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A New Method for Continuous Wave Interference Mitigation in Single-Frequency GPS Receivers

M. R. Mosavi[1] · M. S. Moghaddasi[1] · M. J. Rezaei[1]

Abstract This paper proposes a new method for jamming suppression in the global positioning system (GPS) receivers. The proposed mitigation technique is based on cascading the adaptive finite impulse response (FIR) filter and the approximate conditional mean (ACM) filter. Adaptive FIR filter puts a notch at the interference frequency, but it causes self-noise in presence of high power jammers. ACM filter is an effective filter for estimating the jamming with known autoregressive process model parameters. However, its performance is degraded in the cases of high power interferences. Cascading these two filters has two advantages: first, collaboration of these two filters can help removing high power jammers. Second, the depth of notch can be limited by a threshold in presence of high power jammers to prevent self-noise effect. In such a case, the ACM filter can help to remove the remaining jamming effects on GPS signal. The performance of proposed method is analyzed in presence of single and multiple continuous wave interferences. Experimental results show the benefits of proposed method on terms of signal to noise power ratio improvement and mean square error compared to previous methods.

✉ M. R. Mosavi
M_Mosavi@iust.ac.ir

M. S. Moghaddasi
Salamat1989@elec.iust.ac.ir

M. J. Rezaei
Mj_Rezaei@iust.ac.ir

[1] Department of Electrical Engineering, Iran University of Science and Technology, Narmak, Tehran 16846-13114, Iran

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1 Introduction

Global positioning system (GPS) is a code division multiple access (CDMA) system that employs direct sequence spread spectrum (DSSS) signals. Spread spectrum systems are inherently able to mitigate the intentional interference (jamming) and non-intentional interference. However, as the jammer power increases, the spreading gain may be insufficient to suppress the interference. Therefore, many anti-jamming techniques have been employed for interference mitigation in GPS receivers [1].

There are different anti-jamming methods for GPS receivers which are mainly classified in three groups: adaptive antennas based methods, time–frequency filtering and adaptive filtering based methods [1–3].

Adaptive antennas based methods employ spatial and temporal filters to overcome both wideband and narrowband jammers. The main drawbacks of these methods are their computational complexity and their need for additional tools [4–7].

Time–frequency representation of a signal gives useful information in both time and frequency domains simultaneously [8]. Time–frequency filters are mainly used for narrowband jamming mitigation. However, some of them can be combined with an antennas array to overcome wideband jamming [5]. There are different tools for time–frequency filtering: short time Fourier transform (STFT) [9], filter banks [10], wavelet transform (WT) [11, 12] and subspace projection [13]. The STFT and the filter banks cannot model most of the non-stationary signals because they use fixed windows. The WT overcomes STFT and filter bank drawback by controlling the tradeoff between frequency and time. However, it needs to be tuned optimally for each kind of jamming [14].

Adaptive filtering based methods like adaptive notch filters [15–18], Kalman filter [19], neural network based predictors [19, 20], approximate conditional mean (ACM) filter [21, 22] and augmented state ACM (ASACM) filter [23, 24] are formed based on jamming estimation in frequency domain or time domain. These techniques can be implemented easily in GPS digital signal processors (DSPs) and therefore, they are applicable. Adaptive filtering methods can be used for suppression of continuous wave interferences (CWIs) and other kinds of narrowband jamming signals. However, adaptive filtering methods suffer from some disadvantages. Kalman and ACM filters do not show appropriate performance when a priori knowledge of the jamming model parameters is not available. ASACM filter computational complexity increases exponentially in the cases of multi-tone CWI rejection. Also this filter must be combined with a discrete wavelet transform (DWT) filter to overcome jamming attacks with jammer to signal power ratio (JSR) more than 35 dB [19–24]. In the notch filters, a zero is placed on a unit circle with a phase equal to the jammer instantaneous frequency (IF) to perfectly remove the jamming [15]. However, a filter of such zero characteristics creates a significant amount of self-noise due to the large correlation introduced across the different chips of the pseudo-noise (PN) sequence [16]. In the adaptive notch filters, the depth of excision notch filters is adjusted respect to the jammer power and thus, in the cases of high power jammers, a deep notch is needed for the perfect jamming removal. Hence, adaptive notch filters introduce self-noise which drastically reduces the receiver signal to noise power ratio (SNR) in high power jamming rejection cases [15–18].

This paper proposes an anti-jamming method based on cascading an adaptive notch filter with an ACM filter. Proposed method overcomes ACM and adaptive notch filter main drawbacks by sharing responsibility of jamming mitigation between these two filters. Similarly, it overcomes self-noise effect of notch filter by using an ACM filter and also

putting a limitation on depth of notch. As the proposed method can suppress jammers with JSR up to 60 dB, it can overcome ASACM filter limitation, too.

This paper is organized as follows. We first introduce the received signal structure and the concept of adaptive notch filter and ACM filter in Sect. 2. In Sect. 3, we explain proposed jamming mitigation technique based on cascading notch filter and ACM filter. Experimental results are given in Sect. 4 to show the improvement of the GPS receiver performance in different jamming scenarios. Finally, we present conclusion in Sect. 5.

