

# A Coalitional and Two-Level Game Based Approach for Spectrum Leasing in MIMO Cooperative Cognitive Radio Networks

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**Abstract** In this work, we studied the resource allocation problem in a spectrum leasing scenario in MIMO cooperative cognitive radio networks (MIMO-CCRN) with multi channels using a coalitional game based approach. It is proved that the best case is to group all primary users (PUs) and secondary users (SUs) in a set, where each PU is able to lease its unused spectrum to all SUs using a time-division based strategy and the SUs relay the primary signals, in return. As a result of such grouping, the grand coalition is formed. Then, the stability of the proposed coalitional game based approach is analyzed using the core concept. Afterwards, using the achievements of the first part, we investigate a practical scenario where the data rate of the primary system, falls below a minimum data rate requirement, and alternatively, the PU resorts to the leasing of its unused spectrum to SUs. However, it is guaranteed that the SUs perform as cooperatively as to meet the rate requirement of the primary system. Moreover, the benefits of cooperating with the primary system may not be exploited by the secondary system efficiently, if the SUs compete over the resources. We propose a two-level game based on the bargaining games to tackle the aforementioned problems. More specifically, we model the interactions between the primary and the secondary systems as the first level cooperative game and we utilize the bargaining games as the second level of the proposed framework to share the benefits of spectrum leasing among the cooperating SUs in a fair and efficient manner. The simulation results confirm the theoretical achievements.

**Keywords** Cognitive radio networks · MIMO systems · Cooperative communications · Spectrum leasing · Coalitional games · Bargaining games

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## 1 Introduction

Cognitive radio (CR) and multi-input multi-output (MIMO) communications have been proposed as promising solutions to improve spectrum utilization and efficiency in wireless networks. Dynamic and opportunistic spectrum access allows secondary users (SUs) to communicate on temporarily idle or underutilized frequencies. Moreover, MIMO systems enhance spectral efficiency by having a multi-antenna node simultaneously transmit multiple data streams. In newly emerging systems and standards (e.g., WiMAX, 4G Advanced-LTE, IEEE 802.16e) MIMO communications has been as a core feature. TV white bands have also been approved by the FCC for opportunistic, secondary use [1]. As a result, the current trend followed by the research society incorporates recent innovations of the two technologies into a single system.

The non-participation of primary users (PUs), as assumed in the traditional dynamic spectrum access schemes, is inefficient in terms of utilizing the spectrum. As a remedy, we can allow PUs to proactively manage the amount of secondary activity in their licensed band [2]. In [2, 3], a framework for dynamic spectrum leasing was proposed, where the PUs are rewarded for allowing SUs to operate in their licensed spectrum. Thus, the PUs have an incentive to allow SUs to access the spectrum whenever possible to the maximum extent. The spectrum leasing problem in a decentralized cognitive radio model was studied in [4], whereby a PU leases its bandwidth for a fraction of time to a network of independent SUs in exchange for cooperation.

The issue of resource allocation in MIMO CR networks was explored in [5–8]. The authors in [5] presented a low complexity semi-distributed algorithm for resource allocation in MIMO-OFDM based CR networks, using game theory approach, the strong duality in convex optimization and the primal decomposition method. In [6], the authors extended the pricing concept to MIMO-OFDM based CR networks and presented two iterative algorithms for resource allocation in such systems. In order to obtain an optimal subcarrier pairing, relay assignment and power allocation in MIMO-OFDM based CCRNs, the dual decomposition technique was recruited in [7] to maximize the sum-rate subject to the interference temperature limit of the PUs. Moreover, because of high computational complexity of the optimal approach, a suboptimal algorithm was further proposed in [7] and [8].

The optimal resource allocation in MIMO cognitive radio networks with heterogeneous secondary users, centralized and distributed users, was investigated in [9]. The authors in [10] modeled the problem of joint relay selection and power allocation in MIMO-OFDM based CCRNs as a two-level cooperative game problem with two objectives. The first objective is to assign each weak SU to one of the relays (rich SUs) through solving a problem achieved by a non-transferable utility coalition graph game, and the second one is to jointly allocating available channels to the SUs such that no subchannel is allocated to more than one SU and simultaneously optimize the transmit covariance matrices of nodes based on the Nash bargaining solution, which is the second level of the game.

The resource allocation problem in a spectrum leasing scenario in cooperative cognitive radio networks (CCRN) was addressed from various aspects, such as channel allocation for secondary system, relay selection, and power allocation [11, 14]. In [11], the joint problems of subchannel allocation, relay selection, and relay strategy in a multi-channel CCRN were considered and a unified framework based on Nash Bargaining Solutions was further proposed. When there exist one PU and multiple SUs in a single-channel system, the resource allocation problem for spectrum leasing was formulated as a Stackelberg game in

[12, 13]. One of contributions of this work is to consider the existence of multiple channel and multiple PUs in the system which complicates the analysis and we further utilize the coalitional game theory to study the behavior of the various users in the system. The authors solved the problem of resource allocation in a spectrum leasing scenario in CCRN in [14], where the CR users allocate the whole their transmission power in a portion of transmission frame to relay the primary signals. In return, the PU pairs lease their unused portion of transmission frame to the SUs.

In this work, we firstly study a coalitional game-based approach for resource allocation in a multi-channel cooperative cognitive radio network with multiple PUs and SUs. We form the grand coalition through grouping all PUs and SUs in a set, where each PU can lease its spectrum to all SUs in a time-division manner. Meanwhile, the SUs help the data transmission of PUs by relaying, in return. Through validating that the solution concept of the coalitional game (the *core*) is nonempty, we prove that the grand coalition is stable in the proposed scenario. Therefore, we conclude that from the coalitional game-based point of view, the optimal case for the spectrum leasing scenario in MIMO cooperative cognitive radio networks (MIMO-CCRN) is to put all the PUs and the SUs in a coalition.

Afterwards, we study a MIMO-CCRN system with practical assumptions like the minimum rate demand of the PUs. We aim at maximizing simultaneously the data rate of the PUs and the SUs. Meanwhile, the interactions among the SUs to access the leased channels is modeled using the bargaining games to allocate the spectrum in a fair and optimal manner. In a brief, we propose a two-level cooperative game based approach to tackle the aforementioned problems. To provide more clarification, in the proposed framework, the interactions between the PU and the SUs are modeled as the first level bargain game while the second level bargain game is used to formulate the SUs decision making process on spectrum sharing. We analyze the optimal actions of the PU and the SU and derive the theoretic results for the one-PU one-SU scenario. We further extend the achievements for the one-PU multi-SU scenario using a revised numerical searching algorithm and then, we prove its convergence.

## 2 System Model and Basic Assumptions

### 2.1 System Setup

We consider a single-cell network where the  $N_{PU}$  primary users (PUs) communicate with the primary base station (PBS), while  $N_{SU}$  pairs of SUs are seeking vacant channels for transmitting data to their respective secondary receivers (SRs). It is assumed that the PUs transmit to the PBS taking advantage of the SUs cooperation and there is no direct link between the PUs and the PBS. All users are assumed to be equipped with multiple antennas and without losing the generality, it is further assumed that all the users are equipped with  $N$  antennas. We denote the set of PUs and SUs by  $\mathcal{N}_{PU} = \{1, \dots, N_{PU}\}$  and  $\mathcal{N}_{SU} = \{1, \dots, N_{SU}\}$ , respectively. Without losing the generality and for ease of exposition, we assume that the  $i$ -th PU communicates with PBS using the  $i$ -th subchannel only, and the set of all PUs' subchannels in the primary network is denoted by  $\mathcal{N}_F = \{1, \dots, N_{PU}\}$ . The secondary transmitters can only communicate their respective receivers using the subchannels leased from a group of PUs, and as a result, a coalition is formed between the PUs and the SUs.

As described earlier, we firstly aim at studying a coalitional game based method to resource allocation in a multi-channel cooperative cognitive radio network with multiple PUs and SUs. A coalition  $\mathcal{S} = \mathcal{S}_{PU} \cup \mathcal{S}_{SU}$  is a subset of  $\mathcal{N}_{PU} \cup \mathcal{N}_{SU}$  in which multiple PUs and SUs cooperate, where  $\mathcal{S}_{PU} \subseteq \mathcal{N}_{PU}$  and  $\mathcal{S}_{SU} \subseteq \mathcal{N}_{SU}$ . The coalition  $\mathcal{N}_{PU} \cup \mathcal{N}_{SU}$  is called the grand coalition. A simple example of the system model with two PUs and two SUs is shown in Fig. 1. According to Fig. 1, PU 1 and SU 1 may form a coalition  $\mathcal{S}_1$  and PU 2 and SU 2 form another coalition  $\mathcal{S}_2$ , or all of them form a grand coalition. The benefits of the grand coalition for PUs and SUs are summarized in the following:

- It will be possible for the PUs to select different relay nodes and thereby, PUs gain more revenues.
- Forming the grand coalition enables the SUs to obtain more opportunities to access subchannels leased by different PUs.

