Wireless Pers Commun (2017) 96:2603–2619
DOI 10.1007/s11277-017-4314-0

# Performance Analysis of Downlink Linear Precoding in Massive MIMO Systems Under Imperfect CSI

**Adil Israr[1] · Zahid Rauf[1] · Jan Muhammad[1] · Faisal Khan[1]**

Published online: 18 May 2017


**Abstract** Demands of wireless data traffic, throughput, the number of wireless mobile connections and devices will always increase. In addition, the concern about energy consumption is also growing for wireless communication systems. Massive MIMO system is a new emerging research area to resolve these issues. In this paper, the performance of Massive MIMO downlink including linear precoding is evaluated. Spectral efficiency through achievable rate and energy efficiency through transmit power of ZF and MRT linear precoding is investigated under practical limitations, such as imperfect CSI, less complexity processing and inter user interference. Since ZF and MRT precoding can balance the system performance and complexity. Different channel estimation values are considered in order to compare the performance of these precoding techniques in the given system. The achievable rate and the downlink transmit power of ZF and MRT precoding techniques are derived, analyzed and compared under the same conditions and assumptions. Several scenarios are considered to investigate these performance parameters. It is found that when the ratio of BS antennas and number of users is large, ZF is better than MRT while when the ratio is quite small it makes MRT better than ZF for the same conditions.

**Keywords** Massive MIMO · Precoding · Zero-forcing (ZF) · Maximum ratio transmission (MRT) · Achievable rate · Downlink transmit power

✉ Adil Israr
adil.israr@buitms.edu.pk

Zahid Rauf
zahid.rauf@buitms.edu.pk

Jan Muhammad
jan.muhammad@buitms.edu.pk

Faisal Khan
faisal.khan@buitms.edu.pk

[1] Faculty of Information and Communication Technology, Balochistan University of Information Technology, Engineering and Management Sciences, Quetta, Pakistan

## 1 Introduction

The data traffic has grown exponentially during the last years, because of the dramatic growth of many wireless data consuming devices such as smartphones, laptops, tablets, etc. It is expected that global mobile data traffic by 2018 will increase 15.9 Exabyte per month, which is about 6 times increase over 2014 [1]. In addition, by 2018 the number of mobile devices and connections are expected to grow to 10.2 billion [1]. The increasing demand of these wireless data consuming devices and wireless mobile connection also enhancing the demand of more throughputs and the concerns about energy consumption. Thus, future wireless comunication has to satisfy the three main demands, high throughput; serving many users simultaneously; and less energy consumption [2].

In order to meet this demand new technologies are required which will be able to provide optimal quality of service, spectral efficient, link reliable and energy efficient system in future wireless communication. The demand for 5G is already increasing because it is expected that 5G will be able to resolve the huge capacity and connectivity challenges brought by the increasing mobile traffic and data usage [3]. To increase the spectral efficiency, a well-known way is using multiple antennas at the transceivers. MIMO (multiple input multiple output) technique attracted the researchers over the past decade.

The efforts in order to utilize multiplexing gain have been shifted from MIMO to multiuser MIMO (MU-MIMO), where a multiple-antenna base station (BS) serves several users simultaneously. In MU-MIMO system, even with users having single antenna, spatial multiplexing gain can be obtained [4]. MU MIMO not only reaps all the benefits of MIMO systems but it also overcomes its limitations. The more degrees of freedom can be offered if the BS is equipped with more antennas, hence it results a huge sum throughput [5]. This high signal dimensions make the signal processing techniques prohibitively complex. So the main question is whether with low-complexity signal processing, the huge multiplexing gain can be obtained. Marzetta [5] showed that the simple linear processing becomes nearly optimal if number of BS antennas is large as compared with the number of users.

Massive MIMO, a system in which transmitter uses large number of antennas about hundred or more and serving many users simultaneously in the same time frequency resource [5]. Massive MIMO has been considered as a promising technology for next generation wireless systems and to address a significant challenges in 5G [6, 7]. The design and analysis of massive multi user MIMO systems is a fairly novel research area that is attracting substantial interest [8–13]. Before putting Massive MIMO into practice, several issues still need to be solved despite the much research work on it [19–26]. If base station communicates in same frequency or time resources with the users, the higher data rates can be obtained [14]. Inter user interference is a problem in data transmission in downlink channel of massive MIMO and require more transmit power, thus, interference cancellation at BS is required. Precoding methods are important to pre-subtract interference from users in transmitter side before sending it through the channel [15]. Precoding is one of the hot and active research topics of massive MIMO [16, 17]. A practical approach that has received considerable attention due to its simplicity is represented by linear precoding [18]. Proper signal processing can be performed at the transmitter to separate the multiple users in space if CSI is available at the transmitter (CSIT) [19]. Exact channel state information, or perfect CSIT, is ideal, but in real scenario, acquiring perfect CSIT is difficult in a severe fading channel [15]. The authors in [8] presented various challenges for massive MIMO in next generation wireless systems, especially to scale up the number of transmit antennas. Among them precoding is one of the hot and active research topic for massive MIMO.

Especially for imperfect channel state information where estimation errors play vital role [16, 17]. The performance of linear and nonlinear precoding methods in faded environment investigated in [15] showed that how different precoding techniques improve the bit error rate using perfect CSI. It is found that implementation of nonlinear precoding methods require a tremendous computational complexity at both BS and user equipment (UEs). The performance of massive MIMO system in single cell downlink with different precoding schemes under perfect CSI is analyzed in [20]. The authors used the same value of signal to interference to noise ratio for each precoders which should be avoided for good analysis because each precoding has different signal-to-interference-plus-noise (SINR).

The matrix and vector normalization for ZF and MRT precoding is compared in [21]. The ergodic performance of these precodings with perfect CSI in a cell boundary users scenario is investigated. In [22], the authors investigated the performance of downlink massive MIMO with ZF precoding in term of outage probability and bit error ratio using perfect CSI. System performance using perfect CSI with ZF and MRT is analyzed in [23]. The power efficiency on the uplink of massive MIMO system under imperfect and perfect CSI using minimum mean square error (MMSE), ZF and maximal ratio combining (MRC) receivers is analyzed in [24]. In [25], the authors analyzed and compared the performance of eigen beaforming (BF) and RZF in term of achievable data rate in presence of channel estimation errors in a multi cell downlink scenario. The uplink achievable rate of massive MIMO incorporated the ZF and MRC receivers is evaluated in Ricean fading channels with both imperfect and perfect CSI in [26].

