confirm we need all four techniques to be performed simultaneously to maintain the performance of the CSS strategy for different SNR values.

Furthermore, these four techniques have four key parameters (the number of cooperative SUs, number of clusters, and two IW fusion rule parameters), which affect CRN's throughput. Therefore, by deriving a closed-form expression for the CRN's throughput, we obtain the optimal values of these parameters. As a result, we proposed a CSS strategy for different SNR values, which simulation results confirm its effectiveness. For example, in one usual case with imperfect channels, the average improvement of the throughput is more than 11% in the CRN, regardless of required protection for the PU.

Appendix 1

This appendix provides the proof of (7). Toward this, we solve the optimization problem in (1) for cluster m, $1 \leq m \leq M$, then we extend it to other clusters.

Consider an arbitrary cluster, m, whose number of members has the following relationship with numbers of other clusters' members,

$$n_m = N - n_q - \sum_{\substack{i=1 \\ i \neq m,q}}^{M} n_i, \quad 1 \leq q \leq M, \; q \neq m. \tag{45}$$

In this case, MSDE in (1) can be rewritten in the following form

$$MSDE(\vec{N}_M) = \frac{2}{(M^2 - M)} \left\{ \sum_{\substack{j=1 \\ j \neq m,q}}^{M} \left[E_q(n_q) - E_j(n_j)\right]^2 + \sum_{\substack{i=1 \\ i \neq m,q}}^{M} \left[E_m(n_m) - E_i(n_i)\right]^2 + \left[E_q(n_q) - E_m(n_m)\right]^2 + X \right\}. \tag{46}$$

where $X = \sum_{\substack{i=1 \\ i \neq m,q}}^{M-1} \sum_{\substack{j=i+1 \\ j \neq m,q}}^{M} \left[E_i(n_i) - E_j(n_j)\right]^2$. Substituting (45) into (46), the constraint of (1) is satisfied (i.e. $\sum_{j=1}^{M} n_j = N$). Therefore, (1) converts to the following unconstrained case:

$$\min_{\substack{n_q \\ 1 \leq q \leq M, q \neq m}} \quad MSDE(n_q) = \frac{2}{(M^2 - M)} \times \left\{ \sum_{\substack{j=1 \\ j \neq m,q}}^{M} \left[E_q(n_q) - E_j(n_j)\right]^2 + \sum_{\substack{i=1 \\ i \neq m,q}}^{M} \left[E_m\left(N - \sum_{\substack{i=1 \\ i \neq m,q}}^{M} n_i - n_q\right) - E_i(n_i)\right]^2 + \left[E_q(n_q) - E_m\left(N - \sum_{\substack{i=1 \\ i \neq m,q}}^{M} n_i - n_q\right)\right]^2 + X \right\} \tag{47}$$

In (47), $MSDE(n_q)$ is a function with discrete variable n_q. Hence, we define

$$D(n_q) = MSDE(n_q + 1) - MSDE(n_q). \tag{48}$$

Substituting $MSDE(n_q)$ into (48), and using (45), we obtain

$$\begin{aligned} D(n_q) = &\left(E_q(n_q + 1) - E_q(n_q)\right)\left(H_1(n_q) + H_1(n_q + 1)\right) \\ &+ \left(E_m(n_m - 1) - E_m(n_m)\right)\left(H_2(n_q) + H_2(n_q + 1)\right), \end{aligned} \tag{49}$$

where

$$H_1(n_q) = (m - 1)E_q(n_q) - E_m(n_m) - \sum_{\substack{i=1 \\ i \neq m,q}}^{M} E_i(n_i), \tag{50}$$

$$H_2(n_q) = (m - 1)E_m(n_m) - E_q(n_q) - \sum_{\substack{i=1 \\ i \neq m,q}}^{M} E_i(n_i). \tag{51}$$

According to (2)–(6), we know that $E_q(n_q)$ increases by increasing n_q. So, we can define the following two cases:

Case 1 If $E_q(n_q + 1) \leq E_m(n_m - 1)$ for $\forall q, \forall m,\ q \neq m$, then $H_1(n_q + 1) \leq 0$ and $H_2(n_q + 1) \geq 0$. Furthermore, from (50) and (51), we have $H_1(n_q) < H_1(n_q + 1)$ and $H_2(n_q + 1) < H_2(n_q)$. Hence, in this case, by looking at (49), we have $D(n_q) < 0$.

Case 2 If $E_q(n_q - 1) \geq E_m(n_m + 1)$ for $\forall q, \forall m,\ q \neq m$, then $H_1(n_q - 1) \geq 0$ and $H_2(n_q - 1) \leq 0$. Furthermore, from (50) and (51), $H_1(n_q - 1) < H_1(n_q)$ and $H_2(n_q) < H_2(n_q - 1)$. Hence, in this case, by looking at (49), we have $D(n_q - 1) > 0$.

