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An Analytical Discourse on Strong Edge Coloring for Interference-free Channel Assignment in Interconnection Networks

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Abstract A strong edge coloring of a graph G is a proper edge coloring in which no two edges of the same color lie within distance 2 from each other. The minimum number of colors required for strong edge coloring of a graph G is called strong chromatic index and is denoted by $\chi'_s(G)$. Channel assignment problems are closely related with strong edge coloring problem where the colors represent frequencies. In wireless networks, assigning channels or frequencies to the links between transceivers (vertices) to avoid interference can be modelled as a strong edge coloring problem. In this paper, we determine the exact values of strong chromatic indices of interconnection networks namely butterfly network, Benes network, hypertree network and honeycomb network.

Keywords Chromatic index · Strong chromatic index · Induced matching · Butterfly network · Benes network · Hypertree network · Honeycomb network

1 Introduction

Graph coloring is a well known and well examined area of graph theory that has numerous applications. A proper edge-coloring of a graph G is an assignment of colors to the edges of G such that no two adjacent edges share the same color. The chromatic index of G is the littlest number of colors utilized in the proper-edge coloring of G. The distance between two edges e and f, is the number of edges in a shortest path between an endpoint of e and an endpoint of f. A strong edge coloring of a graph G is a proper edge coloring in which no two edges of the same color lie within distance 2 from each other. The strong chromatic index of G is the minimum number of colors required for strong edge coloring of G. Each color class of a strong edge coloring of G is an induced matching M in G [4]. That is,

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strong edge coloring of G is a coloring of edges of G in which edges of the same color form a matching M where no two edges of M are joined by an edge of G. Strong edge-colouring has stimulating applications, specifically for channel assignment in mobile multi-hop radio networks [15] and in cellular networks [8].

2 An Overview of the Paper

The concept of strong edge coloring was introduced by Erdős and Nešetřil around 1985. The problem of determining the strong chromatic index of a graph is proved to be *NP–complete* [10]. A basic conjecture formulated by Erdős and Nešetřil [3] states that $\chi'_s(G) \leq \frac{5}{4}\Delta^2$ for every graph G with maximum degree Δ. Also, it is conjectured by Faudree et al. [4] that every bipartite graph of maximum degree Δ has a strong edge coloring with Δ^2 colors. Togni [19] obtained strong chromatic indices of d-dimensional grids and some toroidal grids. He also gave approximate results on the strong chromatic index of generalized hypercubes. In 2011, Chand and Liu [2] determined some cubic graphs with strong chromatic index 6. Strong chromatic index of planar graphs was studied by Hudak et al. [7]. The bound given by Hudak for planar graphs was improved by Bensmail et al. in [1]. Strong edge coloring is still in research on effective grounds in wireless networks.

Being one of the fields in wireless networks, channel assignment problems emerge in a wide assortment of certifiable circumstances. The traditional channel assignment problem is to assign a radio channel which is a nonnegative integer to every transmitter of a remote system. The channels assigned to adjacent transmitters fulfil some partition constraints in order to keep away from interference. The thought is to minimize the number of frequencies and build up obstruction free communication in remote systems. In 1980, Hale [6] planned such channel assignment problems in network engineering into a graph coloring problem where the edges of a graph signify the communication channels, and two transmitters are nearby if they are very close to one another. Moreover, interference levels might be powerful to the point that even the distinctive frequencies utilized at “close” communication channels might meddle. Thus, in 1988, Roberts [16] presented a variation of the classical channel assignment problem in which “close” communication channels within distance 2 must get diverse frequencies. Persuaded by this issue, Griggs and Yeh [5] made an interpretation of the issue into the problem of graph theory, where two transmitters are “close” in the network if they are of distance 2 in the graph.

This problem can be regarded as a problem in strong edge coloring where two edges within distance 2 from each other of a graph must get different colors (frequencies). In strong edge coloring we reuse the colors of the edges which are from two distance apart. It helps us to reduce the number of new frequencies and avoid frequency interference. It has lot of applications in interconnection networks especially in telecommunications as there is frequency interference problem. Due to the increased persuasion of strong edge coloring in interconnection networks, we examine the strong chromatic indices of butterfly network, Benes network, hypertree network and honeycomb network.

3 Strong Chromatic index of Butterfly Network

Butterfly graphs are characterized as the basic graph of FFT networks which can perform the Fast Fourier Transforms productively. The butterfly network comprises of a series of switch stages and interconnection designs, which permits n inputs to be associated with n

