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Power Optimization in MIMO-OFDM Systems with Mixed Orthogonal Frequency Division and Space Division Multiple Access Scheme

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Abstract In recent years, energy efficiency has become a critical metric for green system design and drawn universal attention. In this paper, we propose a new radio resource allocation algorithm to minimize the power consumption of the base station for the multi-input multi-output orthogonal frequency division multiplexing (MIMO-OFDM) systems with different quality of service constraints of users. The discontinuous transmission (DTX) technique is applied in the systems with mixed orthogonal frequency division access and space division multiple access. Radio resources in time, frequency and space domain are optimally allocated to users in two steps: In the first step, the active time in the DTX mode is estimated under the assumption that every user experiences slow time-varying fading in an OFDM frame. In the second step, subcarriers, bits and power are allocated in the active time slots. Although the proposed algorithm is suboptimal due to high computation complexity, simulation results show that the proposed algorithm can reduce the power consumption of the base station and improve resource utilization.

Keywords MIMO-ODFM · DTX · Power allocation · SDMA

1 Introduction

With the rapid development of mobile communication services and the increasing of the number of mobile terminals, the users' requirements for high-speed data transmission are growing rapidly, followed by a continuous increase in energy consumption. According to [1], the energy consumption of information and communication industry causes about 2 %

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of world CO_2 emissions yearly and brings huge economic pressure to the network operators, which has drawn universal attention. The energy consumption in the wireless communications, especially the transmitting power cost by the base stations of the cellular systems, is closely related to the physical layer technologies used by the systems, where MIMO and OFDM as two key technologies in LTE have received much attention in recent years. OFDM divides channel into a number of orthogonal subcarriers, which reduces the mutual interference between subcarriers and can effectively inhibit the frequency selective fading. Compared to single-input single-output (SISO), MIMO offers higher diversity which can potentially lead to a multiplicative increase in capacity. For these reasons, energy efficiency in MIMO-OFDM wireless communication systems has become a hot topic in the last decade [2–5].

Among the energy efficiency techniques for wireless communication systems, DTX technique is to stop transmitting the radio signals when there is no data to be transmitted, thus having the potential to become one of the most effective means for reducing the system power consumption. In recent years, a few researches have tried to adopt the DTX technique to optimize the power consumption of base stations [6, 7]. However, in these researches, the base station enters sleep mode only when all of the users in the corresponding cell are non-active, which is very rare in practice. For this reason, most of the existing works [2–5] do not consider this technology in system power optimization. For example, in [2], users are classified into different groups by their spatial separability. This scheme facilitates mitigating the interference among users and thereby reduces the power consumption. Reference [3] uses the Lagrange duality method to optimize the system power, etc. In fact, it is not necessary for the base station to enter sleep mode only when all users are non-active. Because we can burst data of users in an OFDM frame in several time slots and put the base station into sleep mode during remaining time while users' delay requirements are still maintained. To the best of our knowledge, only reference [8] uses the above method to optimize system power. However, the role of MIMO played in [8] is only to increase the users' transmission bit rate instead of multiplexed users, and users' multiple access technique is still OFDMA. We will show that using mixed OFDMA and SDMA instead of OFDMA can further reduce the power consumption of the base station. This conclusion is based on the fact that realizing SDMA can improve the system throughput without increasing the power consumption of the MIMO system [9], if an appropriate number of multiplexed users are selected on the same subcarrier, which makes the gain of multiuser diversity greater than the loss of antenna diversity. Under appropriate resource allocation policies, we will prove that the converse fact is also true, which means mixed OFDMA and SDMA technique consumes less power than OFDMA technique under the same requirements of transmission bit rate. Motivated by above facts, this paper presents a DTX power optimization method in MIMO-OFDM system with mixed OFDMA and SDMA, which adds SDMA into MIMO-OFDM system and combines appropriate resource allocation algorithm to reduce the power consumption of base stations.

In our method, we burst data of users in several time slots and put the base station into sleep mode during remaining time in an OFDM frame to reduce system power. In the active time slots, we realize SDMA by precoding which makes one subcarrier can be used by different users simultaneously and design a new subcarriers, bits and power allocation algorithm to achieve a balance between transmission bit rate and power consumption. More specifically, the proposed algorithm comprises two steps: (a) the first step is to estimate the active time of the DTX mode and then determine the users' bit rate requirements in the active time slots. (b) The second step is to allocate subcarriers, bits and power in the active time slots.

2 System Model

We consider the downlink of a point-to-multipoint MIMO-OFDM system composed by one base station (BS) and several users. The number of transmit antennas on the BS is N_T, and the number of receive antennas is N_R for all users' mobile terminals (MTs). The number of users is K, and the number of subcarriers is M. We add SDMA into MIMO-OFDM system, which means the subcarrier can be shared by different users simultaneously. However, due to limitation of space resource, the number of multiplexed users is limited, and the maximum number of multiplexed users on the same subcarrier is denoted as K_m, which is depended on N_T and N_R and will be discussed in Sect. 2.2. The OFDM frame comprises T time slots and M subcarriers. Hence, there are totally $M \times T$ resource units. And the frame structure is illustrated in Fig. 1. $\boldsymbol{H}_{k,m,t}$ denotes the MIMO channel matrix on each resource unit, with user index $k = 1,\ldots,K$, subcarrier index $m = 1,\ldots,M$, time slot index $t = 1,\ldots,T$. B_k is the bits needed to be transmitted by user k in each OFDM frame. We assume that the delay requirement for user k can be met if B_k bits are successfully transmitted in an OFDM frame. Based on this assumption, we can burst data of users in an OFDM frame in several time slots and put the base station into sleep mode during remaining time while users' delay requirements are still maintained, thus the power consumption is reduced.