2 Adaptive Filters for CWI Suppression

Before introducing the concept of mitigation filters, we need to know about the received signal $r(t)$ characteristics. As illustrated in Eq. (1), $r(t)$ is the combination of GPS signal $s(t)$, noise $n(t)$ and jamming signal $j(t)$. In anti-jamming applications, the combination of GPS signal and noise has to be retrieved from the received signal. The GPS signal $s(t)$ is a DSSS binary phase shift keying (BPSK) modulated signal [23] which is depicted in Eq. (2). Noise signal $n(t)$ is a white Gaussian noise with noise level σ^2. In this paper, we assumed that the jammer is in the class of CWI. CWI is one of the main classes of the jamming signals which include all signals that can be represented by pure sinusoids.

$$r(t) = s(t) + n(t) + j(t) \tag{1}$$

where $s(t)$ indicates the GPS L1 signal that is made from course acquisition (C/A) code sequence with chip rate *1.023* MHz and the transmitted navigation data which is a binary data with duration $T = 20$ ms. The GPS L1 carrier frequency is *1575.42* MHz. The transmitted signal may be represented as:

$$s(t) = \sum_{k} b(k)m(t - kT_b) \tag{2}$$

where $b(k)$ is the binary data sequence. T_b is the bit duration and it is equal to $L\tau_c$. L is the number of PN chips per message bit, and τ_c is the chip interval. $m(t)$ is the DSSS signal and it is represented as [23]:

$$m(t) = \sum_{k=0}^{L-1} c(k)q(t - k\tau_c) \tag{3}$$

where $c(k)$ is the kth PN code generator output sequence, and $q(t)$ is the rectangular chip pulse with duration τ_c.

2.1 Adaptive Notch Filter

A useful tool for removing jammers which are in the class of CWI, is the finite impulse response (FIR) notch filter. It is simple to implement on most DSPs because its calculations can be done by looping a single instruction. It also can easily be designed to be linear phase. FIR filters can be implemented using few bits and therefore the designer practical problems to solve non-ideal arithmetic would be few. Moreover, FIR filter can be implemented on fixed-point DSPs because it can be implemented using fractional arithmetic. Hence, it is a good choice for single-frequency GPS receivers. An interference mitigation method based on an adaptive three-coefficient FIR filter has been proposed in

[15]. This method processes the received signal by a short length time varying FIR filter. The notch is placed at the jammer frequency. The depth of notch is controlled by parameter a. The value of this parameter is determined so that the receiver SNR gets the minimum value. The concept of a simple adaptive three-coefficient FIR filter that can suppress a single CWI is introduced in the following [15]:

$$H(z) = z^{-1}(z - ae^{-j\omega_0})(1 - az^{-1}e^{j\omega_0}) = 1 - 2az^{-1}\cos\omega_0 + a^2 z^{-2} \quad (4)$$

where the parameter a represents the amplitude of the filter zero. The depth of notch is determined by its value. ω_0 is the jammer frequency and the notch is placed on it. Inverse Z transform of $H(z)$ gives the filter impulse response:

$$h(k) = \delta(k) - (2a\cos\omega_0)\delta(k-1) + a^2\delta(k-2) \quad (5)$$

The receiver SNR is calculated based on Eq. (6):

$$SNR_0 = \frac{(E\{y\})^2}{\sigma_y^2} = \frac{L^2 h_0^2}{L(1+\sigma_n^2)\sum_{k=0}^{N} h_k^2 - L h_0^2 + \sigma_{j0}^2} \quad (6)$$

where y is the output of the receiver correlator, σ_{j0} is the power of jammer at the output of correlator, and h_k for $k = 1, 2, \ldots, N$, are the filter coefficients. In this case, the filter coefficients are $h_0 = 1$, $h_1 = -2a\cos(\omega_0)$, and $h_2 = a^2$.

Consider a single CWI of the form $j(k) = A\sin(k\omega_0 + \varphi)$, where A and φ are the interference amplitude and phase, respectively. The correlator output due to this jammer is depicted in Eq. (7) [15]:

$$\sigma_{j0}^2 \approx \frac{LA^2}{2}[(1-a)^4 + 2a(1-a)^2(1-\cos 2\omega_0)] \quad (7)$$

The receiver SNR can be calculated by replacing filters coefficient from Eq. (5), and σ_{j0} from Eq. (7) into Eq. (6):

$$SNR_0 = \frac{L}{(1+\sigma^2)(1+a^4+4a^2\cos^2\omega_0) - 1 + A^2(1-a)^2\left[\frac{1}{2}+\frac{a^2}{2}-a\cos 2\omega_0\right]} \quad (8)$$

The maximum SNR can be achieved by minimizing the denominator of Eq. (8). If we write the denominator of Eq. (8) as a function of a, then $f'(a)$ can be calculated from Eq. (9). Now the optimized a can be computed from $f'(a) = 0$.

$$\begin{aligned} f'(a) = {} & a^3(4 + 4\sigma^2 + 2A^2) + a^2(-3A^2 - 3A^2\cos 2\omega_0) \\ & + a[8(1+\sigma^2)\cos^2\omega_0 + 2A^2 + 4A^2\cos 2\omega_0] - (A^2 + A^2\cos 2\omega_0) \end{aligned} \quad (9)$$

2.2 ACM Filter

The ACM filter has a structure similar to the standard Kalman–Bucy filter. Since a wide class of jamming signals can be modeled as an autoregressive process (AR), the ACM filter is used to estimate such these kinds of interferences [23]. The ACM filter is developed by Masreliez [21] to estimate the state of a system with Gaussian state noise and non-Gaussian measurement noise [22]. The nature of the nonlinearity in the ACM filter makes it suitable to deal with spread-spectrum GPS signals. The nonlinearities take the form of a soft

decision feedback, which seeks to remove the GPS signal from the estimation of the narrowband interference [24].