However, most of the above work considered perfect channel state information which is hard to obtain in practical scenarios. The difference of our work with respect to [20, 21, 23, 27] is that the effect of estimation error is considered which was neglected in these works. Due to estimation error consideration, the analysis will be different and more practical. While in [24, 26], channel estimation was considered for the perfromance of uplink channel and in [25] BF and RZF precoding was compared in the presence of imperfect CSI. Whereas, we consider downlink channel with ZF and MRT precoding.

In this paper, the performance of Massive MIMO downlink including linear precoding is evaluated. Spectral efficiency in terms of achievable rate and energy efficiency in terms of transmit power of zero force (ZF) and maximum ratio transmission (MRT) linear precoding is investigated under imperfect channel state information (CSI). Since ZF and MRT precoding can balance the system performance and complexity. Different channel estimation values are considered in order to compare the performance of these precoding techniques in the given system. The achievable rate and the downlink transmit power of ZF and MRT precoding techniques are derived, analyzed and compared under the same conditions and assumptions. Several scenarios are considered to study these performance parameters.

The paper makes the following specific contributions:

- We consider a massive MIMO system where the number of BS antennas $M$ and number of users $K$ grow large and examine the achievable rates of ZF and MRT precoding techniques for large $M$ and large $K$.
- We also derive and evaluate downlink transmit power of these precoding techniques and perform the comparison by investigating the effect of large $M$ and $K$ on it.
- We explore the impact of imperfect CSI on achievable rates and downlink transmit power of ZF and MRT precoding techniques.

The remainder of this paper is organized as follows. Section 2 describes the system & channel model. Section 3 derives achievable rate and downlink transmit power of ZF and

MRT. In Sect. 4, we provide a set of simulation results, while Sect. 5 concludes the main results of this paper.

*Notation* Throughout the paper, matrices are denoted by uppercase boldface letters while vectors are expressed in lowercase boldface letters. $\mathbf{X}^H$, $\mathbf{X}^T$ and $\mathbf{X}^{-1}$ are used to denote conjugate transpose, transpose, and inverse of $\mathbf{X}$, respectively. Moreover, $[\mathbf{X}_{ij}]$ gives the $(i, j)$th entry of $\mathbf{X}$ and $\mathbf{Z} \in \mathbb{CN}(a, b)$ denotes that $\mathbf{Z}$ is a complex Gaussian matrix with mean $a$ and variance $b$.

## 2 System and Channel Model

The downlink of a multi user massive MIMO system is considered where the BS (base station) and each MS (mobile station) are equipped with $M$ and $N$ antennas, respectively, and BS serves $K$ single antenna users ($N = 1$). The multiuser massive MIMO system is shown in Fig. 1

The downlink transmission occurs in two phases: training phase and downlink data transmission phase [28]. The base station estimates the CSI from $K$ users based on the received pilot sequences in the uplink during the training phase. The base station uses this CSI and linear precoding schemes to process the transmit data [27].

### 2.1 Channel Estimation

The channel estimation techniques depend upon the system duplexing mode either time division duplex (TDD) or frequency division duplex (FDD) [29]. In this work we assume TDD mode, that the channel is estimated at the BS via uplink pilots, assuming channel reciprocity. Uplink and downlink channels are reciprocal in TDD because it uses the same frequency spectrum for the uplink and downlink transmissions but with different time slots. $M$ is large in massive MIMO therefore, TDD operation is preferable in Massive MIMO [29]. The downlink and uplink channels can not be perfectly reciprocal in practice because of hardware chains mismatch. But with proper calibration [30–32], this non-reciprocity can be removed. In this work, it is assumed that the hardware chain calibration is perfect. It is assumed that the channel characteristics do not change and are constant for $T$ symbol

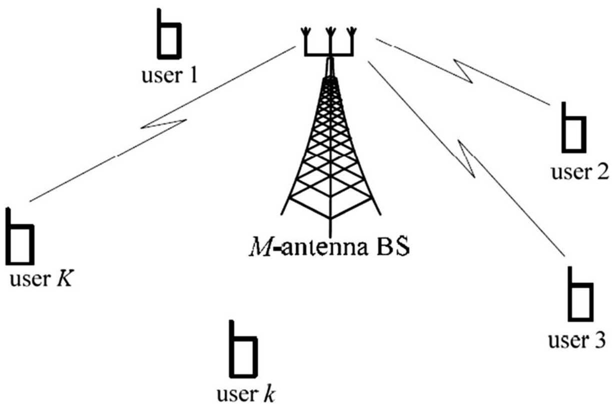


**Fig. 1** Multiuser massive MIMO systems, here, M-antenna BS serves K single-antenna users in the same time-frequency resource

durations. There are two phases during each coherence interval [29]. In the first phase, during part $t$ of the coherence interval the training or pilot sequences are transmitted to estimate the channel of each user. During the second phase, the data of all $K$ users is simultaneously transmitted to the BS. The channel estimates obtained during first phase are then used by the BS to detect the transmitted signals. Due to channel reciprocity in TDD system, the channel estimated at the base station in the uplink can be used to precode the transmit symbols in the downlink. Estimated channel may contain estimation errors and therefor it is not perfect which is given as

$$\mathbf{H} = \bar{\mathbf{H}} + \mathbf{E} \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{K \times M}$ is a Rayleigh fading MIMO channel with independent and identically distributed zero mean and unit variance complex Gaussian entries. $\bar{\mathbf{H}}$ is the imperfect CSI or estimated channel matrix obtained through minimum mean square error (MMSE) channel estimation [33] and $\mathbf{E} = \mathbf{H} - \bar{\mathbf{H}}$ is the error matrix. $\bar{\mathbf{H}}$ and $\mathbf{E}$ are uncorrelated. Each element of error matrix $\mathbf{E}$ is i.i.d zero mean complex Gaussian random variables with variance $\sigma_e^2 = MMSE = \epsilon\{[\mathbf{H}]_{ij} - [\bar{\mathbf{H}}_{ij}]^2\}$

Each element of $\bar{\mathbf{H}}$ is independent and identically distributed as $\mathbb{CN}(0, 1 - \sigma_e^2)$ while the error is complex white Gaussian with variance $\sigma_e^2$.