2.1 Power Model

The power consumption models of the BS in most existing researches [2, 3, 10] are represented by the transmitting power only. Fixed circuit power consumption was not considered in those models. In fact, hardware components of the BS like power amplifier and antenna interface consume a large share of the overall power of the BS [11, 12]. In the power model considered in this paper, power consumption of the BS comprises two parts: transmitting power and circuit power. Due to the DTX technique, a BS can enter sleep

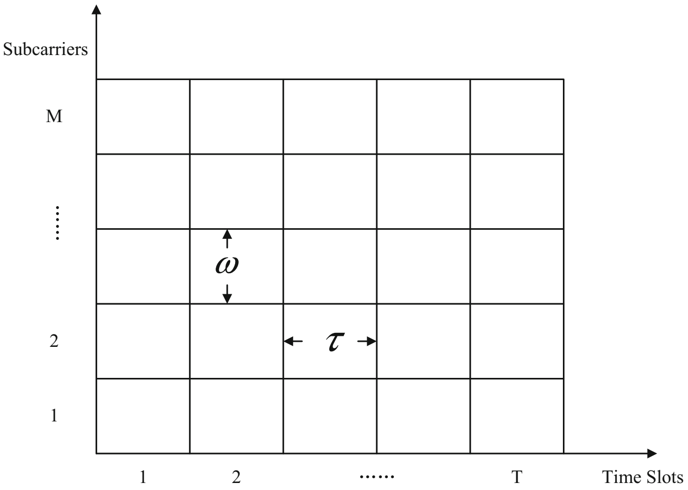

Fig. 1 OFDM frame structure

mode so that the BS only consumes circuit power when there is no transmitting task. For simplicity, we assume that there is no delay incurred by putting a BS into DTX mode.

More specifically, the power model used in this paper is described by following parameters: (a) P_o is the circuit power in active mode. (b) P_T is the transmitting power. (c) P_S is the circuit power in sleep mode. d) Δp is the power dependence factor which denotes the power loss when the power consumption of BS converts into the transmitting power. In active mode, the power consumption of the BS is $P_o + \Delta pP$. In sleep mode, the power consumption of the BS is P_S, and $P_S < P_o$. Assuming the four parameters are time-invariant, we can get the following power consumption model:

$$P(P_T) = \begin{cases} P_o + \Delta p P_T & \mathit{if}\ P_T > 0 \\ P_S & \mathit{if}\ P_T = 0 \end{cases} \tag{1}$$

The above power model indicates that the BS in sleep mode consumes less power than in the active mode. However, due to the data delay constraint, the less active time slots an OFDM frame has, the higher transmission bit rate is required by the BS in an active time slot, which results in more power consumption of the BS in the active time slots. Therefore we should choose appropriate number of active time slots to reduce the total power consumed by the BS. In the next section, we will begin with the analysis of the transmission bit rate that system physical layer can support and then present the power allocation method.

2.2 Transmission Bit Rate of DTX MIMO-OFDM System with Mixed Orthogonal Frequency Division and Space Division Multiple Access Technique

In MIMO-OFDM system, one subcarrier can be transformed into several parallel subchannels by adopting precoding matrix and receive matrix which are obtained by singular-value decomposition (SVD) of channel matrix. And all the sub-channels can be used to transmit data. The capacity of one subcarrier, denoted as R, is given by

$$R = \sum_{l=1}^{\eta} \log_2\left(1 + \frac{p_l s_l}{N_o}\right) \tag{2}$$

where p_l is the power assigned to the sub-channel l; s_l is the channel gain of sub-channel l; η is the rank of the MIMO channel gain matrix; N_o is the noise power.

If the BS transmits data only in the active time slots, the transmission bits on one subcarrier in each OFDM frame, denoted as B_c, is given by

$$B_c = \omega\tau \sum_{t=1}^{T_{Active}} \sum_{l=1}^{\eta_t} \log_2\left(1 + \frac{p_{t,l} s_{t,l}}{N_o}\right) \tag{3}$$

where ω is the bandwidth of one subcarrier; τ is the time slot duration; T_{Active} is the number of active time slots; $p_{t,l}$ is the power assigned to the sub-channel l in the time slot t; $s_{t,l}$ is the channel gain of sub-channel l in the time slot t; η_t is the rank of the MIMO channel gain matrix in the time slot t.

The transmission bits of user k in each OFDM frame, denoted as B_{uk}, is given by

$$\begin{aligned} B_{uk} &= \omega\tau \sum_{t=1}^{T_{Active}} \sum_{m=1}^{M} \sigma_{k,m,t} \sum_{l=1}^{\eta_{k,m,t}} \log_2\left(1+\frac{p_{k,m,t,l}s_{k,m,t,l}}{N_o}\right) \\ s.t. \quad & \sum_{k=1}^{K} \sigma_{k,m,t} \le 1 \quad \forall m,t \end{aligned} \tag{4}$$

where $\sigma_{k,m,t}$ can only be 1 or 0 indicating whether subcarrier m is used by user k in the time slot t; $s_{k,m,t,l}$ and $p_{k,m,t,l}$ are the channel gain and assigned power on the lth sub-channel $(1 \le l \le \eta_{k,m,t})$ of subcarrier m used by user k in the time slot t respectively. $\eta_{k,m,t}$ is the rank of the MIMO channel gain matrix of subcarrier m used by user k in the time slot t. The constraint $\sum_{k=1}^{K} \sigma_{k,m,t} \le 1 \quad \forall m,t$ means that a subcarrier only can be used by one user in each time slot. However, if the mixed OFDMA and SDMA technique is used, this constraint will not hold, and the power consumption will subsequently reduced.

When the SDMA is introduced into MIMO-OFDM system, severe interference problem will occur as multiple users simultaneously occupy the same subcarrier. Dirty paper coding (DPC) [13] is the optimal method to remove the interference, but it is difficult to be implemented due to its high computation complexity. In this paper, we use zero-forcing linear block diagonalization (ZF-LBD) [14] to solve the interference problem. ZF-LBD can separate users who occupy the same subcarrier by precoding.