Our objective at the first part of this work is to validate the above two statements using the the coalitional game theory [15].

subsectionCoalition-based Transmission Conventions

Before proceeding to the game theoretic formulation and analysis, we describe the cooperation mechanism and assumptions in the following:

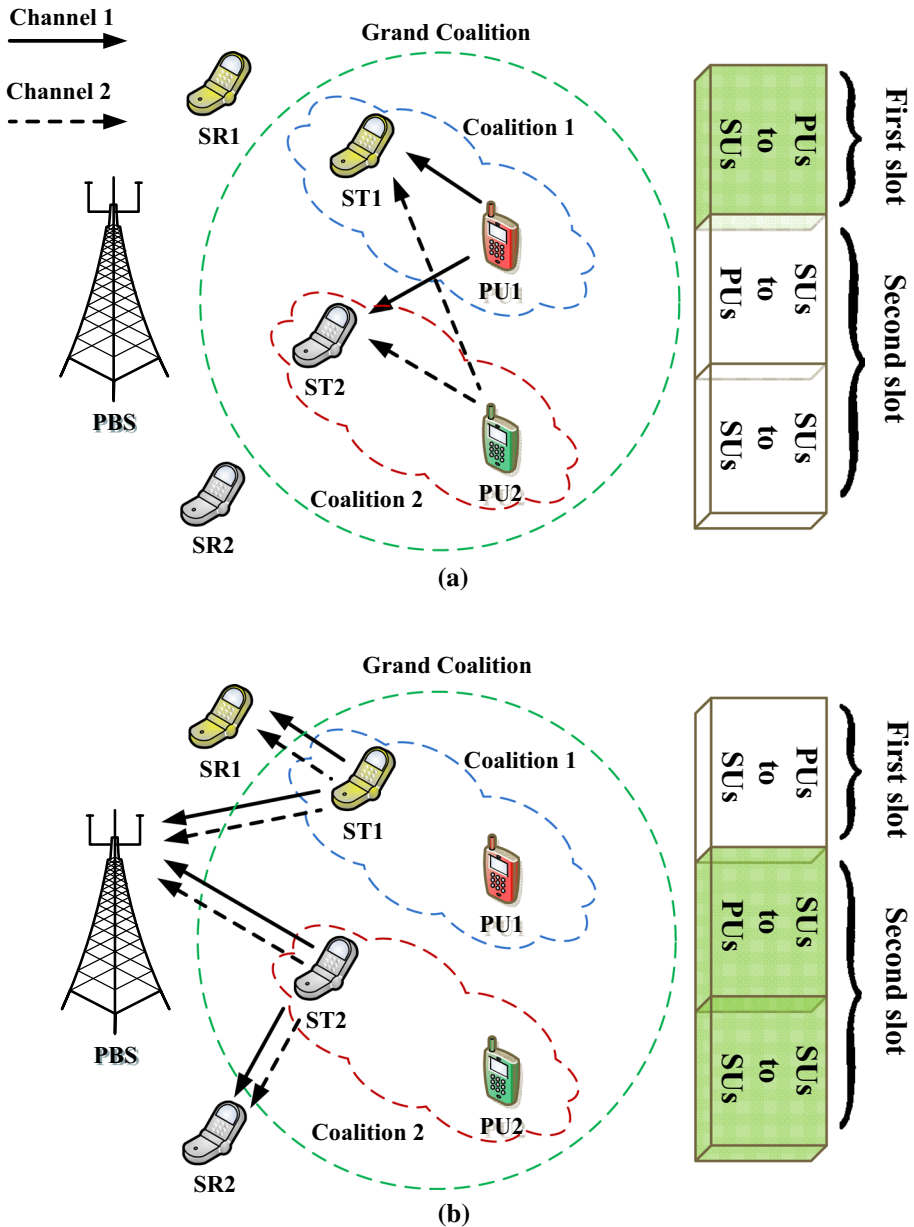
1. It is possible for the SUs to use different subchannels at any given time.
2. In the formed coalition, each PU has only one available subchannel to lease to the SUs. When an SU relays the PU *i*s traffic, it uses the subchannel *i* leased by PU *i*.
3. Cooperative transmissions for PUs and SUs are realized in a time-division fashion. As shown in Fig. 1, the first time slot is for PUs transmission and the second time slot for SUs transmission. The second time slot is further divided into two parts, i.e., SUs relaying PUs' data and transmitting their own data. In the second time slot, the fraction of time that SU *j* relays PU *i*'s traffic (using subchannel *i*) and transmits its own traffic using subchannel *l* are denoted by  $\beta_{j,i}$  and  $\alpha_{j,l}$ , respectively, where  $0 \leq \beta_{j,i} \leq 1$  and  $0 \leq \alpha_{j,l} \leq 1$  for all  $i \in \mathcal{S}_{PU}, j \in \mathcal{S}_{SU}$ , and  $l \in \mathcal{S}_F$ .
4. The cooperation strategy in the proposed scenario is Amplify-and-forward (AF). Meanwhile, we assume flat Rayleigh fading for all channels which is invariant within each time-slot. The PBS uses maximal ratio combining to combine signals from the direct link and cooperative links. The PBS is also aware of the channel state information (CSI).

## 2.2 Utility Function

When PU *m* transmits to the PBS taking advantage of the cooperation of SU *k*, the received signal at the PBS can be written as

$$\mathbf{y}_m = \mathbf{G}_k \mathbf{A}_{m,k} \mathbf{H}_{m,k} \mathbf{x}_m + \mathbf{G}_k \mathbf{A}_{m,k} \mathbf{z}_k + \mathbf{w}_m \tag{1}$$

where  $\mathbf{x}_m \in \mathbb{C}^{N \times 1}$  denotes the transmit signal of PU *m*;  $\mathbf{H}_{m,k}$  and  $\mathbf{G}_k$  are channel matrices from PU *m* to SU *k* and from SU *k* to PBS, respectively;  $\mathbf{A}_{m,k}$  denotes the amplification matrix at SU *k* for the signals of PU *m*; The noise contribution at SU *k* and the PBS are denoted by  $\mathbf{z}_k \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_N)$  and  $\mathbf{w}_m \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_N)$ , respectively, where we have assumed the same noise power at all SUs and PBS. Due to the availability of the full CSI at all nodes, we can use the singular value decomposition of the channel matrices to determine transmit and receive beamforming matrices at all nodes. Since these matrices are



**Fig. 1** The system model for the spectrum leasing in MIMO-CCRN from a coalitional game based point of view and the transmission slots: **a** secondary transmitters (STs) receive primary signals In the first time slot; **b** In the second time slot, STs cooperate with PUs or transmit signals to their respective secondary receivers (SRs)

unitary they do not change the statistics of the channel and therefore preserve the mutual information and error performance of this link. The singular value decomposition of the channel matrices is given by

$$\begin{aligned} \mathbf{H}_{m,k} &= \mathbf{U}_{m,k} \mathbf{A}_{m,k} \mathbf{V}_{m,k}^H \\ \mathbf{G}_k &= \mathbf{S}_k \Gamma_k \mathbf{D}_k^H \end{aligned} \tag{2}$$

Using the appropriate beamforming, the resulting transmit signal at PU  $m$  is thus given by  $\mathbf{x}_m = \mathbf{V}_{m,k} \tilde{\mathbf{x}}_m$  and the amplification matrix has the structure  $\mathbf{A}_{m,k} = \mathbf{D}_k \Pi_{m,k} \mathbf{U}_{m,k}^H$ . The PBS multiplies the received signal in (1) by  $\mathbf{S}_k^H$  such that it can be expressed as

$$\tilde{\mathbf{y}}_m = \mathbf{S}_k^H \mathbf{y}_m = \Gamma_k \Pi_{m,k} \mathbf{A}_{m,k} \tilde{\mathbf{x}}_m + \Gamma_k \Pi_{m,k} \tilde{\mathbf{z}}_k + \tilde{\mathbf{w}}_m \tag{3}$$

where  $\tilde{\mathbf{z}}_k = \mathbf{U}_{m,k}^H \mathbf{z}_k$  and  $\tilde{\mathbf{w}}_m = \mathbf{S}_k^H \mathbf{w}_m$  denote the equivalent noise contributions.

Up till now, we have no restriction on the structure of the gain matrix  $\Pi_{m,k}$ . The optimal structure of  $\Pi_{m,k}$  is found through the maximization of the achievable data rate of PU  $m$  which is given by

$$R_{m,k}^P = \beta_{k,m} \log \det \left( \mathbf{I}_N + \frac{1}{\sigma^2} \left( \mathbf{I}_M + \Gamma_k \Pi_{m,k} \Pi_{m,k}^H \Gamma_k^H \right)^{-1} \Gamma_k \Pi_{m,k} \mathbf{A}_{m,k} \Phi_m \mathbf{A}_{m,k}^H \Pi_{m,k}^H \Gamma_k^H \right) \tag{4}$$

where we have assumed that transmit signals Gaussian with  $\Phi_m = E\{\tilde{\mathbf{x}}_m \tilde{\mathbf{x}}_m^H\}$ . Owing to Hadamard inequality, a requirement for maximizing the determinant in (4) is that the rows (or columns) of the matrix within the determinant have to be orthogonal [16]. Using this condition and the observation that all matrices in (4) are diagonal, we pick the gain matrix  $\Pi_{m,k}$  also as a diagonal matrix. This leads to a diagonal overall matrix in the determinant which maximizes (4). Therefore, we define  $\Pi_{m,k}$  as

$$\Pi_{m,k} = \text{diag}\{\pi_{m,k,1}, \dots, \pi_{m,k,N}\} \tag{5}$$

with

$$\pi_{m,k,n} = \sqrt{\frac{P_{k,n}^s}{P_{m,n}^p \lambda_{m,k,n} + \sigma^2}} \tag{6}$$

where  $P_{m,n}^p$  and  $P_{k,n}^s$  denote the transmit power of PU  $m$  and cooperating SU  $k$  on the  $n$ -th spatial subchannel, respectively.  $\lambda_{m,k,n}$  represents the  $n$ -th diagonal element of  $\mathbf{A}_{m,k}$ . Therefore, the achievable data rate of PU  $m$ , taking advantage of the cooperation of SU  $k$ , can be written as