CSI is imperfect if it contains the estimation errors. Channel estimation error can be modeled by specifying the channel estimation error matrix elements using i.i.d zero mean complex Gaussian variables [33]. Hence the imperfect CSI or estimated CSI ($\bar{\mathbf{H}}$) using minimum mean square error (MMSE) channel estimation is [33]

$$\bar{\mathbf{H}} = \sqrt{1 - \sigma_e^2}\mathbf{H} + \sigma_e \mathbf{E} \tag{2}$$

The parameter $\sigma_e$ is used to compute the accuracy of channel estimation. When $\sigma_e = 0$, it means that there is no channel estimation error and channel information is perfect while $\sigma_e = 1$ indicates it is a complete failure of channel estimation and estimation totally contains estimation error matrix and it is complete imperfect CSI.

## 2.2 Downlink Data Transmission

The base station uses the channel estimates obtained from channel estimation phase to process the signals before transmitting them to $K$ users and the base station uses linear precoding techniques. Using linear precoding the received signal $\mathbf{y} \in \mathbb{C}^{K \times 1}$ will be

$$\mathbf{y} = \bar{\mathbf{H}}\mathbf{W}\mathbf{x} + \mathbf{n} \tag{3}$$

where $\mathbf{x} \in \mathbb{C}^{M \times 1}$ is the transmit signal from the base station, $\mathbf{W} \in \mathbb{C}^{M \times K}$ is a linear precoding matrix and $\mathbf{n} \in \mathbb{C}^{K \times 1}$ is the AWGN (additive white Gaussian noise) with zero-mean and variance.

The received signal by $k$th user using linear precoding technique is given by

$$y_k = \bar{\mathbf{h}_k}\mathbf{w}_k x_k + \sum_{i=1, i \neq k}^{k} \bar{\mathbf{h}_k}\mathbf{w}_i x_i + n_k \tag{4}$$

$\bar{\mathbf{h}_k}\mathbf{w}_k x_k$ is the desired signal, $\sum_{i=1, i \neq k}^{k} \bar{\mathbf{h}_k}\mathbf{w}_i x_i$ is the interference from the other users and $\mathbf{n}$ is the noise.

The received signal to interference plus noise ratio of the of the $k$th user can be expressed as [20]

$$SINR_k = \frac{|\bar{\mathbf{h}}_k \mathbf{w}_k|^2}{\sum_{i=1,i\neq k}^{k} |\bar{\mathbf{h}}_k \mathbf{w}_i|^2 + 1} \tag{5}$$

## 2.3 Linear Precoding

In this work, we consider ZF and MRT linear precoding techniques.

### *2.3.1 ZF Precoding*

ZF precoding is a linear precoding technique which cancel out the inter user interference at each user. This precoding is assumed to implement a pseudo-inverse of the channel matrix. ZF precoding at BS is given as [20]

$$\mathbf{W} = \beta\left\{\bar{\mathbf{H}}^H(\bar{\mathbf{H}}\bar{\mathbf{H}}^H)^{-1}\right\} \tag{6}$$

where $\beta$ is a scaling factor to satisfy the transmit power constraint and it is given as

$$\beta = \sqrt{\frac{P_d}{tr(\mathbf{A}\mathbf{A}^H)}} \tag{7}$$

where $\mathbf{A} = \bar{\mathbf{H}}^H(\bar{\mathbf{H}}\bar{\mathbf{H}}^H)^{-1}$ and $P_d$ is the downlink transmit power.

The $K$th user $SINR_k^{zf_{csi}}$ under Imperfect CSI for large values of $M$ and $K$ is given as (see Appendix 1)

$$SINR_k^{zf_{csi}} = \left\{\frac{(\alpha-1)(1-\sigma_e^2)P_d}{(P_d\sigma_e^2)+1}\right\} \tag{8}$$

where $\alpha = \frac{M}{K}$

### *2.3.2 MRT Precoding*

MRT Maximizes signal gain and SNR at the intended user. MRT precodning at BS is given as [20]

$$\mathbf{W} = \beta\{\mathbf{H}^H\} \tag{9}$$

where $\beta$ is same as (8), but $\mathbf{A} = \mathbf{H}^H$

The $K$th user $SINR_k^{MRT_{csi}}$ under Imperfect CSI for large values of $M$ and $K$ is given as (see Appendix 2)

$$SINR_k^{MRT_{csi}} = \frac{(1-\sigma^2)P_d\alpha}{(P_d+\sigma^2)+1} \tag{10}$$

## 3 Performnce Analysis

The system performance under imperfect CSI is analysed using achievable rate and downlink transmit power.

### 3.1 Achievable Rate

Achievable rate is the one method to quantify the system performance. The achievable rate is followed by Shannon theorem. This theory tells the maximum rate which the transmitter can transmit over the channel [34]

$$C = log_2(1 + SNR) \quad (\text{bits/s/Hz}) \tag{11}$$

In MU-Massive MIMO downlink system, the transmitter transmits multiple data streams to each user simultaneously and selectively with CSI [35], so transmitter must know this CSI. With MU-MIMO interference among the users and additive noise remains a common factor. Therefore, $k$th user will experience the inference of the different users and the additive noise [15]. If the power of the desired signal, interference and the noise is defined as $S_n, I_n$ and $N_n$ respectively, then the SINR at the $k$th user becomes [34]

$$SINR_k = \left(\frac{S_n}{I_n + N_n}\right) \tag{12}$$

The achievable rate for $K$ number of users is given as [20]

$$C_k = Klog_2\{1 + SINR_k\} \tag{13}$$

From (13) the achievable rate with ZF precoding is calculated as

$$C_k^{zf} = Klog_2\left\{1 + SINR_k^{zf_{csi}}\right\} \tag{14}$$

Substituting (8) into (14), we get;

$$C_k^{zf} = Klog_2\left(1 + \frac{(\alpha - 1)(1 - \sigma_e^2)P_d}{(P_d\sigma_e^2) + 1}\right) \tag{15}$$

The achievable rate with MRT precoding from (13) can be calculated as

$$C_k^{MRT} = Klog_2\left\{1 + SINR_k^{MRT_{csi}}\right\} \tag{16}$$

Substituting (10) into (16), we get

$$C_k^{MRT} = Klog_2\left(1 + \frac{(1 - \sigma^2)P_d\alpha}{(P_d + \sigma^2) + 1}\right) \tag{17}$$

### 3.2 Downlink Transmit Power

The dowlink transmit power makes the system energy efficient or inefficient. Since the energy efficiency of the system depends on transmit power, with increasing dowlink transmit power, higher capacity can be achieved. but it dereases the energy efficiency of the system [27]. The system becomes more energy efficient when less transmit power is required to achieve the targeted information rate. Here we calculated the dowlink transmit power of ZF and MRT to obtain the same achievable rate for the system under consideration.