We assume there are up to K_m users occupying the subcarrier m in the downlink case. The receive signal on subcarrier m denoted as $\boldsymbol{y_m}$, is given by

$$\boldsymbol{y_m} = \boldsymbol{H_m x_m} + \boldsymbol{n_m} \tag{5}$$

where $\boldsymbol{x_m}$ denotes the transmit signal on subcarrier m, and is given by $\boldsymbol{x_m} = \sum_{k=1}^{K_m} \boldsymbol{x_{k,m}} = \sum_{k=1}^{K_m} \boldsymbol{T_{k,m} b_{k,m}}$. $\boldsymbol{T_{k,m}}$ is the precoding matrix of user k on the subcarrier m. $\boldsymbol{b_{k,m}}$ is the actual transmitted data of user k on the subcarrier m. $\boldsymbol{H_m} = \left[\boldsymbol{H}_{1,m}^T, \boldsymbol{H}_{2,m}^T, \boldsymbol{H}_{3,m}^T, \ldots \boldsymbol{H}_{K_m,m}^T\right]$ is the $K_m N_R \times N_T$ MIMO channel on the subcarrier m. $\boldsymbol{y_m} = \left[\boldsymbol{y}_{1,m}^T, \boldsymbol{y}_{2,m}^T, \boldsymbol{y}_{3,m}^T, \ldots \boldsymbol{y}_{K_m,m}^T\right]$ is the $K_m N_R \times 1$ received signal vector. $\boldsymbol{n_m}$ is the $K_m N_R \times 1$ noise vector on the subcarrier m whose elements are independently identical distributed (i.i.d.) zero mean complex Gaussian random variables.

The received signal for user k on the subcarrier m, denoted as $\boldsymbol{y_{k,m}}$, is given by

$$\boldsymbol{y_{k,m}} = \boldsymbol{H_{k,m} x_m} + \boldsymbol{n_{k,m}} = \boldsymbol{H_{k,m} x_{k,m}} + \sum_{i \ne k}^{K_m} \boldsymbol{H_{k,m} x_{i,m}} + \boldsymbol{n_{k,m}} \tag{6}$$

$\sum_{i \ne k}^{K_m} \boldsymbol{H_{k,m} x_{i,m}}$ represents the interference to user k due to the other $K_m - 1$ users and should be eliminated. Since $\boldsymbol{b_{k,m}}$ is the user data which should not be zero, $\sum_{i \ne k}^{K_m} \boldsymbol{H_{k,m} x_{i,m}} = \boldsymbol{H_{k,m}} \sum_{i \ne k}^{K_m} \boldsymbol{T_{i,m} b_{i,m}} = \boldsymbol{0}$ implies $\boldsymbol{H_{k,m} T_{i,m}} = \boldsymbol{0}$, $i \ne k$. Therefore, the precoding matrix of user k on the subcarrier m, denoted as $\boldsymbol{T_{k,m}}$, should satisfy

$$\boldsymbol{H_{i,m} T_{k,m}} = \boldsymbol{0} \quad i \ne k \tag{7}$$

which means the column of $\boldsymbol{T_{k,m}}$ belongs to the intersection of the null space of $\boldsymbol{H_{i,m}}$ $\quad i \ne k$ [9].

Hence, the BS can eliminate interference between users by precoding when the channel state information (CSI) is known. The transmission bits of user k in each OFDM frame in MIMO-OFDM system with mixed OFDMA and SDMA, denoted as B^*_{uk}, is modified as

$$\begin{aligned} B^*_{uk} &= \omega\tau \sum_{t=1}^{T_{Active}} \sum_{m=1}^{M} \sigma_{k,m,t} \sum_{l=1}^{\eta_{k,m,t}} \log_2\left(1 + \frac{p_{k,m,t} s_{k,m,t}}{N_o}\right) \\ s.t. &\qquad \sum_{k=1}^{K} \sigma_{k,m,t} \le K_m \quad \forall m, t \end{aligned} \tag{8}$$

Equation (8) indicates that one subcarrier can be shared by K_m users simultaneously which is the multiplexing gain given by the space resource. Then we will present the upper bound of K_m and this upper bound will be the constraint of the power optimization problem in the next section.

Defining $\tilde{\boldsymbol{H}}_{\boldsymbol{k,m}}$ as

$$\tilde{\boldsymbol{H}}_{\boldsymbol{k,m}} = \left[\boldsymbol{H}^{\boldsymbol{T}}_{\boldsymbol{1,m}}, \boldsymbol{H}^{\boldsymbol{T}}_{\boldsymbol{2,m}} \cdots \boldsymbol{H}^{\boldsymbol{T}}_{\boldsymbol{k-1,m}}, \boldsymbol{H}^{\boldsymbol{T}}_{\boldsymbol{k+1,m}}, \ldots, \boldsymbol{H}^{\boldsymbol{T}}_{\boldsymbol{K_m,m}}\right]^{\boldsymbol{T}} \tag{9}$$

and performing SVD on $\tilde{\boldsymbol{H}}_{\boldsymbol{k,m}}$, it follows

$$\tilde{\boldsymbol{H}}_{\boldsymbol{k,m}} = \tilde{\boldsymbol{U}}_{\boldsymbol{k,m}} \tilde{\boldsymbol{S}}_{\boldsymbol{k,m}} \tilde{\boldsymbol{V}}^{\boldsymbol{H}}_{\boldsymbol{k,m}} = \tilde{\boldsymbol{U}}_{\boldsymbol{k,m}} \tilde{\boldsymbol{S}}_{\boldsymbol{k,m}} \left[\tilde{\boldsymbol{V}}^{\boldsymbol{1}}_{\boldsymbol{k,m}} \tilde{\boldsymbol{V}}^{\boldsymbol{0}}_{\boldsymbol{k,m}}\right]^{\boldsymbol{H}} \tag{10}$$