Furthermore, by using the properties of discrete-variable functions, solution of (47), $\hat{n}_q, q = 1, \ldots, m - 1, m + 1, \ldots, M$, must satisfy the following inequalities

$$D\left(\hat{n}_q\right) \geq 0, \tag{52}$$

$$D\left(\hat{n}_q - 1\right) \leq 0. \tag{53}$$

Therefore, $E_q\left(\hat{n}_q + 1\right) > E_m(\hat{n}_m - 1)$ and $E_q\left(\hat{n}_q - 1\right) < E_m(\hat{n}_m + 1)$ for $\forall q, \forall m,\ q \neq m$ are the necessary conditions for solution of (47), which can rewrite as

$$\left\lfloor E_q^{-1}(E_m(\hat{n}_m - 1))\right\rfloor \leq \hat{n}_q \leq \left\lceil E_q^{-1}(E_m(\hat{n}_m + 1))\right\rceil, 1 \leq q \leq M, q \neq m. \tag{54}$$

where $\hat{n}_m = N - \sum_{\substack{i=1\\ i \neq m,q}}^{M} \hat{n}_i - \hat{n}_q$. To obtain the exact value of $\hat{n}_m$, we add M-1 inequalities (54) together as follows

$$\sum_{\substack{q=1\\ q\neq m}}^{M} \left\lfloor E_q^{-1}(E_m(\hat{n}_m - 1))\right\rfloor \leq N - \hat{n}_m \leq \sum_{\substack{q=1\\ q\neq m}}^{M} \left\lceil E_q^{-1}(E_m(\hat{n}_m + 1))\right\rceil, 1 \leq m \leq M. \tag{55}$$

Hence, we get (7) that we wanted.

Appendix 2

In this appendix, we derive (23) and (24). By using $\vec{\boldsymbol{X}}_k^m$, $\vec{\boldsymbol{A}}_k^a$, $\vec{\boldsymbol{B}}_k^b$ and $\vec{\boldsymbol{C}}_k^c$ defined in Theorem 1, the probability of $x_k^m = 1$ for $y \in \{0, 1\}$ is given as

$$\begin{aligned}
\Pr\left\{x_k^m = 1|H_y\right\} &= \Pr\left\{ w_m \sum_{i=1}^{k-1} \hat{x}_{i,k}^m + d_k^m \geq \psi_{IW}(m,k) \middle| H_y \right\} \\
&= \sum_{a=0}^{2^{k-1}-1} \Pr\left\{ w_m \sum_{i=1}^{k-1} A_i^a + d_k^m \geq \psi_{IW}(m,k) \middle| \vec{\boldsymbol{X}}_k^m = \vec{\boldsymbol{A}}_k^a \right\} \Pr\left\{\vec{\boldsymbol{X}}_k^m = \vec{\boldsymbol{A}}_k^a \middle| H_y\right\} \\
&= \sum_{a=0}^{2^{k-1}-1} \left(1 - F_k\left(\vec{\boldsymbol{A}}_k^a \middle| H_y\right)\right) \Pr\left\{\vec{\boldsymbol{X}}_k^m = \vec{\boldsymbol{A}}_k^a \middle| H_y\right\},
\end{aligned} \tag{56}$$

where $F_k\left(\vec{\boldsymbol{A}}_k^a \middle| H_y\right)$ is the CDF of random variable d_j^m, which is defined in (27).

Now, we evaluate the pmf of $\vec{\boldsymbol{X}}_k^m$, $\Pr\left\{\vec{\boldsymbol{X}}_k^m = \vec{\boldsymbol{A}}_k^a \middle| H_y\right\}$, as

$$\Pr\left\{\vec{\boldsymbol{X}}_k^m = \vec{\boldsymbol{A}}_k^a \middle| H_y\right\} = \sum_{b=0}^{2^{k-1}-1} \Pr\left\{\vec{\boldsymbol{X}}_k^m = \vec{\boldsymbol{A}}_k^a \middle| \vec{\boldsymbol{Z}}_k^m = \vec{\boldsymbol{B}}_k^b\right\} \Pr\left\{\vec{\boldsymbol{Z}}_k^m = \vec{\boldsymbol{B}}_k^b \middle| H_y\right\}, \tag{57}$$

where $\vec{\boldsymbol{Z}}_k^m = \left\{x_1^m, x_2^m, \ldots, x_{k-1}^m\right\}$ is the vector of decisions that transmitted from $k-1$ SUs. The elements of $\vec{\boldsymbol{X}}_k^m$ are conditionally independent for a given $\vec{\boldsymbol{Z}}_k^m$. So,