Downlink Transmit Power with ZF from (15) can be calculated as

$$ln2(\frac{C_k}{K}) = ln\left(1 + \frac{(\alpha - 1)(1 - \sigma_e^2)P_d}{(P_d\sigma_e^2) + 1}\right) \tag{18}$$

By taking exponential on both sides, we get

$$e^{ln2(\frac{C_k}{K})} = 1 + \left(\frac{(\alpha - 1)(1 - \sigma_e^2)P_d}{(P_d\sigma_e^2) + 1}\right) \tag{19}$$

But $e^{ln2} = 2$

$$\therefore 2^{(\frac{C_k}{K})} = 1 + \left(\frac{(\alpha - 1)(1 - \sigma_e^2)P_d}{(P_d\sigma_e^2) + 1}\right) \tag{20}$$

Hence, the total downlink transmit power with ZF under imperfect CSI is given as

$$P_d^{ZF} = \frac{\left(2^{(\frac{C_k}{K})} - 1\right)}{(\alpha - 1)(1 - \sigma_e^2) - \sigma_e^2\left(2^{(\frac{C_k}{K})} - 1\right)} \tag{21}$$

Similary downlink transmit power with MRT using (17) can be written as

$$ln2\left(\frac{C_k}{K}\right) = ln\left(1 + \frac{(1 - \sigma^2)P_d\alpha}{(P_d + \sigma^2) + 1}\right) \tag{22}$$

Taking exponetial on both sides

$$e^{ln2\left(\frac{C_k^{MRT}}{K}\right)} = 1 + \frac{(1 - \sigma^2)P_d\alpha}{(P_d + \sigma^2) + 1} \tag{23}$$

Finally, the total downlink transmit power with MRT under imperfect CSI is given as

$$P_d^{MRT} = \frac{\sigma_e^2\left(2^{(\frac{C_k}{K})} - 1\right) + \left(2^{(\frac{C_k}{K})} - 1\right)}{(1 - \sigma_e^2)\alpha - \left(2^{(\frac{C_k}{K})} - 1\right)} \tag{24}$$

## 4 Simulation Results

The performance of massive MU-MIMO downlink system with ZF and MRT precoding over Rayleigh fading channel is analyzed in terms of achievable rate and downlink transmit power. Using the theoretical results of Sect. 3, Matlab simulation is performed to provide the numerical results. Three different scenarios are considered to analyze achievable rate and downlink transmit power.

### 4.1 Achievable Rate

To analyze the achievable rate it is assumed that downlink power of 0 dB is equally divided among all users.

#### 4.1.1 First Scenario

During the first scenario, we change the number of antennas $M$ from 20 to 200 while keep the number of users fixed, $K = 20$ with channel estimation error of 0.3.

Figure 2 shows the achievable rate verus the number of BS antennas $M$ for both ZF and MRT under perfect and imperfect CSI. From Fig. 2, it is clear that the as $M$ increases the achievable rate also increases for both precoding techniques. Achievable rate for ZF increases rapidly with $M$ while for MRT it increases slowly. The performance of both precoding techniques degrades in presence of estimation error. This is because when the number of BS antennas $M$ increases without limit, uncorrelated noise, fast fading and intra-cell interference tend to vanish. Therefore, improvement in achievable rate is greater with large number of $M$. Moreover, this also increases the value of signal to noise ratio (SNR) which makes the performance of ZF better than MRT. Since ZF performs better at high SNR [26].

#### 4.1.2 Second Scenario

During the second scenario, the $M$ is kept fixed while we increase $K$. We set the $M = 200$ and increase $K$ from 20 to 200 with channel estimation error of 0.3.

Figure 3 shows the achievable rate verus the number of users $K$ for both ZF and MRT under perfect and imperfect CSI. From Fig. 3, it is observed that as $K$ increases, the achievable rate for ZF starts to decrease and it is concave function of $K$, while it keeps on