where $\tilde{\boldsymbol{U}}_{\boldsymbol{k,m}}$ and $\tilde{\boldsymbol{V}}_{\boldsymbol{k,m}}$ are unitary matrices whose columns are left and right singular vectors of $\tilde{\boldsymbol{H}}_{\boldsymbol{k,m}}$, respectively. $\tilde{\boldsymbol{S}}_{\boldsymbol{k,m}}$ is a diagonal matrix which contains the singular value of $\tilde{\boldsymbol{H}}_{\boldsymbol{k,m}}$. The columns of $\tilde{\boldsymbol{V}}^1_{\boldsymbol{k,m}}$ and $\tilde{\boldsymbol{V}}^0_{\boldsymbol{k,m}}$ correspond to the nonzero and zero singular values of $\tilde{\boldsymbol{H}}_{\boldsymbol{k,m}}$, respectively. Therefore, $\tilde{\boldsymbol{V}}^0_{\boldsymbol{k,m}}$ is the null space of $\tilde{\boldsymbol{H}}_{\boldsymbol{k,m}}$ [3].

Defining $\tilde{\boldsymbol{H}}_{\boldsymbol{k,m}}$ as

$$\hat{\boldsymbol{H}}_{\boldsymbol{k,m}} = \boldsymbol{H}_{\boldsymbol{k,m}} \tilde{\boldsymbol{V}}^{\boldsymbol{0}}_{\boldsymbol{k,m}} = \hat{\boldsymbol{U}}_{\boldsymbol{k,m}} \hat{\boldsymbol{S}}_{\boldsymbol{k,m}} \hat{\boldsymbol{V}}^{\boldsymbol{H}}_{\boldsymbol{k,m}} \tag{11}$$

where $\hat{\boldsymbol{U}}_{\boldsymbol{k,m}}$ and $\hat{\boldsymbol{V}}_{\boldsymbol{k,m}}$ are unitary matrices whose columns are left and right singular vectors of $\tilde{\boldsymbol{H}}_{\boldsymbol{k,m}}$, respectively. $\hat{\boldsymbol{S}}_{\boldsymbol{k,m}}$ is a diagonal matrix which contains the singular value of $\tilde{\boldsymbol{H}}_{\boldsymbol{k,m}}$. Then we can define the precoding matrix as $\boldsymbol{T}_{\boldsymbol{k,m}} = \tilde{\boldsymbol{V}}^0_{\boldsymbol{k,m}} \hat{\boldsymbol{V}}_{\boldsymbol{k,m}}$ and the receive matrix as $\boldsymbol{R}_{\boldsymbol{k,m}} = \hat{\boldsymbol{U}}_{\boldsymbol{k,m}}$ [14]. The final input–output relationship for user k on the subcarrier m can be expressed as

$$\begin{aligned} \boldsymbol{y}_{\boldsymbol{k,m}} &= \boldsymbol{R}^{\boldsymbol{H}}_{\boldsymbol{k,m}} \left(\boldsymbol{H}_{\boldsymbol{k,m}} \boldsymbol{x}_{\boldsymbol{k,m}} + \boldsymbol{n}_{\boldsymbol{k,m}}\right) \\ &= \boldsymbol{R}^{\boldsymbol{H}}_{\boldsymbol{k,m}} \left(\boldsymbol{H}_{\boldsymbol{k,m}} \boldsymbol{T}_{\boldsymbol{k,m}} \boldsymbol{b}_{\boldsymbol{k,m}} + \boldsymbol{n}_{\boldsymbol{k,m}}\right) \\ &= \hat{\boldsymbol{S}}_{\boldsymbol{k,m}} \boldsymbol{b}_{\boldsymbol{k,m}} + \boldsymbol{R}^{\boldsymbol{H}}_{\boldsymbol{k,m}} \boldsymbol{n}_{\boldsymbol{k,m}} \end{aligned} \tag{12}$$

$\hat{\boldsymbol{S}}_{\boldsymbol{k,m}}$ is the channel gain matrix after the precoding. Therefore, $\hat{\boldsymbol{H}}_{\boldsymbol{k,m}}$ can be viewed as the equivalent MIMO channel matrix for user k on the subcarrier m. From (9), (10), it follows that the dimension of $\tilde{\boldsymbol{V}}^0_{\boldsymbol{k,m}}$ is $N_T \times n$, where $n = N_T - (K_m - 1)N_R$ if $\boldsymbol{H}_{\boldsymbol{k,m}}\ i \neq k$ is full rank. To guarantee the existence of precoding matrix, n must be greater than 0. It follows that the upper bound of K_m is

$$K_m = \lfloor N_T/N_R + 1 \rfloor \tag{13}$$

where $\lfloor x \rfloor$ is the nearest integer smaller than x.

From (13) we can see that one subcarrier can be multiplexed by up to $\lfloor N_T/N_R + 1 \rfloor$ users simultaneously in MIMO-OFDM system with mixed OFDMA and SDMA. In the next section, we will solve the power optimization problem under the condition given by (13) and some other QoS constraints of users.

3 Optimal Resource Allocation

In this section the power optimization problem is mathematically modeled. The objective of the model is to minimize the power consumption of the BS under the constraints of the users by finding the optimal number of active time slots and allocating radio resource to users.