$$R_{m,k}^P = \sum_{n=1}^N \beta_{k,m} \log \left( 1 + \frac{P_{m,n}^p a_{m,k,n} P_{k,n}^s b_{k,n}}{1 + P_{m,n}^p a_{m,k,n} + P_{k,n}^s b_{k,n}} \right) \tag{7}$$

where  $a_{m,k,n} = \frac{\lambda_{m,k,n}}{\sigma^2}$  and  $b_{k,n} = \frac{\gamma_{k,n}}{\sigma^2}$ .  $\gamma_{k,n}$  denotes the  $n$ -th diagonal element of  $\Gamma_k$ . Hence, the achievable data rate of PU  $m$  can be expressed as

$$R_m^P = \sum_{k \in \mathcal{S}_{SU}} R_{m,k}^P \tag{8}$$

In case of leasing the subchannel of PU  $m$  to SUs, the cost of PU  $m$ 's spectrum leasing can be modeled as the total fraction of time that its subchannel is used by SUs, i.e.,

$$\mu_m = \sum_{k \in \mathcal{S}_{SU}} \alpha_{k,m} \tag{9}$$

Therefore, the utility of PU  $m$  can be written as

$$U_m^P = F(R_m^P) + c_m^P - G(\mu_m) \tag{10}$$

where  $F(\cdot)$  is a concave function which maps the achievable data rate of the PU to a utility gain,  $G(\cdot)$  is a convex function which maps the cost of each PU to a utility loss, and  $c_m^P = \eta \sum_{k \in S_{SU}} \alpha_{k,m}$  denotes the amount of payment of SUs to PU  $m$  with  $\eta$  representing the price of frequency use per unit time. The achievable data rate of SU  $k$  can be expressed as

$$R_k^S = \sum_{j \in S_f} R_{k,j}^S \tag{11}$$

where  $R_{k,j}^S = \alpha_{k,j} R_{k,j}$  represents the maximum data rate of the  $k$ -th SU link using the  $j$ -th channel and using the appropriate transmit and receive beamforming in each SU link, we have,

$$R_{k,j} = \sum_{n=1}^N \log \left( 1 + \frac{P_{k,n}^s \theta_{k,j,n}}{\sigma^2} \right) \tag{12}$$

where  $\theta_{k,j,n}$  denotes the  $n$ -th eigen-value of the channel matrix in the  $k$ -th SU link over the  $j$ -th channel. Then, the utility of the  $k$ -th SU link can be expressed as

$$U_k^S = L(R_k^S) - c_k^S \tag{13}$$

where  $L(\cdot)$  is a concave function to project the rate to the revenues and  $c_k^S = \eta \sum_{k \in S_{SU}} \alpha_{k,j}$  denotes the payment of the  $k$ -th SU to the PUs.

In the next section, we recruit the coalitional cooperative games to formulate and analyze the performance of the proposed system model.

### 3 Coalitional Cooperative Game Based Formulation

#### 3.1 Concepts and Game-Theoretic Problem Formulation

A classification of the coalitional games can be based on the transferability of the utility of the game [15]. Therefore, a coalitional game can be either with transferable utility (TU) or with non-transferable utility (NTU). The coalitional game in which the coalition value can be apportioned arbitrarily between the coalition members is called a TU game. Similarly, when the apportioning of the coalition value depends on the joint actions of the members in the coalition, the resulting game is referred to as an NTU game. As will be proved, since we define the coalition value as the sum of utilities generated by PUs and SUs in the coalition, a TU game applies to our considered network. Moreover, as will be shown, the considered network in this paper fulfills the properties of a canonical coalitional game, e.g., grand coalition. Thus, we formulate the cognitive radio network as a canonical coalitional game with TU. In what follows, we present some coalitional game-related terminologies firstly, and then we prove that this coalitional game satisfies the desired properties that ensure a stable grand coalition and a fair payoff allocation.

A coalitional game with TU comprises of a set of players who form cooperative groups and the coalition value which correlates each nonempty subset of players with a number [15]. First of all, we provide the definition of the coalition value for the considered game.

**Definition 1** For the proposed canonical coalitional game with TU, the coalition value  $v(\mathcal{S})$  of a coalition  $\mathcal{S}$  is the maximum sum utility of PUs and SUs in  $\mathcal{S}$  and depends only on the actions of the PUs and SUs in  $\mathcal{S}$  (not those in  $\mathcal{N}_{PU} \cup \mathcal{N}_{SU} \setminus \mathcal{S}$ ).

Recruiting Definition 1 and using the utility definitions in (10) and (13), the coalition value  $v(\mathcal{S})$  is the solution to the following convex optimization problem:

$$\begin{aligned}
 v(\mathcal{S}) \triangleq & \max \sum_{m \in \mathcal{S}_{PU}} U_m^P + \sum_{k \in \mathcal{S}_{SU}} U_k^S \\
 \text{s.t. (C1):} & \sum_{j \in \mathcal{S}_F} \alpha_{k,j} + \sum_{m \in \mathcal{S}_{PU}} \alpha_{k,m} = 1 \\
 \text{(C2):} & \sum_{k \in \mathcal{S}_{SU}} (\alpha_{k,j} + \beta_{k,m}) = 1 \\
 \text{(C3):} & 0 \leq \alpha_{k,j} \leq 1, 0 \leq \beta_{k,m} \leq 1
 \end{aligned} \tag{14}$$

where  $U_m^P$  and  $U_k^S$  the utility functions for PU  $m$  and SU  $k$  are given in (10) and (13), respectively. We require the set of time fractions  $\alpha = \{\alpha_{k,j} | k \in \mathcal{S}_{SU}, j \in \mathcal{S}_F\}$  and  $\beta = \{\beta_{k,m} | k \in \mathcal{S}_{SU}, m \in \mathcal{S}_{PU}\}$  to solve the problem in (14). Note that  $\alpha$  is in  $U_m^P$  and  $U_k^S$ ; while  $\beta$  exists in  $U_m^P$ . The first constraint in (14), (C1), makes sure that each SU works in either relay or access mode but not simultaneously. The second constraint in (14), (C2), guarantees that the total fraction of time that an SU in coalition  $\mathcal{S}$  accesses the spectrum will not exceed one. Moreover, (C3) is to guarantee a feasible joint action of coalition  $\mathcal{S}$  [17]. If the solution to problem in (14) is not feasible, then  $v(\mathcal{S}) \triangleq -\infty$ .

### 3.2 The Core of the Game

As stated earlier, for a coalitional game with TU, the coalition value can be divided arbitrarily among the participants in the coalition. A feasible payoff allocation policy for a canonical coalitional game guarantees that if a subset of players separates from the grand coalition, at least one PU or SU in the separated subset has a utility worse than that in the grand coalition. To obtain a feasible payoff allocation policy, we recruit the core solution concept [15] as defined in the following:

**Definition 2** The core of the proposed coalitional game is the set of feasible payoff allocation vectors  $\mathbf{y} = [y_1^P, \dots, y_{N_{PU}}^P, y_1^S, \dots, y_{N_{SU}}^S]$  (where  $y_m^P$  and  $y_k^S$  represent the payoff value of PU  $m$  and SU  $k$ , respectively) for the grand coalition and is called an imputation if  $\sum_{m \in \mathcal{S}_{PU}} y_m^P + \sum_{k \in \mathcal{S}_{SU}} y_k^S = v(\mathcal{N}_{SU} \cup \mathcal{N}_{PU})$  and  $y_i \geq v(\{i\}), \forall i \in (\mathcal{N}_{SU} \cup \mathcal{N}_{PU})$ . The core  $\mathcal{C}$  is the set of imputations for which  $\sum_{m \in \mathcal{S}_{PU}} y_m^P + \sum_{k \in \mathcal{S}_{SU}} y_k^S \geq v(\mathcal{S})$  for all  $\mathcal{S} \subseteq \mathcal{N}_{PU} \cup \mathcal{N}_{SU}$ , i.e.,

$$\begin{aligned}
 \mathcal{C} = & \left\{ \mathbf{y} : \sum_{m \in \mathcal{S}_{PU}} y_m^P + \sum_{k \in \mathcal{S}_{SU}} y_k^S = v(\mathcal{S}_{SU} \cup \mathcal{S}_{PU}) \text{ and} \right. \\
 & \left. \sum_{m \in \mathcal{S}_{PU}} y_m^P + \sum_{k \in \mathcal{S}_{SU}} y_k^S \geq v(\mathcal{S}), \forall \mathcal{S} \subseteq \mathcal{N}_{PU} \cup \mathcal{N}_{SU} \right\}
 \end{aligned} \tag{15}$$



According to Definition 2, if the core exists, it offers a set of stable payoff allocations because no coalition  $S \subseteq \mathcal{N}_{PU} \cup \mathcal{N}_{SU}$  has the incentives to refuse the proposed payoff allocation and leave the grand coalition to form a separate coalition instead.