The order p, AR model of the narrow band interference is depicted in Eq. (10) [25]:

$$j(k) = \sum_{n=1}^{p} a(n)j(k-n) + e(n) \tag{10}$$

where $a(n)$ for $n = 1,2, \ldots, p$ are the AR parameters and they are known for the GPS receiver. $e(n)$ is a white Gaussian process. In this section, it is assumed that the jammer frequency is known. Therefore, the received signal $r(k)$ can be modeled by the following state space representation:

$$X(k) = \Phi X(k-1) + w(k) \tag{11}$$

$$r(k) = HX(k) + v(k) \tag{12}$$

where $X(k)$ is the state vector, $w(k)$ is the Gaussian process, Φ is the state transition matrix, and $v(k)$ is the observation noise. $X(k)$, $w(k)$, $v(k)$, Φ and matrix H are depicted in Eqs. (13), (14), (15), (16) and (17), respectively [25, 26].

$$X(k) = [j(k-p+1) \quad \ldots \quad j(k-1) \quad j(k)]^T \tag{13}$$

$$w(k) = [0 \quad 0 \quad \ldots \quad e(k)]^T \tag{14}$$

$$v(k) = s(k) + n(k) \tag{15}$$

$$\Phi = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a(1) & a(2) & \ldots & a(p-1) & a(p) \end{bmatrix} \tag{16}$$

$$H = [0 \quad 0 \quad \ldots \quad 1] \tag{17}$$

As the last component of the state vector is the interference $j(k)$, the interference can be obtained by estimating the state. Then it can be subtracted from the received signal. If the observation noise was Gaussian then the Kalman filter would be the optimal estimator. But in this case, the observation noise $v(k)$, is the summation of two independent random variables, the GPS signal $s(k)$ takes on $+1$ or -1 with equal probability and white noise $w(k)$ is Gaussian. Hence, observation noise density is the weighted sum of two Gaussian densities. In such a case, the ACM filter is a better choice than the Kalman filter. With the assumption that the state prediction density $p(X(k)|R^{k-1})$ is Gaussian with mean $\bar{X}(k)$ and covariance matrix M_k, the ACM filter is derived [22]. It must be noted that the R^k, denotes the set of observations recorded up to time k. The filtered estimate $\hat{X}(k)$ and its conditional covariance $P(k)$ are obtained recursively by the following equations [22]:

$$\hat{X}(k) = \bar{X}(k) + M(k)H^T g_k(r(k)) \tag{18}$$

$$P(k) = M(k) - M(k)H^T G_k(r(k))HM(k) \tag{19}$$

$$M(k) = \Phi P(k-1)\Phi^T + Q(k-1) \tag{20}$$

$$\bar{X}(k) = \Phi\hat{X}(k-1) \tag{21}$$

where $Q(k)$ is the covariance matrix of the process noise and $Q(k) = E\{w(k)w^T(k)\}$. As mentioned before, the state prediction density $p(X(k)|R^{k-1})$ assumed to be Gaussian. Since v_k is independent of R^{k-1}, the observation prediction density can be expressed as:

$$
\begin{aligned}
p(r(k)|R^{k-1}) = \frac{1}{2}[&N_{(HM_kH^T+\sigma_n^2)}(r(k) - H\bar{X}(k) - 1) \\
&+ N_{(HM_kH^T+\sigma_n^2)}(r(k) - H\bar{X}(k) + 1)]
\end{aligned}
\tag{22}
$$

Nonlinearities arising from the non-Gaussian observation noise are denoted by $g(k)$ and $G(k)$:

$$
\begin{aligned}
g_k(r(k)) &= -\left[\frac{\partial p(r(k)|R^{k-1})}{\partial r(k)}\right] \cdot \left[p(r(k)|R^{k-1})\right]^{-1} \\
&= \frac{1}{(HM_kH^T + \sigma_n^2)}\left[r(k) - H\bar{X}(k) - \tanh(\frac{r(k) - H\bar{X}(k)}{HM_kH^T + \sigma_n^2})\right]
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
G_k(r(k)) &= \frac{\partial g_k(r(k))}{\partial r(k)} \\
&= \frac{1}{(HM_kH^T + \sigma_n^2)}\left[1 - \frac{1}{(HM_kH^T + \sigma_n^2)} \cdot \sec h^2\left(\frac{r(k) - H\bar{X}(k)}{HM_kH^T + \sigma_n^2}\right)\right]
\end{aligned}
\tag{24}
$$

3 Proposed Cascade Filter

As it was mentioned before, for the case of high power jamming mitigation, adaptive notch filter results in receiver SNR reduction despite the depth of notch is controlled by optimized parameter a. The optimum a in such these cases is unity. Choice of this value for a, results in maximum depth of notch and it results in self-noise effect. Moreover, the ACM filter does not mitigate strong jamming signals perfectly.

To overcome the disadvantages of discussed filters, we propose a cascade of adaptive notch and ACM filter for jamming suppression. A simplified block diagram of the anti-jamming cascade filter is shown in Fig. 1. The goal of the cascade filter is to retrieve the original signal $s(k)$ from the jammed signal $r(t)$ for a broad range of JSR.