Mathematically, the power optimization problem can be expressed as

$$\begin{aligned} &\min_{T_{Active},p_{k,m,t,l},\sigma_{k,m,t}} \left[P = \frac{1}{T}\left(\sum_{t=1}^{T_{Active}} (P_o + \Delta p P_t) + \sum_{t=1}^{T_{Sleep}} P_s \right) \right] \\ &s.t. \qquad B_k \le \omega\tau \sum_{t=1}^{T_{Active}} \sum_{m=1}^{M} \sigma_{k,m,t} \sum_{l=1}^{\eta_{k,m,t}} \log_2\left(1 + \frac{p_{k,m,t,l} s_{k,m,t,l}}{N_o \Gamma}\right) \\ &\qquad\qquad P_t = \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{l=1}^{\eta_{k,m,t}} p_{k,m,t,l} \\ &\qquad\qquad \sum_{k=1}^{K} \sigma_{k,m,t} \le K_m \quad \forall m,t \\ &\qquad\qquad p_{k,m,t,l} \ge 0 \quad \forall k,m,l,t \end{aligned} \tag{14}$$

where T_{Active}, T_{Sleep} and T are the number of active time slots, the number of sleep time slots, and the total number of time slots in an OFDM frame respectively; $T_{Sleep} + T_{Active} = T$. Γ is the SNR gap which is given by $\Gamma = -\ln(5\,\text{BER})/1.5$ for an uncoded M quadrature amplitude modulation (M-QAM) with a specified BER [15].

Optimal subcarrier and power allocation is known to be a complex problem for a single time slot in frequency-selective fading channels [2–5]. In this paper, we add two degrees of freedom into the optimization problem by considering DTX and SDMA and allocate radio resource in time, frequency and space domain, which makes the optimization problem more complex. Consequently, we divide the proposed resource optimization algorithm into two steps: the first step is to estimate the active time of the DTX mode and then determine the users' bit rate requirements in the active time slots. The second step is to allocate subcarriers, bits and power in each active time slot.

3.1 Estimate the Number of Active Time Slots

We assume that the transmit signals experience slowly time-vary fading channel in an OFDM frame which means $\boldsymbol{H}_{k,m,t} = \boldsymbol{H}_{k,m}$. The problem of selecting suitable time slots as the active ones transforms to the problem of estimating the number of active time slots in an OFDM frame. From (14), we can see that, to estimate the optimal number of active time slots, the transmitting power of each active time slot $P_t(t = 1 \cdots T_{Active})$ should be known. The transmitting power is related to the transmission bit rate of users. Hence, estimating the number of active time slots starts from discussing the required bit rate of the users to meet their delay constraints.

Because the channel is stable in an OFDM frame, the required bit rate for user k to meet the delay constraint, denoted as R_k, is invariant in each active time slot. R_k is given by

$$R_k = \frac{B_k}{\tau T_{Active}} \tag{15}$$

From (8), the transmission bit rate supported by the system physical layer with mixed OFDMA and SDMA, denoted as R_{uk} for user k in a active time slot, is given by

$$\begin{aligned} R_{uk} &= \omega \sum_{m=1}^{M} \sum_{l=1}^{\eta_{k,m}} \sigma_{k,m} \log_2\left(1 + \frac{p_{k,m,l} s_{k,m,l}}{N_o \Gamma}\right) \\ s.t. \quad & \sum_{k=1}^{K} \sigma_{k,m} \le K_m \quad \forall m \end{aligned} \tag{16}$$

where $\sigma_{k,m}$ can only be 1 or 0 indicating whether subcarrier m is used by user k; $s_{k,m,l}$ and $p_{k,m,l}$ are the channel gain and assigned power on the lth sub-channel $(1 \le l \le \eta_{k,m})$ of subcarrier m used by user k, respectively. $\eta_{k,m}$ is the rank of the MIMO channel gain matrix of subcarrier m used by user k.

For simplicity, we replace each sub-channel gain $s_{k,m,l}$ with the average gain of all sub-channels assigned to the user. Denoted as s_k, the average gain is given by $S_k = \left(\sum_{m=1}^{M} \sum_{l=1}^{\eta_{k,m}} S_{k,m,l} \sigma_{k,m}\right) / \left(\sum_{m=1}^{M} \eta_{k,m} \sigma_{k,m}\right)$. The minimum power consumption of the BS is occurred when the power is uniformly distributed in each sub-channel. Hence, (16) is modified as

$$\begin{aligned} R_{uk} &= \omega M_k \log_2\left(1 + \frac{p_k s_k}{M_k N_o \Gamma}\right) \\ s.t. \quad & \sum\nolimits_{k=1}^{K} \sigma_{k,m} \le K_m \quad \forall m \end{aligned} \tag{17}$$

where

$$M_k = \sum_{m=1}^{M} \eta_{k,m} \sigma_{k,m} \tag{18}$$

and p_k denotes the total power assigned to user k. From (17), p_k is given by

$$p_k = \frac{M_k \left[2^{\left(\frac{R_{uk}}{\omega M_k}\right)} - 1 \right]}{G_k} \tag{19}$$

where $G_k = s_k / \Gamma N_o$. To satisfied the transmission bit rate requirement for each user, R_{uk} should satisfy

$$R_{uk} \ge R_k \quad k = 1, 2, \ldots, K \tag{20}$$

Therefore, to achieve the minimum transmitting power of the BS, the transmission bit rate should equal to the required bit rate, and P_t can be expressed as

$$P_t = \sum_{k=1}^{K} p_k = \sum_{k=1}^{K} \frac{M_k\left[2^{\left(\frac{R_k}{\omega M_k}\right)} - 1\right]}{G_k} \tag{21}$$

Substituting (15) into (21), P_t can be modified as

$$P_t = \sum_{k=1}^{K} \frac{M_k\left[2^{\left(\frac{B_k}{\omega M_k \tau T_{Active}}\right)} - 1\right]}{G_k} \tag{22}$$

From (18), (22), we can see that P_t is related to the number of active time slots T_{Active} and the allocation of subcarriers, i.e. $\sigma_{k,m}$. We adopt a simple algorithm (Sub-algorithm 1) to allocate subcarriers in active time slots, which makes P_t only dependent on the number of active time slots. The Sub-algorithm 1 is only used to estimate the number of active time slots. More complex and efficient subcarrier allocation algorithms are presented in Sub-algorithms 2 and 3, which will be discussed in the next section.