In the following, we prove that the proposed canonical coalitional game with TU has a nonempty core. At first, we find the dual problem of (14) using the duality theorem [18]. Using the transformation of the primal problem in (14) to the dual problem, the relation of the solution set between the dual problem and the primal problem can be obtained. In other words, the optimal solution of the dual problem is the upper bound of the optimal solution of the primal problem [19]. Meanwhile, the solution set of the dual problem can be shown to be a subset of the core, as will be proved in Theorem 1. The Lagrangian functions of PUs and SUs can be expressed as

$$f_{m,j}(\delta, \rho, \tau) = \max_{R_m^p \geq 0, \mu_m \geq 0, c_m^p \geq 0, \forall m \in \mathcal{S}_{PU}} (F(R_m^p) + c_m^p - G(\mu_m) + \delta_m R_m^p + \rho_m c_m^p + \tau_m \mu_m) \tag{16}$$

$$g(\psi, \varphi) = \max_{c_k^s \geq 0, R_k^s \geq 0, k \in \mathcal{S}_{SU}} (L(R_k^s) - c_k^s + \psi_k R_k^s + \varphi_k c_k^s) \tag{17}$$

where  $\delta_m, \rho_m, \tau_m, \psi_k$  and  $\varphi_k$  are the Lagrange multipliers. Therefore, the dual problem of the problem in (14) for  $\mathcal{S} = \mathcal{S}_{PU} \cup \mathcal{S}_{SU}$  is formulated as

$$\begin{aligned} \mathbf{D}(\mathcal{S}) : \min_{\alpha, \beta} & \sum_{m \in \mathcal{S}_{PU}} \sum_{k \in \mathcal{S}_F} (f_{m,j}(\delta, \rho, \tau) + \omega_{m,j}) + \sum_{k \in \mathcal{S}_{SU}} (g(\psi, \varphi) + \lambda_k) \\ \text{s.t.} & \rho_j + \psi_k R_{k,j}^S + \eta \tau_j + \eta \varphi_k + \sum_{m \in \mathcal{S}_{PU}} \omega_{m,j} \geq 0, \forall k \in \mathcal{S}_{SU}, j \in \mathcal{S}_F \\ & \delta_m R_{k,m}^P + \lambda_k + \sum_{j \in \mathcal{S}_F} \omega_{m,j} \geq 0, \forall k \in \mathcal{S}_{SU}, m \in \mathcal{S}_{PU} \\ & \varphi_k, \lambda_k, \psi_k \geq 0, \forall k \in \mathcal{S}_{SU} \\ & \rho_j, \tau_j \geq 0, \forall j \in \mathcal{S}_F \\ & \omega_{m,j} \geq 0, \forall m \in \mathcal{S}_{PU}, j \in \mathcal{S}_F \\ & \delta_m \geq 0, \forall m \in \mathcal{S}_{PU} \end{aligned} \tag{18}$$

where  $\omega_{m,j}$  and  $\lambda_k$  are also the Lagrange multipliers, derived from the transformation of the problem in (14) into its dual problem in (18). Let  $\mathcal{D}$  denote the set of optimal solutions of the dual problem in (18) for  $\mathcal{S}_{PU} = \mathcal{N}_{PU}$  and  $\mathcal{S}_{SU} = \mathcal{N}_{SU}$ , and let

$$\begin{aligned} \mathcal{O} = \{ & \mathbf{y}^* : y_m^{P^*} = f_{m,j}(\delta^*, \rho^*, \tau^*) + \omega_{m,j}^*, m \in \mathcal{N}_{PU}, j \in \mathcal{S}_F \\ & y_k^{S^*} = g(\psi^*, \varphi^*) + \lambda_k^*, k \in \mathcal{N}_{SU} \\ & \text{for } (\delta^*, \rho^*, \tau^*, \omega^*, \psi^*, \varphi^*, \lambda^*) \in \mathcal{D} \} \end{aligned} \tag{19}$$

correspond to the dual payoff generated by the solution of the dual problem. The dual payoff is the set of the optimal payoff allocation of PUs and SUs. In the following theorem, we prove that the core of the proposed coalitional game with TU is nonempty.

**Theorem 1** *The core of the proposed canonical coalitional game with TU is nonempty and  $\mathcal{O} \subseteq \mathcal{C}$ .*

**Proof** The set  $\mathcal{O}$  is nonempty, because the set  $\mathcal{D}$  is nonempty. We consider an arbitrary  $\mathbf{y}^* \in \mathcal{O}$  which corresponds to some  $(\delta^*, \rho^*, \tau^*, \omega^*, \psi^*, \varphi^*, \chi^*) \in \mathcal{D}$ . Therefore,  $\sum_{m \in \mathcal{S}_{PU}} y_m^{P^*} + \sum_{k \in \mathcal{S}_{SU}} y_k^{S^*}$  is the optimal value of the objective function of  $\mathbf{D}(\mathcal{S})$ . The objective function of (14) is concave, because  $F(\cdot)$  and  $L(\cdot)$  are concave, and  $G(\cdot)$  is convex. Moreover, the constraints (C1)–(C3) are linear.  $\mathbf{D}(\mathcal{S})$  is the dual of the problem in (14) for each  $\mathcal{S} \subseteq \mathcal{N}_{PU} \cup \mathcal{N}_{SU}$ . Therefore, based on the strong duality [18], the duality gap is zero. Hence,  $\sum_{m \in \mathcal{S}_{PU}} y_m^{P^*} + \sum_{k \in \mathcal{S}_{SU}} y_k^{S^*} = v(\mathcal{N}_{PU} \cup \mathcal{N}_{SU})$ . We only need to show that  $\sum_{m \in \mathcal{S}_{PU}} y_m^{P^*} + \sum_{k \in \mathcal{S}_{SU}} y_k^{S^*} \geq v(\mathcal{S})$  for all  $\mathcal{S} \subseteq \mathcal{N}_{PU} \cup \mathcal{N}_{SU}$ . Assume that (14) is feasible. Then, strong duality states that  $v(\mathcal{S})$  equals the optimal value of the objective function of  $\mathbf{D}(\mathcal{S})$ . The sub-vectors  $(\delta, \rho, \tau, \omega, \psi, \varphi, \chi)$  consisting of the components of  $(\delta^*, \rho^*, \tau^*, \omega^*, \psi^*, \varphi^*, \chi^*)$  in  $\mathcal{S}$  satisfy the constraints of the dual problem in (18). Thus,  $\sum_{m \in \mathcal{S}_{PU}} y_m^{P^*} + \sum_{k \in \mathcal{S}_{SU}} y_k^{S^*} = v(\mathcal{N}_{PU} \cup \mathcal{N}_{SU})$  is the value of the objective function of the problem in (18) for the above feasible solution. It follows that the optimal value of the objective function of the problem in (18) is a lower bound for  $\sum_{m \in \mathcal{S}_{PU}} y_m^{P^*} + \sum_{k \in \mathcal{S}_{SU}} y_k^{S^*}$ . Therefore,  $\mathbf{y}^* \in \mathcal{C}$ .  $\square$

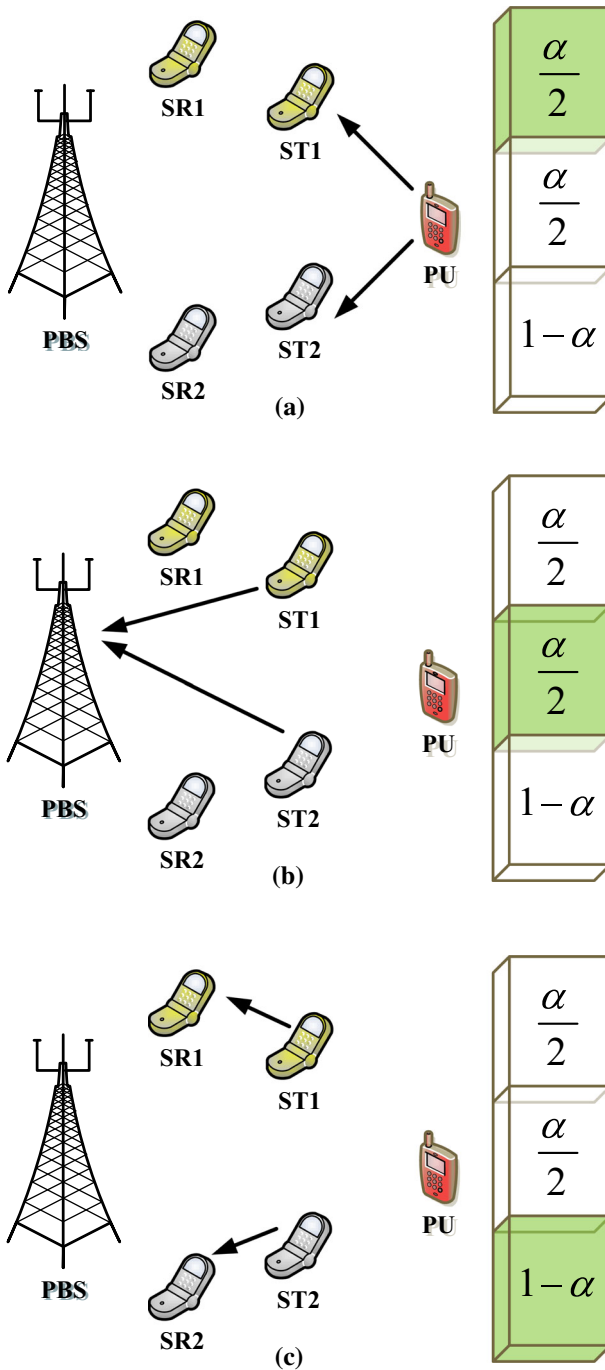
Based on Theorem 1, the optimal payoff vector  $\mathbf{y}^*$  lies in the core and can be achieved by solving the dual problem  $\mathbf{D}(\mathcal{N}_{PU} \cup \mathcal{N}_{SU})$ . In other words, the solution of the dual problem  $\mathbf{D}(\mathcal{N}_{PU} \cup \mathcal{N}_{SU})$  provides the optimal payoff allocation of the PUs,  $y_m^{P^*}$ , and the optimal payoff allocation of the SUs,  $y_k^{S^*}$ , in the core.

#### 4 A Two-Level Cooperative Game-Based Approach for Spectrum Leasing in MIMO-CCRN

In this section, we study the interactions among the SUs and PUs from a bargaining game-based point of view. We consider a single-PU multi-SU scenario and formulate the cooperative interactions between the PU and SUs as the first-level bargain game, while the interactions among the SUs is modeled as the second-level bargain game. In what follows, we derive the optimal actions of the PU and SU in a single-PU single-SU scenario, firstly. Then, a revised numerical searching algorithm is proposed for the single-PU multi-SU case and the convergence of the proposed scheme is proved.