In the Fig. 1, $\hat{s}$ explains the noise free signal that estimated by cascade filter. As illustrated in Fig. 1, the first block of the cascade filter is the adaptive notch filter. The
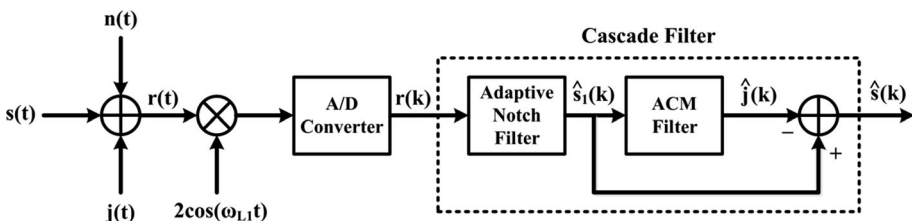

**Fig. 1** Block diagram of the GPS anti-jamming system

input of this block is $r(k)$ [digitized version of $r(t)$ in Eq. (1)]. We limit the depth of notch by a threshold $\delta-$. We perform numerous tests to determine the near optimum threshold for each kind of jammer (single-tone CWI and multi-tone CWI), separately. Hence, the parameter a, did not get unity value even for high power jammers.

Optimum performance of the proposed cascade filter belongs to the contribution of each filter in suppressing jammers. So, we defined three working modes for the first filter based on parameter a value: First mode for low values of a, second mode for middle values of a and third mode for high values of a. we use experimental value of the notch depth a_{exp} in the proposed algorithm:

$$a_{\exp} = \begin{cases} a & a < 0.2 \\ k_m \cdot a & 0.2 \leq a < 0.8 \\ \delta & a \geq 0.8 \end{cases} \quad (25)$$

In the first mode, the optimum calculated a, is used as the notch depth. In the second mode, the depth of notch is adjusted by modification factor k_m. Hence, the depth of notch in the second mode is set to k_m*a based on trial experiments and the best value for k_m factor is 0.5. In the third mode, the depth of notch is limited by threshold $\delta = 0.4$. Applying this threshold limits the self-noise effect of deep notch filter.

It must be noted that the a_{exp} [Eq. (25)] is derived based on the experimental results. The results of the Fig. 2 are used for the extraction of Eq. (25). For the extraction of the Fig. 2 and subsequently for the derivation of the Eq. (25) a set of experiments are performed. In the each experiment, an FIR notch filter with optimum depth of notch [optimum parameter a from Eq. (9)] is employed to reject the jamming signal. The mean square error (MSE) of the retrieved signal is calculated in each experiment [based on Eq. (27)]. The experiments are done for both single-tone and multi-tone CWI rejection. It is obvious from Fig. 2 that the increase of the MSE is occurred in two phases. In the first phase (a = 0.2–0.8), the slop of the MSE growth is low. Hence, we share the responsibility of the jamming mitigation between the FIR and ACM filter, equally. In the second phase (a = 0.8–1.0), the slope of the MSE growth is high. Hence, we limit the depth of notch filter to overcome the self-noise effect of the FIR filter. The $k_m = 0.5$ was chosen for two reasons: first, to avoid inconsistency in the lines 2 and 3 of the Eq. (25); Second, trail experiments are proven that equal division of duty to deal with the jamming in the $0.2 < a < 0.8$ between notch and ACM filters is a proper decision.
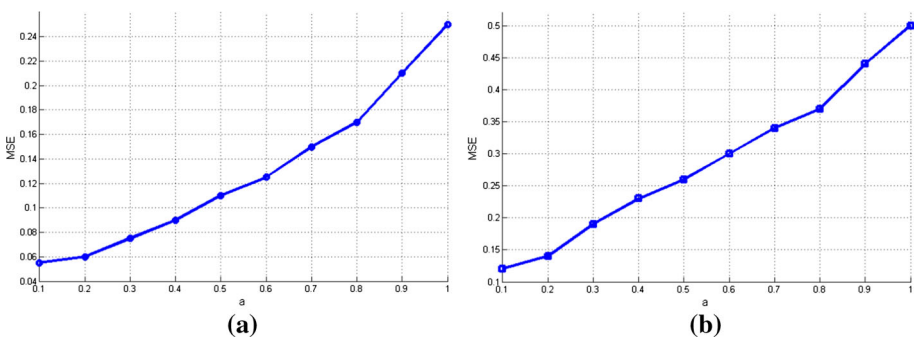
Fig. 2 MSE for different parameter a values **a** single-tone CWI mitigation and **b** multi-tone CWI mitigation

As it was depicted in Fig. 1, the second block of the cascade filter is the ACM filter. The input of this block is $\hat{s}_1(k)$. In the proposed filter, the $r(k)$ in Eq. (12) is replaced by $\hat{s}_1(k)$. The residual jammer effect is mitigated by second block of the cascade filter. As a portion of jammer effect is removed by first block, the ACM filter input is a GPS signal which contaminated by a low power jammer. Therefore, the ACM filter can show its optimum performance.

It is illustrated in Fig. 3 that in the presence of effective jammers, the GPS signal is not amplified sufficiently in the receiver. Hence, the thermal noise observation would be dominated and characteristics of the signal would be hidden in the noise. Therefore, our proposed anti-jamming method removes the signal $j(t)$ in order to retrieve the GPS signal. Then the estimated signal can be fed to acquisition and tracking units of the GPS receiver.

The proposed anti-jamming method is a pre-correlation technique and can be implemented as a plug-in filter in the GPS receiver. Furthermore, the proposed jammer canceller comprises entirely digital signal processing. As the cascade filter can be implemented on low cost digital signal processors, proposed technique is a practical real-time and low power anti-jamming method for GPS receivers.