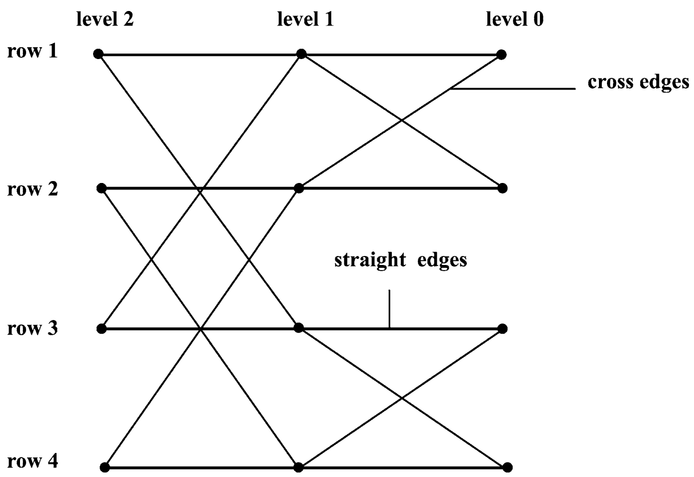


Fig. 1 *BF*(2)

outputs. The Benes network consists of back-to-back butterflies. As butterfly is known for FFT, Benes is known for permutation routing [11]. The butterfly and Benes networks are essential multistage interconnection networks, which have appealing topologies for communication networks [11]. The most commonly utilized networks as part of the routing literature and in parallel computers are butterflies. It is a bounded-degree derivative of the hypercube which aims at overcoming few disadvantages of hypercube. It is non Hamiltonian. It is hierarchically recursive [11].

The n-dimensional butterfly network $BF(n)$ has $(n+1)2^n$ vertices and $n2^{n+1}$ edges. The set V of nodes of an n-dimensional butterfly network correspond to pairs (x, j), where j is the level or dimension of a node $0 \leq j \leq n$ and x is an n-bit binary number that denotes the row of the node. The edge connecting nodes (x, j) and (x', j') is called a row edge if $x = x'$ and is called a cross edge if x and x' are unequal. Denote the row edges between two consecutive levels j and $j-1$ by e_{ji}, $1 \leq j \leq n$ and $1 \leq i \leq 2^n$. Denote the cross edges from level j to level $j-1$ (left to right) by e_i^j, $1 \leq i \leq 2^{n-1}$ and the cross edges from level $j-1$ to level j (right to left) by E_i^{j-1}, $1 \leq i \leq 2^{n-1}$. Figure 1 shows a $BF(2)$network.

3.1 Proposition

For $BF(1)$, $\chi'_s(G) = |E(G)| = |V(G)|$.

Proof $BF(1)$is a cycle C_4. For strong edge coloring, all the four edges of C_4 should be assigned with distinct colors. This completes the proof.

3.2 Lemma

For $BF(2)$, $\chi'_s(G) = 8$.

Proof We apply greedy algorithm to color all edges of $BF(2)$. Let $c(e)$denote the colour of the edge $e \in E$.

We shall first assign colors to the row edges of $BF(2)$.

The row edges between levels 1 and 0 can be assigned with the colors as follows.

$$c(e_{1i}) = \begin{cases} c_{11,} & i \equiv 1(\text{mod})2 \\ c_{12,} & i \equiv 0(\text{mod})2 \end{cases}$$

The row edges between levels 2 and 1 can be assigned with the colors as follows.

$$c(e_{2i}) = \begin{cases} c_{21,} & 1 \leq i \leq 2 \\ c_{22,} & 3 \leq i \leq 4 \end{cases}$$

Now we proceed to assign colors to the cross edges. The cross edges $\{e_i^1,\ 1 \leq i \leq 2^{n-1}\}$ connecting levels 1 and 0 are assigned with color a_1 and the cross edges $\{e_i^2,\ 1 \leq i \leq 2^{n-1}\}$ connecting levels 2 and 1 are assigned with color a_2. The cross edges $\{E_i^0,\ 1 \leq i \leq 2^{n-1}\}$ connecting levels 0 and 1 are assigned with color a_3 and the cross edges $\{E_i^1,\ 1 \leq i \leq 2^{n-1}\}$ connecting levels 1 and 2 are assigned with color a_4. Totally 8 colors are used for both the row edges and cross edges. We can see that the above coloring scheme gives a strong edge coloring. There are four cycles of length 4 in $BF(2)$of which two cycles lie between the levels 1 and 0, the other two cycles lie between levels 2 and 1. Each cycle C_4 requires 4 colors for strong edge coloring. We note that the two cycles between any two consecutive levels are disjoint and are at two distance apart. This enables us to assign the same 4 colors for the two cycles C_4 between the two consecutive levels. But the colors assigned for the cycles between the levels 1 and 0 are distinct from the colors assigned for the cycles between the levels 2 and 1. Thus the coloring scheme defined above optimally colors $BF(2)$using 8 colors.

3.3 Theorem

The strong chromatic index of $BF(n)$, $n \geq 3$ is 12.

Proof First we assign colors to the edges of $BF(3)$ between levels 0 and 3. This process includes two steps.