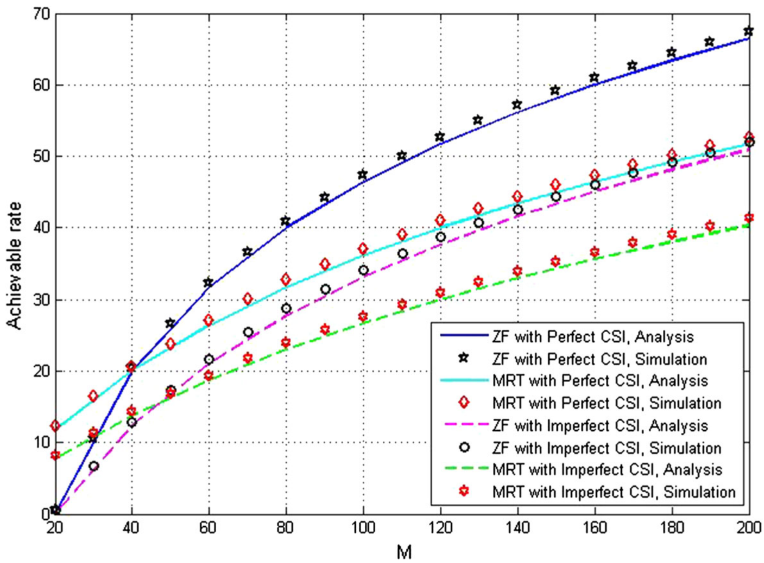


**Fig. 2** Achievable rate of ZF and MRT for different number of antennas keeping the number of users constant

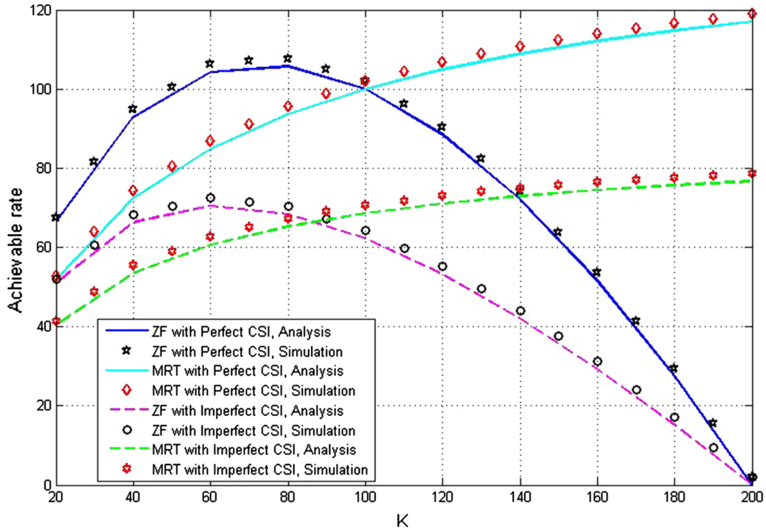


**Fig. 3** Achievable rate of ZF and MRT for different number of users while keeping the number of antennas constant

increasing for MRT. The findings are similar for both Perfect and imperfect CSI. As long as $M \gg K$, the performance of ZF is much better than MRT. For larger value of $K$, performance of MRT is better than ZF. This is due to the fact that, ZF works well at high SNR while MRT performs well at low SNR [26]. When $K$ is large it increases the intra-cell interference which causes the SNR to decrease and hence the performance of ZF degrades.

### *4.1.3 Third Scenario*

In this scenario effect of channel imperfectness is analyzed on achievable rate. For achievable rate the channel estimation error are considered from 0 to 1. Achievable rate under different estimation errors is investigated when (1) ratio of $M$ and $K$ is not large, (2) ratio of $M$ and $K$ is large enough.

Figure 4 shows the achievable rate for ZF and MRT with different channel estimation errors. It is observed from Fig. 4 that the achievable rate for both precoding techniques decreases with estimation errors but the decrease in achievable rates of ZF is more as compared to MRT with increase in estimation error. Further, the MRT achievable rate is greater than ZF when $M$ and $K$ are closer to each other, while ZF achievable rate is much higher when $M$ is large enough than $K$. This is because $P_d$ is kept constant while estimation error noise increases that causes the achievable rates of both precodings to decrease. Moreover, when ratio of $M$ and $K$ is large, SNR value is high that causes performance of ZF superior over MRT. This is further validated by reducing $M$ to smaller value, that caused the SNR value to go low and performance of MRT is better.

## 4.2 Downlink Transmit Power

To analyze the downlink transmit power it is assumed that the total achievable data rate of 15 bits/s/Hz is equally shared among the users.

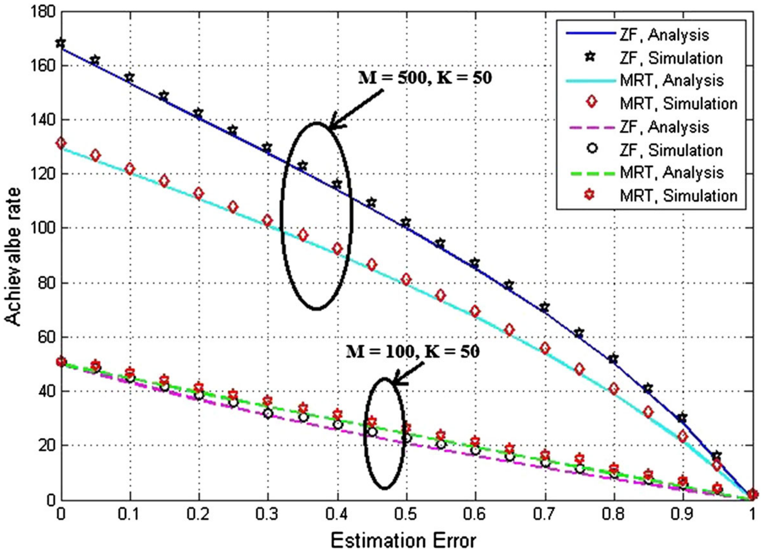


**Fig. 4** Achievable rate of ZF & MRT verus channel estimation error

### 4.2.1 First Scenario

To observe the downlink transmit power we change $M$ from 20 to 200 and choose $K = 10$ with channel estimation error of 0.3 during the first scenario.

Figure 5 shows the downlink transmit power (dB) versus the number of BS antennas $M$ for both ZF and MRT under perfect and imperfect CSI with channel estimation error of 0.3. It is obseved from Fig. 5 that the downlink transmit power decreases as $M$ increases for both precoding techniques. But ZF is more power efficient in this case as compare to MRT. This is because when $M$ is small, $P_d$ has not been cut down so much. However, as $M$ gets larger, $P_d$ is cut down more.

### 4.2.2 Second Scenario

During the second scenario, downlink transmit power of ZF and MRT precoding is analyzed for $M = 200$ and $K$ from 20 to 200 with channel estimation error of 0.3.

Figure 6 shows the downlink transmit power (dB) versus the number of user $K$ for both ZF and MRT under perfect and imperfect CSI with channel estimation error of 0.3. It is observed from Fig. 6 that as the $K$ increases the downlink power of ZF keeps on increasing rapidly while the downlink power of MRT decreases slowly. With the increase of $K$ the downlink transmit power of ZF is quite large as compare to MRT. This is due to the same fact that, ZF works well at high SNR while MRT performs well at low SNR [26]. When $K$ is large, value of SNR is low and performance of MRT is better.

### 4.2.3 Third Scenario

For downlink transmit power, we change the channel estimation errors from 0 to 1. Downlink transmit power under different estimation errors is investigated when (1) ratio of $M$ and $K$ is not large, (2) ratio of $M$ and $K$ is large enough.