As depicted in Fig. 2, the PU communicates with the PBS using the licensed band, while  $N_{SU}$  SUs are hunting for vacant channels to transmit data to their respective secondary receivers (SRs). We assume that the PU has a minimum rate demand denoted by  $R_{min}^P$ . Moreover, it is further assumed that the communications between the PU and the PBS does not meet the minimum rate requirement of the PU and thereby, the PU will recruit the existing SUs, equipped with cognitive radio capabilities, for cooperative transmission in order to meet the minimum rate demand. The SUs are also interested in cooperating with the PU through relaying the signals transmitted from the PU to the PBS. The SUs are granted a portion of the transmission frame to transmit their signals from secondary transmitters (STs) to their intended receivers (SRs), in return, as shown in Fig. 2. After the SUs assist the PU, they bargain to share the granted bandwidth using the Time Division Multiple Access (TDMA) strategy. Similar to the previous section, we focus our discussions on Amplify-and-Forward (AF) strategy.

According to Fig. 2, the PU allocates  $(1 - \alpha)$  fraction of its transmission frame to SUs. It is presumed that the PU is able to achieve its required QoS by using only  $\alpha$  fraction of its transmission frame (for  $\alpha \in [0, 1]$ ). This is the origin of the so-called spectrum holes that



**Fig. 2** The modified system model for the spectrum leasing scenario in MIMO-CCRN with a single PU: **a** The PU transmits to the secondary transmitters (STs); **b** STs relay the primary signals to the PBS; **c** STs transmit to their respective secondary receivers (STs)

leads to the spectrum under-utilization. Therefore, at the beginning of each transmission frame, the PU decides that it can free-up up to an  $(1 - \alpha)$  fraction of its transmission frame. As shown in Fig. 2, the cooperative transmission for the PU consists of two phases. In the first phase, the PU transmits its signals to the SU. The bandwidth that the PU retains for its own use is divided into two sub-slots with equal length. In the first sub-slot, the PU transmits to the SUs, and then the SUs relay the primary data to the PBS in the second sub-slot. In the following, we study the proposed spectrum leasing scenario.

### 4.1 Two-level cooperative game-based Problem Formulation

Recall from (8) that the utility function of the PU (the achievable data rate of PU), as a result of the cooperation of SU  $k$ , can be written as

$$U^P = \sum_{k \in \mathcal{N}_{SU}} \sum_{n=1}^N \frac{\alpha}{2} \log \left( 1 + \frac{P_n^p a_{k,n} d_{k,n} P_{k,n}^s b_{k,n}}{1 + P_n^p a_{k,n} + d_{k,n} P_{k,n}^s b_{k,n}} \right) \tag{20}$$

where  $a_{k,n} = \frac{\lambda_{k,n}}{\sigma^2}$  and  $b_{k,n} = \frac{\gamma_{k,n}}{\sigma^2}$ ;  $\lambda_{k,n}$  and  $\gamma_{k,n}$  denotes the  $n$ -th eigen-value of the channel matrices from PU to SU  $k$  and from SU  $k$  to PBS, respectively;  $P_n^p$  and  $P_{k,n}^s$  denote the transmit power of the PU and cooperating SU  $k$  on the  $n$ -th spatial subchannel, respectively; We further assume that the SU  $k$  uses only part of its power in each spatial mode  $n$ , i.e.,  $d_{k,n} P_{k,n}^s$  ( $0 \leq d_k \leq 1$ ), to relay primary signal. Using (12), the utility function of the  $k$ -th SU can be expressed as

$$U_k^S = \sum_{n=1}^N \left( e_k \log \left( 1 + \frac{P_{k,n}^s \theta_{k,n}}{\sigma^2} \right) - \frac{\alpha}{2} c_s d_{k,n} P_{k,n}^s \right) \tag{21}$$

where  $e_k$  is the fraction of time in the third sub-slot which is allocated to the  $k$ -th SU to transmit to its intended receiver;  $c_s$  is the price of per unit power for the SUs.

Therefore, the PU should determine the fraction of time which is supposed to be granted to the SUs,  $\alpha$ , and each SU  $k$  in the secondary system should find the fraction of its cooperative transmission power on the  $n$ -th spatial subchannel to relay primary signal,  $d_{k,n}$ . Moreover, each SU  $k$ , is supposed to determine the fraction of the granted spectrum it can use for its own transmission,  $e_k$ . To stimulate the cooperation between the PU and the SUs, as well as the cooperation among the SUs, we propose to cast such a decision making process into a two-level bargain framework.

Before formulating the problem as a cooperative game, we introduce the Nash Bargaining Solution (NBS) briefly. A special type of cooperative games are bargaining games, where players negotiate/bargain their actions/strategies to reach an agreement with guaranteed minimum payoffs. The agreement is associated with a utility vector  $\mathbf{u} = [u_1, \dots, u_N]$ , where  $u_i$  is the utility of player  $i$  and there are  $N$  players in the game. Let  $d_i$  and  $D_i$  denote the action and action space for player  $i$ , respectively ( $d_i \in D_i$ ). The utility  $u_i$  is a function of the action vector  $\mathbf{d} = [d_1, \dots, d_N]$ . The utility space  $U$  is the set of all possible payoff allocations  $\mathbf{u}$  that result from all possible action vectors  $\mathbf{d}$ . It is also possible that no agreement is reached after bargaining, a situation referred to as a *disagreement point*. A disagreement point is associated with a utility vector  $\mathbf{u}_0$ , which consists of minimum payoffs that players insist on having.

Given the variety of outcomes, Nash [20] suggested to, instead of study all possible outcomes, specify characteristics or axioms of one or several outcomes that we expect and

find how to drive the bargaining process to that agreement point. Nash proposed following axioms that describe a Nash Bargaining Solution (NBS) [20], denoted by  $S(\mathbf{u}, \mathbf{u}_0)$ :

- A NBS is Pareto optimal, i.e. there is no other solution that two or more players can simultaneously be better off.
- At the NBS, all players are guaranteed their minimum payoffs that they insist on at the beginning of the bargaining process.
- A NBS is symmetric, meaning that all players have the same priority. Specifically, if all users have identical action space  $D_i$ , then they have the same utility/payoff at the NBS.
- If the action spaces of players shrink, leading to a shrunken utility space  $\tilde{U}$ , the old NBS  $S(\mathbf{u}, \mathbf{u}_0)$  remains optimal provided that  $S(\mathbf{u}, \mathbf{u}_0)$  remains feasible in the new utility space  $\tilde{U}$ . This property ensures that eliminating solutions that would not have been selected does not affect the NBS.

Given the above properties, the problem is whether a unique NBS exists, and how to find such a unique NBS. Nash proved the following theorem that answers these three key questions [20]:

**Theorem 2** *If the utility space  $U$  is upper-bounded, closed and convex, then there exists a unique NBS which is the solution of the following problem:*

$$\mathbf{u} = \arg \max_{\{\mathbf{u} \in U\}} \prod_{i=1}^N (u_i - u_{0,i}) \tag{22}$$

In the first level of the resource allocation game and with the target of maximizing the social welfare of the secondary system, we model all SUs in the secondary system all together which bargain with the PU, i.e.,  $U^S = \sum_{k \in \mathcal{N}_{SU}} U_k^S$ . Therefore, the game between the PU and the SUs, also known as the first level bargain game, can be formulated as  $\mathcal{G}_1(U^P, U^S; u_p^0, u_s^0)$ , where  $u_p^0$  and  $u_s^0$  are the disagreement point of the PU and the SUs, respectively. The disagreement point of PU should be set as  $R_{\min}^P$ . Besides, the goal of the SUs is to get the opportunity to access the spectrum. Then, the SUs wish to cooperate if their gain from rate increasing by spectrum leasing is not less than the cost from power consumption. Therefore, the disagreement point of the secondary system has to be set to  $u_s^0 = 0$ . At last, the first level Nash bargaining based game can be formulated as

$$\begin{aligned} \mathcal{G}_1 : \max_{\alpha, d, k, n} & (U^P - u_p^0)(U^S - u_s^0) \\ \text{s.t.} & U^P \geq u_p^0, U^S \geq u_s^0 \end{aligned} \tag{23}$$

The benefits of the cooperation with the PU should be allocated among the SUs in a fair fashion. Thus, another Nash bargaining game, known as the second level game, should be played among the SUs, which is defined as  $\mathcal{G}_2(U_k^S; u_{s,k}^0 | k \in \mathcal{N}_{SU})$ , with  $u_{s,k}^0 = 0$  for all  $k \in \mathcal{N}_{SU}$ , being the disagreement point of the SUs. Similarly, the solution of  $\mathcal{G}_2$  can be achieved through solving the following problem

$$\begin{aligned} \mathcal{G}_2 : \max_{e_k} & \prod_{k \in \mathcal{N}_{SU}} (U_k^S - u_k^S) \\ \text{s.t.} & \sum_{k \in \mathcal{N}_{SU}} e_k = 1 - \alpha \end{aligned} \tag{24}$$

### 4.2 Solution of the Two-Level Game

In this section, we explore the interactions between the PU and the SUs and among the SUs. First of all, the case of single-PU single-SU is considered and the optimal actions of the PU and the SU are derived. Afterwards, the multi-user scenario is considered and a numerical searching algorithm is proposed and its convergence is proved.

#### 4.2.1 Single-PU Single-SU Case

In the following, we prove that the NBS for the single-PU single-SU bargaining game is unique.