The Sub-algorithm 1 is shown in Table 1, in which N_k is the number of subcarriers assigned to user k; B_k is the bits needed to be transmitted by user k in each OFDM frame; $U \backslash \{k^*\}$ means k^* is removed from the set U. Due to SDMA, one subcarrier can be used by K_m users simultaneously, thus we can give one subcarrier to the K_m users with lowest N_k/B_k (the ratio of assigned subcarriers to user's transmission bit rate requirement). However, one subcarrier should not be allocated to the same user twice or more.

Once the subcarrier allocation is determined, the transmitting power of the BS is only dependent on the number of active time slots. From (14), (22), defining $u_T = T_{Active}/T$,

Table 1 Sub-algorithm 1

```
step1: $N_k = 0 \ \forall k$;
step2: for $m = 1 : M$
            $U = \{1, 2 \ldots K\}$;
            for $i = 1 : K_m$
                $k^* = \arg\min_k \left\{ \frac{N_k}{B_k}, k \in U \right\}$;
                $U = U \backslash \{k^*\}$;
                $\sigma_{k^*,m} = 1$;
                $N_{k^*} = N_{k^*} + 1$;
            end for
       end for
```

which is consecutive between 0 and 1, the total power consumption of the BS can be expressed as

$$P = u_T\left(P_o + \Delta p\sum_{k=1}^{K}\frac{M_k\left[2^{\left(\frac{B_k}{\omega M_k \tau T u_T}\right)} - 1\right]}{G_k}\right) + (1 - u_T)P_s \tag{23}$$

Lemma 1 *The power consumption function P is strict convex in u_T.*

Proof

$$\begin{aligned}\frac{\partial^2 P}{(\partial u_T)^2} &= \frac{\partial\left(P_o + \Delta p\sum_{k=1}^{K} M_k/G_k\left(2^{\frac{\alpha_k}{u_T}} - 1\right) - P_s - \frac{\alpha_k}{u_T}\Delta p\sum_{k=1}^{K} M_k/G_k \ln 2\left(2^{\frac{\alpha_k}{u_T}}\right)\right)}{\partial u_T}\\ &= \Delta p\sum_{k=1}^{K}\left[\frac{\alpha_k^2 M_k}{u_T^3 G_k}(\ln 2)^2 2^{\frac{\alpha_k}{u_T}}\right] > 0\end{aligned} \tag{24}$$

where $\alpha_k = B_k/T\omega\tau M_k$. The partial second derivative of P with respect to u_T is positive. Thus the power consumption function P is strict convex in u_T. □

From Lemma 1, we can obtain the optimal u_T which minimizes the power consumption of the BS by one-dimensional search. Then the number of active time slots is given by

$$T_{Active} = round(u_T T) \tag{25}$$

where $round(x)$ is the nearest integer to x. Especially, when $round(u_T T) = 0$, $T_{Active} = 1$.

3.2 Subcarriers and Power Allocation in the Active Time Slots

In the above section we obtain the number of active time slots and the transmission bit rate requirement for each user in the active time slots. This section describes subcarrier and power allocation in the active time slots. The power optimization problem, in each active time slot, is expressed as

$$\begin{aligned}\min_{\{p_{k,m,l},\sigma_{k,m}\}} \quad & \Delta p\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{\eta_{k,m}} p_{k,m,l} + P_o\\ s.t. \quad & \sum_{m=1}^{M}\sigma_{k,m} r_{k,m} > R_k \quad \forall k\\ & p_{k,m,l} \geq 0 \quad \forall k, m, l\\ & \sum_{k=1}^{K}\sigma_{k,m} \leq K_m \quad \forall m\\ & r_{k,m} = \omega\sum_{l=1}^{\eta_{k,m}}\log_2\left(1 + \frac{p_{k,m,l}s_{k,m,l}}{\Gamma N_o}\right)\end{aligned} \tag{26}$$

where R_k is expressed by (15).

The problem posed by (26) is computationally intractable [3, 16, 17]. Therefore, we divide this problem into two steps. The first step is to allocate subcarriers by Sub-algorithm 2 without considering SDMA, i.e. subcarrier only can be used by one user. In the second step, Sub-algorithm 3 grabs the benefit of SDMA by iteratively changing the set of users on each subcarrier by to further reduce the power consumption of the BS. After that, the power is allocated by the Water-filling algorithm.

The Sub-algorithm 2 is shown in Table 2, in which $D \backslash \{i\}$ means removing i from the set D; $\Delta P_{k,m}$ represents the decrease of the transmit power after that subcarrier m is assigned to user k which is calculated by the Water-filling algorithm. In the first step, each user chooses its best subcarrier. And in the second step, each of the remaining subcarriers is allocated to the user who can achieve maximum decrease of the transmitting power.

The Sub-algorithm 3 is shown in Table 3, in which U_m represents the set of users on subcarrier m and has $N = \sum_{i=1}^{K_m} C_K^i$ possible combination of users which are denoted as U_m^n, $n = 1\ldots N$; $P_{\min}$ is the minimum transmit power of BS. $P(U_1 \ldots U_m \ldots U_M)$ is the transmitting power of the BS obtained by the Water-filling algorithm when the sets of users on subcarriers are $U_1 \ldots U_m \ldots U_M$. In the first step, $U_1 \ldots U_m \ldots U_M$ is initialized by Sub-algorithm 2. And in the second step, Sub-algorithm 3 iteratively changes the set of users on each subcarrier. Note that the sets of users on other subcarriers are not changed simultaneously in each iteration step. After M iteration steps, the minimum power consumption of the BS will be achieved.