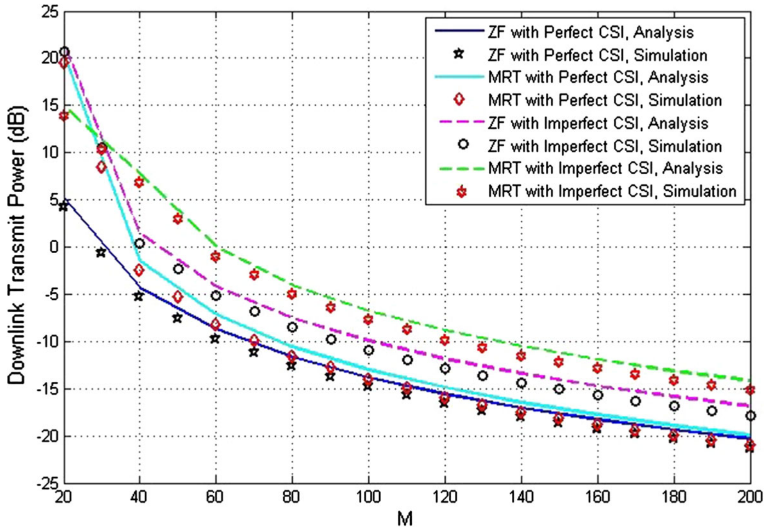


**Fig. 5** Downlink transmit power of ZF and MRT for different number of antennas keeping the number of users constant

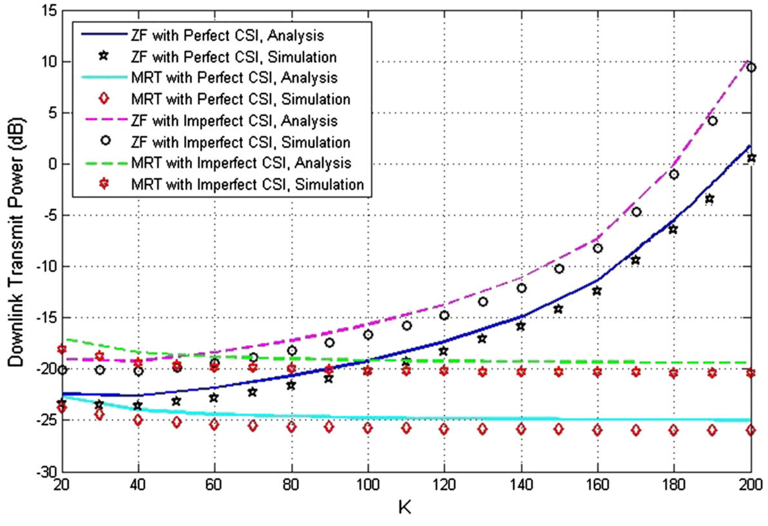


**Fig. 6** Downlink transmit power of ZF and MRT for different number of users keeping the number of antennas constant

Figure 7 shows the downlink transmit power for ZF and MRT with different estimation errors. From Fig. 7 it is clear that downlink transmit power of both precoding techniques increases with estimation error. It is also observed that when $M$ is not quits large, MRT needs less power to transmit the same number of bits as compare to ZF regardless of the estimation error. When we increase the number of antennas from 100 to 500 for same estimation error model, ZF becomes more power efficient for higher estimation error. This

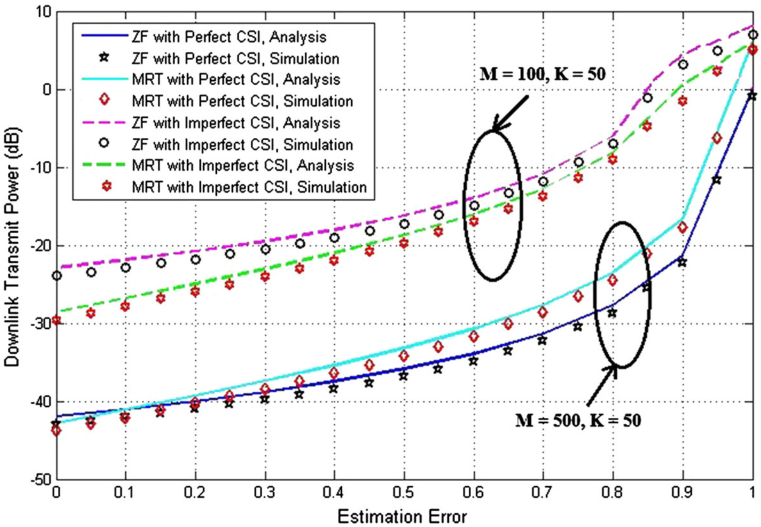


**Fig. 7** Downlink transmit power of ZF and MRT for different estimation errors, (1) when ratio of $M\&K$ is not large, (2) when ratio of $M\&K$ is large

is because ZF performs better than MRT at high SNR. When $M$ is large, value of SNR is high. When $M$ is small, SNR is low, as a result MRT performs better.

## 5 Conclusion

This paper provides performance of MU-Massive MIMO system with ZF and MRT precoding under imperfect CSI. The key performance parameters are achievable rate and downlinks transmit power which is theoretically derived for imperfect CSI. The simulation results show that performance of ZF is better when $M$ is much larger than $K$, in this case ZF will achieve higher data rate and will be more power efficient as compare to MRT. But when the ratio of $M$ and $K$ is not large, performance of MRT is superior over ZF. The effect of increasing $K$ is more on the performance of ZF while MRT performance is robust in this case. The effect of channel estimation errors is more on ZF as compare to MRT. Hence, we can say that ZF performs better when BS antennas are large as compare to users, while MRT performs better even when BS antennas are small.

## Appendix 1

The desired signal power $S_k$, interference $I_k$ and Noise $n_k$ is

$$
\begin{aligned}
|S_k|^2 &= \frac{P_d}{tr(\bar{\mathbf{A}}\bar{\mathbf{A}}^{H})} \\
|I_k|^2 &= P_d \sigma_e^2
\end{aligned}
$$

and

$$|n_k|^2 = n = 1$$

By substituting the valuse of $S_k, I_k$ and $n_k$ in (13), the SINR of the $k$th user is given as

$$SINR_k^{zf_{csi}} = \frac{P_d}{tr(\bar{\mathbf{A}}\bar{\mathbf{A}^H})}\left(\frac{1}{P\sigma_e^2 + 1}\right) \tag{25}$$

$$SINR_k^{zf_{csi}} = \frac{P_d}{(P\sigma_e^2 + 1)tr(\bar{\mathbf{A}}\bar{\mathbf{A}}^H)} \tag{26}$$

When the value of $M$ and $K$ is large [23], then

$$\frac{1}{tr(\mathbf{A}\mathbf{A}^H)} \approx \text{Diversity order of ZF precoding}$$

The diversity order measures the number of independent paths over which the data is received [23]

$$\text{Diversity order of ZF precoding} = \alpha - 1$$

where $\alpha = \frac{M}{K}$

$$\therefore \frac{P_d}{tr(\bar{\mathbf{A}}\bar{\mathbf{A}^H})} = \frac{(M-K)(1-\sigma_e^2)P_d}{K} \tag{27}$$

Putting the values of (27) in (26) and after some manipulations, we obtain the result of (9).