(1) Color all row edges
(2) Color all cross edges

Step (1)

- Between the levels 1 and 0, the row edges can be assigned with the colors as follows.
$$c(e_{1i}) = \begin{cases} c_{11,} & 0 \leq k \leq \frac{2^n - 2}{2}, & i = 2k+1 \\ c_{12}, & & i = 2k+2 \end{cases}.$$
- Between the levels 2 and 1, the row edges can be assigned with the colors as follows.
$$c(e_{2i}) = \begin{cases} c_{21}, & 0 \leq k \leq \frac{2^n - 4}{4}, & 4k+1 \leq i \leq 4k+2 \\ c_{22}, & & 4k+3 \leq i \leq 4k+4 \end{cases}$$
- Between the levels 3 and 2, the row edges are assigned with the colors as follows.
$$c(e_{3i}) = \begin{cases} c_{31}, & 0 \leq k \leq \frac{2^n - 8}{8}, & 8k+1 \leq i \leq 8k+4 \\ c_{32}, & & 8k+5 \leq i \leq 4k+8 \end{cases}$$

Step (2)

- The cross edges $\{e_i^j, 1 \le i \le 2^{n-1}, 1 \le j \le 3\}$ from the level j to the level $j-1$ are assigned with the color a_j.
- The cross edges $\{E_i^{j-1}, 1 \le i \le 2^{n-1}, 1 \le j \le 3\}$ from the level $j-1$ to the level j are assigned with the color a_{j+3}.

The above coloring algorithm defines a strong edge coloring of $BF(3)$.

For $n > 3$, $BF(n)$ consists of more than 3 levels. When $j \ge 4$, the row edges between the levels 4 and 3 are at two distance apart from the row edges between the levels 1 and 0. Therefore the colors assigned for the row edges between levels 1 and 0 can be reused for the row edges between levels 4 and 3. Similarly the row edges between the levels 2 and 1 are at two distance apart from the row edges between the levels 5 and 4 and so on. Applying this idea to the row edges of $BF(n)$, $n > 3$, we have the following coloring algorithm.

For the row edges between the levels j and $j-1, j \ge 4$, we discuss the following cases.

- When $j \equiv 1(\text{mod}3)$, $c(e_{ji}) = \begin{cases} c_{11}, & 0 \le k \le \dfrac{2^n - 2^j}{2^j}, 2^jk + 1 \le i \le 2^jk + 2^{j-1} \\ c_{12}, & 2^jk + 2^{j-1} + 1 \le i \le 2^jk + 2^j \end{cases}$
- When $j \equiv 2(\text{mod})3$, $c(e_{ji}) = \begin{cases} c_{21}, & 0 \le k \le \dfrac{2^n - 2^j}{2^j}, 2^jk + 1 \le i \le 2^jk + 2^{j-1} \\ c_{22}, & 2^jk + 2^{j-1} + 1 \le i \le 2^jk + 2^j \end{cases}$
- When $j \equiv 3(\text{mod})3$,$c(e_{ji}) = \begin{cases} c_{31}, & 0 \le k \le \dfrac{2^n - 2^j}{2^j}, 2^jk + 1 \le i \le 2^jk + 2^{j-1} \\ c_{32}, & 2^jk + 2^{j-1} + 1 \le i \le 2^jk + 2^j \end{cases}$

Now we proceed to color cross edges of $BF(n)$, $n > 3$.

The cross edges between the levels j and $j-1$, $4 \le j \le n$ can be colored as follows.

- The cross edges $\left\{e_i^j, 1 \le i \le 2^{n-1}, j \equiv 1(\text{mod})3\right\}$ between the levels j and $j-1$, are assigned with the color a_1.
- The cross edges $\left\{e_i^j, 1 \le i \le 2^{n-1}, j \equiv 2(\text{mod})3\right\}$ between the levels j and $j-1$, are assigned with the color a_2.
- The cross edges $\left\{e_i^j, 1 \le i \le 2^{n-1}, j \equiv 3(\text{mod})3\right\}$ between the levels j and $j-1$ are assigned with the color a_3.
- The cross edges $\left\{E_i^j, 1 \le i \le 2^{n-1}, j \equiv 1(\text{mod})3\right\}$ between the levels $j-1$ and j are assigned with the color a_4.
- The cross edges $\left\{E_i^j, 1 \le i \le 2^{n-1}, j \equiv 2(\text{mod})3\right\}$ between the levels $j-1$ and j are assigned with the color a_5.
- The cross edges $\left\{E_i^j, 1 \le i \le 2^{n-1}, j \equiv 3(\text{mod})3\right\}$ between the levels $j-1$ and j are assigned with the color a_6.

The above coloring algorithm gives a strong edge coloring of $BF(n)$using 12 colors. Figure 2, illustrates the strong edge coloring of $BF(3)$.

4 Strong Chromatic Index of Benes Network

In this section, we find the strong chromatic index of Benes network. Benes network consists of back to back butterflies. An n-dimensional Benes network has $2n + 1$ levels, each level with 2^n nodes [11]. The middle level of the Benes network is shared by these