**Theorem 3** *There exists a unique NBS for the single-PU single-SU bargaining game, and the solution is given by*

$$\begin{cases} \alpha^* = \frac{AC + B(C + D)}{2A(C + D)}, \mathbf{d}^* = \arg \min \sqrt{\frac{C + D}{A}} & \text{if } \alpha_L \leq \alpha_M \leq \alpha_H \text{ and } X(\mathbf{d}^*) < \sqrt{\frac{C}{B}} \\ \alpha^* = 1, \mathbf{d}^* = \mathbf{0} & \text{otherwise} \end{cases} \tag{25}$$

where:

$$A = \frac{1}{2} \sum_{n=1}^N \log \left( 1 + \frac{P_n^p a_n d_n P_n^s b_n}{1 + P_n^p a_n + d_n P_n^s b_n} \right) \tag{26}$$

$$B = u_0^p \tag{27}$$

$$C = \sum_{n=1}^N \log \left( 1 + \frac{P_n^s \theta_n}{\sigma^2} \right) \tag{28}$$

$$D = \frac{1}{2} \sum_{n=1}^N c_s d_n P_n^s \tag{29}$$

*Proof* For the single-PU single-SU scenario, the utility functions (20) and (21) can be written as

$$U^p = \sum_{n=1}^N \frac{\alpha}{2} \log \left( 1 + \frac{P_n^p a_n d_n P_n^s b_n}{1 + P_n^p a_n + d_n P_n^s b_n} \right) \tag{30}$$

$$U_k^s = \sum_{n=1}^N \left( (1 - \alpha) \log \left( 1 + \frac{P_n^s \theta_n}{\sigma^2} \right) - \frac{\alpha}{2} c_s d_n P_n^s \right) \tag{31}$$

Therefore, the following problem should be solved:

$$\begin{aligned}
 \mathcal{G}_1 : \max_{\alpha, d_{k,n}} & \left( \sum_{n=1}^N \frac{\alpha}{2} \log \left( 1 + \frac{P_n^p a_n d_n P_n^s b_n}{1 + P_n^p a_n + d_n P_n^s b_n} \right) - u_0^p \right) \\
 & \left( \sum_{n=1}^N \left( (1 - \alpha) \log \left( 1 + \frac{P_n^s \theta_n}{\sigma^2} \right) - \frac{\alpha}{2} c_s d_n P_n^s \right) \right) \\
 \text{s.t.} & \sum_{n=1}^N \frac{\alpha}{2} \log \left( 1 + \frac{P_n^p a_n d_n P_n^s b_n}{1 + P_n^p a_n + d_n P_n^s b_n} \right) \geq u_0^p \\
 & \sum_{n=1}^N \left( (1 - \alpha) \log \left( 1 + \frac{P_n^s \theta_n}{\sigma^2} \right) - \frac{\alpha}{2} c_s d_n P_n^s \right) \geq 0
 \end{aligned} \tag{32}$$

□

At first, we consider the fraction of SU power for cooperating with the PU in the  $n$ -th spatial mode,  $d_n$  for  $n = 1, \dots, N$ , as a constant and attempt to find  $\alpha$ . Then, we can rewrite (32) using (26)–(29), according to the following:

$$\begin{aligned}
 \mathcal{G}_1 : \max_{\alpha, d_{k,n}} & (\alpha A - B) \left( (1 - \alpha) C - \frac{\alpha}{2} D \right) \\
 \text{s.t.} & \alpha A \geq B \\
 & (1 - \alpha) C \geq \frac{\alpha}{2} D
 \end{aligned} \tag{33}$$

By setting the first derivative of (33) to zero, we can solve (33) and summarize the results according to the following:

$$\alpha^* = \alpha^*(\mathbf{d}) = \begin{cases} 1, & \text{if } \alpha_H < \alpha_L \\ \alpha_L, & \text{if } \alpha_M < \alpha_L \\ \alpha_M, & \text{if } \alpha_L \leq \alpha_M \leq \alpha_H \\ \alpha_H, & \text{if } \alpha_H < \alpha_M \end{cases} \tag{34}$$

where  $\mathbf{d} = [d_1, \dots, d_N]$ ,  $\alpha_L = \frac{B}{A}$ ,  $\alpha_M = \frac{AC+B(C+D)}{2A(C+D)}$ , and  $\alpha_H = \frac{C}{C+D}$ . Afterwards, by replacing (34) into (33) we can find the optimal value of  $\mathbf{d} = [d_1, \dots, d_N]$ ,  $\mathbf{d}^*$ , under different conditions. If  $\alpha_H < \alpha_L$ ,  $\alpha_M < \alpha_L$  and  $\alpha_H < \alpha_M$ , then  $\alpha^* = 1$  and  $\mathbf{d}^* = \mathbf{0}$ , meaning that the PU will not lease its spectrum to the SU. As a result, no cooperation will take place between the PU and the SU. However, if  $\alpha_L \leq \alpha_M \leq \alpha_H$ , by replacing  $\alpha^* = \alpha_M = \frac{AC+B(C+D)}{2A(C+D)}$  into (33), and assuming that  $X^2 = \frac{C+D}{A}$ , we have

$$\begin{aligned}
 \max_{\mathbf{d}} & \frac{1}{4} \left( \frac{C}{X} - BX \right)^2 \\
 \text{s.t.} & 0 < X < \sqrt{\frac{C}{B}}
 \end{aligned} \tag{35}$$

For  $X \in \left( 0, \sqrt{\frac{C}{B}} \right]$ ,  $f(X) = \frac{1}{4} \left( \frac{C}{X} - BX \right)^2$  is a decreasing function of  $X$ . In this case, (35) is equivalent to the following:

$$\begin{aligned}
 \max_{\mathbf{d}} & X(\mathbf{d}) \\
 \text{s.t.} & 0 < X(\mathbf{d}) < \sqrt{\frac{C}{B}}
 \end{aligned} \tag{36}$$

Ultimately, the optimum value of  $\mathbf{d}, \mathbf{d}^*$ , can be achieved by

$$\mathbf{d}^* = \begin{cases} \arg \min \sqrt{\frac{C+D}{A}} & \text{if } X(\mathbf{d}^*) < \sqrt{\frac{C}{B}} \\ 0 & \text{otherwise} \end{cases} \tag{37}$$

Finally, the NBS for the single-PU single-SU scenario can be summarized as

$$\begin{cases} \alpha^* = \frac{AC + B(C + D)}{2A(C + D)}, \mathbf{d}^* = \arg \min \sqrt{\frac{C + D}{A}} & \text{if } \alpha_L \leq \alpha_M \leq \alpha_H \text{ and } X(\mathbf{d}^*) < \sqrt{\frac{C}{B}} \\ \alpha^* = 1, \mathbf{d}^* = \mathbf{0} & \text{otherwise} \end{cases} \tag{38}$$

### 4.2.2 Single-PU Multi-SU Case

After finding the optimal solution of the single-PU single-SU case, we consider the single-PU multi-SU case. In order to derive the optimal solution for such a scenario, we first calculate the solution of  $\mathcal{G}_2$ , i.e.,  $e_k$ , by assuming that  $\alpha$  and  $\mathbf{d}_k$ , for all  $k \in N_{SU}$ , are known.

**Theorem 4** *If  $\alpha$  and  $\mathbf{d}_k$ , for all  $k \in N_{SU}$ , are given, the optimal solution of  $\mathcal{G}_2$  is*

$$e_k^* = \frac{1}{N_{SU}} \left[ (1 - \alpha) - \frac{\alpha}{2} \sum_{i \in N_{SU}} \frac{c_s \mathbf{d}_i \mathbf{P}_i^S}{R_i^S} \right] + \frac{\alpha c_s \mathbf{d}_k \mathbf{P}_k^S}{2 R_k^S} \tag{39}$$

where  $R_i^S = \sum_{n=1}^N \log \left( 1 + \frac{P_{i,n}^S \theta_{i,n}}{\sigma^2} \right)$ ,  $\mathbf{d}_i = [d_{i,1}, \dots, d_{i,N}]$  and  $\mathbf{P}_i^S = [P_{i,1}^S, \dots, P_{i,N}^S]^T$ .

**Proof** The Lagrangian function of the problem in (24) can be written as

$$L = \prod_{k \in N_{SU}} \left( e_k R_k^S - \frac{\alpha}{2} c_s \mathbf{d}_i \mathbf{P}_i^S \right) - \lambda \left( \sum_{k \in N_{SU}} e_k - 1 + \alpha \right) \tag{40}$$

where  $\lambda$  is the Lagrange multiplier. We set to zero the first order derivative of (40) with respect to  $e_j$ , for all  $j \in N_{SU}$ , we have

$$\frac{\partial L}{\partial e_j} = \prod_{k \in N_{SU}, k \neq j} \left( e_k R_k^S - \frac{\alpha}{2} c_s \mathbf{d}_i \mathbf{P}_i^S \right) R_j^S - \lambda = 0 \tag{41}$$

By some simple manipulation, (41) can be rewritten as

$$\prod_{k \in N_{SU}} \left( e_k R_k^S - \frac{\alpha}{2} c_s \mathbf{d}_i \mathbf{P}_i^S \right) = \frac{\lambda}{R_j^S} \left( e_j R_j^S - \frac{\alpha}{2} c_s \mathbf{d}_j \mathbf{P}_j^S \right) \tag{42}$$

for all  $j \in N_{SU}$ . Then, it can be concluded that

$$e_k - \frac{\alpha}{2R_k^S} c_s \mathbf{d}_i \mathbf{P}_i^S = e_j - \frac{\alpha}{2R_j^S} c_s \mathbf{d}_j \mathbf{P}_j^S \tag{43}$$

for all  $k, j \in N_{SU}$ . Finally, the optimal value of  $e_k, e_k^*$ , can be expressed as



$$e_k^* = \frac{1}{N_{SU}} \left[ (1 - \alpha) - \frac{\alpha}{2} \sum_{i \in \mathcal{N}_{SU}} \frac{c_s \mathbf{d}_i \mathbf{P}_i^S}{R_i^S} \right] + \frac{\alpha c_s \mathbf{d}_k \mathbf{P}_k^S}{2 R_k^S} \tag{44}$$