## Appendix 2

Received Signal at the $k$ $th$ user is given as

$$y_k = \beta\bar{\mathbf{h}_k}\bar{\mathbf{h}_k^H}x_k + \sum_{i=1, i\neq k}^{k}\beta\bar{\mathbf{h}_k}\bar{\mathbf{h}_i^H}x_i + \sum_{i=1}^{k}\beta\bar{\mathbf{h}_i^H}\mathbf{E}_i x_i + n_k \tag{28}$$

Therefore, SINR of MRT precoding under imperfect CSI is

$$SINR_k^{mrt_{csi}} = \frac{\beta^2|\mathbf{h}_k\mathbf{h}_k^H|^2(1-\sigma^2)}{\beta^2\left[\sum_{i=1, i\neq k}^{k}|\mathbf{h}_k\mathbf{h}_i^H|^2(1-\sigma^2) + \sigma^2\right] + 1} \tag{29}$$

where $'\beta'$ for imperfct CSI is given as

$$\beta = \sqrt{\frac{P_d}{MK(1-\sigma^2)}} \tag{30}$$

But from [36]

$$\frac{1}{K}\sum_{i=1, i\neq k}^{k}|\mathbf{h}_k\mathbf{h}_i^H|^2 = \mathbf{E}|\mathbf{h}_k\mathbf{h}_i^H|^2 = M \tag{31}$$

and

$$\mathbf{h}_k \mathbf{h}_k^H = M \tag{32}$$

Substituting (30), (31) and (32) in (29) and after some manipulations, we obtain the result of (11)

## References


1. Index, C. V .N. (2014). Global mobile data traffic forecast update, 2013–2018. *UR l*:http://www.cisco.com/c/en/us/solutions/collateral/service-provider/visual-networking-index-vni/white_paper_c11-520862.html. Visited on 14 May 2014.
2. Marzetta, T. L. (2015). Massive mimo: An introduction. *Bell Labs Technical Journal*, *20*, 11–22.
3. Andrews, J. G., Buzzi, S., Choi, W., Hanly, S. V., Lozano, A., Soong, A. C., et al. (2014). What will 5g be? *IEEE Journal on Selected Areas in Communications*, *32*(6), 1065–1082.
4. Gesbert, D., Kountouris, M., Heath, R. W, Jr., Chae, C.-B., & Sälzer, T. (2007). Shifting the mimo paradigm. *IEEE Signal Processing Magazine*, *24*(5), 36–46.
5. Marzetta, T. L. (2010). Noncooperative cellular wireless with unlimited numbers of base station antennas. *IEEE Transactions on Wireless Communications*, *9*(11), 3590–3600.
6. Qiao, D., Wu, Y., & Chen, Y. (2014). Massive mimo architecture for 5g networks: Co-located, or distributed?. In: *2014 11th international symposium on wireless communications systems (ISWCS)*. IEEE, pp. 192–197.
7. Boccardi, F., Heath, R. W., Lozano, A., Marzetta, T. L., & Popovski, P. (2014). Five disruptive technology directions for 5g. *IEEE Communications Magazine*, *52*(2), 74–80.
8. Rusek, F., Persson, D., Lau, B. K., Larsson, E. G., Marzetta, T. L., Edfors, O., et al. (2013). Scaling up mimo: Opportunities and challenges with very large arrays. *IEEE Signal Processing Magazine*, *30*(1), 40–60.
9. Larsson, E., Edfors, O., Tufvesson, F., & Marzetta, T. (2014). Massive mimo for next generation wireless systems. *IEEE Communications Magazine*, *52*(2), 186–195.
10. Hoydis, J., Ten Brink, S., & Debbah, M. (2013). Massive mimo in the ul/dl of cellular networks: How many antennas do we need? *IEEE Journal on Selected Areas in Communications*, *31*(2), 160–171.
11. Shepard, C., Yu, H., Anand, N., Li, E., Marzetta, T., Yang, R., & Zhong, L. (2012). Argos: Practical many-antenna base stations. In *Proceedings of the 18th annual international conference on Mobile computing and networking*. ACM, pp. 53–64.
12. Mohammed, S. K., & Larsson, E. G. (2013). Per-antenna constant envelope precoding for large multi-user mimo systems. *IEEE Transactions on Communications*, *61*(3), 1059–1071.
13. Hoydis, J., Ten Brink, S., & Debbah, M. (2011). Massive mimo: How many antennas do we need?. In *2011 49th Annual Allerton conference on communication, control, and computing (Allerton)*. IEEE, pp. 545–550.
14. Jose, J., Ashikhmin, A., Marzetta, T. L., & Vishwanath, S. (2011). Pilot contamination and precoding in multi-cell TDD systems. *IEEE Transactions on Wireless Communications*, *10*(8), 2640–2651.
15. Mohan, K.J., Gogoi, O., & Gogoi, P. (2014). Interference cancellation in massive mimo base stations with certain precoding techniques in faded environment. In *2014 international conference on signal processing and integrated networks (SPIN)*. IEEE, pp. 795–800.
16. Huh, H., Caire, G., Papadopoulos, H. C., Ramprashad, S., et al. (2012). "Achieving" massive mimo" spectral efficiency with a not-so-large number of antennas. *IEEE Transactions on Wireless Communications*, *11*(9), 3226–3239.
17. Prabhu, H., Edfors, O., Rodrigues, J., Liu, L., & Rusek, F. (2014). A low-complex peak-to-average power reduction scheme for OFDM based massive mimo systems. In: *2014 6th international symposium on communications, control and signal processing (ISCCSP)*. IEEE, pp. 114–117.
18. Björnson, E., Bengtsson, M., & Ottersten, B. (2014). Optimal multiuser transmit beamforming: A difficult problem with a simple solution structure. arXiv preprint arXiv:1404.0408.
19. Lee, J., Han, J.-K., & Zhang, J. (2009). Mimo technologies in 3GPP ITE and ITE-advanced. *EURASIP Journal on Wireless Communications and Networking*, *2009*, 3.
20. Selvan, V., Iqbal, M. S., & Al-Raweshidy, H. (2014). Performance analysis of linear precoding schemes for very large multi-user mimo downlink system. In *2014 fourth international conference on innovative computing technology (INTECH)*. IEEE, pp. 219–224.