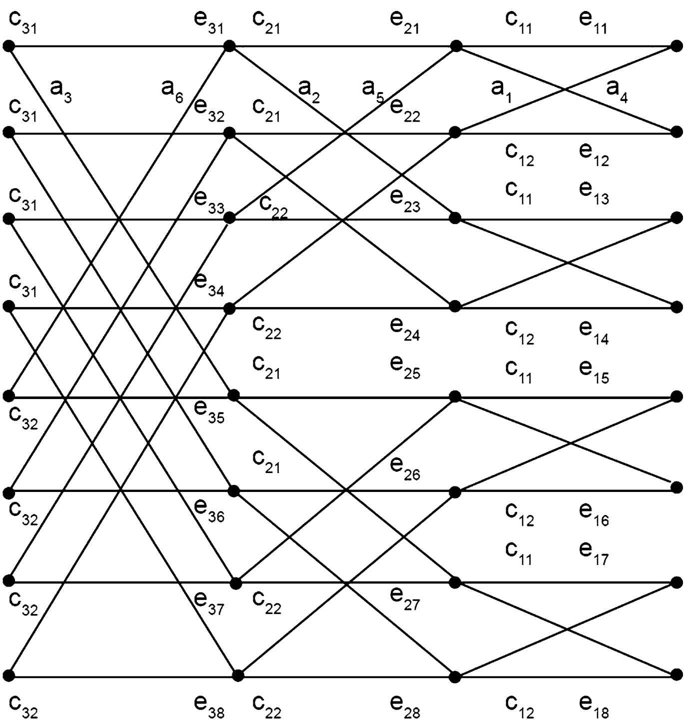


Fig. 2 *BF*(3)

butterflies [11]. An n-dimensional Benes network is denoted by $B(n)$. For convenience, we call the butterfly network lying between level n and level $2n$ of $B(n)$ as left $BF(n)$ and the butterfly network lying between level n and level 0 of $B(n)$as right $BF(n)$. An illustration is given in Fig. 3.

We use the same notation for left $BF(n)$ as given in Sect. 3. The edges of right $BF(n)$can be labeled as follows. Denote the row edges in right $BF(n)$ between level $n-j+1$ and level $n-j$, $1 \le j \le n$ by $e'_{(n-j+1)i}$, $1 \le j \le n$ and $1 \le i \le 2^n$. Denote the cross edges from level $n-j+1$ to level $n-j$ (left to right) by e'^{n-j+1}_i, $1 \le i \le 2^{n-1}$ and the cross edges from level $n-j$ to level $n-j+1$ (right to left) by E'^{n-j}_i, $1 \le i \le 2^{n-1}$.

4.1 Proposition

The strong chromatic index of $B(1)$ *is 8.*

Proof $B(1)$consists of two cycles C_4. Each cycle requires 4 colors for strong edge coloring. Hence $\chi'_s\,(B(1)) = 8$.

4.2 Lemma

The strong chromatic index of $B(2)$ *is 12.*

Proof $B(2)$ consists of two copies of $BF(2)$. The coloring algorithm given in Lemma 3.2 can be used to color the edges of left $BF(2)$ lying between the level 2 and level 4.

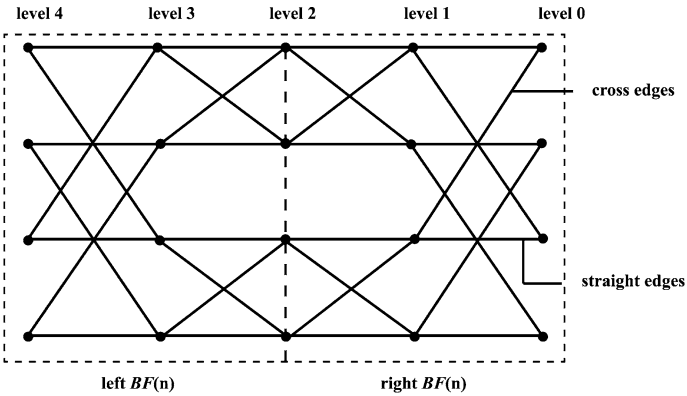


Fig. 3 $B(2)$

Let us consider right $BF(2)$. The row edges between level 1 and level 0 are assigned with the colors as follows.

$$c(e'_{1i}) = \begin{cases} c_{21,} & 1 \le i \le 2 \\ c_{22,} & 3 \le i \le 4 \end{cases}$$

Also the row edges between level 2 and level 1 are assigned with the colors as follows.

$$c(e'_{2i}) = \begin{cases} c_{31,} & i \equiv 1 \bmod 2 \\ c_{32,} & i \equiv 0 \bmod 2 \end{cases}$$

The cross edges between level 2 and level 1 are assigned with color a_5 and the cross edges between the level 1 and 0 are assigned with the color a_6. The cross edges between level 1 and level 2 are assigned with color a_2 and the cross edges between level 0 and level 1 are assigned with color a_1. This gives a strong edge coloring of $B(2)$using 12 colors.

4.3 Theorem

For $n \geq 3$, $\chi'_s\,(B(n)) = 12$.

Proof We apply the coloring scheme given in Theorem 3.3, for left $BF(n)$. Now the right $BF(n)$ can be colored as follows. We note that the right $BF(n)$ lies between level n and level 0. We color the edges of right $BF(n)$ in two phases.

(1) color all the row edges.
(2) color all the cross edges.