$$\begin{aligned}\Pr\left\{\vec{X}_k^m = \vec{A}_k^a \middle| \vec{Z}_k^m = \vec{B}_k^b\right\} &= \prod_{j=2}^{k-1} \Pr\left\{\hat{x}_{j,k}^m = A_j^a | x_j^m = B_j^b\right\} \\ &= \prod_{j=1}^{k-1} (\mathrm{P}_{e1})^{\left|A_j^a - B_j^b\right|} (1 - P_{e1})^{\left|1 - A_j^a - B_j^b\right|},\end{aligned} \tag{58}$$

where $P_{e1} = \Pr\left\{A_j^a \neq B_j^b \middle| x_j^m = B_j^b, \hat{x}_{j,k}^m = A_j^a\right\} = P_e^{in}\left(SNR_{in}^m\right)$. Furthermore, x_j^m depends on $j-1$ previous SUs' decisions, $\vec{Z}_j^m$. So, we have

$$\Pr\left\{\vec{Z}_k^m = \vec{B}_k^b \middle| H_y\right\} = \Pr\left\{x_1^m = B_1^b \middle| H_y\right\} \prod_{j=2}^{k-1} \Pr\left\{x_j^m = B_j^b \middle| \vec{Z}_j^m = \vec{B}_j^b, H_y\right\}. \tag{59}$$

Now,

$$\begin{aligned}\Pr\left\{x_j^m = 1 \middle| \vec{Z}_j^m = \vec{B}_j^b, H_y\right\} &= \sum_{c=0}^{2^{j-1}-1} \Pr\left\{x_j^m = 1 \middle| \vec{X}_j^m = \vec{C}_j^c, \vec{Z}_j^m = \vec{B}_j^b, H_y\right\} \Pr\left\{\vec{X}_j^m = \vec{C}_j^c \middle| \vec{Z}_j^m = \vec{B}_j^b\right\} \\ &= \sum_{c=0}^{2^{j-1}-1} \left\{\Pr\left\{x_j^m = 1 \middle| \vec{X}_j^m = \vec{C}_j^c, H_y\right\} \prod_{i=1}^{j-1} (P_{e2})^{\left|B_i^b - C_i^c\right|} (1 - P_{e2})^{1 - \left|B_i^b - C_i^c\right|}\right\} \\ &= \sum_{c=0}^{2^{j-1}-1} \left\{\left(1 - F_j\left(\vec{C}_j^c \middle| H_y\right)\right) (P_{e2})^{\sum_{i=1}^{j-1}\left|B_i^b - C_i^c\right|} (1 - P_{e2})^{j-1-\sum_{i=1}^{j-1}\left|B_i^b - C_i^c\right|}\right\}\end{aligned} \tag{60}$$

and

$$\begin{aligned}\Pr\left\{x_j^m = 0 \middle| \vec{Z}_j^m = \vec{B}_j^b, H_y\right\} &= \sum_{c=0}^{2^{j-1}-1} \left\{\left(1 - \Pr\left\{x_j^m = 1 \middle| \vec{X}_j^m = \vec{C}_j^c, H_y\right\}\right) \prod_{i=1}^{j-1} (P_{e2})^{\left|B_i^b - C_i^c\right|} (1 - P_{e2})^{1 - \left|B_i^b - C_i^c\right|}\right\} \\ &= 1 - \sum_{c=0}^{2^{j-1}-1} \left\{\left(1 - F_j\left(\vec{C}_j^c \middle| H_y\right)\right) (P_{e2})^{\sum_{i=1}^{j-1}\left|B_i^b - C_i^c\right|} (1 - P_{e2})^{j-1-\sum_{i=1}^{j-1}\left|B_i^b - C_i^c\right|}\right\} \\ &= 1 - \Pr\left\{x_j^m = 1 \middle| \vec{Z}_j^m = \vec{B}_j^b, H_y\right\}\end{aligned} \tag{61}$$

Using $p_j\left(\vec{B}_k^b \middle| H_y\right)$ defined in (26), and (60) and (61), we can rewrite (59) as

$$\Pr\left\{\vec{Z}_k^m = \vec{B}_k^b \middle| H_y\right\} = \prod_{j=1}^{k-1} p_j\left(\vec{B}_j^b \middle| H_y\right)^{B_j^b} \left(1 - p_j\left(\vec{B}_j^b \middle| H_y\right)\right)^{1-B_j^b}. \tag{62}$$

Substituting (62) into (56), then we get the same formula for $y = 0$ and $y = 1$ as (23) and (24) that we wanted.

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