Table 2 Sub-algorithm 2

```
step1: $D = \{1, \ldots, M\}$;
        for $k = 1:K$
            for $m = 1:M$
                $\sigma_{k,m} = 0$;
            end for
        end for
        for $k = 1:K$
            $i = \arg\max_m \left\{ \prod_{l=1}^{\eta_{k,m}} s_{k,m,l} , m \in D \right\}$;
            $\sigma_{k,i} = 1$;
            $D = D \backslash \{i\}$;
        end for
step2: for $m = 1:M$
            if $(m \notin D)$
                continue;
            else
                calculate $\Delta P_{k,m}$ , $k = 1, \ldots K$;
                $i = \arg\max_k \{\Delta P_{k,m}\}$;
                $\sigma_{i,m} = 1$;
                $D = D \backslash \{m\}$;
            end if
        end for
```

Table 3 Sub-algorithm 3

```
step1: initialize $U_m, \forall m$;
step2: $P_{\min} = P(U_1 \ldots U_M)$;
        for $m = 1 : M$
            for $n = 1 : N$
                if $P_{\min} > P(U_1 \ldots U_m^n \ldots U_M)$
                    $P_{\min} = P(U_1 \ldots U_m^n \ldots U_M)$;
                    $U_m = U_m^n$;
                end
            end
        end
```

4 Simulation Results

In this section, simulation results are presented to demonstrate the performance of the proposed power allocation algorithm. We assume that each user's subcarrier signal undergoes identical Rayleigh fading independently. And the average channel gain of each subcarrier obeys the distribution of N(1,1). Each user has the same BER requirement and transmission bit rate requirement. The simulation parameters are given in Table 4.

Figure 2 shows the power consumption of the BS versus the number of active time slots. We can see that, for any transmission bit rate requirement, there is always an optimal number of active time slots which minimizes the power consumption of the BS. The reason is that the number of active time slots is inversely proportional to the sleep time of the BS and proportional to the transmitting power of the BS in active time slots. And it is also observed that the optimal number of active time slots changes with the increasing of

Table 4 Simulation parameters

Variable	Explanation	Value
K	Number of users	10
M	Number of subcarriers	50
T	Number of time slots	10
N_T	Number of transmit antennas	4
N_R	Number of receive antennas	2
P_o	Circuit power in active mode	18 W
Δp	Load-dependence factor	4.7
P_s	Circuit power in sleep mode	15 W
τ	Duration of time slot	1 ms
ω	Subcarrier bandwidth	20 kHZ
N_o	Power of noise	10^{-4} W
BER	Bit error rate	0.001

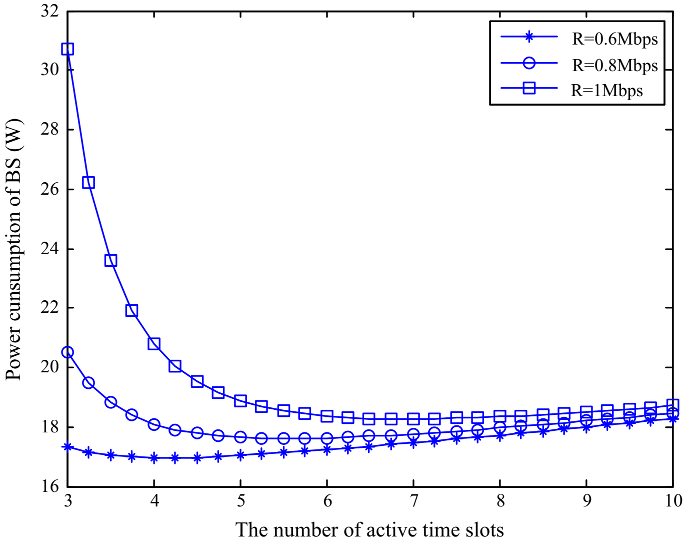

Fig. 2 Power consumption of the BS versus the number of active time slots

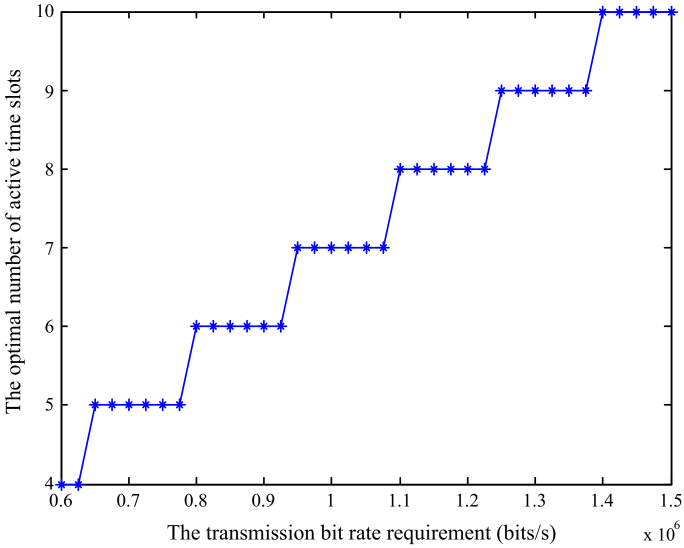

Fig. 3 The optimal number of active time slots versus the transmission bit rate requirement

transmission bit rate requirement. Nevertheless, the optimal number of active time slots can be obtained by one-dimensional search if Sub-algorithm 1 used in the proposed scheme. The optimal number of active time slots versus the transmission bit rate requirement is showed in Fig. 3.