Step (1) The row edges between the levels $n - j + 1$ and $n - j$, $1 \leq j \leq n$ can be colored as follows.

$$\text{when } j \equiv 1(\text{mod})3, c(e'_{(n-j+1)i}) = \begin{cases} c_{31}, & 0 \le k \le \dfrac{2^{n-1} - 2^{j-1}}{2^{j-1}}, \quad 2^j k + 1 \le i \le 2^j k + 2^{j-1} \\ c_{32}, & 2^j k + 2^{j-1} + 1 \le i \le 2^j k + 2^j \end{cases}$$

$$\text{when } j \equiv 2(\text{mod})3, c(e'_{(n-j+1)i}) = \begin{cases} c_{21}, & 0 \le k \le \dfrac{2^{n-1} - 2^{j-1}}{2^{j-1}}, \quad 2^j k + 1 \le i \le 2^j k + 2^{j-1} \\ c_{22}, & 2^j k + 2^{j-1} + 1 \le i \le 2^j k + 2^j \end{cases}$$

$$\text{When } j \equiv 3(\text{mod})3, c(e'_{(n-j+1)i}) = \begin{cases} c_{11}, & 0 \le k \le \dfrac{2^{n-1} - 2^{j-1}}{2^{j-1}}, 2^j k + 1 \le i \le 2^j k + 2^{j-1} \\ c_{12}, & 2^j k + 2^{j-1} + 1 \le i \le 2^j k + 2^j \end{cases}$$

Step (2)

- The cross edges from the level $n - j + 1$ to the level $n - j$, $1 \le j \le n$ can be colored as follows.
- When $j \equiv 1(\text{mod})3$, the cross edges $\{e'^{n-j+1}_i : 1 \le i \le 2^{n-1}, 1 \le j \le n\}$ between the two consecutive levels $n - j + 1$ and $n - j$, are assigned with the color a_3.
- When $j \equiv 2(\text{mod})3$, the cross edges $\{e'^{n-j+1}_i : 1 \le i \le 2^{n-1}, 1 \le j \le n\}$ between the two consecutive levels $n - j + 1$ and $n - j$ are assigned with the color a_2.
- When $j \equiv 3(\text{mod})3$, the cross edges $\{e'^{n-j+1}_i : 1 \le i \le 2^{n-1}, 1 \le j \le n\}$ between the two consecutive levels $n - j + 1$ and $n - j$ are assigned with the color a_1.
- When $j \equiv 1(\text{mod})3$, the cross edges $\{E'^{n-j+1}_i : 1 \le i \le 2^{n-1}, 1 \le j \le n\}$ between the two consecutive levels $n - j$ to $n - j + 1$, are assigned with the color a_6.
- When $j \equiv 2(\text{mod})3$, the cross edges $\{E'^{n-j+1}_i : 1 \le i \le 2^{n-1}, 1 \le j \le n\}$ between the two consecutive levels $n - j$ to $n - j + 1$, are assigned with the color a_5.
- When $j \equiv 3(\text{mod})3$, the cross edges $\{E'^{n-j+1}_i : 1 \le i \le 2^{n-1}, 1 \le j \le n\}$ between the two consecutive levels $n - j$ to $n - j + 1$, are assigned with the color a_4.

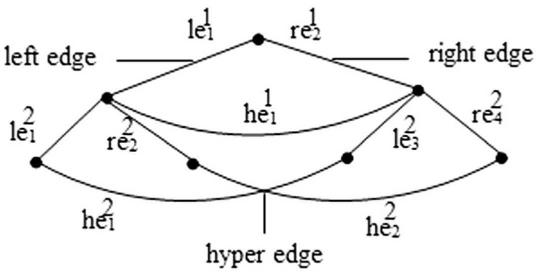
The proposed coloring algorithm bequeaths an optimum strong edge coloring using 12 colors.

5 Strong Chromatic Index of Hypertree Network

The basic skeleton of hypertree is a parallel tree structure. The hypertree consolidates the best features of the binary tree and the hypercube. The structure of binary tree has some flaws. Message traffic density through single vertices turns out to be high and it results in significant queuing delay at these vertices. One approach to reduce the blockage and lessen the results of a failure of vertex or edge is to provide extra edges.

We denote an n-level hypertree by $HT(n)$. It has $2^{n+1} - 1$ vertices and $3(2^n - 1)$ edges [14]. The topmost vertex in the tree is called the root. Every vertex has two children namely left child and right child. For convenience, we call the edge incident to left child by left edge and right child by right edge respectively. The extra edges that connect two vertices at the same level are called as hyper edges. The level of a vertex is defined to be the distance from the root vertex to that vertex. For any edge $e = xy$ of $HT(n)$, let x be child of y. The level of e is defined to be the level of y. Denote the left edge at any level k by le^k_i, $1 \le i \le 2^k$, $i \equiv 1(\bmod 2)$, $1 \le k \le n$, right edge at any level k by re^k_i, $1 \le i \le 2^k$, $i \equiv 0(\bmod 2)$, $1 \le k \le n$ and hyper edge at any level k by he^k_i, $1 \le i \le 2^{k-1}$, $1 \le k \le n$. An illustration is given in Fig. 4.

Fig. 4 *HT*(2)



5.1 Proposition

(1) $\chi'_s(HT_1) = 3$.
(2) $\chi'_s(HT_2) = 8$.

Proof To prove (1), let us consider the hypertree *HT*(1). We find that the hypertree *HT*(1)is a cycle C_3. The three edges of C_3 should be assigned with three distinct colors for strong rainbow edge coloring. Thus $\chi'_s(HT_1) = 3$.