□

Now, the optimal solution of  $\mathcal{G}_1$ , i.e.  $\alpha^*$  and  $\mathbf{d}_k^*$ , should be found. Using (44), the total utility of the SUs can be expressed as

$$U^S = \sum_{k \in \mathcal{N}_{SU}} U_k^S = \left( \frac{1 - \alpha}{N_{SU}} - \frac{\alpha}{2 N_{SU}} \sum_{i \in \mathcal{N}_{SU}} \frac{c_s \mathbf{d}_i \mathbf{P}_i^S}{R_i^S} \right) \sum_{k \in \mathcal{N}_{SU}} R_k^S \tag{45}$$

By replacing (45) into (23), the following complex non-linear optimization problem is obtained:

$$\begin{aligned} \max_{\alpha, \mathbf{d}_k} & \left( \sum_{n=1}^N \frac{\alpha}{2} \log \left( 1 + \frac{P_n^p a_n d_n P_n^s b_n}{1 + P_n^p a_n + d_n P_n^s b_n} \right) - u_0^p \right) \left( \left( \frac{1 - \alpha}{N_{SU}} - \frac{\alpha}{2 N_{SU}} \sum_{i \in \mathcal{N}_{SU}} \frac{c_s \mathbf{d}_i \mathbf{P}_i^S}{R_i^S} \right) \sum_{k \in \mathcal{N}_{SU}} R_k^S \right) \\ \text{s.t.} & \sum_{n=1}^N \frac{\alpha}{2} \log \left( 1 + \frac{P_n^p a_n d_n P_n^s b_n}{1 + P_n^p a_n + d_n P_n^s b_n} \right) \geq u_0^p \\ & \left( \frac{1 - \alpha}{N_{SU}} - \frac{\alpha}{2 N_{SU}} \sum_{i \in \mathcal{N}_{SU}} \frac{c_s \mathbf{d}_i \mathbf{P}_i^S}{R_i^S} \right) \sum_{k \in \mathcal{N}_{SU}} R_k^S \geq 0 \end{aligned} \tag{46}$$

It is possible to obtain a closed-form solution for the problem in (46). Therefore, we propose a numerical searching algorithm which recruits the sequential unconstrained minimization technique (SUMT).

Let  $\mathbf{z} = [\alpha, \mathbf{d}_1, \dots, \mathbf{d}_{N_{SU}}]^T$ . Then (46) can be reformulated in the form of the following:

$$\begin{aligned} \min & v(\mathbf{z}) \\ \text{s.t.} & w_k(\mathbf{z}) \geq 0, \quad k = 1, 2, \dots, 3N_{SU} + 4 \end{aligned} \tag{47}$$

Let  $\mathcal{Z}_1 = \{\mathbf{z} | w_k(\mathbf{z}) > 0, k = 1, \dots, 3N_{SU} + 4\}$ . Then, it would be possible to solve (47) through sequentially solving a series of unconstrained optimization problems in the form of

$$\min_{\mathbf{z} \in \mathcal{Z}_1} \phi(\mathbf{z}, t_k) \tag{48}$$

where  $\phi(\mathbf{z}, t_k) = v(\mathbf{z}) + \varphi(\mathbf{z}, t_k)$ , and  $\varphi(\mathbf{z}, t_k) = -t_k \sum_{k=1}^{3N_{SU}+4} \ln(w_k(\mathbf{z}))$  is the barrier item,  $k$  is a positive integer and  $\{t_k\}$  is a positive descending series with  $\lim_{k \rightarrow \infty} t_k = 0$ . The steps to find the solution of (47), i.e.,  $\mathbf{z}$ , is presented in Table 1 (Algorithm 1).

In order to run Algorithm 1, we need a strictly interior point to start the iteration. In Algorithm 2, we propose a scheme to obtain a strictly interior point. Therefore, in order to solve (47), the first step is to call Algorithm 2. Then, using the initial point found in Algorithm 2, Algorithm 1 is recruited to solve the reformulated optimization problem in (48). It is noteworthy that in each round of iteration in Algorithm 1 and Algorithm 2, a modified Broyden-Fletcher-Goldfarb-Shanno (MBFGS) method [21] is used to solve the associated unconstrained optimization problems. The point that Algorithm 1 is converged to is announced as the optimal solution of (47). In Theorem 5, the convergence of the proposed algorithm is proved formally (Table 2).

**Table 1** Algorithm 1 (Interior Point Method to solve (48))

**Initializations:**

- 1) Set an initial point  $\mathbf{z}^{(0)} \in \mathcal{Z}_1$  (a strictly interior point)
- 2) Set an initial barrier factor  $t_1$ .
- 3) Set an initial contraction coefficient  $\delta \in (0, 1)$ .

**Stopping Condition:**

- 1) Specify  $\varepsilon$ .
- 2) Compute  $E = |\varphi(\mathbf{z}^{(m)}, t_m)| = \left| t_m \sum_{k=1}^{3N_{SU}+4} \ln(w_k(\mathbf{z}^{(m)})) \right|$ .

4) **If**  $E < \varepsilon$  then STOP

**Else** CONTINUE

**Iterations:**

Iteration Count:  $m = 1$ .

**While** *Stopping Condition* is not satisfied **do**

- 1) Using  $\mathbf{z}^{(m-1)}$  as starting point, solve the unconstrained optimization problem in (48) and save the solution as  $\mathbf{z}^{(m)}$ .
- 2) Set  $t_{m+1} = \delta t_m$ .
- 3)  $m = m + 1$ .

**End While**

**Table 2** Algorithm 2 (Searching for A Strictly Interior Initial Point for Algorithm 1)

**Initializations:**

- 1) Set an initial barrier factor  $t_1$ .
- 2) Set an initial contraction coefficient  $\delta \in (0, 1)$ .
- 3) Set a maximum iteration number  $m_{\max}$ .
- 4) Set  $\mathbf{z}^{(0)} = [0.5, \dots, 0.5]^T$ .

**Iterations:**

Iteration Count:  $m = 1$ .

- 1) Construct sets  $\mathcal{P}_m$  and  $\mathcal{Q}_m$  as

$$\mathcal{P}_m = \{k | w_k(\mathbf{z}^{(m-1)}) \leq 0, 1 \leq k \leq 3N_{SU} + 4\}$$

$$\mathcal{Q}_m = \{k | w_k(\mathbf{z}^{(m-1)}) > 0, 1 \leq k \leq 3N_{SU} + 4\}$$

- 2) **If**  $\mathcal{P}_m = \emptyset$  **do**

$\mathbf{z}^{(m-1)}$  is returned as a strictly interior point and the algorithm ends.

**Else** Continue to the algorithm.

- 3) Construct the barrier function  $\phi(\mathbf{z}, t_m) = - \sum_{k \in \mathcal{P}_m} w_k(\mathbf{z}) + t_k \sum_{k \in \mathcal{Q}_m} \ln(w_k(\mathbf{z}))$  Denote

$\mathcal{R}_m = \{\mathbf{z} | w_k(\mathbf{z}) > 0, k \in W_m\}$ . Use  $\mathbf{z}^{(m-1)}$  as the starting point and solve the unconstrained optimization problem  $\min_{\mathbf{z} \in \mathcal{R}_m} \phi(\mathbf{z}, t_m)$  And solve the solution as  $\mathbf{z}^{(m)}$ .

- 4) Set  $m = m + 1$ .

- 5) **If**  $m > m_{\max}$  **do**

The algorithm terminates and no feasible point is announced.

**Else**  $t_{m+1} = \delta t_m$  and go to Step (1).

**Theorem 5** *Algorithm 1 is convergent to at least a local optima of the proposed two-level game.*

*Proof* Based on Algorithm 1, we know that all the iterations of MBFGS are limited within the open ball with center  $\mathbf{x} = [0.5, \dots, 0.5]^T$  and radius  $R = 0.5$ , i.e.,

$$\mathcal{B}(\mathbf{y}) = \{\mathbf{y} \in \mathbb{R}^{N_{SU}N+1} \mid \|\mathbf{y} - \mathbf{x}\| < R\} \quad (49)$$

We refer to the functions in the iterative step of Algorithm 1 and step 3 of Algorithm 2 as the target functions. It is easy to verify that the target functions are continuously differentiable within the open ball. Since both the starting point and the domain of the target function are within this ball, the target function is continuously differentiable within the domain. According to [22], the target function is Lipschitz continuous. Then, according to [21], the MBFGS is globally convergent. According to [23], as long as the method for solving the unconstrained optimization problem, i.e., MBFGS, is globally convergent, Algorithm 1 is guaranteed to converge at least to a local optimum of the original bargaining game.  $\square$

Similarly, we can also prove that Algorithm 2 will converge to a feasible solution if the feasible region is not empty.