4 Experimental Results

As shown in Fig. 4, A test setup is provided to examine the proposed cascade filter. It consists of a GPS receiver, a combiner, two GPS antennas, a spectrum analyzer, an RF signal generator, and a computer. RF signal generator is employed to generate jamming signals. Generated jamming signal and GPS signal are combined in a combiner. Then the combined signal is fed to the GPS receiver. The received RF signal is band-pass filtered, amplified and down converted to IF in GPS Receiver. Then this signal is converted to digital form in the 2-bits A/D converter of the receiver.

A GPS software receiver is employed to speed up our numerous test scenarios. The GPS software block diagram is illustrated in Fig. 5. Digital IF is fed to anti-jammer block (cascade filter) and the anti-jammer output is sent to acquisition and tracking units of the GPS software receiver. Digital IF center frequency is *1.405* MHz. Our GPS receiver A/D converter sampling frequency is *5.714* MHz. Therefore, the corresponding value in the GPS software receiver is set to *5.714* MHz.

Proposed cascade filter has been tested with two measured datasets. In the first data set, the single-tone CWI is added to the received signal. In the second data set, the multi-tone CWI is added to the received GPS signal. The performance of the proposed filter is expressed in terms of number of acquired satellites, SNR improvement, and MSE.

Fig. 3 Frequency domain of GPS signal, thermal noise and single-tone CWI
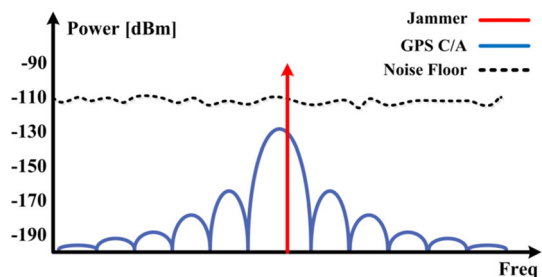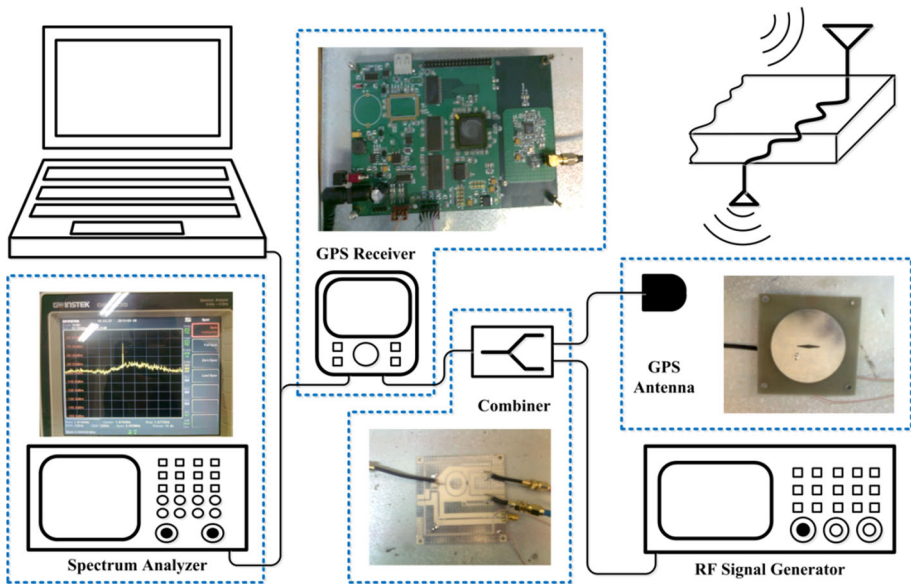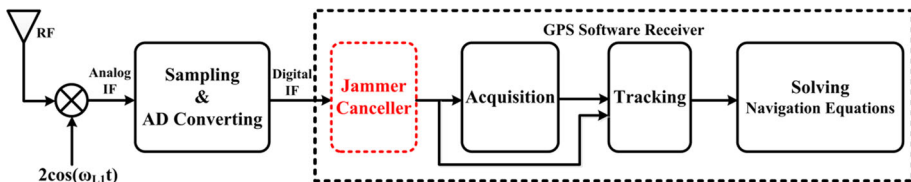
Fig. 4 The laboratory platform scheme

Fig. 5 GPS software block diagram

$$\text{SNR improvement} = \frac{E(|r(k) - s(k)|^2)}{E(|\hat{s}(k) - s(k)|^2)} \tag{26}$$

$$MSE = \frac{1}{N} \sum_{k=1}^{k=N} [\hat{s}(k) - s(k)]^2 \tag{27}$$

In the presence of an effective jamming signal, less than four satellites can be acquired by the acquisition unit. Therefore, the navigation process cannot be performed by the GPS receiver. Hence, the ability of any anti-jamming method for retrieving missed satellites must be evaluated at first.

4.1 Single-Tone CWI Mitigation

The first data set is composed from the received GPS signal and the single-tone CWI. The received signal that is saved in a test interval contains $N = 62,854$ samples. The considered single-tone CWI is sinusoidal signal. Its frequency is selected from *0.4* to *2.4* MHz

range. The frequency domain digital IF of original signal and jammed signal (JSR = 50 dB) are shown in Figs. 6a and 7a, respectively. Acquisition results (number of acquired satellites) of these two signals are also depicted in Figs. 6b and 7b, respectively. As the employed jammer (RF signal generator) could not generate clear single-tone, we observed multiple peaks (instead of a single peak) in the jammed signal spectrum. Figure 7c and d are presented the frequency domain digital IF and the acquisition results of estimated signal.