To prove (2), let us consider the hypertree *HT*(2). Left and right edges at level 1 can be colored as $c(le_1^1) = c_1$ and $c(re_2^1) = c_2$. Similarly level 2 edges can be colored as $c(le_1^2) = c_3$, $c(re_2^2) = c_4$, $c(le_3^2) = c_5$ and $c(re_4^2) = c_6$. Moreover, the hyper edges at level 1 are assigned with color c_7 and the hyper edges at level 2 are assigned with color c_8. This gives an optimum strong edge coloring using 8 colors.

5.2 Theorem

For $n \geq 3$, *the strong chromatic index of HT*(n) *is 12.*

Proof We color the edges of *HT*(n)in two phases.

(1) color the left and right edges
(2) color the hyper edges

Step (1) Left and right edges at level k, $1 \leq k \leq 3$ can be colored as stated below

- Left and right edges at level 1 can be colored as follows.

$$c(le_1^1) = c_1, \quad c\left(re_2^1\right) = c_2.$$

- Left and right edges at level 2 can be colored as follows. $c(le_1^2) = c_3$, $c(re_2^2) = c_4$, $c(le_3^2) = c_5$, $c(re_4^2) = c_6$.
- Left and right edges at level 3 can be colored as follows.
  $c(le_i^3) = c_7$ for $1 \leq i \leq 2^{k-1}$, $i \equiv 1(\text{mod}2)$ and $c(re_i^3) = c_8$ for $1 \leq i \leq 2^{k-1}$, $i \equiv 0(\text{mod}2)$
  $c(le_i^3) = c_9$ for $2^{k-1} + 1 \leq i \leq 2^k$, $i \equiv 1(\text{mod}2)$ and $c(re_i^3) = c_{10}$ for $2^{k-1} + 1 \leq i \leq 2^k$, $i \equiv 0(\text{mod}2)$.

For $n > 3$, the left and right edges at level $k \geq 4$ can be colored as follows.

- When $k \equiv 0(\text{mod}4)$, $c(le_i^k) = c_1$ for $1 \leq i \leq 2^{\,k-1}$, $i \equiv 1(\text{mod}2)$ and $c(re_i^k) = c_2$ for $1 \leq i \leq 2^{\,k-1}$, $i \equiv 0(\text{mod}2)$. Similarly $c(le_i^k) = c_3$ for $2^{k-1} + 1 \leq i \leq 2^k$, $i \equiv 1(\text{mod}2)$ and $c(re_i^k) = c_4$ for $2^{k-1} + 1 \leq i \leq 2^k$, $i \equiv 0(\text{mod}2)$.

- When $k \equiv 1(\text{mod}4)$, $c(le_i^k) = c_5$ for $1 \le i \le 2^{k-1}$, $i \equiv 1(\text{mod}2)$ and $c(re_i^k) = c_6$ for $1 \le i \le 2^{k-1}$, $i \equiv 0(\text{mod}2)$. Also $c(le_i^k) = c_7$ for $2^{k-1} + 1 \le i \le 2^k$, $i \equiv 1(\text{mod}2)$ and $c(re_i^k) = c_8$ for $2^{k-1} + 1 \le i \le 2^k$, $i \equiv 0(\text{mod}2)$.
- When $k \equiv 2(\text{mod}4)$, $c(le_i^k) = c_3$ for $1 \le i \le 2^{k-1}$, $i \equiv 1(\text{mod}2)$ and $c(re_i^k) = c_4$ for $1 \le i \le 2^{k-1}$, $i \equiv 0(\text{mod}2)$. Moreover $c(le_i^k) = c_1$ for $2^{k-1} + 1 \le i \le 2^k$, $i \equiv 1(\text{mod}2)$ and $c(re_i^k) = c_2$ for $2^{k-1} + 1 \le i \le 2^k$, $i \equiv 0(\text{mod}2)$.
- When $k \equiv 3(\text{mod}4)$, $c(le_i^k) = c_7$ for $1 \le i \le 2^{k-1}$, $i \equiv 1(\text{mod}2)$ and $c(re_i^k) = c_8$ for $1 \le i \le 2^{k-1}$, $i \equiv 0(\text{mod}2)$. Furthermore $c(le_i^k) = c_5$ for $2^{k-1} + 1 \le i \le 2^k$, $i \equiv 1(\text{mod}2)$ and $c(re_i^k) = c_6$ for $2^{k-1} + 1 \le i \le 2^k$, $i \equiv 0(\text{mod}2)$.

Step (2) Hyper edges at level k can be colored as follows.

If $k \equiv 1(\text{mod}2)$, $c(he_i^k) = c_{11}$ for $1 \le i \le 2^{k-1}$ and if $k \equiv 0(\text{mod}2)$, $c(he_i^k) = c_{12}$ for $1 \le i \le 2^{k-1}$.

The above coloring algorithm gives a strong edge coloring of $HT(n)$using 12 colors. Figure 5 illustrates the strong edge coloring of $HT(3)$.