21. Lim, Y.-G., Chae, C.-B., & Caire, G. (2013). Performance analysis of massive mimo for cell-boundary users. *arXiv preprint*arXiv:1309.7817.
22. Zhao, L., Zheng, K., Long, H., & Zhao, H. (2014). Performance analysis for downlink massive mimo system with ZF precoding. *Transactions on Emerging Telecommunications Technologies*, *25*(12), 1219–1230.
23. Parfait, T., Kuang, Y., & Jerry, K. (2014). Performance analysis and comparison of ZF and MRT based downlink massive mimo systems. In *2014 sixth international conference on ubiquitous and future networks (ICUFN)*. IEEE, pp. 383–388.
24. Ngo, H. Q., Larsson, E. G., & Marzetta, T. L. (2013). Energy and spectral efficiency of very large multiuser mimo systems. *IEEE Transactions on Communications*, *61*(4), 1436–1449.
25. Hoydis, J., Ten Brink, S., & Debbah, M. (2012). Comparison of linear precoding schemes for downlink massive mimo. In *2012 IEEE international conference on communications (ICC)*. IEEE, pp. 2135–2139.
26. Zhang, Q., Jin, S., Wong, K.-K., Zhu, H., & Matthaiou, M. (2014). Power scaling of uplink massive mimo systems with arbitrary-rank channel means. *IEEE Journal of Selected Topics in Signal Processing*, *8*(5), 966–981.
27. Bjornson, E., Sanguinetti, L., Hoydis, J., & Debbah, M. (2014). Designing multi-user mimo for energy efficiency: When is massive mimo the answer?. In *2014 IEEE wireless communications and networking conference (WCNC)*. IEEE, pp. 242–247.
28. Yin, X., Yu, X., Liu, Y., Tan, W., & Chen, X. (2013). Performance analysis of multiuser mimo system with adaptive modulation and imperfect CSI. In *IET international conference on information and communications technologies (IETICT 2013)*. IET, pp. 571–576.
29. Ngo, H. Q. (2015). *Massive MIMO: Fundamentals and system designs* (Vol. 1642). Linköping: Linköping University Electronic Press.
30. Larsson, E. G., Edfors, O., Tufvesson, F., & Marzetta, T. L. (2014). Massive MIMO for Next Generation Wireless Systems. *IEEE Communications Magazine*, *52*(2), 186–195.
31. Nishimori, K., Cho, K., Takatori, Y., & Hori, T. (2001). Automatic calibration method using transmitting signals of an adaptive array for TDD systems. *IEEE Transactions on Vehicular Technology*, *50*(6), 1636–1640.
32. Rogalin, R., Bursalioglu, O. Y., Papadopoulos, H. C., Caire, G., & Molisch, A. F. (2013). Hardware-impairment compensation for enabling distributed large-scale mimo. In *Information theory and applications workshop (ITA)*. IEEE, pp. 1–10.
33. Frigyes, I., Bitó, J., & Bakki, P. (2008). *Advances in mobile and wireless communications: Views of the 16th IST mobile and wireless communication summit*. Berlin: Springer.
34. Madhow, U. (2008). *Fundamentals of digital communication*. Cambridge: Cambridge University Press.
35. Marzetta, T. L., & Hochwald, B. M. (2006). Fast transfer of channel state information in wireless systems. *IEEE Transactions on Signal Processing*, *54*(4), 1268–1278.
36. Ngo, H. Q., Larsson, E. G., & Marzetta, T. L. (2013). Massive mu-mimo downlink tdd systems with linear precoding and downlink pilots. In *2013 51st Annual Allerton conference on communication, control, and computing (Allerton)*. IEEE, pp. 293–298.

**Adil Israr** received B.S. and M.S. degrees in Telecommunication Engineering from Balochistan University of Information Technology, Engineering & Management Science (BUITEMS), Quetta, Pakistan, in 2009 and 2015 respectively with distinctions. Currently he is serving as Assistant Professor in Department of Telecommunication Engineering, BUITEMS, Quetta, Pakistan. His research interests lies in digital and wireless communications, 5G and beyond 5G technologies including Massive MIMO and 3D MIMO techniques, Spectrum sharing management, Advanced channel coding, Channel modeling for 5G, channel estimation, transmission strategies in communication systems, mmWave communications.

**Zahid Rauf** received his BE degree in Information Technology from Hamdard University, Islamabad Campus, Pakistan, the M. Sc. Degree in Electrical and Electronic Engineering from University of Leeds, U.K, and the PhD degree in Electrical and Electronic Engineering from University of Canterbury, New Zealand in 2013. He is currently an associate professor in the Department of Electrical Engineering, Balochistan University of Information Technology, Engineering and Management Sciences (BUITEMS), Quetta, Pakistan. He is also serving as the Director of Quality Enhancement and Accreditation (QE&A) at this university. His research interests span many topics within digital wireless communications, including MIMO, Multiuser Communication Systems, Channel Estimation, Equalization and Low-complexity Iterative Detection Algorithms, Cooperative Relaying Networks, Optimal and Adaptive Resource Allocation in Wireless Networks, and Cross-layer Design and Analysis (primarily Physical and Link layer). He is also a Member of IEEE.

**Jan Muhammad** received the B.S. degree in computer engineering from NED University of Engineering & Technology, Karachi, Pakistan in 1997 and M.S. in computer science from BUITEMS, Quetta, Pakistan in 2006. He received the Ph.D. degree in computer science from University of Glasgow, UK in 2012. He is working as Associate Professor in the department of Computer Engineering in BUITEMS, Quetta, Pakistan with research focused in Distributed Systems. He has a teaching experience of 15 years and has been teaching a broad range of undergrad and graduate level courses including: Computer System Architecture, Microprocessors & Assembly Language, Operating Systems, Database Management Systems, Programming Fundamentals and Research Methodology. His current work focuses on cloud based learning management systems and distributed systems security (Cloud and Grid security).

**Faisal Khan** received BS degree in Software Engineering and MS degree in Telecommunications in 2005 and 2007, respectively, from Bahria University, Karachi. He received MS and Ph.D. degrees in Electrical and Computer Engineering from the Georgia Institute of Technology, USA in 2012. He is currently working as Associate Professor at the Faculty of Information and Communication Technology at BUITEMS University, Pakistan. He has been working at BUITEMS since October 2007. His research interests include ad hoc networks, device-to-device communications and network security. Awards and distinctions to his credit include Fulbright Ph.D. scholarship award 2009–2012, Bahria University scholarship award 2006, distinction Bahria University 2005, and best teacher award FICT BUITEMS 2008.