It is illustrated in Fig. 7c that the jamming signal is removed from the GPS signal spectrum. Figure 7b shows that there is only one visible satellite in the presence of the single-tone CWI. However, Fig. 7d reveals that the proposed anti-jamming technique can retrieve five satellites.

Performance of cascade filter is analyzed in terms of SNR improvement and MSE for different JSR values. The JSR value is varied from 40 to 60 dB by the steps of 5 dB. The SNR improvement for estimated GPS signal is presented in Fig. 8a. Figure 8b illustrates the MSE progress of proposed filter compared to adaptive notch filter [15], ACM filter [22].

It is shown in Fig. 8b that for the JSRs lower than 44 dB, there are small differences between MSE curves of proposed filter, ACM filter and notch filter. It is due to this fact that the single ACM or Notch filter has a good performance in the presence of low power jamming signals.

4.2 Multi-Tone CWI Mitigation

Multi-tone CWI is composed of three sinusoidal signals with frequencies of *1.4 MHz*, *1.9 MHz* and *2.4 MHz* as depicted in Fig. 9a. In the first step, adaptive notch filter block processes multiple-tones based on their power (the most powerful jamming tone is processed at first). In the second step, the output of the first block is fed to the ACM filter block. ACM filter block processes multiple-tones based on their power, too. Acquisition results of jammed signal with multi-tone CWI is shown in Fig. 9b. The results show that no satellite can be acquired by the GPS receiver in presence of multi-tone CWI with JSR = 60 dB. The frequency domain digital IF and the acquisition results of estimated signal are presented in Fig. 9c and d, respectively.
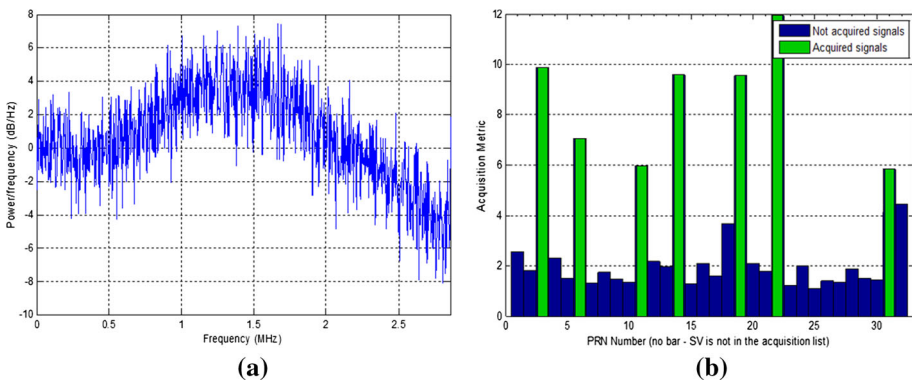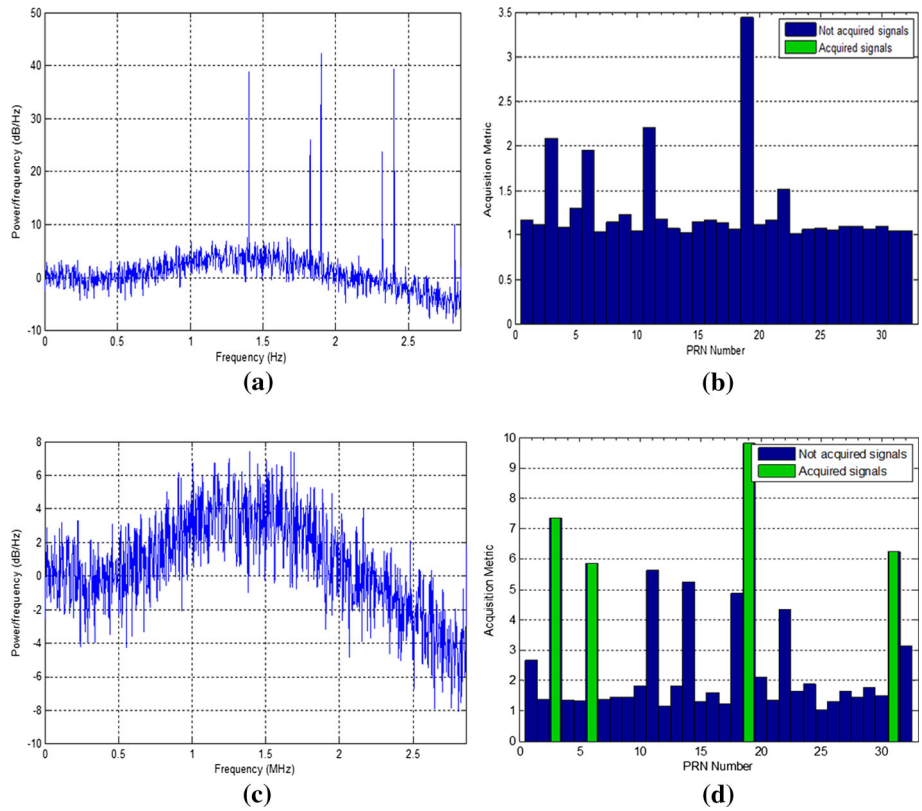