6 Strong Chromatic Index of Honeycomb Network

Honeycomb network $HC(n)$can be constructed from hexagons in different ways. The most straightforward approach to characterize them is to consider the segment of the hexagonal tessellation which is inscribed in a given convex polygon [18]. This network is convenient to model the base stations of wireless networks. Honeycomb networks are mostly used in computer graphics [9] and cellular phone base stations [13]. Honeycomb networks are better as far as degree, diameter, aggregate number of connections, cost and the bisection width than mesh connected planar graphs. Stojmenovic [18] has examined the topological characteristics of honeycomb networks and routing in honeycomb networks.

The number of vertices and edges of $HC(n)$ are $6n^2$ and $9n^2 - 3n$ respectively [12]. The diameter is $4n - 1$.We use the level numbering scheme proposed by the Sharieh et al. [17] for the honeycomb networks. Each row edge in $HC(n)$is identified by $e_{i,j}$, where i represents the row number in which the edge exists, j represents the position of the edge in the row. Denote the vertical edge along column by $V_{p,q}$, where p represents the column number in which the edge exists, q represents the position of the edge in the column. A strip in $HC(n)$ is a series of hexagons aligned in a line between the rows R_i and R_{i+1}, $1 \le i \le 2n - 1$ such that any two consecutive hexagons will have a vertical edge in

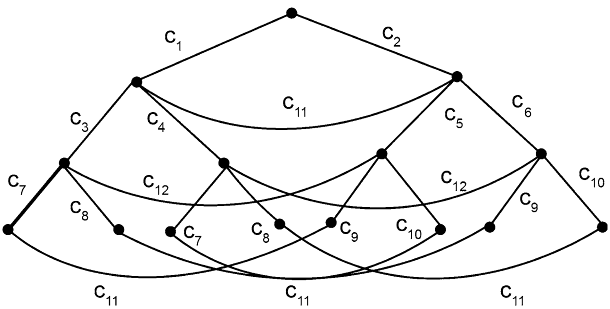


Fig. 5 Strong edge coloring of $HT(3)$

common. The strip between the rows R_i and R_{i+1}, $1 \leq i \leq 2n-1$ can be denoted by $S_{i,i+1}$. Figure 6 shows a $HC(2)$ network.

To find the strong chromatic index of $HC(n)$, we apply the concept of antipodal edges. Two edges are antipodal in Gif the distance between them is diameter of G. We find that each hexagon $HC(1)$is a cycle C_6 and opposite edges in C_6 are antipodal edges. In addition, the distance between two antipodal edges is two. Hence antipodal edges receive the same color. This technique will be useful in proving the following results.

6.1 Proposition

The strong chromatic index of $HC(1) = 3$.

Proof In Fig. 7, e_1 and e_4, e_2 and e_5, e_3 and e_6 are antipodal edges. We find that $d(e_i, e_{i+3}) = 2 = diam(HC(1))$for $1 \leq i \leq 3$. Thus $c(e_1) = c(e_4) = c_1$, $c(e_2) = c(e_5) = c_2$ and $c(e_3) = c(e_6) = c_3$. Hence $HC(1)$ can be colored with 3 colors for strong edge coloring.

6.2 Theorem

For $n \geq 2$, $\chi_s'(HC(n)) = 6$.

Proof To color the edges of $HC(n)$, we consider each strip in $HC(n)$and the edges in it. Each strip consists of a series of hexagons aligned in a line. Since each hexagon is a cycle C_6, opposite edges in C_6 are antipodal edges. As the distance between a pair of antipodal edges in each cycle C_6 is two, both the edges receive the same color. We first find the antipodal edges in each hexagon and this will be helpful in proving our result. To do this, we consider the brick representation of $HC(n)$. Figure 8 is a subgraph of $HC(n)$.

Choose the strip consisting of maximum number of hexagons. The strip $S_{i+1,i+2}$ consists of maximum number of hexagons. The strip $S_{i+1,i+2}$ is formed from rows R_{i+1} and R_{i+2} consisting of a series of hexagons aligned in a line. We first color the row edges in R_{i+1}. The edges $e_{i+1,1}$, $e_{i+1,2}$, $e_{i+1,3}$, $e_{i+1,4}$ are assigned with the colors c_1, c_2, c_3, c_4 respectively. We shall first examine the antipodal edges of each hexagon in $S_{i+1,i+2}$. We find that the edge $e_{i+1,1}$ of R_{i+1} and the edge $e_{i+2,2}$ of R_{i+2} are antipodal and thus $c(e_{i+1,1}) = c(e_{i+2,2})$.Similarly the edges $e_{i+1,2}$ and $e_{i+2,1}$are antipodal and therefore $c(e_{i+1,2}) = c(e_{i+2,1})$.By a similar argument the edges $e_{i+1,3}$ and $e_{i+2,4}$ are antipodal. Also the edges $e_{i+1,4}$ and $e_{i+2,3}$ are antipodal. As a result $c(e_{i+1,3}) = c(e_{i+2,4})$ and $c(e_{i+1,4}) = c(e_{i+2,3})$. All