In Fig. 4, the number of users is $K = 4$ and other parameters are in Table 4. Figure 4 compares the power consumption of three power optimization schemes:

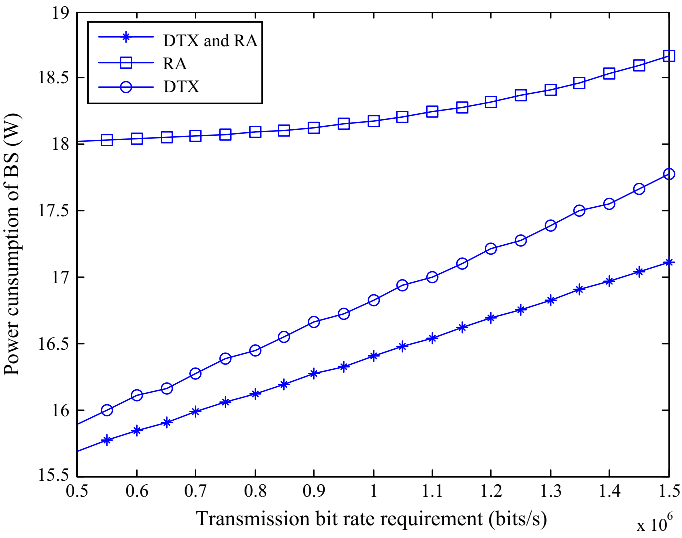

Fig. 4 Power consumption of the BS with three power optimizing themes

(a) RA only: Sub-algorithm 2 and Sub-algorithm 3.
(b) DTX only: DTX and Sub-algorithm 1.
(c) Joint DTX and RA: The algorithm proposed in this paper.

We can see that the performance of the scheme with DTX only is better than that of the scheme with RA only and the gap between the two schemes decreases continuously with the increase of transmission bit rate requirements. The scheme with joint DTX and RA has the least power consumption at any transmission bit rate.

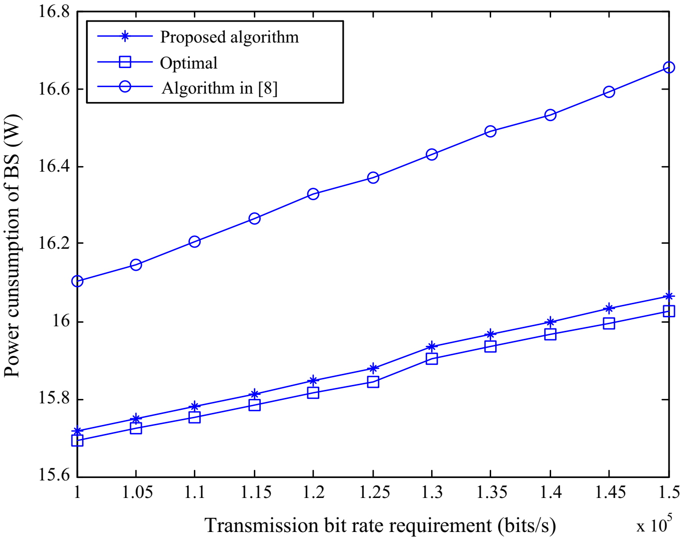

Fig. 5 Power consumption comparison among the proposed algorithm, the algorithm in [8] and the optimal solution

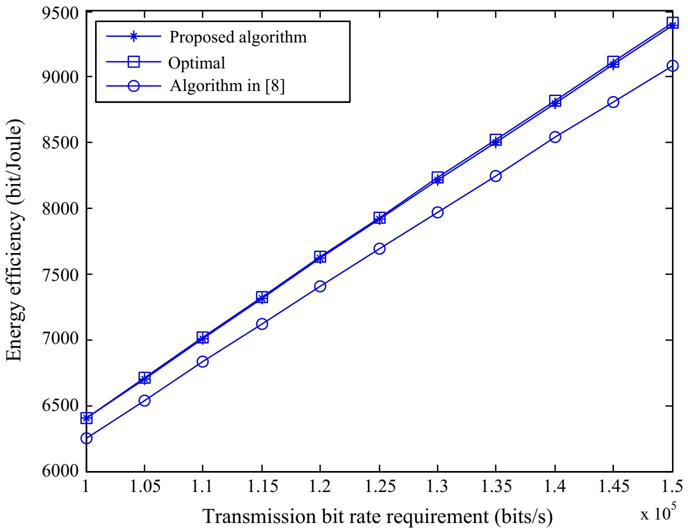

Fig. 6 Energy efficiency comparison among the proposed algorithm, the algorithm in [8] and the optimal solution

For simplicity, in Figs. 5 and 6, the number of users is $K = 4$, the number of subcarriers is $M = 4$ and other parameters are listed in Table 4. The power consumption and energy efficiency comparison among the proposed algorithm, the algorithm in [8] and the optimal solution are shown in Figs. 5 and 6, respectively. The optimal solution is obtained through the exhaustion method. Due to DTX and SDMA, the proposed algorithm can allocate radio resource in time, frequency and space domain while the algorithm in [8] only can allocate radio resource in time and space domain, which leads to the loss of flexibility in resource allocation. Although, the implement of SDMA increases the system complexity, which makes us can only obtain a suboptimal allocation of the resources, the proposed algorithm still performs better than the algorithm in [8] and is close to the optimal solution in terms of power consumption and energy efficiency.

5 Conclusions

We propose a new radio resource allocation algorithm to minimize the power consumption of the base station for the MIMO-OFDM systems. The DTX is applied in the systems with mixed OFDMA and SDMA. Radio resources in time, frequency and space domain are optimally allocated to users to minimize the power consumption of the BS and guarantee the QoS requirements of the users. Specifically, the resource allocation is divided in two steps: estimate the active time in the DTX mode and allocate subcarriers, bits and power in the active time slots. Simulation results show that the proposed algorithm can reduce the power consumption of the base station and improve the utilization of resource.

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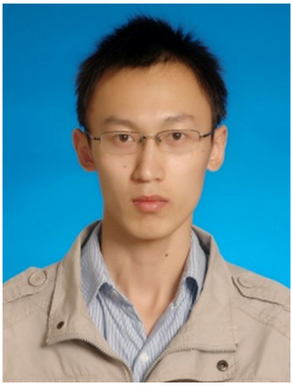

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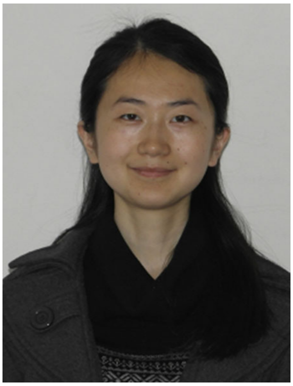

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