Fig. 6 **a** Frequency domain digital IF of the received signal (without jammer) and **b** acquisition results of the received signal (without jammer)
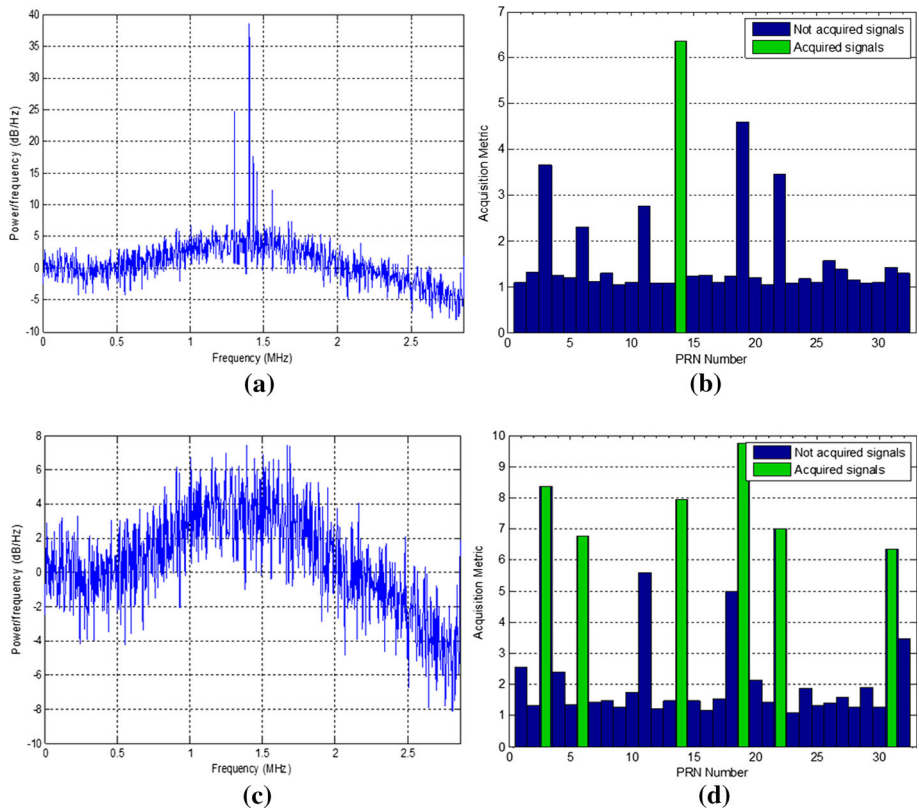
Fig. 7 **a** Frequency domain digital IF of the jammed signal (single-tone CWI, JSR = 50 dB), **b** acquisition results of the jammed signal (single-tone CWI, JSR = 50 dB), **c** frequency domain digital IF of the estimated signal (in presence of single-tone CWI) and **d** acquisition results of the estimated signal (in presence of single-tone CWI)
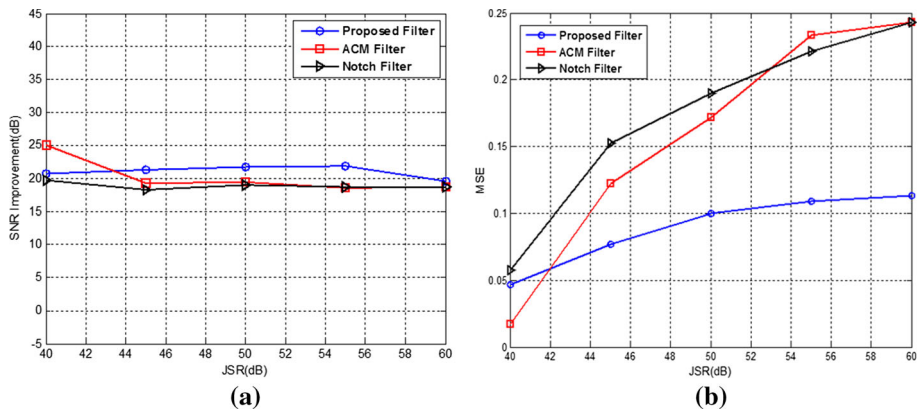
Fig. 8 **a** SNR improvement versus JSR for the estimated signal (single-tone CWI) and **b** MSE Vs. JSR for the estimated signal (single-tone CWI)
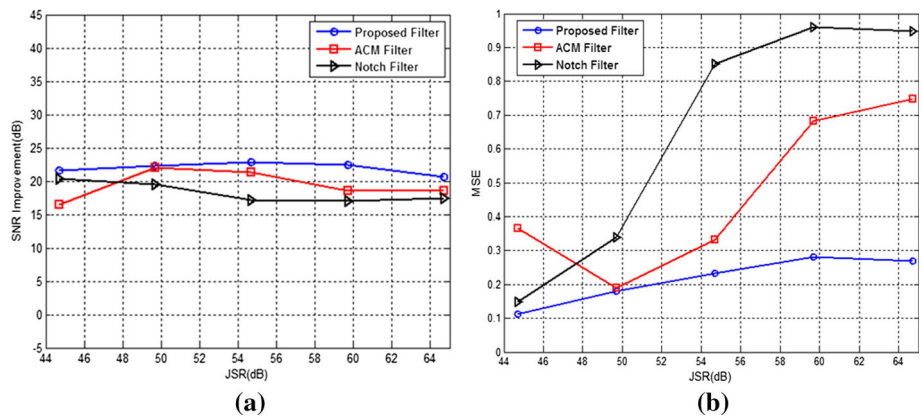
Fig. 9 **a** Frequency domain digital IF of the jammed signal (multi-tone CWI, JSR = 60 dB), **b** acquisition results of the jammed signal (multi-tone CWI, JSR = 60 dB), **c** frequency domain digital IF of the estimated signal (in presence of multi-tone CWI) and **d** acquisition results of the estimated signal (in presence of multi-tone CWI)