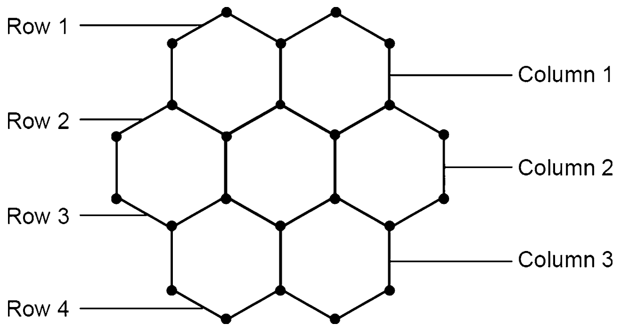


Fig. 6 $HC(2)$

Fig. 7 *HC*(1)

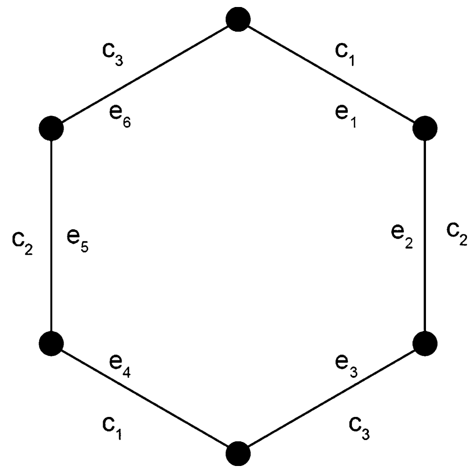
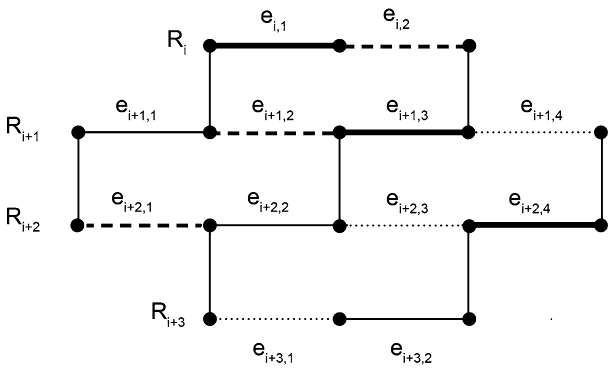


Fig. 8 Subgraph of *HC*(*n*)



the row edges in R_{i+1} and R_{i+2} have been assigned with colors. Next we color the row edges above R_{i+1} of the strip $S_{i+1,i+2}$. We find from the above figure that the row R_i lies above R_{i+1}. Also the row R_i lies in the strip $S_{i,i+1}$.

The strip $S_{i,i+1}$ consists of a series of hexagons formed from rows R_i and R_{i+1}. In $S_{i,i+1}$, the edges $e_{i,2}$ and $e_{i+1,2}$ lie on the same hexagon and are antipodal. Therefore they receive the same colors. Since $e_{i+1,2}$ is already assigned with color c_2, $e_{i,2}$ also receives the color c_2. Also the edges $e_{i,1}$ and $e_{i+1,3}$ lie on the same hexagon and are antipodal. Hence they receive the same colors.

Next we color the row edges below the strip $S_{i+1,i+2}$. The strip adjacent and that which lies below $S_{i+1,i+2}$ is $S_{i+2,i+3}$. The row that is common to $S_{i+1,i+2}$ and $S_{i+2,i+3}$ is R_{i+2}. The row edges in R_{i+2} has already been assigned with colors. Hence the antipodal edges in each hexagon of the strip $S_{i+2,i+3}$ receive the same colors.

After coloring all the row edges, we proceed to color the vertical edges. Now the vertical edges $V_{p,q}$, p odd, are assigned with the color c_5 and p even, are assigned with the color c_6. Thus the above coloring scheme is the strong edge coloring of Fig. 8. In order to find the strong chromatic index of $HC(n)$,we apply this idea repeatedly. This completes the proof. An illustration is given in Fig. 9.

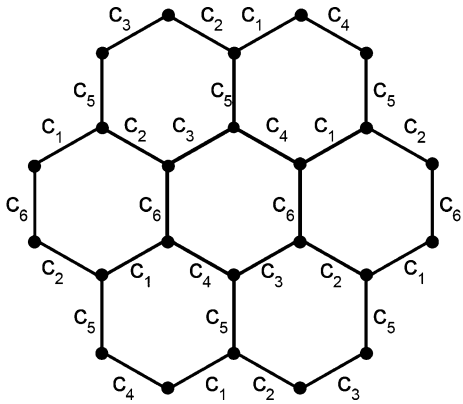


Fig. 9 Strong edge coloring of *HC*(2)

7 Conclusion

In this paper, the exact values of strong chromatic indices of the interconnection networks, namely butterfly network, Benes network, hypertree network and honeycomb network have been obtained. The strong chromatic indices for butterfly network, Benes network and hypertree network are 12 respectively. In the case of honeycomb network $\chi'_s(G)$ is found to be 6. We also note that the coloring algorithms use the least possible number of colors. The problem remains open for other interconnection networks such as cube connected cycles, shuffle exchange, De Bruijin, star and pancake networks.

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