## 5 Performance Evaluation

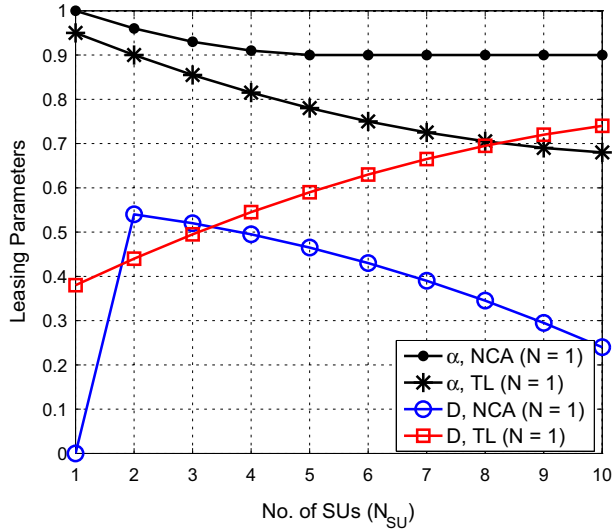
In this section, we evaluate the performance of the proposed algorithms are using simulations. We do a static system level simulation as a single cell. The simulation assumptions are as follows:

- The area covers  $2 \text{ km} \times 2 \text{ km}$  and the users are uniformly distributed in the area.
- All users are assumed to be equipped with the same number of antennas, denoted by  $N$ .
- The minimum rate requirement of the PU is set to  $2 \text{ bps} / \text{Hz}$ .
- The noise power,  $\sigma^2$ , is set to  $10^{-6} W / \text{Hz}$ .

The performance evaluation tools are the leasing parameters (such as  $\alpha$  and  $d_{k,n}$ 's, for all  $k \in \mathcal{S}_{SU}$  and all spatial modes), the outage probability of the primary link and the total rate of all SUs. The performance evaluation is realized through comparing the evaluation tools when the proposed two-level game based approach (TL) is recruited, when the primary transmitter directly communicates primary receiver (direct) and when the non-cooperative approach (NCA) proposed in [24] is recruited. In NCA, a portion of the primary spectrum is leased in return for the cooperation of SUs, and the leased spectrum is non-cooperatively shared among the SUs.

The superiority of the proposed scheme over NCA approach is shown in Fig. 3, where the leasing parameters, i.e., the portion of primary spectrum retained for the PU communications ( $\alpha$ ) and the average portion of power assigned by the SUs to relay primary signals,  $D = \frac{1}{N_{SU}N} \sum_{k \in \mathcal{N}_{SU}} \sum_{n=1}^N d_{k,n}$ . To provide more clarifications, the leasing parameters,  $\alpha$  and  $D$ , indicate the eagerness of the users for cooperating. More specifically, it can be deduced from the smaller  $\alpha$  and the larger  $D$  that the PU and SUs are highly motivated for taking part in the proposed cooperative two-level game. According to Fig. 3, when  $N_{SU} = 1$ , the leasing parameters of the NCA are  $\alpha = 1$  and  $D = 0$ , meaning that the PU and the SU are not cooperating. However, the proposed two-level game based approach results

**Fig. 3** The leasing parameters versus number of SUs

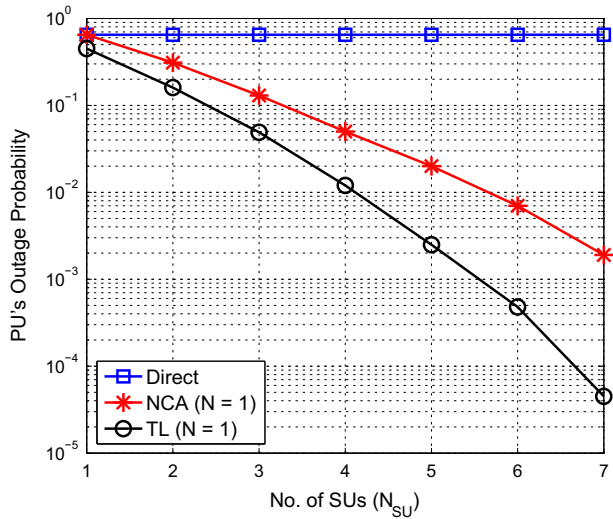


in  $\alpha = 0.95$  and  $D = 0.378$ , which shows that the PU and the SU cooperate successfully. The superiority of the proposed approach (TL) over the spectrum leasing approach in [24] (NCA) can be attributed to more power that the SUs own for bargaining with the PU and further, sharing the profits of cooperation in a cooperative manner among themselves. When  $N_{SU} > 1$ , although the cooperation between the PU and the SUs takes place as a result of NCA, but the enthusiasm of the PU for cooperating with SUs does not change significantly, because of the selfish nature of the game in [24], while the SUs eagerness for cooperating diminishes as the number of the SUs grows. Opposed to NCA, the proposed two-level game approach leads in augmenting the motivations of the PU and the SUs to cooperate with each other, as the number of SUs increases. Another significant observation is that as a result of the proposed scheme, the cooperative spectrum leasing is guaranteed to happen between PU and the SUs.

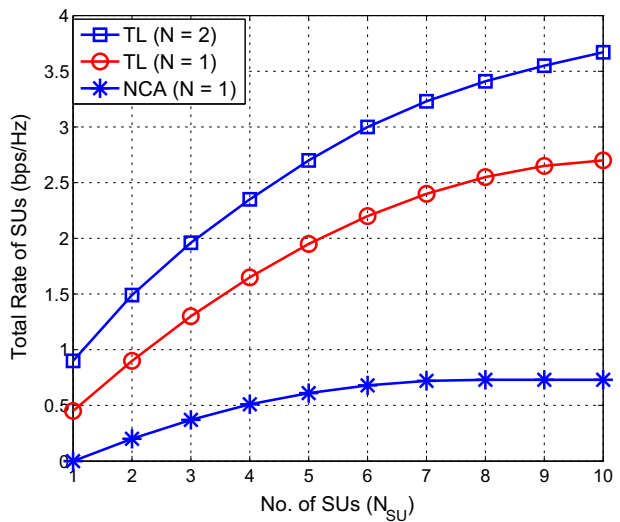
The outage probability of the primary user is another important tool for evaluating the performance of the proposed scheme. As shown in Fig. 4, the worst outage probability is achieved when the PU decides not to cooperate with the SUs. The cooperation of the SUs with the PU in the leasing scenario presented in [24] (NCA) reduces the outage probability. However, in comparison with NCA, our proposed spectrum leasing scheme causes a more significant reduction in the outage probability of the PU. The proposed two-level game based approach (TL) motivates the SUs to allocate a higher portion of their transmission power for relaying the primary signal and consequently, the outage probability performance of the proposed scheme is better than others.

The total rate of SUs in terms of the number of SUs for different number of the antennas and different schemes is presented in Fig. 5. Similar to previous discussions, it can be inferred from Fig. 5 that for the single-SU case, NCA does not motivates the SU enough to participate in PU transmissions and get access to the leased resources, in return. However, the proposed two-level game based approach (TL) persuades the SU to relay the primary signals and benefit from the leased resources to increase its data rate. As the number of existing SUs in the system increases, the total rate of the system implemented based on NCA does not escalate significantly, because of the competition among the SUs for sharing

**Fig. 4** The outage probability of the primary user



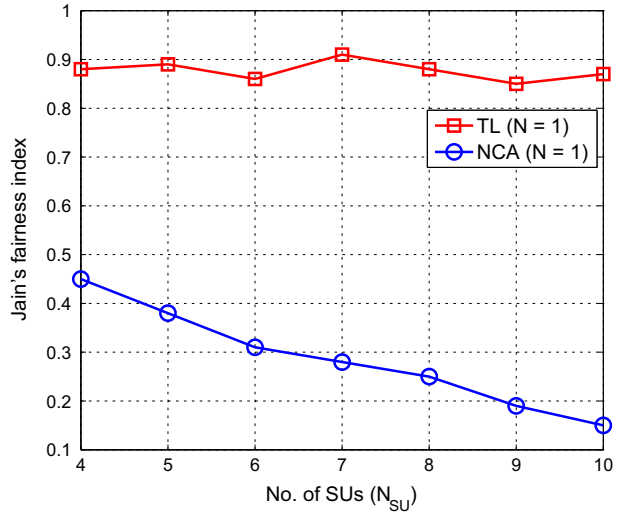
**Fig. 5** The total rate of SUs



the leased spectrum. Meanwhile, the proposed two-level game based scheme which takes advantage of the cooperative games, provides a much better sharing of the leased spectrum among the SUs and thereby higher sum rate for the SUs. As another point, Fig. 5 suggests that deploying more antennas in all users, results in higher data rates expectedly.

In order to determine whether SUs are receiving a fair share of system resource, Jain's fairness measure as a widely used fairness measure is used. Fig. 6 depicts the Jains fairness indices of the different algorithms. Evidently and expectedly, the proposed two-level cooperative game based scheme achieves significantly better fairness than NCA. As the number of SUs increases, fairness indices under NCA decreases. However, the proposed approach maintain quite stable fairness for different network sizes.

**Fig. 6** Fairness index for different approaches



## 6 Conclusions

In this work, we studied the resource allocation problem in a spectrum leasing scenario in MIMO-CCRN with multi channels using a coalitional game based approach. After proving that all PUs and SUs should be grouped in a set and as a result of such grouping, the grand coalition is formed, we analyzed the stability of the proposed coalitional game based approach using the core concept. Afterwards, we explored a practical scenario where the data rate of the primary system, falls below a minimum data rate requirement, and alternatively, the PU resorts to the leasing of its unused spectrum to SUs. However, it is guaranteed that the SUs perform as cooperatively as to meet the rate requirement of the primary system. Moreover, the benefits of cooperating with the primary system may not be exploited by the secondary system efficiently, if the SUs compete over the resources. We propose a two-level game based on the bargaining games to tackle the aforementioned problems. The simulation results confirm the theoretical achievements.

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