Fig. 10 **a** SNR improvement versus JSR for the estimated signal (multi-tone CWI) and **b** MSE versus JSR for the estimated signal (multi-tone CWI)

Figure 9c reveals that all the jamming components are removed from the received signal spectrum. Acquisition result of the retrieved signal (Fig. 9d) shows that the proposed anti-jamming algorithm can retrieve four satellites. Therefore, it can suppress multi-tone jamming signals with JSR up to 60 dB.

The SNR improvement for estimated signal by proposed filter, adaptive notch filter [15] and ACM filter [22] is presented in Fig. 10a. Figure 10b illustrates the MSE progress of proposed filter compared to adaptive notch filter [15], ACM filter [22].

Figures 8a and 10a demonstrate that proposed cascade filter has a good improvement over the adaptive notch filter [15] and ACM filter [22], in term of SNR improvement. From Figs. 8b and 10b it is observed that the MSE is reduced considerably by the proposed filter. In order to provide a better comparison, Tables 1, 2, 3 and 4 are presented.

Table 1 SNR improvement of proposed cascade filter compared to adaptive notch filter and ACM filter (single-tone CWI mitigation)

JSR (dB)	Notch filter [15]	ACM filter [22]	Proposed filter
40	19.78	25.11	20.72
45	18.29	19.27	21.27
50	19	19.43	21.78
55	18.73	18.5	21.82
60	18.65	18.66	19.59

Table 2 SNR improvement of proposed cascade filter compared to adaptive notch filter and ACM filter (multi-tone CWI mitigation)

JSR (dB)	Notch filter [15]	ACM filter [22]	Proposed filter
45	20.41	16.53	21.62
50	19.54	22.06	22.32
55	17.2	21.3	22.84
60	17.06	18.54	22.41
65	17.45	18.49	20.56

Table 3 MSE of proposed cascade filter compared to adaptive notch filter and ACM filter (single-tone CWI mitigation)

JSR (dB)	Notch filter [15]	ACM filter [22]	Proposed filter
40	0.05	0.01	0.04
45	0.15	0.12	0.07
50	0.18	0.17	0.10
55	0.22	0.23	0.10
60	0.24	0.24	0.11

Table 4 MSE of proposed cascade filter compared to adaptive notch filter and ACM filter (multi-tone CWI mitigation)

JSR (dB)	Notch filter [15]	ACM filter [22]	Proposed filter
45	0.14	0.36	0.11
50	0.34	0.19	0.17
55	0.85	0.33	0.23
60	0.96	0.68	0.28
65	0.94	0.74	0.26

Tables 1 and 2, indicate that in term of SNR improvement, proposed cascade filter has an average improvement of 16 and 13 % over the adaptive notch filter and ACM filter, respectively. Comparison of second and fourth columns of the Table 3 to those of the Table 4 demonstrates that the MSE value is increased in the cases of multi-tone CWI rejection. This is due to the fact that in these cases, the number of employed FIR notch filters increases proportional to the tones of jamming. Hence, the self-noise imposed by the rejection filter increases too. Tables 3 and 4, indicate that in term of MSE, proposed filter has an average improvement of 51 and 46 % over the adaptive notch filter and ACM filter, respectively.

5 Conclusions

A new cascade filter for CWI suppression in GPS receiver was proposed in this paper. The proposed filter structure was based on cascading a three coefficient adaptive notch filter with an ACM filter. Furthermore, a limitation on depth of notch was imposed to proposed filter. It reduced self-noise effect of notch filter. Therefore, the remaining jammer effect was mitigated by an ACM filter. The cascade filter was well suited for low cost and low power applications because of its simplicity and its applicability on low cost processors. The proposed cascade filter was applied to two experimental data sets: real recorded GPS signals contaminated by single-tone CWI and multi-tone CWI. The results showed that the cascade filter has a considerable progress over the adaptive notch filter and ACM filter in terms of SNR improvement and MSE. The use of fuzzy systems to determine the contribution of the notch filter and ACM filter in suppressing jammers is left for future work.

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M. R. Mosavi received his B.Sc., M.Sc., and Ph.D. degrees in Electronic Engineering from Iran University of Science and Technology (IUST), Tehran, Iran, in 1997, 1998, and 2004, respectively. He is currently faculty member of Department of Electrical Engineering of IUST as professor. He is the author of more than 250 scientific publications on journals and international conferences. His research interests include circuits and systems design.

M. S. Moghaddasi received her B.Sc. degree in Electronic Engineering from Iran University of Science and Technology (IUST), Tehran, Iran, in 2011. She is currently a M.Sc. student in the Department of Electrical Engineering at IUST. Her research interests include GPS safety and signal processing.

M. J. Rezaei received his B.Sc., and M.Sc. degrees in Electronic Engineering from Iran University of Science and Technology (IUST), Tehran, Iran, in 2009, and 2011, respectively. He is currently a Ph.D. student in the Department of Electrical Engineering at IUST. His research interests include systems design and signal processing and GPS security.